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Preprint

RECOMBINATION BETWEEN ELECTRONS AND ATOMIC IONS
PART 2: OPTICALLY THICK PLASMAS

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CORRIGENDUM TO CLM - P 10

Would you please make the following alterations to this Preprint:-

Page 4, first line,

Delete 'n(c)' and substitute 'n(2)'.

Page 10, Table 4, line 4,

Delete '$_Q(2)$ in cm^{-3}' and substitute '$n_Q(2)$ in cm^{-3}'.

Page 13, Table 7, line 6 under '32000',

Delete '1.1^{-20}' and substitute '1.1^{-29}'.

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RECOMBINATION BETWEEN ELECTRONS AND ATOMIC IONS

PART II. OPTICALLY THICK PLASMAS

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ABSTRACT

The effect of self-absorption on the rate of collisional-radiative recombination is investigated using the statistical theory developed in an earlier paper. Detailed calculations are carried out on hydrogen ion plasmas. The following cases are treated: case (i), plasma optically thick towards lines of the Lyman series; case (ii), plasma optically thick towards lines of all series; case (ic) and (iic), plasma as in cases (i) and (ii) respectively but also optically thick towards the Lyman continuum.

It is found that self-absorption reduces the recombination coefficient even in the low electron density limit.

CONTENTS

1. Introduction

A recent paper* (Bates, Kingston and McWhirter, 1962) describes a method by which it is possible to calculate the rate of loss of atomic ions from a plasma due to the complex of interacting collisional and radiative processes occurring. In the case of hydrogen ions these processes are as follows, p or q denoting the principal quantum number of the state of the hydrogen atom occupied: three-body recombination

$$H^+ + e + e \rightarrow H(p) + e; \tag{1}$$

and the inverse collisional ionization

$$H(p) + e \rightarrow H^+ + e + e; \tag{2}$$

superelastic collisions

$$H(p) + e \rightarrow H(q) + e, \ q < p; \tag{3}$$

and inelastic collisions

$$H(p) + e \rightarrow H(q) + e, \ q > p; \tag{4}$$

downward cascading

$$H(p) \rightarrow H(q) + h\nu; \tag{5}$$

and upward transitions by line absorption

$$H(q) + h\nu \rightarrow H(p); \tag{6}$$

radiative recombination

$$H^+ + e \rightarrow H(p) + h\nu \tag{7}$$

and photo-ionization

$$H(p) + h\nu \rightarrow H^+ + e; \tag{8}$$

and elastic collisions involving a change in the azimuthal quantum number.

It was assumed in paper I that if $\sigma(\lambda)$ is the optical thickness of the plasma at wave length λ then

$$\sigma(\lambda) \ll 1 \tag{9}$$

throughout the spectrum so that processes (6) and (8) may be neglected in the absence of a source of radiation other than the plasma itself. Complications arise in cases where this assumption is not valid during all stages of the decay. There is an infinite variety of such cases: thus $\sigma(\lambda)$, which is of course a rapidly varying function of λ, depends on the number density of hydrogen atoms in each state, on the widths of the spectral features and on the linear dimensions of the plasma. A detailed general treatment is scarcely feasible. It will however, be shown that the position may be clarified by considering a few special cases: (i) a hydrogen ion plasma for which (9) is satisfied except in the Lyman lines where instead

$$\sigma(\lambda) \gg 1; \tag{10}$$

(ii) a hydrogen ion plasma for which (9) is satisfied except in the lines of any of the series of hydrogen where instead (10) is satisfied; (ic) and (iic) plasmas as in cases (i) and (ii) respectively but with (10) also satisfied through the Lyman continuum.

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*This paper is referred to as I hereafter

2. Lyman Radiation Absorbed

The treatment of collisional-radiative recombination developed in paper I for the case of an optically thin plasma is statistical. It requires that the mean thermal energy is much less than the first excitation energy and that

$$n(c) \gg 10^{14 + w} \text{ cm}^{-3} \tag{11}$$

where n(c) is the number density of free electrons and where w is given by

$$2^w = T/1000 \tag{12}$$

T being the temperature in degrees Kelvin. If these requirements are met (as they are in most plasmas of interest) a quasi-equilibrium distribution of population through the excited levels is established effectively instantaneously. In this quasi-equilibrium n(p), the number density of hydrogen atoms in any excited level p, is very small in the sense that

$$n(p),\ p \neq 1,\ \ll n(1) \text{ or } n(c). \tag{13}$$

On equating the rates at which atoms enter and leave level p ($\neq 1$) a set of linear equations was obtained from which n(p), $p \neq 1$, was calculated in terms of n(1), n(c) and parameters characteristic of the elementary processes listed above. Knowing n(p), $p \neq 1$, the time derivative $\dot{n}(1)$ could readily be determined; and since the quasi-equilibrium ensures that

$$\dot{n}(H^+) = -\dot{n}(1) \tag{14}$$

hence so also could be the <u>collisional-radiative decay coefficient</u> γ defined according to

$$\dot{n}(H^+) = -\gamma n(H^+)n(c) \tag{15}$$

This may be put in the form

$$\dot{n}(H^+) = -\{\alpha n(H^+)n(c) - Sn(1)n(c)\} \tag{16}$$

where in contrast to γ which is of a composite nature α and S are simple rate coefficients in that they are positive quantities dependent only on n(c), T and various atomic parameters. The quantity α was given the name <u>collisional-radiative recombination coefficient</u> and the quantity S the name <u>collisional-radiative ionization coefficient</u>.

The procedure outlined in the preceding paragraph must be modified if there is self-absorption. In case (i) of §1 the Lyman lines are completely absorbed so that the downward radiative transitions from any level p to level 1 are balanced by the reverse upward transitions. It is therefore necessary to remove all terms involving the spontaneous transition probabilities A(p,1) from the set of linear equations governing the quasi-equilibrium. Cognisance must also be taken of the fact that this makes level 2 effectively stable with respect to radiative transitions. In general it does not remain permissible to group this level with the other excited levels. Condition (13) was therefore relaxed to

$$n(p),\ p \neq 1 \text{ or } 2,\ \ll \{n(1) + n(2)\} \text{ or } n(c); \tag{17}$$

the set of linear equations was solved for n(p), $p \neq 1$ or 2, with n(2), as well as n(1) and n(c), taken as known; and relation (14) was replaced by

$$\dot{n}(H^+) = -\dot{n}(1) - \dot{n}(2) \tag{18}$$

The collisional-radiative decay coefficient γ introduced in (15) may now be expressed as the sum

$$\gamma = \gamma_1 + \gamma_2 \tag{19}$$

in which γ_1 and γ_2 are such that

$$\dot{n}(1) = \gamma_1 n(H^+)n(c) \tag{20}$$

and

$$\dot{n}(2) = \gamma_2 \, n(H^+)n(c). \qquad (21)$$

Referring to paper I it may be seen that (20) and (21) may be written as

$$\dot{n}(1) = a_1 n(H^+)n(c) + P_{21} n(2)n(c) - R_1 n(1)n(c) \qquad (22)$$

and

$$\dot{n}(2) = a_2 n(H^+)n(c) + P_{12} n(1)n(c) - R_2 n(2)n(c) \qquad (23)$$

a_1, a_2, P_{12}, P_{21}, R_1 and R_2 like α and S of (16), being positive quantities dependent only on n(c), T and various atomic parameters. The computed values of the six new rate coefficients are given in Tables 1 and 2.

