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ELECTROSTATIC INSTABILITIES IN A CONFINED PLASMA HEATED BY HIGH ENERGY ION BEAMS

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ELECTROSTATIC INSTABILITIES IN A CONFINED PLASMA HEATED BY HIGH ENERGY ION BEAMS

by

M.J. Houghton

ABSTRACT

A detailed analysis is made of the electrostatic instabilities associated with the heating of a confined plasma by means of a high energy, tenuous, neutral beam ionized in the plasma; particular emphasis is laid on toroidal reactor parameters.

It is concluded that high growth rate instabilties may lead to faster heating of the background ions than expected classically, but only at low electron temperatures, i.e. $T_e < \frac{m_e}{m_i} W_H$ [W_H = directed beam energy]. The non flute modes grow faster than the flute modes, but are more easily stabilized by both shear and finite energy spread in the injected beam. The shear requirement, for stability of all modes, is

$$\frac{2\pi \lambda_B}{L_s} > 1$$

where L_s = shear length, and λ_B = effective Debye length of the injected beam component.

The above criterion is unlikely to be satisfied in future toroidal experiments or in a reactor; however the presence of these electrostatic instabilities may not be very serious from the containment point of view.

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C O N T E N T S

I. INTRODUCTION

Previous work[1] at Culham Laboratory on neutral injection concluded that heating confined plasma by an energetic beam of some transiently neutralized species [e.g. D, T] could become an important feature of the next generation of toroidal experiments, and eventually of reactor systems. In particular it considered injection into a stellarator type reactor to produce temperatures ~ 10 keV in a plasma with density ~ 10^{14} cm^{-3}. The beam is required to heat a cold dense plasma from an initial temperature ~ 100 eV up to the point at which alpha particle heating takes over[1] ~ 10 keV. In this report we study the electrostatic plasma instabilities associated with the ionized and subsequently trapped beam with particular reference to the above reactor system, but also with an appended calculation appropriate to an ORMAK type experiment.

The ionization cross-sections and ionized beam profiles were studied in reference (1) and indicated a suitable directed beam energy of about 1 MeV. Typical relaxation times for a charged test particle undergoing small angle Coulomb deflections have also been calculated, from the Fokker-Planck equation, and used to estimate the time scale for thermalization of such a beam. For example, an injected 1 MeV beam will heat a typical reactor plasma in two stages under the action of dynamical friction; firstly it will heat the electrons which should reach 10 keV on a time scale of ~ 1 sec, the heated electrons relaxing to thermal equilibrium with the plasma ions on a rather faster time scale [see Fig. 5]*.

We expect that collective effects, associated with the injected beam, may be important both from the heating and containment point of view. In this report we make a detailed analysis of possible microinstabilities and leave other plasma effects, such as the generation of large scale electric fields, to a further report[2].

The detailed working has been divided into six separate sections, i.e. II through VII. Firstly, in sections (II) and (III) the form of the ionized particle distribution is considered and hence the dispersion relation for the electrostatic modes derived. Certain basic assumptions are made and the dispersion equation reduced to a more elementary form. Sections (IV) and (V) contain details of the growth rate calculations for the infinite uniform media case, section (V) contains a discussion of electron Landau and ion cyclotron damping with particular reference to background plasma heating. In section (VI) the stabilizing effect of finite energy spread in the injected beam is studied, while section (VII) contains the analysis of finite geometry effects. In particular, plasma boundaries, non-uniformity and shear of the ambient magnetic field are shown to be important and shear stabilization criteria are derived. A summary, conclusions and appended examples then follow.

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*It is assumed that injection is at right angles to the ambient magnetic field, that the resulting ion beam is monoenergetic ~ 1 MeV and the accompanying electron beam gives negligible heating.

II. PARTICLE DISTRIBUTION FUNCTIONS AND THE GENERAL DISPERSION EQUATION

Particle Distributions

The model for the plasma injected beam system consists of a uniform Maxwellian plasma

$$f^{o}(\underline{v}) = \frac{N_o}{\pi^{3/2} v_{T_{(i,e)}}^3} e^{-(v_\perp^2 + v_\parallel^2)/v_{T_{(i,e)}}^2} \qquad (1)$$

(in which $T_i \sim T_e$) in a uniform magnetic field $\underline{B} = (0,0,B)$, into which a neutral beam is injected. We assume the neutral beam is injected, with velocity V_o , at right angles to the ambient magnetic field $\underline{B}$, consequently for the resulting ionized beam we take

$$\underset{\text{(ions only)}}{f^{B}(\underline{v})} = \frac{n_o}{2\pi^2 (\Delta v_{T_B})^2 V_o} \cdot e^{-(v_\parallel/\Delta v_{T_B})^2 - (v_\perp - V_o)^2/(\Delta v_{T_B})^2} . \qquad (2)$$

General Dispersion Equation

We shall consider stability against electrostatic waves of the Harris [3] type. Thus the wave electric field is of the form

$$\underline{E}(\underline{r},t) = -\nabla\phi = \underline{E}_o \, e^{i(\omega t + k_\perp y + k_\parallel z)} \qquad (3)$$

[the notation is standard throughout].

