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Abstract
The resonant layer equations governing tearing and micro-tearing modes in toroidal geometry are formulated and solved for a plasma in the banana regime and for wavelengths in the intermediate collisionality range $\nu_e < \omega_{*e} < \nu_e/\epsilon$. It is shown that stability of such tearing modes is determined by a competition between destabilising trapped electron dissipation and stabilising effects arising from ion magnetisation and collisional broadening of the passing-electron Landau resonance. Analytic stability criteria are derived, but for realistic parameters these modes are predicted to be stable.

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1 Introduction

The discovery by Hazeltine, Dobrott, and Wang (1975) of the importance of the electron temperature gradient for the stability of tearing modes has been responsible for much activity in the development of linear stability theory (Drake *et al* (1977), (1980), (1983); Bussac *et al* (1978); Mahajan *et al* (1979); D'Ippolito *et al* (1980); Rosenberg *et al* (1980); Gladd *et al* (1980); Chang *et al* (1981); Cowley*et al* (1986)) of both long wavelength (gross) and short wavelength (micro) tearing modes. Initially the calculations were performed in plane slab geometry, utilising both kinetic and two-fluid equations. These calculations showed that, if the time dependence of the thermal force term is neglected, tearing modes are strongly stabilised (Drake *et al* (1983); Cowley *et al* (1986) by finite ion orbit effects in the semi-collisional regime ($\omega_* \nu_e \leq k_{\|}^2 v_{Te}^2$). However, kinetic analyses (D'Ippolito *et al* (1980); Rosenberg *et al* (1980); Gladd *et al* (1980); Chang *et al* (1981)) (and two-fluid theory (Hassam (1980)) which included a time-dependent thermal force term in Ohm's law) showed that the tearing layer could provide a strong source of energy. Indeed this could readily overcome the negative Δ' representing the stabilising effect of field line bending from the outer, ideal mhd, region of the mode structure.

The resulting instabilities were suggested (Drake *et al* (1980)) as possible candidates for explaining the anomalous electron thermal loss in Tokamaks. However, the calculations predicted that at the low values of electron collision frequency ν_e, ($\nu_e < \omega_{*e}$, the electron diamagnetic frequency) typical of more recent Tokamak experiments, these micro-tearing modes are linearly stable in slab geometry.

Interest in them was revived by a number of papers which sought to extend the linear theory to lower collisionality, $\nu_e/\epsilon < \omega_{be}$ when trapped particle effects come into play (here $\epsilon = r/R$ is local inverse aspect ratio and $\omega_{be} \simeq Rq/\epsilon^{1/2} v_{Te}$ is the bounce frequency of toroidally trapped electrons). For example Chen, Rutherford and Tang (1977) predicted an extended range of instability at low collision frequency in a calculation which modelled collisional detrapping of electrons with an ingenious, number conserving, Krook collision operator. The Krook operator used by Chen, Rutherford and Tang permitted analytic treatment of both the weakly collisional ($\nu_e/\epsilon < \omega_{*e}$) and intermediate collisional ($\nu_e < \omega_{*e} < \nu_e/\epsilon$) regimes. It is not, however, capable of representing the boundary layer behaviour in velocity space which is an inherent feature of the electron perturbation whenever $\nu_e < \omega_{*e}$. The contribution to the perturbed longitudinal current $\delta J_{\|}$ arising from passing electrons in this boundary layer (just passing electrons) is therefore not obtained from a Krook model operator. This deficiency was rectified in the work of Catto and Rosenbluth (1981), where electron pitch angle scattering is represented by a Lorentz collision term. These authors investigated only the weak collisionality regime ($\nu_e/\epsilon < \omega_{*e}$) where trapped electron dissipation was found to be destabilising. In the present paper we reconsider the intermediate collisional range discussed by Chen, Rutherford and Tang (1977). We obtain solutions of the tearing layer equations using the Lorentz description of electron pitch angle scattering, and derive the dispersion relation for micro-tearing modes in the important parameter range defined by $\nu_e < \omega_{*e} < \nu_e/\epsilon$. This calculation, which complements that of Catto and Rosenbluth, provides a more realistic treatment than that of Chen *et al* (1977) for the intermediate collisional range. For consistency we also calculate collisional effects on the bulk of passing particles where a slab-treatment is possible. Stability is then governed by a competition between the trapped particle effects and the stabilising ion orbit effects (calculated by Cowley *et al* (1986) in the limit $\rho_i \gg \delta$, with δ, the width of the tearing layer), and passing particle collisional effects.

We also note the appearance recently of another group of toroidal calculations (Callen *et al* (1987); Fitzpatrick (1989)) for tearing modes in the banana regime. Since they are based on neo-classical fluid equations these calculations only apply to longer wavelength tearing modes for which the inequality $\nu_e > \omega_{*e}$ is satisfied.

The layout of this paper is as follows. In section 2 we briefly derive the relevant equations and discuss their solution. In section 3, the effect of electron trapping is calculated and in section 4 the effect of collisional broadening of the passing-electron Landau resonance is evaluated. In section 5 we derive the dispersion relation for micro-tearing modes and relate it to that obtained by Catto and Rosenbluth. Finally, in section 6 we discuss the significance of the results.

2 Equations for Micro-tearing modes

In this section we derive the basic equations governing micro-tearing modes in a large aspect ratio, circular cross-section model of a tokamak equilibrium with $\beta \sim 0(\epsilon^2)$. The starting point is the Fokker-Planck equation for each charged species. In configuration space we use the coordinates (r, θ, ϕ) with r a magnetic surface variable having the dimension of length, and θ, ϕ the poloidal and toroidal angles. The poloidal angle θ is chosen so that the magnetic field lines appear straight $\left(\frac{d\phi}{d\theta} = q(r) \text{with } q \text{ the safety factor}\right)$, and the Jacobian for the (r, θ, ϕ) coordinate system is $J = R^2 r/R_0$ where R is the major radius, and R_0 the major radius of the magnetic axis. The equilibrium magnetic field may then be written in the form

$$\mathbf{B} = B_0 R_0 [f(r)\nabla\phi \times \nabla r + g(r)\nabla\phi] \tag{2.1}$$

so that the safety factor is $q(r) = rg/R_0 f$, and the operator $\mathbf{B} \cdot \nabla$ takes the form

$$\mathbf{B} \cdot \nabla = \frac{B_0}{Rq} \left(\frac{\partial}{\partial \theta} + q \frac{\partial}{\partial \phi} \right) \tag{2.2}$$

We take all perturbed quantities to have the form

$$\delta y = y(r, \theta) e^{i(m\theta - n\phi - \omega t)} \tag{2.3}$$

and, since we have assumed $\beta \ll \epsilon$, we neglect the compressional Alfven component, $\delta B_{||}$, of the electromagnetic perturbation which is then described by the electrostatic potential Φ and the longitudinal component of the vector potential $A_{||}$.