Examination of (22) and (23) in conjunction with Tables 1 and 2 shows that

$$\tau(2) \ll \tau(1) \qquad (24)$$

where $\tau(2)$ and $\tau(1)$ are the relaxation times for hydrogen atoms in the levels indicated. Provided

$$\tau(2) \ll \tau(e) \text{ and } \tau(H^+) \qquad (25)$$

where $\tau(e)$ and $\tau(H^+)$ are the relaxation times of the electrons and the hydrogen ions* it follows that n(2) quickly rises to a quasi-equilibrium value

$$n_Q(2) = \frac{a_2 n(H^+) + P_{12} \, n(1)}{R_2} \qquad (26)$$

and that thereafter

$$\dot{n}(H^+) = - \{\alpha^i n(H^+) - S^i n(1)\} n(c) \qquad (27)$$

in which the collisional-radiative recombination coefficient

$$\alpha^i = a_1 + (P_{21}/R_2) a_2 \qquad (28)$$

and the collisional-radiative ionization coefficient

$$S^i = R_1 - (P_{21}/R_2) P_{12}. \qquad (29)$$

When the steady state is reached the number densities in levels 1 and 2 are given by

$$n_S(1) = \frac{a_1 \, R_2 + a_2 \, P_{21}}{R_1 \, R_2 - P_{12} P_{21}} \; n(H^+) \qquad (30)$$

and

$$n_S(2) = \frac{a_2 \, R_1 + a_1 \, P_{12}}{R_1 \, R_2 - P_{12} P_{21}} \; n(H^+) \qquad (31)$$

Values of $n_S(1)$ and $n_S(2)$ are shown in table 3.

The coupled differential equations which describe the decay may be simplified if the degree of ionization is high enough for the number density of normal hydrogen atoms to be well below the steady state value. In this important circumstance (cf. table 3) the terms involving n(1) on the right of (22) and (23) may be omitted without appreciable loss of accuracy leaving

$$\dot{n}(1) = a_1 n(H^+)n(c) + P_{21} n(2)n(c) \qquad (32)$$

and

$$\dot{n}(2) = a_2 n(H^+)n(c) - R_2 n(2)n(c). \qquad (33)$$

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*These times in general differ if n(e) and $n(H^+)$ differ.

The quasi-equilibrium value which n(c) may reach may be taken to be

$$n'_Q(2) = \{a_2/R_2\}n(c) \tag{34}$$

Knowing a_1 and P_{21} (table 1) and a_2 and R_2 (table 2) the course of the decay may readily be computed. Detailed computations are not needed to reveal the general pattern.

For the sake of definiteness attention will hereafter be confined to plasmas in which $n(H^+)$ and n(c) are equal; and for the sake of simplicity it will be further confined to plasmas in which conditions (11) and (13) are satisfied.

Condition (25) requires that

$$n'_Q(2) \ll n(c) \tag{35}$$

In the cool and dense plasmas in which this is violated (cf. table 4) P_{21} is little less than R_2 so that an electron entering level 2 is much more likely to fall to level 1 than to regain the free state. Consequently the recombination coefficient is simply

$$\alpha^i = a_1 + a_2 \tag{36}$$

This happens to be just what (28) would give if it were applied. However the quasi-equilibrium picture on which (28) is based is here incorrect. Instead n(2) increases rapidly during the greater part of the decay.

The computed values of the recombination coefficient α^i of (28) and (36) are given in table 5.

The recombination coefficient α^{ic} in a plasma which is also optically thick towards the Lyman continuum case (ic) of §1, may now be readily obtained since

$$\alpha^{ic} = \alpha^i - \beta(1) \tag{37}$$

where $\beta(1)$ is the coefficient for radiative recombination into level 1 (cf. table 6).

3. All Line Radiation Absorbed

The absorption of all line radiation, case (ii) of § 1, may be represented by supposing that every spontaneous transition probability vanishes. This brings about great simplification in the set of linear equations determining the quasi-equilibrium. Thus it may be seen from (11) of paper 1 that

$$n(p) \sum_{q \neq p} K(p,q) - n_E(p) \sum_{q \neq p} \frac{n(q)}{n_E(q)} K(p,q) \tag{38}$$

$$= n(c) \{K(c,p) + \beta(p)\}, \quad p \neq 1$$

K(c,p), K(p,q) and $\beta(p)$ being the rate coefficients of (1), (3) or (4) and (7) respectively and $n_E(p)$ being the Saha number density of the level indicated. The solutions to (38) are clearly of the form

$$n(p) = n_t(p) + n_r(p) \tag{39}$$

where $n_t(p)$, which is proportional to $n(c)^2$, arises from the K(c,p) term on the right hand side (three body recombination), and where $n_r(p)$, which is proportional to n(c), arises from the $\beta(p)$ term (radiative recombination). It follows immediately that the collisional-radiative recombination coefficient may be expressed as

$$\alpha^{ii} = n(c)k_t + a_r \tag{40}$$

in which the two terms on the right correspond to the two on the right of (39) and k_t and a_r are functions only of the temperature (table 7).

The treatment described depends on the usual assumption that the relaxation time for the excited atoms is much shorter than that for the electrons and ions. This assumption has a rather less wide range of validity than in the case where only the Lyman radiation is absorbed. The populations of the excited levels in the cool and dense plasmas in which it cannot be made are smaller than the quasi-equilibrium populations. In consequence the quasi-equilibrium model leads to the effectiveness of collisional ionization being overestimated. The actual recombination coefficient must therefore be greater than the recombination coefficient given by (40) and table 7. It must also be less than the recombination coefficient in the optically thin case. Thus lower and upper limits to the recombination coefficient sought are known. The difference between these limits in the case of a cool and dense plasma is happily negligible. This argument could also have been used in §2 to justify the use of table 5 at temperatures and densities at which condition (35) is not satisfied.

If the Lyman continuum is also absorbed the recombination coefficient is simply

$$\alpha^{iic} = \alpha^{ii} - \beta(1) = [a_r - \beta(1)] + n(c)k_t \tag{41}$$

The composite term in square brackets is, at high temperatures very much smaller than its two components. It was evaluated directly and is included in table 7.

4. Discussion of Results

A number of processes depending in different ways on the temperature and on the electron density are involved. The pattern of the results is in consequence rather complicated.

Figs. 1a, b, c and d show the collisional-radiative recombination coefficients α (plasma optically thin), α^i (plasma optically thick towards Lyman lines and α^{ii} (plasma optically thick towards lines of all series) at selected values of n(c), together with the coefficient $\beta(1)$ for radiative recombination into the ground level, plotted against T on a double logarithmic scale. The $\beta(1)$ curve is included because the α^i and α^{ii} curves must tend towards it when T is high. Fig. 2 shows the variation with n(c), at different T, of the same three collisional-radiative recombination coefficients and also of α^{ic} (plasma optically thick towards Lyman lines and continuum) and of α^{iic} (plasma optically thick towards lines of all series and towards Lyman continuum).