Using well known techniques[3] we find the dispersion relation

$$k^2 = D_1(\omega,k) + D_2(\omega,k) \qquad (4)$$

where

$$D_1(\omega,k) = \sum_{\text{species }(i,e)} \frac{2\omega_p^2}{v_T^3}\left\{\frac{\Omega}{k_\parallel}\sum_{n=-\infty}^{+\infty}\left[-Z(-t_n)\,n\,[e^{-\lambda}I_n(\lambda)]\right] + \right.$$

$$\left. + \frac{v_T}{2}\sum_{n=-\infty}^{+\infty}\left[Z'(-t_n)[e^{-\lambda}I_n(\lambda)]\right]\right\} , \qquad (5a)$$

$$t_n \equiv \frac{\omega + n\Omega}{k_\parallel v_T} , \qquad \lambda = \frac{1}{2}\left(\frac{k_\perp v_T}{\Omega}\right)^2 ,$$

and

$$D_2(\omega,k) = \frac{\omega_B^2}{\pi(\Delta v_{T_B})^2 V_o}\sum_{n=-\infty}^{+\infty}\int_0^\infty dv_\perp\, v_\perp\left\{\frac{n\Omega_i}{v_\perp}\left[\frac{\partial}{\partial v_\perp} e^{-(v_\perp - V_o)^2/(\Delta v_{T_B})^2}\right]\frac{\sqrt{\pi}}{k_\parallel}Z(-t_n^B)\right.$$

$$\left. + e^{-(v_\perp - V_o)^2/(\Delta v_{T_B})^2}\left[\frac{2\sqrt{\pi}}{\Delta v_{T_B}}\; Z'(-t_n^B)\right]\right\} J_n^2\left(\frac{k_\perp v_\perp}{\Omega_i}\right) .$$

(5b)

$t_n^B \equiv \dfrac{\omega + n\Omega_i}{k_\parallel \Delta v_{T_B}}$, and ω_B = ion plasma frequency associated with the injected beam, the electron contribution being ignored. $Z(t)$ is the Plasma Dispersion Function tabulated by Fried and Conte[4] i.e.

$$Z(t) \equiv \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{dy\; e^{-y^2}}{y - t} , \qquad Z'(t) \equiv \frac{dZ}{dt} .$$

III. SIMPLIFICATION OF THE DISPERSION EQUATION

The general dispersion equation (4) can be reduced to a more elementary form under the following assumptions.

Basic Conditions

(i) $\omega \simeq m\Omega_i$, unstable modes at or near multiples of the ion gyrofrequency.

(ii) $\lambda_e < 1$. In general the unstable modes have wavelengths which are larger than the electron gyroradius, i.e. $k_\perp^{-1} > \rho_e$. The unstable modes for which this condition can break down are discussed.

(iii) $\frac{\omega}{k_\parallel v_{T_e}} \gg 1$. Unstable modes which are not electron Landau damped. Plasma heating, arising from beam driven modes which are heavily Landau damped by the background electrons, is discussed in section (V). The condition $\frac{m\Omega_i}{k_\parallel v_{T_i}} \gg 1$ is, of course, implied by (i) and (iii) [for $T_i \sim T_e$] and the imaginary contribution from the cold ions to equation (5a) is dominated by the Z , as opposed to the Z', function. This imaginary contribution implies ion cyclotron damping. The onset of this damping of the beam driven modes is also discussed, in section (V), as a possible heating mechanism for the background plasma.

(iv) $\frac{|\omega - m\Omega_i|}{k_\parallel v_{T_i}} \ll 1$. This condition [in equation (5a)] is distinct from the pure flute modes, consequently the dispersion equation for the flute modes has a different, but simpler, form and can easily be incorporated in the general analysis.

(v) $\frac{|\omega - m\Omega_i|}{k_\parallel \Delta v_{T_B}} \gg 1$. This condition implies that the beam is monoenergetic and is practically equivalent to using the delta function form of (2) for the injected beam distribution. Consequently only the high growth rate reactive instabilities, in which all the beam particles take part in driving the unstable modes, appear in the analysis. The weak resonant particle instabilties, which may appear through the imaginary contribution from the Z and Z' functions, are discussed in section (VI).