2.1 The electron perturbation

The perturbed electron distribution function

$$f_e = -\frac{e\Phi}{T_e} F_m + g_e \tag{2.4}$$

is obtained from the drift kinetic equation for electrons in the vicinity of a resonant surface r_s given by $m = nq(r_s)$, namely

$$\left(\omega - \omega_{de} - k_{||} v_{||} - iC + i \frac{v_{||}}{Rq} \frac{\partial}{\partial \theta} \right) g_e = -\frac{eF_m}{T_e} (\omega - \omega_{*e}^T) \left(\Phi - \frac{v_{||} A_{||}}{c} \right) \tag{2.5}$$

where $k_{||} = (m - nq)/Rq \simeq -nq'(r - r_s)/Rq$, $\omega_{*e}^T = \omega_{*e}[1 - \eta_e \left(\frac{3}{2} - \frac{v^2}{v_{Te}^2} \right)]$, with $v_{Te} = (2T_e/m_e)^{1/2}, \eta_e = \partial \ln T_e / \partial \ln n_e$, and $\omega_{*e} = \frac{nT_e}{eB} \frac{\partial \ln n_e}{\partial r}$.

The magnetic drift can be decomposed into radial and geodesic components so that

$$\omega_{de} \simeq -\frac{m}{r} v_{||} \frac{\partial}{\partial r} \left(\frac{v_{||}}{\omega_{ce}} \right) - i \frac{v_{||}}{r} \frac{\partial}{\partial \theta} \left(\frac{v_{||}}{\omega_{ce}} \right) \frac{\partial}{\partial r} \tag{2.6}$$

where $\omega_{cj} = e_j B/m_j$ and the collisional term is represented by a Lorentz model

$$C(g) = \nu_e v_{||} \frac{\partial}{\partial \mu} \frac{\mu v_{||}}{B} \frac{\partial g_e}{\partial \mu} \tag{2.7}$$

with $\nu_e(v) = \nu_e(v_{Te}) \left(\frac{v_{Te}}{v} \right)^3$, μ the magnetic moment and $v_{||} = (v^2 - 2\mu B)^{1/2}$

The width of the tearing layer δ is given by $\omega_{*e} \simeq k_{||} v_{Te}$, i. e. $\delta \simeq \frac{\rho_e L_s}{L_n}$. In order that the geodesic drift term in equation (2.5) is smaller than the bounce term for such a narrow layer we assume $\epsilon^{1/2} > q \frac{L_n}{L_s}$. Then for modes such that $\omega < \nu_e/\epsilon$ and collision frequencies in the banana regime we may bounce-average equation (2.5) to obtain

$$(\omega - < \omega_d > -i < C >) g_t = -e \frac{F_m}{T_e} (\omega - \omega_{*e}^T) < \Phi > \tag{2.8}$$

for trapped electrons and

$$(\omega - <\omega_{de}> - \sigma i k_{\|} < |v_{\|}| > - i < C >) g_\sigma$$
$$= -e\frac{F_m}{T_e}(\omega - \omega_{*e}^T)(<\Phi> - \frac{\sigma}{c} < |v_{\|}| A_{\|} >) \tag{2.9}$$

for passing electrons, where $\sigma = sign(v_{\|})$ and $\frac{\partial g}{\partial \theta} = 0$. The bounce-average operators are defined as

$$< X > = \left(\int_{-\theta_0}^{\theta_0} \frac{d\theta}{|v_{\|}|} X \right) \left(\int_{-\theta_0}^{\theta_0} \frac{d\theta}{|v_{\|}|} \right)^{-1} \tag{2.10}$$

where θ_0 is defined as the turning point for trapped particles and π for passing particles. It is convenient to introduce the quantity

$$G = g + \frac{eF_m}{T_e}\left(\frac{\omega - \omega_{*e}^T}{\omega} \right) \Phi \tag{2.11}$$

and to use the approximations $\Phi \simeq \bar{\Phi}, A_{\|} \simeq \bar{A}_{\|}$ (where $\bar{X} = \frac{1}{2\pi} \oint X d\theta$), which are justified in subsection (2.3) below, to simplify equations (2.8) and (2.9) to the form

$$(\omega - <\omega_{de}> - i < C >) G_t = -\frac{eF_m}{T_e}(\frac{\omega - \omega_{*e}^T}{\omega}) < \omega_d > \Phi \tag{2.12}$$

$$(\omega - <\omega_{de}> - \sigma k_{\|} < |v_{\|}| > - i < C >) G_\sigma$$
$$= -\frac{eF_m}{T_e}(\frac{\omega - \omega_{*e}^T}{\omega})[< \omega_d > \Phi + \sigma < |v_{\|}| > (k_{\|}\Phi - \frac{\omega}{c} A_{\|})] \tag{2.13}$$

Thus only the untrapped electrons respond to the parallel electric field $E_{\|} = -i(k_{\|}\Phi - \omega\frac{A_{\|}}{c})$.

2.2 The ion perturbation

Because of their relatively large Larmor orbits, which typically exceed the width of the tearing layer ($\rho_i > \delta$) a more sophisticated gyro-kinetic treatment of the ion kinetic equation is necessary. For ions, one obtains

$$f_i = +\frac{e\Phi}{T_i} F_m + g_i(X, \theta, \mu, v) e^{iL} \tag{2.14}$$

where $L = \frac{k_\perp v_\perp}{\omega_{ci}} \cos\alpha$, and X is the guiding-centre position $X = r + \frac{v_\perp}{\omega_{ci}} \sin\alpha$ with α the gyro-angle in velocity space. The equation determining $g_i(X)$ is then the gyro-kinetic equation

$$\left(\omega - \omega_{di} - \hat{k}_{\|} v_{\|} - iC_i - iD\frac{\partial^2}{\partial X^2} + i\frac{v_{\|}\partial}{Rq\partial\theta} \right) g_i$$
$$= +\frac{eF_m}{T_i}(\omega - \omega_{*i}^T) \int_{-\infty}^{\infty} dk\, J_0\left(\frac{k_\perp v_\perp}{\omega_{ci}} \right) \left(\hat{\Phi}(k) - \frac{v_{\|}\hat{A}_{\|}(k)}{c} \right) e^{ikX} \tag{2.15}$$

where $\hat{\Phi}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt\, e^{ikt} \Phi(t)$ etc, and $k_\perp = \sqrt{k_\theta^2 + k^2}, \hat{k}_{\|}(X) = -\frac{nq'}{Rq} X, C_i$ is the ion-ion Fokker-Planck collision term, and the diffusion coefficient $D \approx 0(\nu_i \rho_i^2)$. Definitions of

ω_{di} and ω_{*i}^T are the same as for electrons with the appropriate sign changes and temperature dependence. In the limit of interest, $\rho_i >> \delta$, the argument of the Bessel function in the integrand of the right-hand side of equation (2.15) is large, so that the leading approximation for g_i is merely

$$g_i \simeq 0 \tag{2.16}$$

describing an unmagnetized ion response. Cowley, Kulsrud and Hahm (1986), however, developed the next order term in their slab treatment of tearing modes, and we shall use their result later. This requires that $\omega > \nu_i\ \rho_i^2/\delta^2$, and more importantly, that the toroidal geodesic drift term $\omega_{di} \sim \frac{\rho_i}{\delta}\frac{V_i}{R}$ should not exceed the mode frequency, i.e.

$$k_\theta \rho_e > \left(\frac{L_n^2}{RL_s}\right) \tag{2.17}$$

2.3 The electromagnetic equations

Solution of the kinetic equations for electrons and ions permits one to construct expressions for the perturbed charge densities and hence the quasi-neutrality equation

$$\sum_j \rho_j = \sum_j e_j \int d^3v\ f_j = 0 \tag{2.18}$$

and the perturbed current density

$$j_\| = \sum_j\ e_j \int d^3v\ v_\| f_j \tag{2.19}$$

to match to the external mhd solution.