Self-absorption tends to reduce the collisional-radiative recombination coefficient. The fractional reduction R is vanishingly small if n(c) is sufficiently great because in this circumstance radiative processes are unimportant relative to collisional processes irrespective of the optical thickness of the plasma. If n(c) is decreased from a large value, R initially rises and then falls. Contrary to what is usually assumed it does not become zero in the limiting case of an infinitely tenuous plasma (cf. Fig. 1a). The explanation of this is simple. If all radiation from level p is self-absorbed n(p) is clearly proportional to n(c) in plasmas in which n(c) is low enough for three-body processes to be ignored. Since the rate of collisional ionization is then proportional to $n(c)^2$ it never becomes negligible compared with the rate of radiative recombination provided the mean thermal energy is comparable with the ionization potential of the excited atoms rendered effectively meta-stable by the self-absorption. The depression of the true recombination coefficient below the radiative recombination coefficient should be taken into account in theoretical studies on the processes occurring in nebulae.

Consider the trend as $n(c)$ is increased from zero. Clearly R begins to change when three-body effects enter appreciably and therefore begins to change earlier in cool plasmas than in hot plasmas. This should be borne in mind when scrutinizing Figs. 1b, c and d. It is responsible for the rather peculiar nature of the differences between the α, α^{i} and α^{ii} curves.

Fig. 2 requires little comment. However it reveals one unexpected feature. Instead of being monotonically increasing functions of $n(c)$, α^{i} and α^{ic} are initially decreasing of $n(c)$ if T is high*. The decrease occurs because of the behaviour of P_{21}, a_1, a_2 and R_2 of (28): thus P_{21}, the rate coefficient for collisional deactivation from level 2 to level 1, is almost independent of $n(c)$; and though a_1 and a_2, the rate coefficients for recombination to levels 1 and 2 in the absence of atoms in these levels, increase with $n(c)$ they do so more slowly than does R_2, a rate coefficient such that R_2-P_{21} describes the ionization of atoms in level 2 either in a single step or in several steps.

If there were only natural and Doppler broadening, a plasma would be optically thin to a line emitted in a transition from level q to level p if

$$D_{pq} \ll 1 \tag{42}$$

where

$$D_{pq} = \frac{\pi e^2}{2mc(2k \ln 2)^{\frac{1}{2}}} (M/T)^{\frac{1}{2}} \lambda_{pq} f_{pq} n(p) L \tag{43}$$

in which λ_{pq} is the wavelength and f_{pq} is the absorption oscillator strength of the line, L is the length of the path through the plasma and the other symbols have their customary significance. Numerical substitution, with L expressed in cms., yields

$$D_{12} = 6 \times 10^{-12}\ T^{-\frac{1}{2}} n(1) L \tag{44}$$

for the leading member of the Lyman series; and yields

$$D_{23} = 5 \times 10^{-11}\ T^{-\frac{1}{2}} n(2) L \tag{45}$$

for the lead member of the Balmer series. Collision broadening, which is comparable with Doppler broadening if $n(c)$ is of the order of $10^{11}T$ cm^{-3} tends to make condition (42) too severe. Referring to Tables 3 and 4 it may now be seen that laboratory plasmas are likely to become optically thick towards the Lyman lines unless $n(c)$ is low and T is high; but that they are likely to remain optically thin towards the Balmer and other lines unless $n(c)$ is high and T is low. The Lyman continuum is only absorbed if $n(1)L$ exceeds about $2 \times 10^{17} cm^2$.

The best simple procedure is perhaps to take the collisional-radiative recombination coefficient to be α^{i} in all laboratory plasmas. Examining Figs. 1 and 2 with the statements made in the preceding paragraph in mind it may be verified that this cannot introduce serious error.

Acknowledgements

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References

Bates, D.R., Kingston, A.E. and McWhirter, R.W.P., 1962, Proc.Roy.Soc.A. 267, 297, (1962).

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*Because of this feature the curves of Fig. 1 have the appearance of being inaccurately drawn (on the extreme right).

<u>TABLE 1</u>

<u>H^+ ion plasma optically thick towards Lyman lines</u>

<u>Coefficients a_1, P_{21} and R_1 appearing in formula (22)</u>

n(c) (cm^{-3}) \ T(°K)	250	500	1000	2000	4000	8000	16000	32000	64000
				a_1 in cm^3 sec^{-1}					
Lt n(c) → 0	1.0^{-12}	7.2^{-13}	5.1^{-13}	3.6^{-13}	2.5^{-13}	1.7^{-13}	1.2^{-13}	8.0^{-14}	5.2^{-14}
10^{11}	1.1^{-12}	7.2^{-13}	5.1^{-13}	3.6^{-13}	2.5^{-13}	1.7^{-13}	1.2^{-13}	8.0^{-14}	5.2^{-14}
10^{12}	5.9^{-12}	9.0^{-13}	5.1^{-13}	3.6^{-13}	2.5^{-13}	1.7^{-13}	1.2^{-13}	8.0^{-14}	5.2^{-14}
10^{13}		1.7^{-11}	1.0^{-12}	3.8^{-13}	2.5^{-13}	1.7^{-13}	1.2^{-13}	8.0^{-14}	5.2^{-14}
10^{14}			3.4^{-11}	1.5^{-12}	3.0^{-13}	1.8^{-13}	1.2^{-13}	8.1^{-14}	5.2^{-14}
10^{15}				2.3^{-11}	8.9^{-13}	2.2^{-13}	1.3^{-13}	8.3^{-14}	5.3^{-14}
10^{16}					7.0^{-12}	6.1^{-13}	2.0^{-13}	1.0^{-13}	6.1^{-14}
Lt n(c) → ∞	8.1^{-22}n(c)	2.7^{-23}n(c)	8.6^{-25}n(c)	2.5^{-26}n(c)	6.7^{-28}n(c)	4.4^{-29}n(c)	7.6^{-30}n(c)	2.3^{-30}n(c)	8.9^{-31}n(c)
				P_{21} in cm^3 sec^{-1}					
	8.6^{-9}	8.6^{-9}	8.6^{-9}	8.7^{-9}	8.7^{-9}	8.9^{-9}	9.0^{-9}	9.2^{-9}	9.3^{-9}
				R_1 in cm^3 sec^{-1}					
	6.9^{-214}	4.9^{-111}	1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.4^{-11}	1.1^{-9}	8.9^{-9}

The indices give the power of 10 by which the entries in the a_1, P_{21} and R_1 columns must be multiplied.

