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The Interaction of Plasma Rotation and Toroidal Field Ripple in Tokamaks

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Abstract

A simple MHD model of a plasma rotating through the periodic ripple of the toroidal magnetic field of a tokamak is considered. It is shown that the plasma becomes unstable when the equilibrium toroidal rotation velocity exceeds the sound speed. The rippled field is equivalent to the magnetic "wiggler" of a free electron laser. In the plasma case, long wavelength Alfven waves and short wavelength ion acoustic waves are excited.

1 Introduction

Since the toroidal magnetic field in a tokamak is produced by a finite set of coils placed at regular intervals around the torus there is inevitably a periodic but small modulation associated with this field. This toroidal ripple, as it is called, is not usually of significance unless it is resonant with some equilibrium motion. This point was emphasised recently by LAURENT and RAX(1989) who considered the effect of such a resonance associated with fast electrons. The resonance considered by these authors occurs when the electron cyclotron frequency matches the frequency associated with the transit of a fast electron, travelling close to the velocity of light, through one period of the toroidal ripple. This resonance involves MeV electrons and generates high frequency electron cyclotron radiation. The toroidal ripple plays the role of the magnetic "wiggler" field in a free electron laser.

In this paper a related, low frequency resonance will be considered which can be described as a "tokamak free ion laser". The toroidal ripple again forms the wiggler magnetic field but the fast electrons are replaced by a rotation of the bulk plasma in the toroidal direction. Instead of high frequency electromagnetic waves, low frequency Alfvén waves are generated.

This resonance is closely related to the decay of a large amplitude Alfvén wave into another Alfvén wave and an ion acoustic wave (GALEEV and ORAEVSKII(1963)). However, there are important differences in the two cases which will be discussed later in the paper.

2 The Low Frequency Resonance

We shall adopt a rather simple model to describe the resonance and follow LAURENT and RAX(1989) in neglecting the longitudinal part of the toroidal magnetic field ripple and consider only the transverse part. We therefore idealize the problem by considering a straight system with the following magnetic field

$$\underline{B}_0(z) = \hat{z}B_{z0} + B_{\perp 0}(\hat{x}\ \sin\ k_0 z + \hat{y}\ \cos\ k_0 z) \tag{1}$$

in which B_{z0} and $B_{\perp 0}$ are constants. For the case of toroidal field ripple, $B_{\perp 0} \ll B_{z0}$ and k_0 = N/R where N is the number of toroidal field coils and R is the major radius. In addition to the helical equilibrium magnetic field configuration the plasma is also assumed to have a bulk drift v_d in the z - direction. This corresponds to an equilibrium toroidal rotation of the plasma as a whole. In the rest frame of the plasma, the static toroidal field ripple appears as an oscillation of freqency $k_0 v$. This oscillation can then produce a non-linear coupling between longitudinal density fluctuations, with a wave number k, and transverse magnetic field fluctuations of wave number $k \underline{+} k_0$.

Such a coupling, for a magnetic field described by equation (1), has been discussed by LASHMORE-DAVIES and STENFLO (1979) using the one fluid MHD model. Here, the dispersion relation derived in this reference will be used to analyse the possibility of a low frequency resonance with the toroidal ripple. The dispersion relation (eq.(12) of LASHMORE-DAVIES and STENFLO (1979)) can be written.

$$(\omega - kv_d)^2 - k^2c_s^2 = \frac{k}{2\mu_o}\left[(k - k_0 + k_0\frac{v_d^2}{c_A^2})\alpha_+ + (k + k_0 - k_0\frac{v_d^2}{c_A^2})\alpha_-\right] B_{\perp 0}^2 \tag{2}$$

where $\alpha_\pm$ are given by

$$\alpha_\pm = \pm\frac{(k \mp k_0)}{\rho_0 k_0}k_0^2 v_d^2 \quad \left\{1 + \frac{(\omega - kv_d)}{kv_d} \pm \frac{(\omega - kv_d)[\omega - (k \mp k_0)v_d]}{k_0^2 v_d^2}\frac{k_0}{k}\right\}$$

$$\times\left\{[\omega - (k \mp k_0)v_d]^2 - (k \mp k_0)^2 c_A^2\right\}^{-1} \tag{3}$$

It was pointed out, in reference 3 that the dispersion relation given by Eq (2) has unstable solutions for $k \simeq k_0$. In the present context, this corresponds to a density perturbation which is resonant with the wavelength of the toroidal ripple. This is the low frequency resonance which will now be considered in more detail.

Since the resonance involves a coupling between the density perturbation with wave number k and the magnetic field perturbation with wave number $k - k_0$ we may neglect the non-resonant term, proportional to α_- in Eq (2). Making this approximation, Eq (2) can be written.

$$\{(\omega - kv_d)^2 - k^2c_s^2\}\left\{[\omega - (k - k_0)v_d]^2 - (k - k_0)^2 c_A^2\right\}$$

$$\simeq \frac{kB_{\perp 0}^2}{2\mu_0}(k - k_0 + k_0\frac{v_d^2}{c_A^2})(\frac{k - k_0}{\rho_0 k_0}k_0^2 v_d^2\left\{1 + \frac{(\omega - kv_d)}{kv_d} + \frac{(\omega - kv_d)[\omega - (k - k_0)v_d]}{k_0^2 v_d^2}\frac{k_0}{k}\right\} \tag{4}$$