Important constraints on the functional dependence of $\Phi(\theta)$ and $A_\|(\theta)$ are obtained by constructing an equation for $\mathbf{B}\cdot\nabla(j_\|/B)$ from the kinetic equations (2.5) and (2.15). Taking $e \int d^3v$ (2.5) $-e\int d^3v e^{iL}$ (2.15) one finds

$$\frac{1}{Rq}\frac{\partial}{\partial\theta}(j_\|) + i\ k_\| j_\| = in_0\frac{e^2\Phi}{T}(\omega\tau + \omega_{*e}) + ie\int d^3v\omega_{de}\, g_e \tag{2.20}$$

where $\tau = T_e/T_i$ and small ion terms have been neglected in the limit $\rho_i >> \delta$. For a narrow tearing layer the geodesic contribution to ω_{de} is dominant and generates a variation, $\tilde{j}_\|$, about the mean value $\bar{j}_\|$. Estimating g_e from equations (2.12) and (2.13) we find that

$$\tilde{j}_\|/\bar{j}_\| \sim \frac{L_n}{L_s} << 1 \tag{2.21}$$

where $\bar{j}_\|$ follows from integrating equation (2.20) over a period in θ:

$$i\ k_\|\ \bar{j}_\| = in_0\frac{e^2\bar{\Phi}}{T_e}(\omega\tau + \omega_*) + ie\int d^3v\omega_{de}g_e \tag{2.22}$$

Since $\frac{\partial g_e}{\partial \theta} = 0$ in leading order the $\omega_{de}\, g_e$ term can be integrated by parts in θ and the potentially larger geodesic contribution vanishes. Thus in the large aspect ratio limit we can ignore all contributions from ω_{de} and treat $j_{||}$ as independent of θ. Ampères law, which in a narrow tearing layer takes the form

$$\frac{d^2 A_{||}}{dr^2} = -\frac{4\pi}{c} j_{||} \tag{2.23}$$

then implies that $A_{||}$ is also approximately independent of θ.

Returning to quasi-neutrality, equation (2.18) takes the form

$$\frac{ne^2\Phi}{T_e}(1+\tau) = e \int d^3v\, g_e \tag{2.24}$$

because $g_i \simeq 0$ for $\rho_i >> \delta$. Since $\frac{\partial g_e}{\partial \theta} \simeq 0$ the only θ-variation in Φ arises from the $v_{||}/v \sim 0(\epsilon^{1/2})$ region of the velocity space integration affected by trapped particles so that Φ may also be treated as constant. In this approximation Ampère's law and the current continuity equation (2.22) can be treated cylindrically so that equation (2.22) reduces to

$$k_{||} j_{||} = \frac{n_0 e^2}{T_e}(\omega\tau + \omega_{*e})\Phi \tag{2.25}$$

The only toroidal effect arises through the influence of trapped particles on the solution of equations (2.12) and (2.13) for the electron distribution function. The solution of these equations will enable us to compute $j_{||}$ and a conductivity σ

$$j_{||} = \sigma E_{||} \tag{2.26}$$

Eliminating Φ between equations (2.25) and (2.26) we obtain

$$j_{||} = \frac{n_0 e^2(\omega\tau + \omega_{*e})\sigma i\omega}{n_0 e^2(\omega\tau + \omega_{*e}) + i\, k_{||}^2 \sigma T_e} \frac{A_{||}}{c} \tag{2.27}$$

It is convenient to introduce a number of normalisations

$$X = \frac{(r - r_s)}{\delta}, \hat{\omega} = \frac{\omega}{\omega_{*e}}, \hat{\beta} = \frac{4\pi n_0 T_e}{B^2}\left(\frac{L_s}{L_n}\right)^2, \hat{\sigma} = \frac{m_e\, \omega_{*e}}{2n_0 c^2}\sigma \tag{2.28}$$

where $\delta = \frac{\omega_{*e} L_s}{k_\theta V_{Te}}$ is the tearing layer width. Combining Ampère's law (2.23) with (2.27) then yields

$$\frac{d^2 A_{||}}{dX^2} = \frac{-i\hat{\beta}\hat{\omega}(\hat{\omega}\tau + 1)\hat{\sigma}(X) A_{||}}{[iX^2\hat{\sigma} + \hat{\omega}\tau + 1]} \tag{2.29}$$

Provided $\hat{\beta} << 1$ we can treat $A_{||}$ as a constant and integrate this through the tearing layer to obtain the jump in $\frac{1}{A_{||}}\frac{dA_{||}}{dX}$ across this region. Matching this to the corresponding discontinuity, Δ', in the external ideal mhd solutions yields a dispersion relation

$$\delta\Delta' = -i\hat{\beta}\hat{\omega}(\hat{\omega}\tau+1)\int_{-\infty}^{\infty}\frac{\hat{\sigma}dX}{(iX^2\hat{\sigma}+\hat{\omega}\tau+1)} \tag{2.30}$$

In lowest order $\hat{\sigma}$ is the conductivity of a cylindrical collisionless plasma. This can readily be calculated from the appropriate limit of equations (2.12) and (2.13) and has been given previously (for example by Crew *et al* (1982) and Cowley *et al* (1986))

$$\hat{\sigma}_0(X) = \frac{i}{2X^2}\left[Z'(s)(\hat{\omega}-1)+\frac{\eta_e}{2}sZ''(s)\right] \tag{2.31}$$

where $s = \frac{\omega}{k_{\|}v_{Te}} \equiv \frac{\hat{\omega}}{X}$ and Z is the Plasma Dispersion Function. Using this approximation for $\hat{\sigma}$ in the dispersion relation (2.30) leads to an approximately real frequency. In the next section we shall calculate a correction $\hat{\sigma}^{(t)}$, due to trapped particles, and in section 4 a correction $\hat{\sigma}^{(c)}$, due to collisional broadening of the passing Landau resonance. Thus

$$\hat{\sigma}^{(t)} = \hat{\sigma}_0 + \hat{\sigma}^{(t)} + \hat{\sigma}^{(c)} \tag{2.32}$$

and equation (2.31) can be expanded as

$$\begin{aligned}\delta\Delta' &= -i\hat{\beta}\hat{\omega}(\hat{\omega}\tau+1)\int_{-\infty}^{\infty}dX\left\{\frac{\hat{\sigma}_0}{(iX^2\hat{\sigma}_0+\hat{\omega}\tau+1)}\right.\\ &\quad\left.+\frac{(\hat{\omega}\tau+1)(\hat{\sigma}^{(t)}+\hat{\sigma}^{(c)})}{[iX^2\hat{\sigma}_0+\hat{\omega}\tau+1]^2}\right\}\end{aligned} \tag{2.33}$$

In section 5 we shall discuss whether this dispersion relation can describe instability.