TABLE 2

H^+ ion plasma optically thick towards Lyman Lines

Coefficients a_2, P_{12} and R_2 appearing in formula (23)

$n(c)$ (cm^{-3}) \ $T(^oK)$	250	500	1000	2000	4000	8000	16000	32000	64000
				a_2 in $cm^3 sec^{-1}$					
Lt $n(c) \to 0$	3.8^{-12}	2.4^{-12}	1.5^{-12}	9.0^{-13}	5.4^{-13}	3.1^{-13}	1.7^{-13}	9.3^{-14}	4.8^{-14}
10^8	7.8^{-11}	1.2^{-11}	3.2^{-12}	1.3^{-12}	6.4^{-13}	3.3^{-13}	1.8^{-13}	9.4^{-14}	4.8^{-14}
10^9	3.8^{-10}	3.3^{-11}	6.0^{-12}	1.8^{-12}	7.5^{-13}	3.6^{-13}	1.8^{-13}	9.4^{-14}	4.8^{-14}
10^{10}	2.8^{-9}	1.5^{-10}	1.6^{-11}	3.1^{-12}	9.8^{-13}	4.1^{-13}	1.9^{-13}	9.5^{-14}	4.7^{-14}
10^{11}	2.7^{-8}	1.0^{-9}	6.1^{-11}	7.1^{-12}	1.6^{-12}	5.1^{-13}	2.1^{-13}	9.8^{-14}	4.7^{-14}
10^{12}	2.6^{-7}	9.0^{-9}	3.6^{-10}	2.4^{-11}	3.2^{-12}	7.4^{-13}	2.5^{-13}	1.0^{-13}	4.6^{-14}
10^{13}		8.8^{-8}	3.0^{-9}	1.2^{-10}	9.3^{-12}	1.3^{-12}	3.3^{-13}	1.1^{-13}	4.5^{-14}
10^{14}			2.9^{-8}	9.3^{-10}	4.0^{-11}	3.2^{-12}	5.7^{-13}	1.6^{-13}	5.7^{-14}
10^{15}				8.5^{-9}	2.5^{-10}	1.6^{-11}	2.5^{-12}	6.2^{-13}	1.9^{-13}
10^{16}					2.3^{-9}	1.4^{-10}	2.2^{-11}	5.2^{-12}	1.5^{-12}
Lt $n(c) \to \infty$	$2.6^{-19} n(c)$	$8.8^{-21} n(c)$	$2.9^{-22} n(c)$	$8.4^{-24} n(c)$	$2.3^{-25} n(c)$	$1.4^{-26} n(c)$	$2.1^{-27} n(c)$	$5.1^{-28} n(c)$	$1.5^{-28} n(c)$
				P_{12} in $cm^3 sec^{-1}$					
Lt $n(c) \to 0$	6.9^{-214}	4.9^{-111}	1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.4^{-11}	1.1^{-9}	7.3^{-9}
10^{10}	6.9^{-214}	4.9^{-111}	1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.3^{-11}	1.0^{-9}	7.2^{-9}
10^{12}	6.9^{-214}	4.9^{-111}	1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.3^{-11}	1.0^{-9}	6.8^{-9}
10^{14}			1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.2^{-11}	9.2^{-10}	6.0^{-9}
Lt $n(c) \to \infty$	6.9^{-214}	4.9^{-111}	1.3^{-59}	6.9^{-34}	4.9^{-21}	1.3^{-14}	2.2^{-11}	9.1^{-10}	5.9^{-9}
				R_2 in $cm^3 sec^{-1}$					
Lt $n(c) \to 0$	8.6^{-9}	8.6^{-9}	8.6^{-9}	8.7^{-9}	8.7^{-9}	9.5^{-9}	2.1^{-8}	6.8^{-8}	1.4^{-7}
10^{10}	8.6^{-9}	8.6^{-9}	8.6^{-9}	8.7^{-9}	8.7^{-9}	1.0^{-8}	2.7^{-8}	8.3^{-8}	1.6^{-7}
10^{12}	8.6^{-9}	8.6^{-9}	8.6^{-9}	8.7^{-9}	8.8^{-9}	1.3^{-8}	5.4^{-8}	1.6^{-7}	2.8^{-7}
10^{14}			8.6^{-9}	8.7^{-9}	9.9^{-9}	4.5^{-8}	2.1^{-7}	4.9^{-7}	7.6^{-7}
Lt $n(c) \to \infty$	8.6^{-9}	8.6^{-9}	8.6^{-9}	8.7^{-9}	1.0^{-8}	5.2^{-8}	2.3^{-7}	5.2^{-7}	8.0^{-7}

The indices give the power of 10 by which the entries in the a_2, P_{12} and R_2 columns must be multiplied.

TABLE 3

Steady state number densities $n_s(1)$ and $n_s(2)$ in H^+ ion plasmas optically thick towards Lyman lines

n(c) (cm⁻³) \ T(°K)	2000	4000	8000	16000	32000	64000
			$n_s(1)$ in cm^{-3}			
Lt n(c) → 0	$2.1^{29}n(c)$	$4.7^{11}n(c)$	$4.9^{2}n(c)$	$1.4^{-2}n(c)$	$9.3^{-5}n(c)$	$6.6^{-6}n(c)$
10^8	1.7^{37}	4.4^{19}	4.6^{10}	1.4^{6}	9.3^{3}	6.6^{2}
10^9	1.5^{38}	4.0^{20}	4.1^{11}	1.3^{7}	9.1^{4}	6.5^{3}
10^{10}	1.2^{39}	3.3^{21}	3.5^{12}	1.2^{8}	8.9^{5}	6.5^{4}
10^{11}	8.9^{39}	2.4^{22}	2.7^{13}	1.0^{9}	8.5^{6}	6.4^{5}
10^{12}	5.0^{40}	1.3^{23}	1.6^{14}	8.0^{9}	8.0^{7}	6.2^{6}
10^{13}	2.4^{41}	6.0^{23}	7.6^{14}	6.4^{10}	7.5^{8}	6.0^{7}
10^{14}	2.5^{42}	6.3^{24}	7.5^{15}	6.3^{11}	7.4^{9}	6.0^{8}
10^{15}	1.0^{44}	2.7^{26}	2.7^{17}	9.8^{12}	8.3^{10}	6.3^{9}
10^{16}		2.3^{28}	2.2^{19}	4.6^{14}	1.7^{12}	9.0^{10}
Lt n(c) → ∞	$8.9^{13}n(c)^2$	$2.3^{-4}n(c)^2$	$2.2^{-13}n(c)^2$	$4.0^{-18}n(c)^2$	$1.0^{-20}n(c)^2$	$3.0^{-22}n(c)^2$
			$n_s(2)$ in cm^{-3}			
Lt n(c) → 0	$1.6^{4}n(c)$	$2.6^{-1}n(c)$	$7.2^{-4}n(c)$	$2.3^{-5}n(c)$	$2.8^{-6}n(c)$	$6.8^{-7}n(c)$
10^8	1.3^{12}	2.4^{7}	6.6^{4}	2.2^{3}	2.6^{2}	6.5^{1}
10^9	1.2^{13}	2.2^{8}	6.0^{5}	2.0^{4}	2.5^{3}	6.2^{2}
10^{10}	9.6^{13}	1.9^{9}	5.1^{6}	1.7^{5}	2.3^{4}	5.7^{3}
10^{11}	6.9^{14}	1.3^{10}	3.9^{7}	1.4^{6}	1.9^{5}	4.8^{4}
10^{12}	3.9^{15}	7.4^{10}	2.2^{8}	8.1^{6}	1.2^{6}	3.1^{5}
10^{13}	1.8^{16}	3.4^{11}	9.5^{8}	3.7^{7}	5.4^{6}	1.5^{6}
10^{14}	1.9^{17}	3.5^{12}	9.4^{9}	3.4^{8}	4.6^{7}	1.2^{7}
10^{15}	8.1^{18}	1.5^{14}	3.8^{11}	1.2^{10}	1.3^{9}	2.9^{8}
10^{16}		1.3^{16}	3.3^{13}	9.9^{11}	1.0^{11}	2.0^{10}
Lt n(c) → ∞	$7.0^{-12}n(c)^2$	$1.3^{-16}n(c)^2$	$3.2^{-19}n(c)^2$	$9.7^{-21}n(c)^2$	$1.0^{-21}n(c)^2$	$1.9^{-22}n(c)^2$

