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SINGLE PARTICLE MOTION IN AXISYMMETRIC SYSTEMS HAVING A TOROIDAL ELECTRIC FIELD*

by

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A B S T R A C T

Guiding center computations confirm that a parallel electric field gives rise to an enhanced radial drift of blocked particles. The computations also show that the electric field can "unblock" the particles and so terminate the drift.

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A.A. Ware in a recent publication[1] has discussed the motion of blocked[2] particles in axisymmetric fields. Tokamak systems have a rotational transform (ι), generated by a current which is driven by a toroidal electric field. Ware shows that in this case the mirror points of blocked particles drift inwards in minor radius (r) with a velocity given approximately by:

$$V_r = cE_\varphi/B_\theta \qquad \ldots (1)$$

where φ and θ are angles around the major and minor axis of the torus respectively.

In this note we present a computed orbit which shows the effect predicted by Ware; in addition the computation reveals that the electric field increases the velocity ($v_{\parallel}$) of the particle parallel to $\underline{B}$ so that it ceases to be mirrored and becomes a passing particle. Consequently the particle experiences the enhanced radial drift for only a limited time. The motion of the guiding center shown in Figures 1, 2 and 3 is obtained by integrating the non-relativistic, first-order, equations for the velocity of the guiding center perpendicular and parallel to the magnetic field, see for example equations (1.17) and (1.20) of reference 3. The transverse adiabatic invariant μ is assumed to be a constant of the motion.

The simplest field configuration to give the effect predicted by Ware is that of a levitron with an imposed E_φ field having the same direction as the current in the levitated ring. The configuration we have examined consists of: (a) a toroidal field $B_\varphi = 2I_z/R$, produced by a current I_z along the major axis; (b) the field of a circular current filament of radius R_o carrying current I_φ; (c) a toroidal electric field $E_\varphi = V/2\pi R$, where V is a constant. The orbit shown in Fig.1 is that of an electron with energy 2·59 keV and radius $r_L = mcv/eB_0$ (defined in terms of the total velocity v) of 0·2 cms. The Larmor radius in the poloidal field B_θ is $r_{L\theta} = mcv/eB_\theta$ and varies from ~ 5 cms to ~ 2 cms. The value of

$$q = 2\pi/\iota = (r/R)\ (B_\varphi/B_\theta) \qquad \ldots (2)$$

is 2·5 at the beginning of the orbit. Fig.1 is the projection of the orbit on to a plane φ = constant, (R, Z, φ) are cylindrical polar coordinates. Fig.2 shows the orbit of the same particle with no electric field for comparison and Fig.3 shows the same orbits using R and φ as coordinates. Successive reflection points are lettered and the radial inward drift predicted by Ware is apparent. The magnitude of the drift from the orbit is about $1{\cdot}6 \times 10^6$ cms/sec, while the value predicted by equation (1) is $1{\cdot}2 \times 10^6$ cms/sec. This is good agreement since (1) is derived on the assumption that the magnetic surfaces have a circular cross section whereas the levitron magnetic surfaces are distorted circles.

The reflection point in Fig.1 drifts progressively in θ until the particle becomes a passing particle. It is clear from Fig.3 that this occurs because, even in the absence of an electric field, the blocked particle orbit drifts around the major axis. In the presence of an electric field this drift will cause the particle to gain or lose energy, in the example shown the particle gains energy and, since the adiabatic invariant is preserved, the particle becomes passing. This drift around the major axis can arise as a result either, of magnetic field gradients or, of the gradient in the length between mirror points which occurs in a sheared field[4]. In the example shown, because of the r^{-2} dependence of the transform in the levitron field, the shear effect dominates.

The displacement around the major circumference for one complete traversal of the banana orbit is:

$$\Delta_\varphi \approx 2\pi R \, [\alpha_1 \, \iota^{-1}(r_1) - \alpha_2 \, \iota^{-1}(r_2)]$$

where R is the major radius, α_1 and α_2 are the θ separations of the reflection points and r_1 and r_2 are the effective outer and inner minor radii of the banana. For the levitron this becomes

$$\Delta_\varphi \approx R_0^{-1}(I_Z/I_\varphi)(\alpha_1 r_1^2 - \alpha_2 r_2^2) \qquad \ldots (3)$$

where the magnetic surfaces have been assumed to have a circular cross section. The displacements Δ_φ of the computed orbit for the bananas A B C and B C D in Fig.1 are 145 cms and 154 cms respectively. These values compare well with the over estimate of 200 and 210 cms which result from using the radii along $Z = 0$, for r_1 and r_2 in (3).

When the rotational transform in a Tokamak type system decreases with radius the shear drift for both ions and electrons is in such a direction that driving electric field (E_φ) tends to increase the energy, and make the particles passing. If the rotational transform increases outwards, as in a Tokamak with a skin current, the shear drift will reduce the parallel energy and reduce the orbit so that it lies mainly on the outside of the torus (large R). In this region the drift around the torus due to the gradient of the toroidal field is in the direction for particles to gain energy, the banana orbit size may then reach an equilibrium value so that the radial inward drift given by equation (1) can continue.