3 Trapped Particle Contributions

If we retain the trapped particle effects as the only consequence of toroidal geometry , ie ignore the bounce averaged magnetic drifts, equations (2.12) and (2.13) reduce to

$$(\omega - i < C >)G_t = 0 \tag{3.1}$$

for trapped particles and

$$(\omega - \sigma k_{\|} < |v_{\|}| > -i < C >)G_\sigma = -i\frac{eF_m}{T}\left(\frac{\omega-\omega_*^T}{\omega}\right)\sigma < |v_{\|}| > E_{\|} \tag{3.2}$$

for passing particles where we delete electron indices for simplicity. Introducing a pitch angle variable $\lambda = 2\mu/v^2$, the Lorentz collision operator (2. 7) becomes

$$C = 2\frac{\nu(v)}{v^2}v_{\|}\frac{\partial}{\partial\lambda}\frac{\lambda v_{\|}}{B}\frac{\partial}{\partial\lambda}\ ,\ \nu(v) = \nu\left(\frac{2T}{mv^2}\right)^{3/2}, \tag{3.3}$$

Equations(3.1) and (3.2) are solved by an asymptotic matching procedure based on the small parameter ω/ν_{eff} with $\nu_{eff} = \nu/\epsilon$. Fig. 1 illustrates a division of velocity space into seven regions of pitch angle $\zeta = v_{\|}/v$.

In the trapped region I and the two just-passing regions II_L and II_R the effective collision frequency dominates the mode frequency and the equations (3.1) and (3.2) are solved by expansion in ω/ν_{eff} with appropriate boundary conditions at $\zeta = 0$ and matching conditions between passing and trapped regions. The solutions in $II_{L,R}$ must then be matched to those in the regions $III_{L,R}$ where $|\zeta| \sim 0(\sqrt{\nu/\omega})$ and collisions and mode frequency compete. In these regions $|\zeta| >> \epsilon^{1/2}$ and $v_{\|}$ can be regarded as constant along the field line. Finally these solutions must be matched to the regions $IV_{L,R}$ (where $|\zeta| \sim 0(1)$) which are collisionless since $\omega >> \nu$.

Expanding equation (3.1) for $\omega << \nu_{eff}$ and integrating in pitch angle we find

$$G_t = a_t + b_t \int_{\lambda_c}^{\lambda} \frac{d\lambda}{< v_{\|} >} \tag{3.4}$$

in region I, where a_t and b_t are constants and $\lambda_c = 1/B_{max}$. The condition that G_t is finite as $\lambda \to 1/B_{min}$ requires $b_t = 0$. For the passing particles in regions $II_{L,R}$ we find, with $\nu_{eff} >> \omega$,

$$G_\sigma = a_\sigma + \sigma b_\sigma \int_{\lambda_c}^{\lambda} \frac{d\lambda}{< |v_{\|}| >} \tag{3.5}$$

Continuity of G at λ_c implies

$$a_\sigma = a_t \equiv a \tag{3.6}$$

Integrating equations (3.1) and (3.2) through a boundary layer at λ_c leads to

$$\sum_{\sigma=\pm} \frac{\partial G_\sigma}{\partial \lambda} = 2\frac{\partial G_t}{\partial \lambda} \tag{3.7}$$

so that

$$b_+ = b_- \equiv b \tag{3.8}$$

The constants a and b are determined by matching to the solution in regions $III_{L,R}$. In the limit $|\zeta| >> \epsilon^{1/2}$ equation (3.5) takes the form

$$G_\sigma \to a + \frac{2\sigma b}{Bv}\left[-\frac{|v_{\|}|}{v} + \sqrt{\epsilon} I\right] \tag{3.9}$$

where $v_{\|} = v\sqrt{1 - \lambda B_{max}}$ and

$$I = \frac{Bv}{2\sqrt{\epsilon}} \int_0^{\lambda_c} d\lambda \left[\frac{1}{v_{\|}} - \frac{1}{< v_{\|} >}\right] = 0.99 \tag{3.10}$$

In regions $III_{L,R}$ equation (3.2) takes the form

$$[\omega - k_{\|} v \zeta - \frac{i\nu}{2}\frac{\partial^2}{\partial \zeta^2}] G_\sigma = -i\frac{eF_m}{T}\left(\frac{\omega - \omega_*^T}{\omega}\right) v \zeta E_{\|} \tag{3.11}$$

since $|\zeta| \sim \sqrt{\frac{\nu}{\omega}} << 1$. It is convenient to introduce the normalised quantities X and $\hat{\omega}$ defined in equation (2.28) and $u = v/v_T$ and the quantity

$$A(u) = -\frac{eF_m}{T} E_{\|} [\hat{\omega} - 1 - \eta_e(3/2 - u^2)] \frac{v_T}{\omega_*} \tag{3.12}$$

Then the solutions of equation (3.11) which are well-behaved as $|\zeta| \to \infty$ are

$$\begin{aligned} G_\sigma &= -\frac{iA}{X} + y_0 \frac{A}{X} \int_0^\infty dp \exp[-ip(y - y_0) - p^3/3] \\ &\quad + C_\sigma \int_{\Gamma_\sigma} dp \exp[-ip(y - y_0) - p^3/3] \end{aligned} \tag{3.13}$$

where

$$y = k^{-1/3}\zeta \ , \ y_0 = \frac{2\omega k^{2/3}}{\nu} \ ; \ k = \frac{\nu}{2\omega_* X u} \tag{3.14}$$

and the contours Γ_σ are shown in Fig. 2.

For $|\zeta| \to \infty$ the contributions from the contours Γ_σ are exponentially small and the remaining term reduces to the collisionless solutions of regions $\mathrm{IV}_{L,R}$. The constants C_σ are determined by matching the $|\zeta| \to 0$ limit of result (3.13) to the form (3.9). This yields

$$C_\pm = \pi A \frac{y_0}{k^{1/3} X} \epsilon^{1/2} I \ H_i'(iy_0)[B_i'(iy_0) \mp iA_i'(iy_0)] \tag{3.15}$$

where A_i and B_i are Airey functions, $H_i(iy_0)$ is the related function

$$H_i(iy_0) = \frac{1}{\pi} \int_0^\infty dp \ \exp(ip\, y_0 - p^3/3) \tag{3.16}$$

and prime denotes a derivative with respect to iy_0.

The parallel current is calculated by integrating the $v_{\|}$ moment of G_σ, given by equations (3.13) and (3.15), over velocity space. It is clear that the effects of trapped particles are entirely contained in the terms involving $C_\pm$ driven by the matching to the trapped region and this contribution, $j_{\|}^{(t)}$, we compute next. The remaining part of G_σ merely represents the cylindrical result. In the next section we return to the question of the collisional effects arising from such a term.

To compute $j_{\|}^{(t)}$ we interchange the order of integration over pitch angle ζ, represented by an integral over an infinite range for the scaled variable y, with the integrals over p in equation(3.13) (The y-integration $\int_0^\infty dy$ must be treated as

$$\lim_{y \to \infty} \int_0^y dy$$

to allow this interchange.) After considerable manipulation we obtain

$$j_{\|}^{(t)} = \frac{2n_0 e^2}{m_e \omega_{*e}} \hat{\sigma}^{(t)}(X) E_{\|} \tag{3.17}$$

with

$$\hat{\sigma}^{(t)}(X) = 2\pi^{3/2}\frac{\epsilon^{1/2}I}{X^2}\int_0^\infty u^2 du\, e^{-u^2}[\hat{\omega} - 1 + \eta_e(u^2 - 3/2)]y_0[H_i(iy_0)]^2 \tag{3.18}$$

The contribution to Δ' arising from this current is

$$\delta\Delta'^{(t)} = -i\hat{\beta}\hat{\omega}(1+\tau\hat{\omega})^2\int_0^\infty \frac{\hat{\sigma}^{(t)}(X)dX}{[i\hat{\sigma}_0X^2 + 1 + \tau\hat{\omega}]^2} \tag{3.19}$$