The indices give the power of 10 by which the $n_s(1)$ and $n_s(2)$ columns must be multiplied.

TABLE 4

Quasi-equilibrium number density $n'_Q(2)$ of (34) in H^+ ion plasma optically thick towards Lyman Lines

$n(c)$ (cm^{-3}) \ $T(^oK)$	250	500	1000	2000	4000	8000	16000	32000	64000
					$n'_Q(2)$ in cm^{-3}				
Lt $n(c) \to 0$	$4.4^{-4}n(c)$	$2.8^{-4}n(c)$	$1.7^{-4}n(c)$	$1.0^{-4}n(c)$	$6.1^{-5}n(c)$	$3.3^{-5}n(c)$	$8.2^{-6}n(c)$	$1.4^{-6}n(c)$	$3.4^{-7}n(c)$
10^8	9.1^5	1.4^5	3.8^4	1.5^4	7.4^3	3.5^3	8.0^2	1.3^2	3.3^1
10^9	4.4^7	3.9^6	6.9^5	2.1^5	8.6^4	3.8^4	7.7^3	1.2^3	3.2^2
10^{10}	3.5^9	1.6^8	1.8^7	3.6^6	1.1^6	4.1^5	7.2^4	1.2^4	2.9^3
10^{11}	3.2^{11}	1.1^{10}	7.0^8	8.3^7	1.8^7	4.8^6	6.5^5	9.8^4	2.5^4
10^{12}	3.2^{13}	1.0^{12}	4.2^{10}	2.8^9	3.7^8	5.7^7	4.7^6	6.5^5	1.6^5
10^{13}		1.0^{14}	3.5^{12}	1.4^{11}	1.0^{10}	5.4^8	2.5^7	3.3^6	8.2^5
10^{14}			3.4^{14}	1.1^{13}	4.0^{11}	7.2^9	2.7^8	3.2^7	7.5^6
10^{15}				9.8^{14}	2.4^{13}	3.1^{11}	1.1^{10}	1.2^9	2.4^8
10^{16}					2.2^{15}	2.7^{13}	9.5^{11}	9.9^{10}	1.9^{10}
Lt $n(c) \to \infty$	$3.1^{-11}n(c)^2$	$1.0^{-12}n(c)^2$	$3.3^{-14}n(c)^2$	$9.6^{-16}n(c)^2$	$2.1^{-17}n(c)^2$	$2.7^{-19}n(c)^2$	$9.3^{-21}n(c)^2$	$9.8^{-22}n(c)^2$	$1.9^{-22}n(c)^2$

The indices give the power of 10 by which the entries in the $n'_Q(2)$ columns must be multiplied.

TABLE 5

Collisional-radiative recombination coefficient in H^+ ion plasma optically thick towards Lyman lines

$n(c)$ (cm^{-3}) \ T(°K)	250	500	1000	2000	4000	8000	16000	32000	64000
				α^i in $cm^3\ sec^{-1}$					
Lt $n(c) \to 0$	4.8^{-12}	3.1^{-12}	2.0^{-12}	1.3^{-12}	7.9^{-13}	4.6^{-13}	1.9^{-13}	9.3^{-14}	5.5^{-14}
10^8	7.9^{-11}	1.2^{-11}	3.7^{-12}	1.7^{-12}	8.9^{-13}	4.8^{-13}	1.9^{-13}	9.3^{-14}	5.5^{-14}
10^9	3.8^{-10}	3.4^{-11}	6.4^{-12}	2.2^{-12}	1.0^{-12}	5.0^{-13}	1.8^{-13}	9.2^{-14}	5.5^{-14}
10^{10}	2.9^{-9}	1.5^{-10}	1.6^{-11}	3.5^{-12}	1.2^{-12}	5.4^{-13}	1.8^{-13}	9.1^{-14}	5.5^{-14}
10^{11}	2.7^{-8}	1.0^{-9}	6.1^{-11}	7.5^{-12}	1.8^{-12}	6.0^{-13}	1.8^{-13}	8.9^{-14}	5.4^{-14}
10^{12}	2.6^{-7}	8.9^{-9}	3.6^{-10}	2.5^{-11}	3.4^{-12}	6.8^{-13}	1.6^{-13}	8.6^{-14}	5.3^{-14}
10^{13}		8.8^{-8}	3.0^{-9}	1.3^{-10}	9.2^{-12}	6.6^{-13}	1.4^{-13}	8.3^{-14}	5.3^{-14}
10^{14}			2.9^{-8}	9.4^{-10}	3.5^{-11}	8.1^{-13}	1.4^{-13}	8.3^{-14}	5.3^{-14}
10^{15}				8.5^{-9}	2.1^{-10}	3.0^{-12}	2.3^{-13}	9.3^{-14}	5.5^{-14}
10^{16}					1.9^{-9}	2.5^{-11}	1.1^{-12}	2.0^{-13}	7.9^{-14}
Lt $n(c) \to \infty$	$2.6^{-19}n(c)$	$8.8^{-21}n(c)$	$2.9^{-22}n(c)$	$8.4^{-24}n(c)$	$1.9^{-25}n(c)$	$2.4^{-27}n(c)$	$9.1^{-29}n(c)$	$1.1^{-29}n(c)$	$2.7^{-30}n(c)$

The indices give the power of 10 by which the entries in the α^i columns must be multiplied.