Thus computed guiding center orbits in a levitron field demonstrate the radial drift predicted by Ware[(1)] but also show that the electric field can change the particles from blocked to passing so that a particular particle experiences the drift for only a limited time. These effects in the motion of single particles will influence the behaviour of plasmas, but in the plasma case the behaviour will be complicated by collisional effects such as scattering and dynamic friction.

ACKNOWLEDGEMENT

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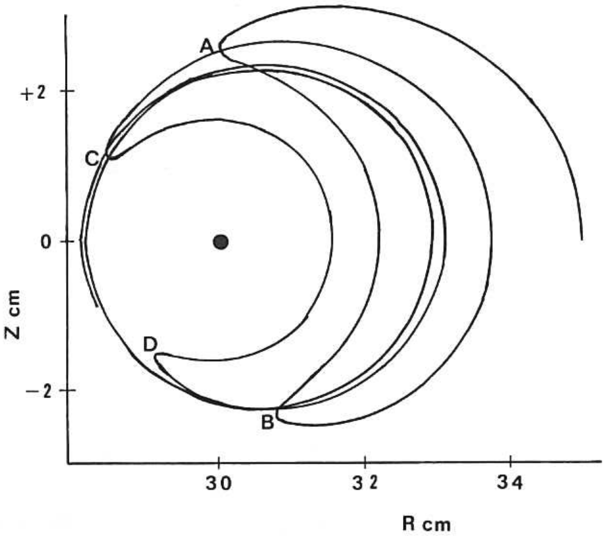

Fig. 1. Blocked particle orbit in a levitron field, the orbit is plotted in a plane ϕ = constant. The circular current filament has radius R_o = 30 cms and carries a current of 1000 amps. The toroidal field strength at R = 30 cms is 1000 G. The particle Larmor radius is 0·2 cms and the initial value of $(v_\parallel/v_\perp)$ is 0·3. The applied electric potential is 200v/turn.

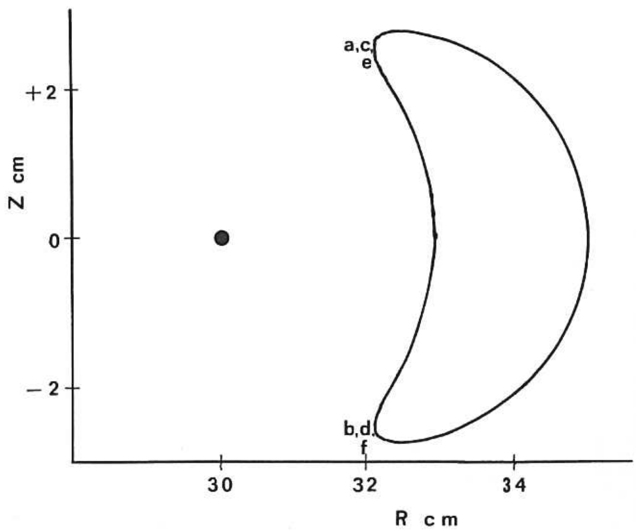

Fig. 2. This orbit is identical to that of Fig. 1 except that there is no applied electric potential.

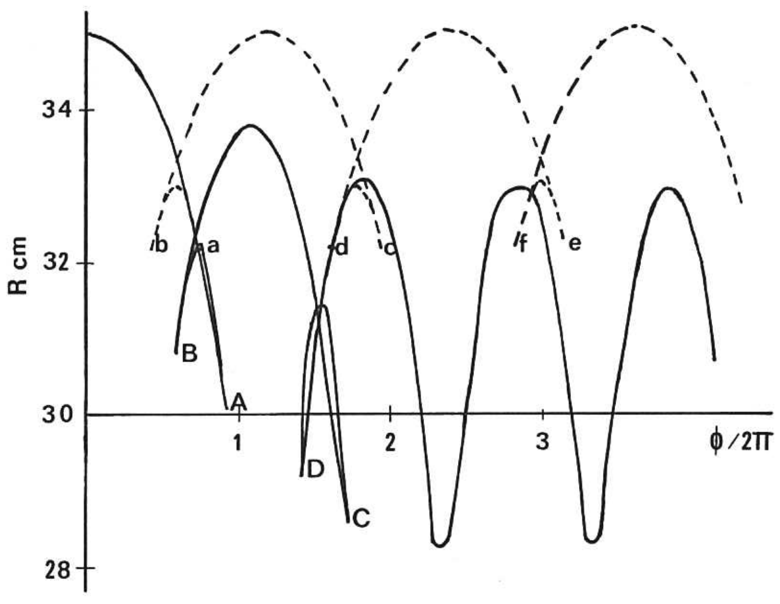

Fig. 3. The orbits of Figs. 1 and 2 plotted using R and ϕ as coordinates, the full curve corresponds to V = 200v/turn and the broken curve to V = 0.

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