The integration over X is dominated by the region $y_0 \sim 0(1)$ ie, $X \sim (\frac{\omega}{\nu})^{1/2} >> 1$. We can therefore used the large X approximation to $\hat{\sigma}_0$, ie $\hat{\sigma}_0 X^2 \simeq -i(\hat{\omega}-1)$. $\Delta'^{(t)}$ involves a double integral over X and u. Changing the integration variable X to y_0 this can be evaluated as

$$\Delta'^{(t)} = -6i\Gamma(5/4)\pi^{3/2}IK\frac{(\hat{\omega} - 1 + \eta_e/4)(1+\tau\hat{\omega})^2}{(1+\tau)^2\hat{\omega}^2}\left(\frac{\nu_e}{2\omega}\right)^{1/2}\hat{\beta} \tag{3.20}$$

where we have used $\nu(u) = \nu_e/u^3$ and

$$K = \int_0^\infty y^{3/2}[H_i'(iy)]^2 dy = \frac{(1+i)}{24\sqrt{2}\pi}\left(\sqrt{3} - \frac{7}{2}ln(2+\sqrt{3})\right) \tag{3.21}$$

(The evaluation of K results from using the representation (3.16) to express it as a triple integral followed by judicious changes of variables.)

4 Cylindrical Collisional Contributions

As mentioned in the previous section the first two terms in the expression (3.13) for G_σ represent cylindrical terms. However they are only valid for $|\zeta| << 1$ since the Lorentz collision term was approximated in equation (3.11). To obtain a solution valid for all ζ we must solve the cylindrical equation

$$[\omega - k_\parallel v\zeta - \frac{i\nu}{2}\frac{\partial}{\partial\zeta}(1-\zeta^2)\frac{\partial}{\partial\zeta}]G = \frac{-ieF_m}{T}\left(\frac{\omega - \omega_*^T}{\omega}\right)v\zeta E_\parallel \tag{4.1}$$

on the continuous interval $|\zeta| < 1$.

We attempt a regular perturbation theory about the collisionless solution so that

$$\begin{aligned} G \;&=\; +\frac{ieF_mE_\parallel}{T\,k_\parallel}\left(\frac{\omega - \omega_*^T}{\omega}\right)\Bigg\{1 - \frac{\omega}{\omega - k_\parallel v\zeta} \\ &\quad - \frac{i\nu}{2(\omega - k_\parallel v\zeta)}\frac{\partial}{\partial\zeta}(1-\zeta^2)\frac{\partial}{\partial\zeta}\left(\frac{\omega}{\omega - k_\parallel v\zeta}\right)\Bigg\} \end{aligned} \tag{4.2}$$

The parallel current can be calculated from the $2\pi\int_0^\infty v^2 dv\int_{-1}^1 d\zeta$ moment of equation (4.1) which leads to the continuity equation

$$\omega n + \frac{k_\parallel j_\parallel}{e} = 0 \tag{4.3}$$

where $n = \int d^3vG$. However, using the approximation (4.2) for G leads to a singularity $\sim (\zeta - \zeta_0)^{-3}$ at $\zeta_0 \equiv \omega/k_{\|}v$ in the integrand.

To resolve this a boundary layer $\Delta\zeta \sim \left[(1-\zeta_0^2)\frac{\nu}{2k_{\|}v}\right]^{1/3}$ around ζ_0 in which collisions are important must be considered. In this region equation (4.1) for G can be solved by Fourier transforming and expanding ζ around ζ_0. Calculating the contribution of the resulting G to the integral over ζ from the vicinity of ζ_0 we find that the only singularity is that corresponding to the collisionless Landau resonance discussed in section 2. The collisional correction $n^{(c)}$ is integrable in ζ yielding

$$n^{(c)} = \frac{-4\pi}{3}\frac{\omega}{k_{\|}^3}\frac{eE_{\|}}{T}\int_0^\infty \frac{dv\,\nu(v)}{(\zeta_0^2-1)^2}\left(\frac{\omega-\omega_*^T}{\omega}\right)F_m \tag{4.4}$$

Clearly this integrand has a singularity $\sim \left(v - \frac{\omega}{k_{\|}}\right)^{-2}$ in v at $\zeta_0 = \omega/k_{\|}v = 1$. In order to discuss the interpretation of this singularity we consider a two-dimensional boundary region in v and ζ near $\zeta = 1$ and $v = \omega/k_{\|}$. In terms of the small parameter $\lambda = \left(\nu/k_{\|}v\right)^{1/2}$ we introduce $W = (1-\omega/k_{\|}v)/\lambda, t = (1-\zeta)/\lambda$. Equation (4.1) then takes the form

$$[i(W-t) - \frac{d}{dt}t\frac{d}{dt}]G = \frac{A}{\lambda}\frac{\left(\frac{\omega}{k_{\|}v_T}\right)\omega_*^2}{\omega k_{\|}v_T} \equiv \frac{\hat{A}}{\lambda} \tag{4.5}$$

for $t > 0$. This can be readily solved by Laplace transform in t for the case $W > 0$. The integration constant is chosen to remove a potential branch point in the Laplace transform $\tilde{G}(s)$ at $s_0 = \exp(-i\pi/4)$, thus ensuring convergent behaviour for $G(t)$ as $t \to \infty$. A solution valid for all W is obtained by analytic continuation from the solution for W > 0 and is given by

$$\tilde{G}(s) = \frac{1}{1+e^{-2\pi i\sigma}}\frac{(s+s_0)^{\sigma-1/2}}{(s-s_0)^{\sigma+1/2}}\int_C \frac{ds}{s}\frac{(s-s_0)^{\sigma-1/2}}{(s+s_0)^{\sigma+1/2}}\frac{\hat{A}}{\lambda} \tag{4.6}$$

where $\sigma = iW/2s_0$ and the contour C is shown in Fig. 3.

To calculate the collisional contribution to n we must subtract $\tilde{G}^{(0)}$ the Laplace transform of the collisionless solution, namely

$$\tilde{G}^{(0)} = \begin{cases} -ie^{Ws}\int_{-\infty}^{s}\frac{ds'}{s'}e^{Ws'}\frac{\hat{A}}{\lambda} & W > 0 \\ i\,e^{-Ws}\int_s^\infty \frac{ds'}{s'}e^{Ws'}\frac{\hat{A}}{\lambda} & W < 0 \end{cases} \tag{4.7}$$

The result is $\tilde{G}^{(c)} \sim W^{-2}$ as $W \to \infty$ so that the integration over W necessary to obtain $n^{(c)}$,

$$n^{(c)} = 2\pi\lambda^2\left(\frac{\omega}{k_{\|}}\right)^3\int_0^\infty dW\tilde{G}^{(c)}(0) \sim 0(\lambda) \tag{4.8}$$

can be evaluated by contour integration in terms of a sum over the poles of $\tilde{G}$, $W_n = (2n+1)e^{-3i\pi/4}$. This sum removes the branch point at s_0 of the integrand in (4.6) and the integral (4.8) vanishes. However, we must calculate the correction from integrating out to a large, but finite, value of W in order to match on to the singularity in the integrand of equation (4.4). This contribution is found to cancel the singularity, allowing us to integrate the integral in (4.4) by parts and disregard the end point contribution from $v = \omega/k_{\|}$.