TABLE 6

Radiative recombination coefficient into ground level

T (^oK)	250	500	1000	2000	4000	8000	16000	32000	64000
$\beta(1)$ in units of $10^{-14}cm^3sec^{-1}$	102	71.7	50.7	35.6	25.0	17.4	12.0	8.02	5.19

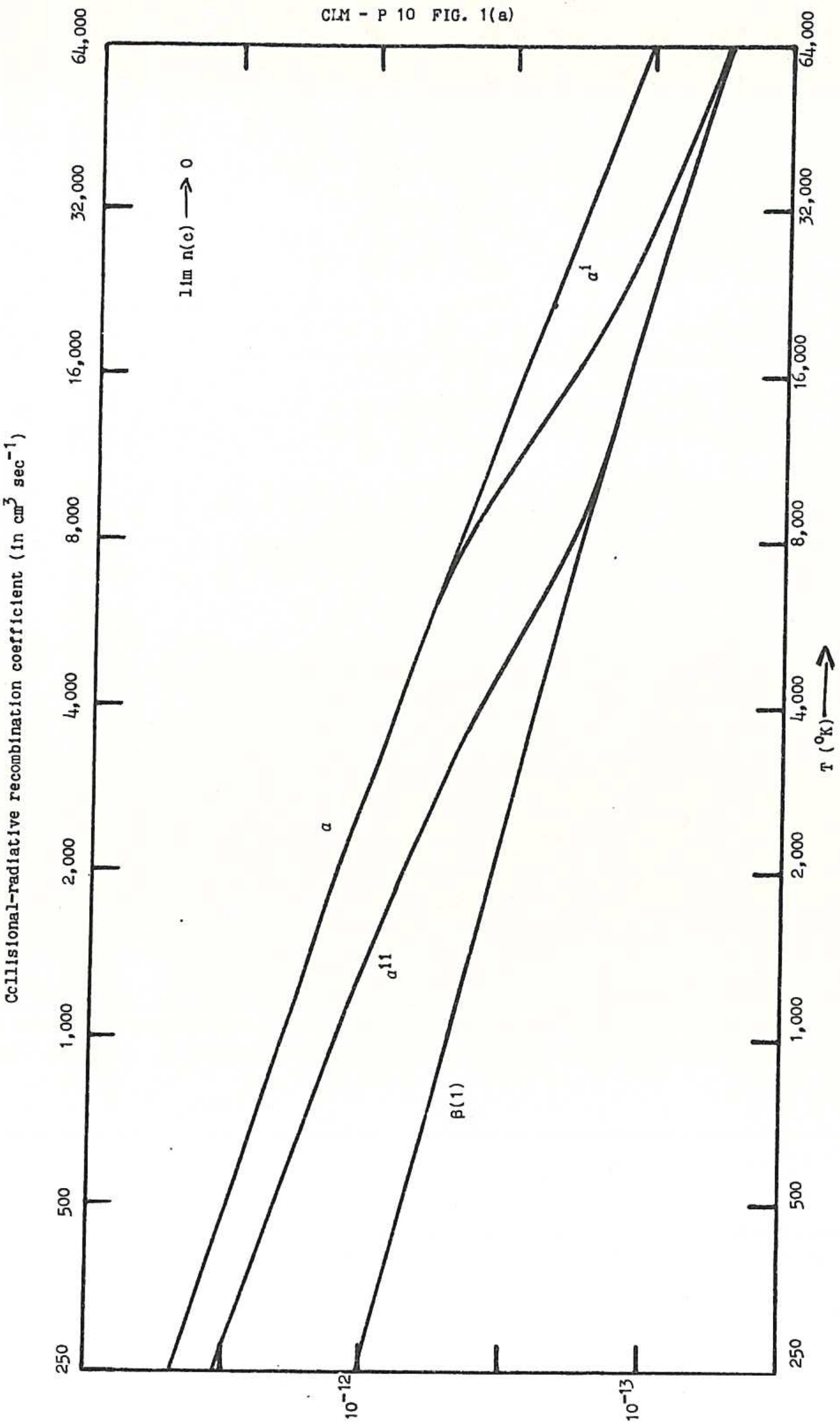
TABLE 7

H^+ ion plasma optically thick towards all lines

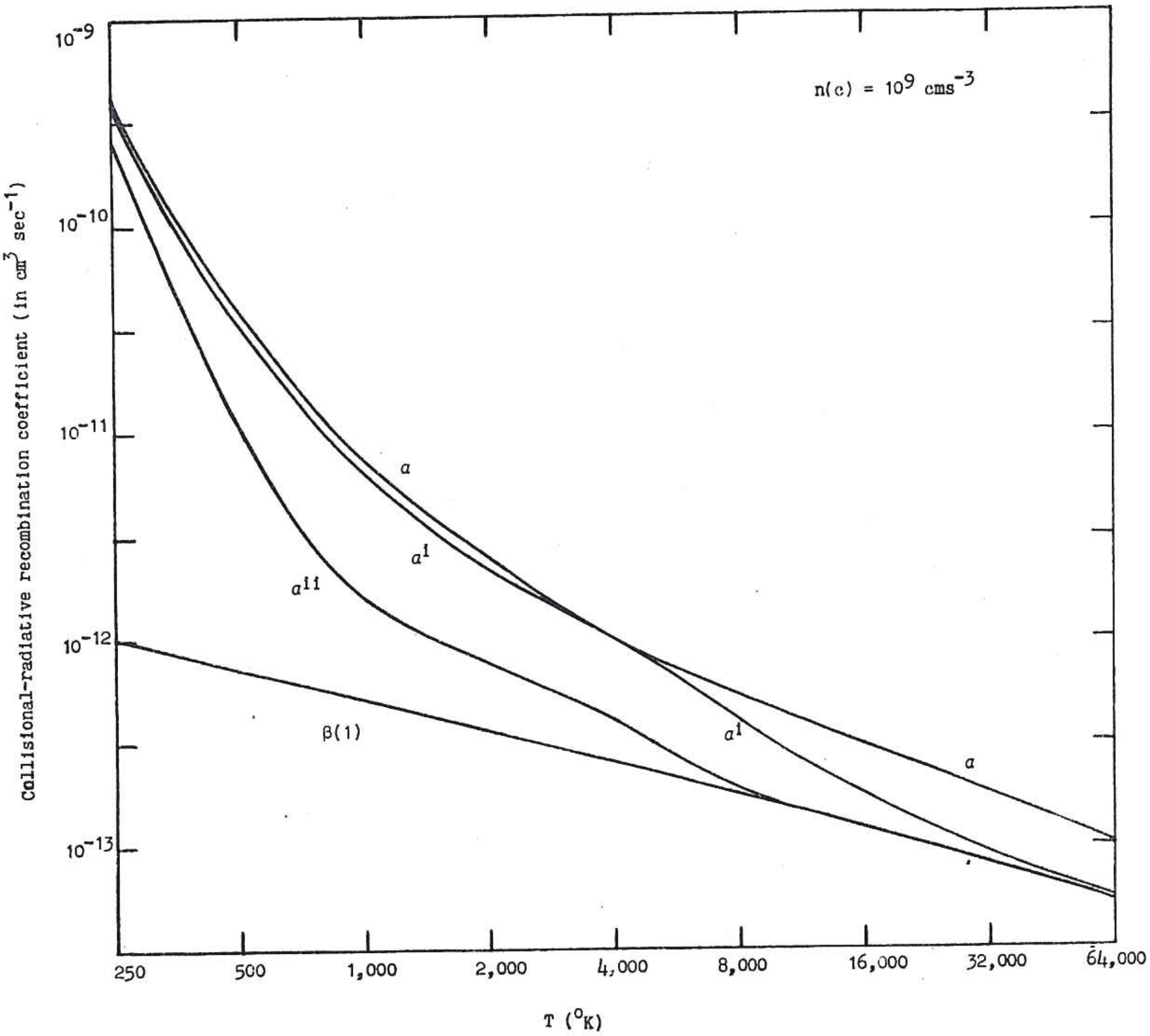
Coefficients k_t and a_r appearing in formula (40) for collisional-radiative recombination coefficient and also difference $a_r - \beta(1)$