A perturbation solution to Eq.(4) can now be obtained assuming

$$\omega = k(v_d - c_s) + \delta\omega \tag{5}$$

with the resonance condition

$$k(v_d - c_s) = (k - k_0)(c_A + v_d) \tag{6}$$

The resonance condition given by Eq (6) determines the unstable wave number k which is given by

$$k = \frac{k_0[1 + (v_d/c_A)]}{[1 + (c_s/c_A)]} \tag{7}$$

Using Eqs (5) and (6), the solution of Eq (4) is given approximately by

$$\delta\omega \simeq \pm\frac{i}{2\sqrt{2}}\frac{B_{\perp 0}}{B_{z0}}k_0 v_d(\frac{v_d - c_s}{c_s})^{1/2}(\frac{v_d - c_s}{v_d}) \tag{8}$$

Thus, the condition for instability is $v_d > c_s$.

The disperson relation given in Eq (2) was obtained from ideal MHD. The effect of dissipation has also been considered (LASHMORE-DAVIES and STENFLO (1981)) by including resistivity and a phenomenological collision frequency . With these additions a threshold condition for instability was obtained which also required $v_d > c_s$. It was shown that for v_d sufficiently close to c_s the amplitude $B_{\perp 0}$ could be arbitrarily small. This result was shown to be due to the fact that the damping of the Alfvén wave vanishes in the limit $v_d = c_s$ since this corresponds to an infinite wavelength mode. In practice, of course, there will be an upper limit to the wavelength of the excited Alfvén wave, which in a torus is simply $2\pi R$. We shall return to this point later.

The interpretation of Eqs (6) - (8) is the following. The resonance condition, Eq (6) describes the coupling between the slow ion acoustic wave and a long wavelength Alfvén wave, both travelling along the axial (toroidal) magnetic field. The waves are coupled by the toroidal ripple and since this is a static magnetic field (in the laboratory frame) the frequencies of the Alfvén and ion acoustic waves are the same. Hence, for a low beta plasma where $c_s^2 \ll c_A^2$ the acoustic wave will have a much shorter wavelength than the Alfvén wave. Since the frequencies are the same it is to be expected that the interaction will couple energy to the longitudinal and transverse modes in equal proportions. The interaction becomes unstable when $v_d > c_s$ since this is the condition for the slow ion acoustic mode to be a negative energy wave.

As already noted, the acoustic wave is excited at a wavelength close to the wavelength of the toroidal ripple. Using Eq (7), the wave number of the excited Alfvén wave, $k - k_0$, is given by

$$k - k_0 = \frac{(v_d - c_s)}{(c_A + c_s)}k_0 \tag{9}$$

In order to be resonant in the torodal geometry of a tokamak, the Alfvén wavelength should satisfy the condition

$$k - k_0 = \frac{n}{R} \tag{10}$$

Using the condition $k_0 = N/R$ and Eq (9), Eq (10) gives

$$\frac{(v_d - c_s)}{c_A} = \frac{n}{N} \tag{11}$$

where c_s has been neglected in comparison with c_A. For given density, temperature and magnetic field, Eq (11) gives a condition on v_d for unstable ion acoustic and Alfvén fluctuations to be resonant simultaneously with the toroidal ripple and the toroidal circumference.

Consider a specific example typical of the parameters obtained in the JET experiment. Thus the ion and electron temperatures are assumed to be equal, $T_e = T_i = 10keV$, the density is taken as $n_0 = 5 \times 10^{19} m^{-3}$, the toroidal magnetic field $B_o = 3.4$ tesla and a pure deuteruim plasma is assumed. Taking $c_s^2 = (T_e + T_i)/m_i$ corresponding to an isothermal gas law and taking $n = 1$ and $N = 32$, appropriate to the 32 toroidal field coils on the JET experiment, Eq (11) gives $v_d = 1.24 c_s$. The frequency of the ion acoustic and Alfvén fluctuations is then approximately 400kHz with a growth rate $\sim$ 1 kHz where $B_{\perp 0}/B_{z0} = 0.01$ which represents an average figure for the amplitude of the ripple on JET. For higher harmonics of the toroidal ripple or larger rotation velocities the growth rate would be higher with correspondingly higher frequencies.

3 Conclusions

In this paper it has been shown that the combination of toroidal rotation and the periodic modulation of the toroidal magnetic field due to the finite number of coils can lead to long wavelength Alfvén waves and short wavelength ion acoustic waves being driven unstable. The condition for instability is that the rotation speed should exceed the sound speed. The underlying mechanism is a low frequency analogue of the free electron laser. Although significant rotation velocities are reached in existing tokamak experiments it appears that the bulk sound speed has not yet been exceeded. The present analysis indicates that significant changes in the discharge behaviour would be expected if this threshold were to be exceeded.

Even if the basic mechanism discussed in this paper is below threshold in existing experiments, the present analysis raises other questions. It was shown in references 3 and 4 that similar instabilities can be driven by oscillating magnetic fields which have a frequency and wave number (ω_0, k_0). The condition for instability is again $\omega_0/k_0 > c_s$. Such oscillating magnetic fields are present in tokamak discharges either as a result of some other instability or due to another heating system, such as ion cyclotron heating. Finally, since the sound speed for a heavier minority or impurity ion would be significantly lower than the sound speed of the bulk plasma it would be worth investigating the possibility of the resonant excitation of other acoustic modes due to the presence of additional ion species.

4 References

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