Calculating $j_{\|}^{(c)}$ from equation (4.3) we obtain a collisional correction $\hat{\sigma}^{(c)}$ to the normalised conductivity

$$\hat{\sigma}^{(c)} = \frac{2}{3\sqrt{\pi}} \frac{\hat{\omega}^2 \omega_{*e}^3 \nu_e}{k_{\|}^4 v_{Te}^4} P \int_0^\infty \frac{dt\, e^{-t}}{\left[\left(\frac{\omega}{k_{\|} v_{Te}}\right)^2 - t\right]} \left\{ \eta_e + t \left[\frac{\hat{\omega} - 1 + 3/2\eta_e}{\left(\frac{\omega}{k_{\|} v_{Te}}\right)^2} - \eta_e \right] \right\} \tag{4.9}$$

The corresponding contribution to Δ' is given by

$$\Delta'^{(c)} = \frac{-64}{3} \frac{i\hat{\beta}}{\sqrt{\pi}} \frac{\nu_e}{\eta_e \omega_{*e}} (1 + \tau\hat{\omega})^2\, I(\hat{\omega}, \eta_e, \tau) \tag{4.10}$$

where

$$I(\hat{\omega}, \eta_e, \tau) = \frac{\eta_e}{16} \int_0^\infty \frac{ds}{Y(s)} s^2 P \int_0^\infty \frac{dt\, e^{-t}}{s^2 - t} \left[\eta_e + t \left(\frac{\hat{\omega} - 1 + 3/2\eta_e}{s^2} - \eta_e \right) \right] \tag{4.11}$$

and we have again introduced $s = \frac{\omega}{k_{\|} v_{Te}} \equiv \frac{\hat{\omega}}{X}$. $Y(s)$ is a function of the plasma dispersion function $Z(s)$

$$Y(s) = \left\{ 1/2 \left[(\hat{\omega} - 1) Z' + \frac{\eta_e}{2} s Z'' \right] - 1 - \hat{\omega}\tau \right\}^2 \tag{4.12}$$

and P represents a principal part integration. Substituting the zero order solution $\hat{\omega}_0 \simeq 1 + \frac{\eta_e}{2}$, discussed in the next section,

$$I = \int_0^\infty \frac{ds\, s^2}{[Z' + sZ'' - \alpha]^2}\; P \int_0^\infty \frac{dt e^{-t}}{s^2 - t} [1 + t(\frac{2}{s^2} - 1)] \tag{4.13}$$

where $\alpha = \frac{4}{\eta_e}(1 + \hat{\omega}_0 \tau)$. A numerical evaluation of $Im(I(\alpha))$ is shown in Fig. 4 indicating that it can be adequately fitted by

$$Im(I(\alpha)) \simeq \frac{8.0}{\alpha^3} \tag{4.14}$$

for $\alpha > 2$

5 Dispersion Relation and Discussion

The effects of trapped particles and collisional modifications to the circulating Landau resonance on the stability of micro-tearing modes can be examined by using equations (2. 33) together with equations (3. 20) and (4.10). First we consider the lowest order solution arising from $\hat{\sigma}_0$. In the work of Crew *et al* (1982) and Cowley *et al* (1986) it was shown that this lowest order solution was real and a good approximation could be obtained by using the large X asymptotic form for $\hat{\sigma}_0$ in the denominator of the integral. The integral could then be integrated analytically to yield

$$\delta\Delta' = -\hat{\beta}\, i\sqrt{\pi}\frac{(1+\tau\hat{\omega})}{\hat{\omega}(1+\tau)}(\hat{\omega} - 1 - \frac{\eta_e}{2}) \tag{5.1}$$

Since $\hat{\beta}/\delta >> 1$ the solution is

$$\hat{\omega} \simeq \hat{\omega}_0 = 1 + \frac{\eta_e}{2} \tag{5.2}$$

Evaluating the corrections from $\hat{\sigma}^{(t)}$ and $\hat{\sigma}^{(c)}$ using this value for $\hat{\omega}$ we obtain a growth rate

$$\begin{aligned}\hat{\gamma} \;=\; & 0.24\left(\frac{\nu_e\epsilon}{\omega_{*e}}\right)^{1/2}\frac{\eta_e(1+\tau\hat{\omega}_0)}{\hat{\omega}_0^{3/2}(1+\tau)} - 0.84\left(\frac{\nu_e}{\omega_{*e}}\right)\frac{(1+\tau)}{(1+\tau\hat{\omega}_0)^2}\eta_e^2 \\ & + \frac{\hat{\omega}_0(1+\tau)}{(1+\tau\hat{\omega}_0)}\frac{\delta\Delta'}{\hat{\beta}\sqrt{\pi}}\end{aligned} \tag{5.3}$$

The trapped electron term is destabilising whereas the collisional broadening of the Landau resonance term is stabilising. For micro-tearing modes $\Delta' \simeq -2k_\theta$ is stabilising and becomes important for short wavelength modes. The stabilising collisional term will tend to dominate in the regime of validity of the analysis ie $\nu_e/\epsilon\omega_{*e} < 1$. However, the trapped electron term in expression (5.3) has similar parametric dependencies to that found by Catto and Rosenbluth (1981)

$$\hat{\gamma}^{(t)} \simeq \frac{0.78\eta_e}{\hat{\omega}^{1/2}}\left(\frac{\epsilon\nu_e}{\omega_{*e}}\right)^{1/2}[ln\left(\frac{128\epsilon\omega}{\nu_e}\right)]^{-1/2} \tag{5.4}$$

if one ignores the weak logarithmic dependence and details of the η_e and τ dependencies. At the transition, $\nu_e/\epsilon\omega_{*e} \sim 1$, the numerical value is also similar to within a factor of about 2. It is therefore reasonable to use the formula throughout the whole range $\nu_e/\omega_{*e} < 1$.

Finally we incorporate the ion orbit effects, parameterised by $\lambda = \hat{\omega}\delta/2\rho_i < 1$ and considered by Cowley *et al* (1986), as another additive perturbation term to yield a complete form for the growth rate

$$\hat{\gamma} = -C_1\lambda\, ln(1/\lambda)\eta_e^2 + C_2\eta_e\sqrt{\frac{\epsilon\nu_e}{\omega_{*e}}} - C_3\eta_e^2\frac{\nu_e}{\omega_{*e}} - C_4\Lambda\left(\frac{\omega_{*e}}{\nu_e}\right) \tag{5.5}$$

where the C_i have weak dependencies on η_e and τ

$$\begin{aligned}
C_1 &= 0.5\frac{[1-\frac{\eta_i}{2}+\tau(1+\frac{\eta_e}{2})]}{(1+\tau)[1+(1+\frac{\eta_e}{2})\tau](1+\frac{\eta_e}{2})} \\
C_2 &= \frac{0.24[1+(1+\frac{\eta_e}{2})\tau]}{(1+\frac{\eta_e}{2})^{3/2}(1+\tau)} \\
C_3 &= \frac{0.84(1+\tau)}{[1+(1+\frac{\eta_e}{2})\tau]^2} \\
C_4 &= \frac{1}{\sqrt{\pi}}\frac{(1+\tau)(1+\frac{\eta_e}{2})}{[1+(1+\frac{\eta_e}{2})\tau]}
\end{aligned} \tag{5.6}$$

and we have made use of $\Delta' = -2k_\theta$ for short wavelength modes to express the field line bending in terms of the parameter Λ

$$\Lambda = \frac{4\epsilon^{3/2}}{\beta_e}\frac{L_n^2}{R^2q^2}s\nu_* \tag{5.7}$$

where $s = rq'/q$ and $\nu_* = (\nu_e Rq/\epsilon^{3/2}v_{Te})$.