T (°K)	250	500	1000	2000	4000	8000	16000	32000	64000
k_t (cm^6 sec^{-1})	2.6^{-19}	8.8^{-21}	2.9^{-22}	8.4^{-24}	1.9^{-25}	2.4^{-27}	9.1^{-29}	1.1^{-20}	2.7^{-30}
a_r (cm^3 sec^{-1})	3.4^{-12}	2.1^{-12}	1.3^{-12}	7.5^{-13}	4.0^{-13}	1.9^{-13}	1.2^{-13}	8.1^{-14}	5.2^{-14}
$a_r - \beta(1)$ (cm^3 sec^{-1})	2.4^{-12}	1.4^{-12}	7.7^{-13}	4.0^{-13}	1.5^{-13}	1.7^{-14}	2.4^{-15}	6.3^{-16}	2.4^{-16}

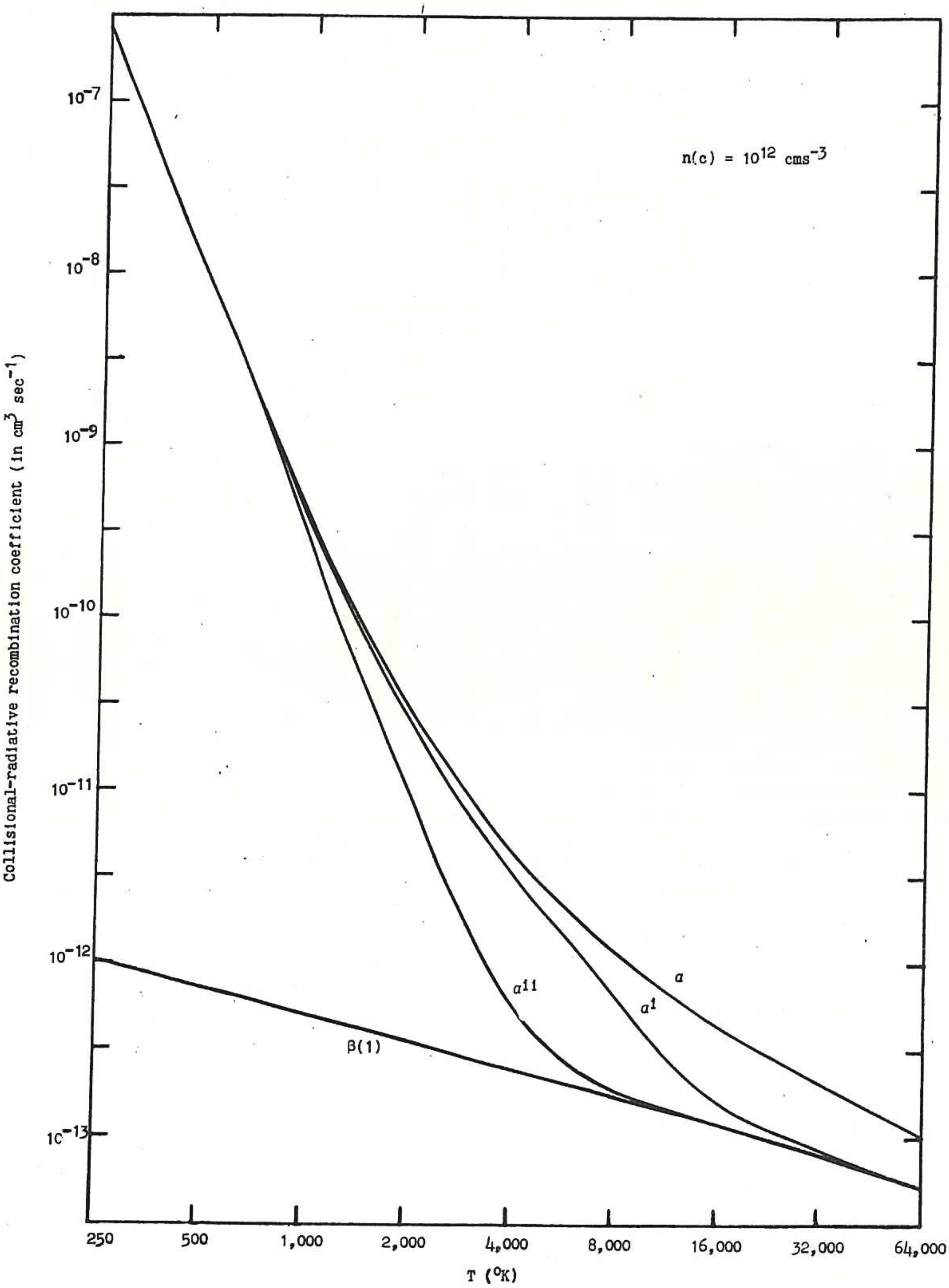
The indices give the power of 10 by which the entries must be multiplied.