Two limiting cases in equation (5.5) can be investigated rather simply. These are (i) the flat temperature case $\eta_i, \eta_e << 1$ and (ii) the flat density case $\eta_e, \eta_i \rightarrow \pm\infty$.

(i) Small η_e.

In the limit $\eta_e \rightarrow 0$, stability is assured, since only the stabilising field line bending term remains in equation (5.5)

(ii) $\eta_e, \eta_i \rightarrow \infty$

The leading approximation for the mode frequency, equation (5.2), given by Crew, Antonsen and Coppi (1982) for finite η_e, must be recalculated for this case. When this is done it is found that (5.2) remains accurate provided $\tau \gtrsim 1$, and ω_0 depends rather weakly on τ for $\tau < 1$. The dispersion relation may be written in the form

$$\frac{\gamma}{(\omega_{*e}^T)} = -\hat{C}_1\lambda ln\lambda + \hat{C}_2\sqrt{\epsilon}\left(\frac{\nu_e}{\omega_{*e}^T}\right)^{1/2} - \hat{C}_3\left(\frac{\nu_e}{\omega_{*e}^T}\right) - \hat{C}_4\left(\frac{\omega_{*e}^T}{\nu_e}\right) \tag{5.8}$$

where

$\omega_{*e}^T \equiv \omega_{*e}\eta_e$

$\hat{C}_1 = \frac{[\tau - \partial lnT_i/\partial lnT_e]}{\tau(1+\tau)}$, $\hat{C}_2 = 0.35\frac{\tau}{1+\tau}$,

$\hat{C}_3 = 3.36\frac{1+\tau}{\tau^2}$ $\hat{C}_4 = 2.26\frac{1+\tau}{\tau}\Lambda$

and the density scale length L_n is replaced by the temperature scale length L_T in both λ and Λ. From the form of $\hat{C}_1$ it is evident that ion orbit effects become destabilising if

$$\frac{dT_i}{dr} > \frac{dT_e}{dr} \tag{5.9}$$

However, because of the larger numerical coefficients associated with the stabilising terms $\hat{C}_3$ and $\hat{C}_4$, it is in general difficult to find a parameter regime in which micro-tearing modes are unstable. Minimising these stabilising contributions over wavelength by the choice

$$\left(\frac{\nu_e}{\omega_{*e}^T}\right) = 0.82(\tau\Lambda)^{1/2} \tag{5.10}$$

equation (5.8) for the growth rate becomes

$$\begin{aligned} \left(\frac{\gamma}{\omega_{*e}^T}\right) &= -\frac{[1 - \frac{\partial T_i}{\partial r}/\frac{\partial T_e}{\partial r}]}{(1+\tau)}\lambda ln(\lambda^{-1}) + 0.32\frac{\tau}{1+\tau}\sqrt{\epsilon}(\tau\Lambda_T)^{1/4} \\ &\quad -5.5\left(\frac{1+\tau}{\tau^2}\right)(\tau\Lambda_T)^{1/2} \end{aligned} \tag{5.11}$$

If the inequality (5.9) is not satisfied the ion orbit effects are stabilising and there is no realistic parameter range in which micro-tearing modes are predicted to be unstable.

Returning to equation (5.5) for finite η_e and η_i, it is an inescapable fact that the trapped electron term driving instability is rather feeble. Short wavelength modes are dominated by field line bending, longer wavelength modes by collisional broadening. As in the $|\eta_e| \to \infty$ limit, instability is only realistically possible when ion orbit effects also destabilise, i. e. when

$$\left[\frac{dT_i}{dr} - \frac{dT_e}{dr}\right] > 2\left[T_e + T_i\right]\frac{dn}{dr} \tag{5.12}$$

6 Conclusion

We have investigated the stability of micro-tearing modes in a realistic parameter range $\nu < \omega_{*e} < \nu_{eff}$ for a toroidal plasma in the banana regime $\nu_{eff} < \omega_{be}$. The presence of trapped particles modifies the collisional slab response in a region of velocity space of width $\sqrt{\nu_e/\omega_{*e}}$ around the trapped region when a Lorentz collision operator is employed. These modifications have been calculated in section 3 by asymptotic matching procedures in velocity space, leading to a contribution to the growth rate $\gamma^{(t)} \sim \epsilon^{1/2}\eta_e\sqrt{\nu_e\omega_{*e}}$. This fits reasonably smoothly onto an expression derived previously by Catto and Rosenbluth (1981) in the complementary regime $\nu_{eff} < \omega_{*e}$.

However, stability is determined by a competition between this effect and stabilising contributions arising from field line bending and ion orbit effects (calculated by Cowley et al (1986)), which dominate at lower values of ν_e/ω_{*e}, and collisional modifications to the circulating electron Landau resonance which dominate at higher values of ν_e/ω_{*e} and have been calculated in section 4. If the stabilising effect of field line bending outside the resonant layer (ie Δ') is neglected short wavelength micro-tearing modes are found to be unstable as reported by Connor et al 1989. However, when all stabilising terms are retained the trapped electron driving terms are too feeble, for typical Tokamak parameters, to outweigh the various damping mechanisms and the dispersion relation (5.5) predicts that micro-tearing modes should be linearly stable.

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Collisionless	Collisional boundary Layer	Just Passing	Trapped	Just Passing	Collisional boundary Layer	Collisionless
IV_L	III_L	II_L	I	II_R	III_R	IV_R

-1 $\quad -\sqrt{\frac{\nu}{\omega}} \quad \epsilon^{1/2} \quad \epsilon^{1/2} \quad \sqrt{\frac{\nu}{\omega}} \quad \zeta \quad 1$

Fig. 1 Regions of pitch angle $\zeta = \frac{v_{\parallel}}{v}$

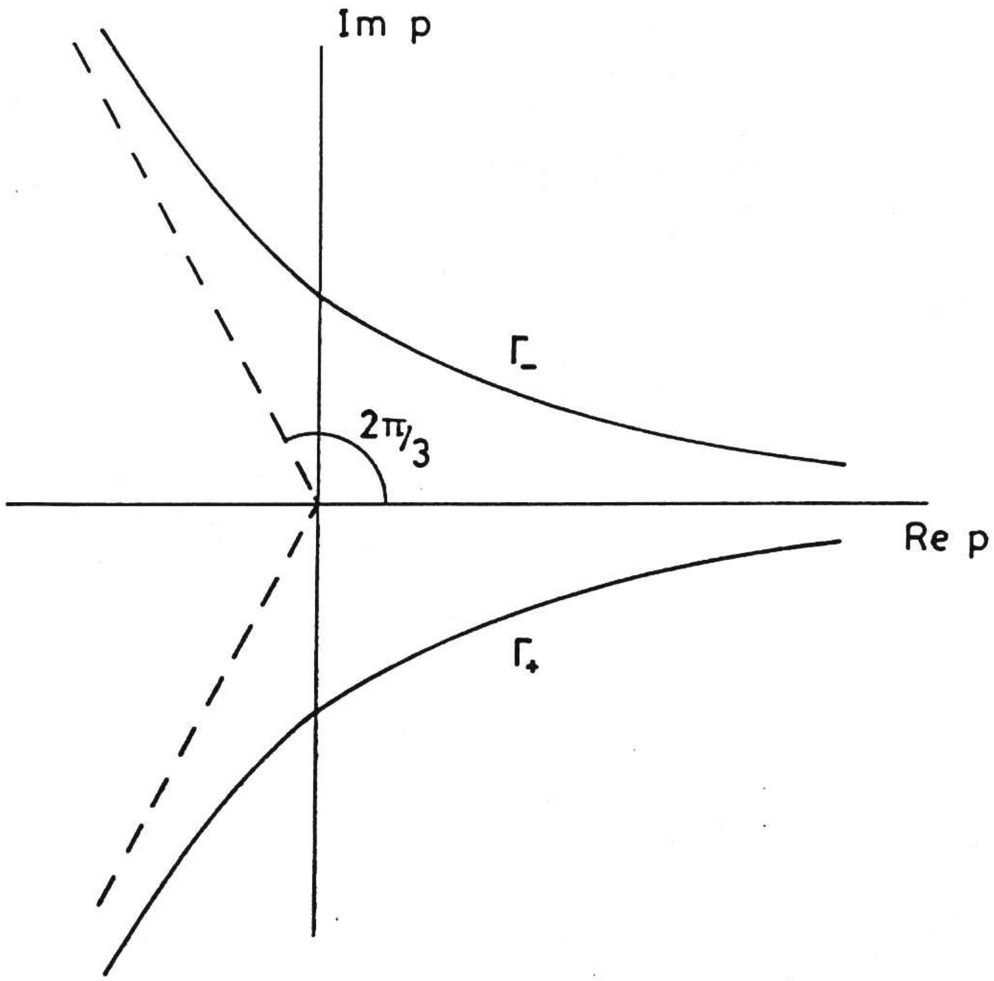

Fig. 2 Contours of Integration for equation (3.13)

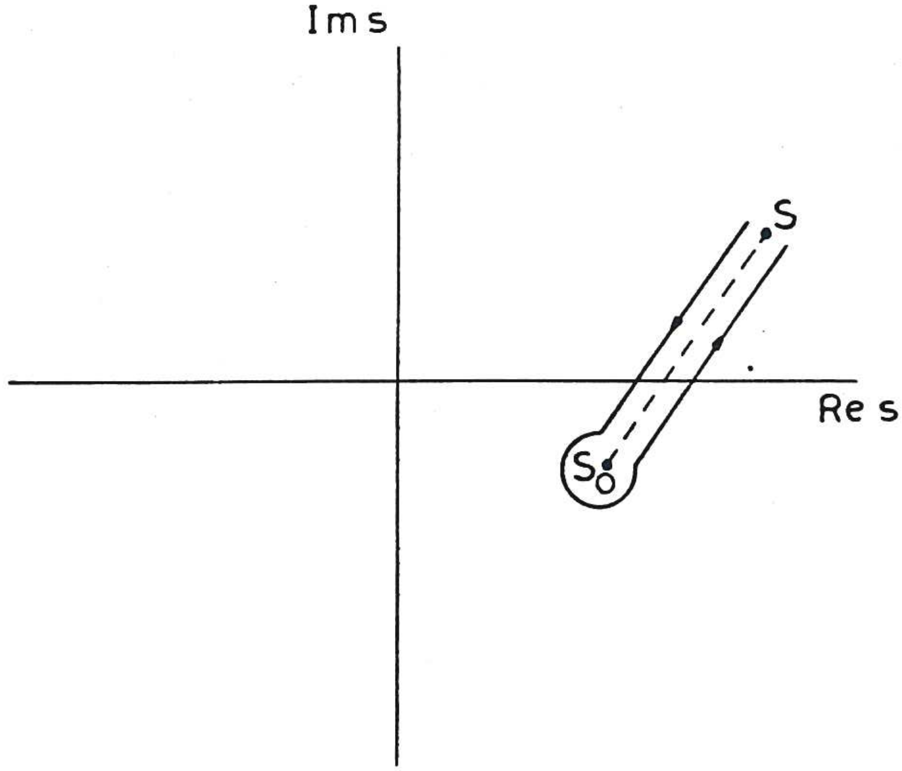

Fig. 3 The contour C in equation (4.6)

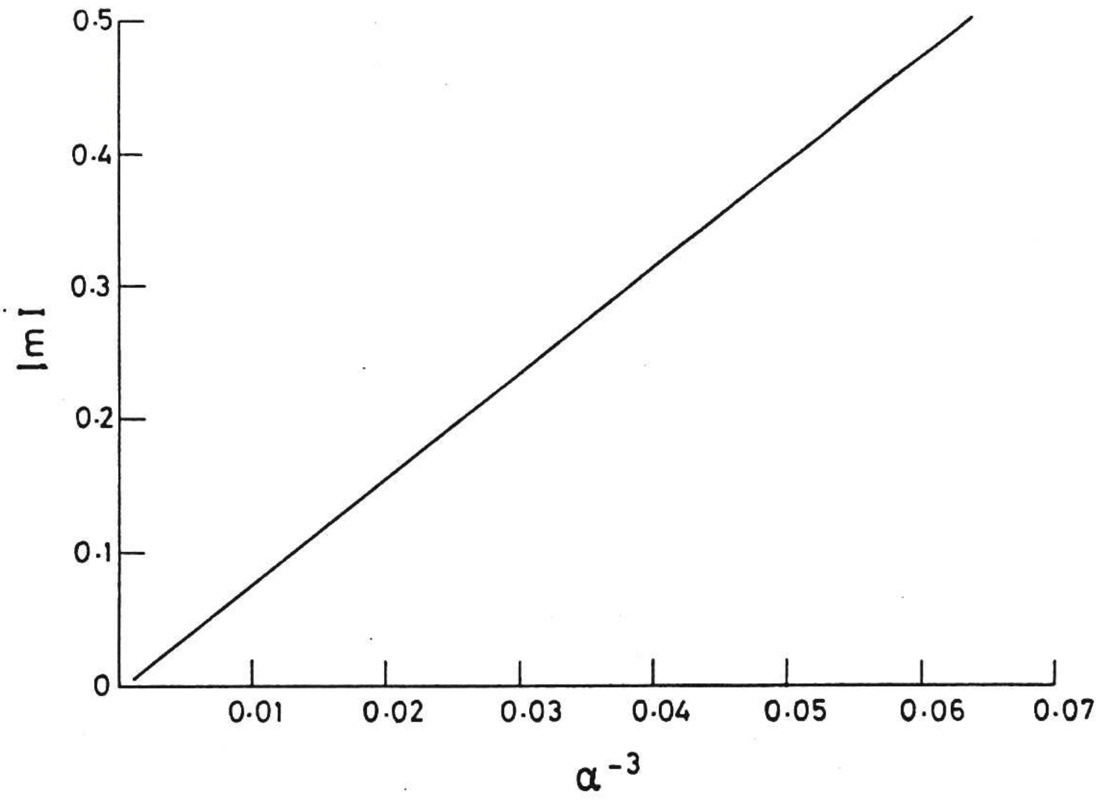

Fig. 4 Imaginary part of integral $I(\alpha)$ in equation 4.13