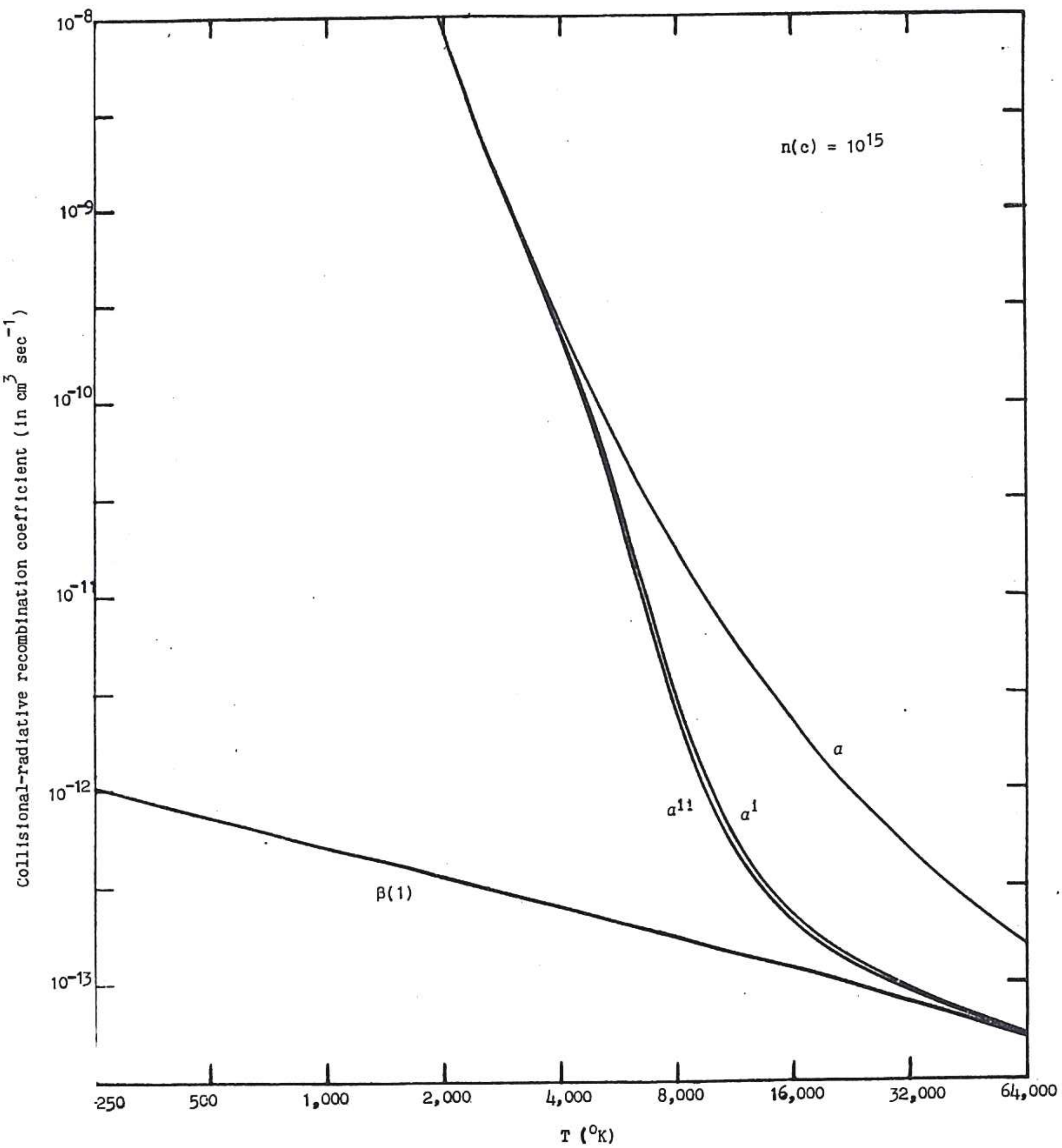
Hydrogen ion collisional-radiative recombination coefficient: α plasma optically thin; $α^i$ plasma optically thick towards lines of Lyman series; $α^{ii}$, plasma optically thick towards lines of all series; β(1), coefficient for radiative recombination into ground level. The electron density n(c) for each set of curves is as indicated.

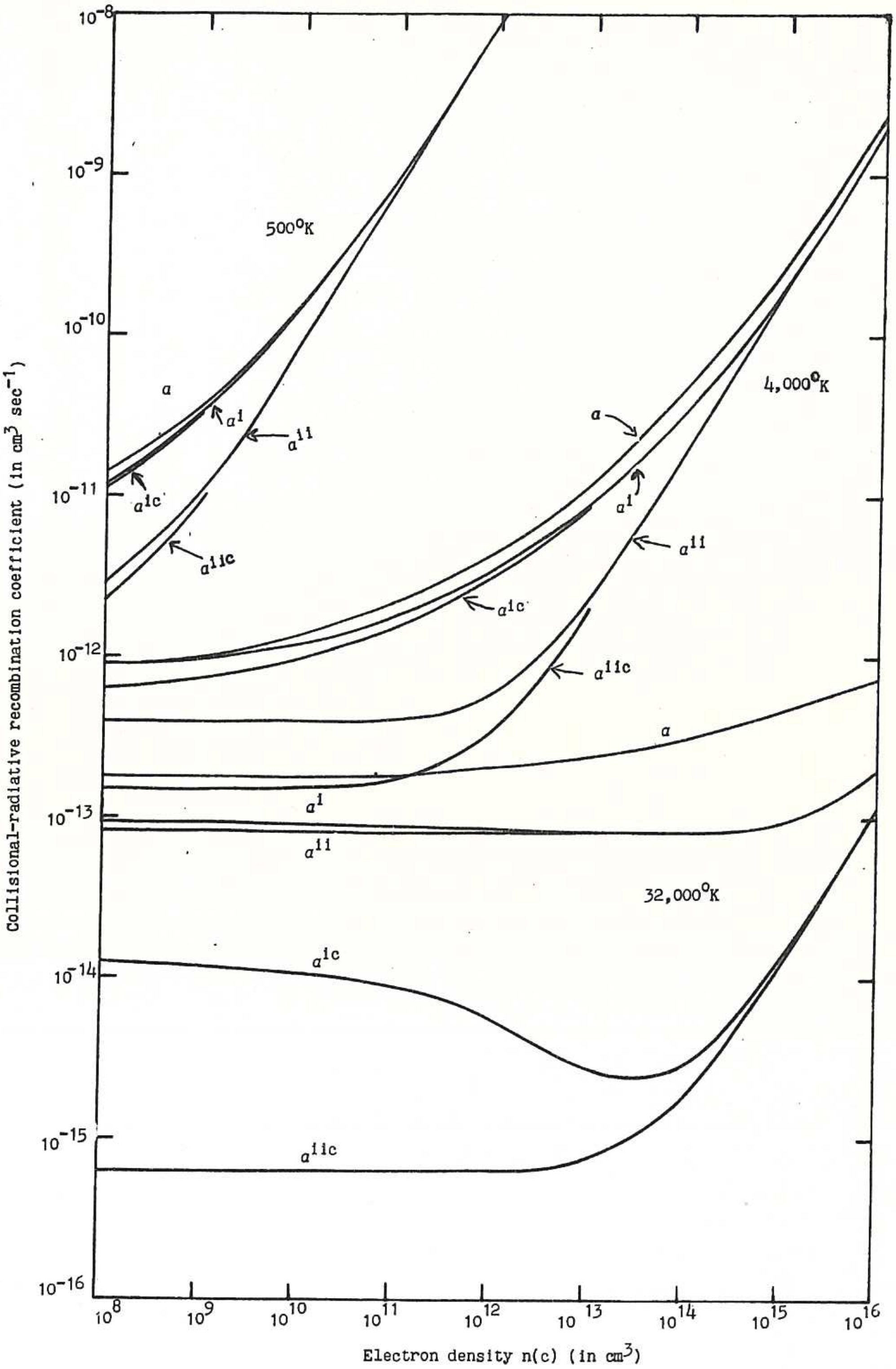

n(e) = 10^9 cms^-3
10^-9
10^-10
10^-11
10^-12
10^-13
Collisional-radiative recombination coefficient (in cm^3 sec^-1)
α
α^I
α^II
β(1)
α^I
α
250
500
1,000
2,000
4,000
8,000
16,000
32,000
64,000
T (°K)

CLM - P 10 FIG. 1(c)



CLM - P 10 FIG. 1(d)





Hydrogen ion collisional-radiative recombination coefficient: α, α^{i} and α^{ii} as in caption to Fig. 1; α^{ic} plasma optically thick towards lines of Lyman series and towards Lyman continuum; α^{iic} plasma optically thick towards lines of all series and towards Lyman continuum. The electron temperature T for each set of curves is as indicated.