For conditions (i) through (v) the general dispersion equation (4) reduces to:-

$$1 + \left(\frac{k_\parallel}{k_\perp}\right)^2 \simeq F_e(\omega) + \sum_{n=1\,(n\neq m)}^{\infty} \left\{ \frac{E_n + \varepsilon_n}{[\omega^2 - (n\Omega_i)^2]} + \frac{D_n + \delta_n}{[\omega^2 - (n\Omega_i)^2]^2} \right\} +$$

$$+ i \left\{ \frac{\omega_{p_i}^2}{\Omega_i^2} \frac{[e^{-\lambda_i} I_m(\lambda_i)]}{\lambda_i} \cdot \left[\frac{m\Omega_i}{k_\parallel v_{T_i}}\right] \right\} + \frac{\varepsilon_m}{[\omega^2 - (m\Omega_i)^2]} + \frac{\delta_m}{[\omega^2 - (m\Omega_i)^2]^2} \tag{6}$$

where

$$F_e(\omega) \simeq \frac{\omega_{p_e}^2}{\omega^2 - \Omega_e^2} + \frac{\omega_{p_e}^2}{\omega^2}\left(\frac{k_\parallel}{k_\perp}\right)^2 \;;\quad E_n \equiv \omega_{p_i}^2\, n^2\, \frac{[e^{-\lambda_i} I_n(\lambda_i)]}{\lambda_i/2} ,$$

$$D_n \equiv 2\omega_{p_i}^2 \left(\frac{k_\parallel}{k_\perp}\right)^2 [\omega^2 + (n\Omega_i)^2]\,[e^{-\lambda_i} I_n(\lambda_i)] .$$

The pure flute mode dispersion equation takes on the simpler form [see (iv)]

$$1 \simeq \frac{\omega_{p_e}^2}{\omega^2 - \Omega_e^2} + \sum_{n=1}^{\infty} \frac{E_n + \varepsilon_n}{[\omega^2 - (n\Omega_i)^2]} . \tag{7a}$$

The beam contribution to (6) and (7a) is through the terms

$$\varepsilon_n \equiv \omega_B^2\, \frac{2n^2}{b}\left[\frac{d}{db} J_n^2(b)\right] \;;\quad \delta_n \equiv \omega_B^2 \left(\frac{k_\parallel}{k_\perp}\right)^2 2[\omega^2 + (n\Omega_i)^2] J_n^2(b)$$

$$\text{where} \quad b \equiv \frac{k_\perp V_o}{\Omega_i} . \tag{7b}$$

Further Simplification

Since $\omega \simeq m\Omega_i$ the dominant beam terms are those with numerators ε_m and δ_m. It can be verified [using (18)] that provided

$$\omega_B < \Omega_i$$

all neighbouring beam terms, e.g. those whose numerators are $\varepsilon_{m \pm 1}$, $\delta_{m \pm 1}$, can be neglected in the dispersion equations (6) and (7a).

Similarly the contribution from the neighbouring background ion terms, e.g. those at $n = m \pm 1$, is of order $\frac{\omega_{p_i}^2}{\Omega_i^2} m \frac{[e^{-\lambda_i} I_m(\lambda_i)]}{\lambda_i}$. The asymptotic form of $[e^{-\lambda_i} I_m(\lambda_i)]$ for the beam driven modes is found in section (V) and it is always exponentially small [see (31)].

The above considerations reduce the dispersion equations (6) and (7a) to the general form:-

$$1 + \left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 = A(\omega) + \frac{\varepsilon_m}{[\omega^2 - (m\Omega_i)^2]} + \frac{\delta_m}{[\omega^2 - (m\Omega_i)^2]^2} \tag{8a}$$

where for the 'long' wavelength modes $\lambda_i = \frac{(k_{\perp}\rho_i)^2}{2} \ll 1$

$$A(\omega) = \frac{\omega_{p_e}^2}{\omega^2 - \Omega_e^2} + \frac{\omega_{p_e}^2}{\omega^2}\left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 + \frac{\omega_{p_i}^2}{\omega^2 - \Omega_i^2} + 0\,[\lambda_e,\ \lambda_i] \tag{8b}$$

and for the 'short' wavelength modes $\lambda_i > 1$

$$A(\omega) = \frac{\omega_{p_e}^2}{\omega^2 - \Omega_e^2} + \frac{\omega_{p_e}^2}{\omega^2}\left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 + 0\left[\lambda_e\ ,\ \frac{1}{\sqrt{\lambda_i}}\right]. \tag{8c}$$

[For example in (8c) the dominant background ion term is approximately $\frac{\omega_{p_i}^2}{2n_1[\omega^2-(n_1\Omega_i)^2]}$ where $n_1 \simeq \sqrt{\lambda_i}$].

Notice that in equations (8b) (8c) the value of $\frac{k_\parallel}{k_\perp}$ may be taken as a zero or finite quantity. However when $\frac{k_\parallel}{k_\perp} > 0$ imaginary terms can arise, these are discussed fully in section (V).

IV. ANALYSIS OF THE UNSTABLE MODES

We take $k_\parallel \lesssim k_\perp$ throughout and then simplify the analysis by treating (8a) in two distinct limits. In the first limit

$$\varepsilon_m^2 \gg [1 - A(\omega)]\ \delta_m \tag{9}$$

the resulting dispersion equation then describes what are essentially flute like modes [e.g. $\delta_m \equiv 0$ if $\frac{k_\parallel}{k_\perp} \equiv 0$]. The second limit is

$$\varepsilon_m^2 \ll [1 - A(\omega)]\ \delta_m \ . \tag{10}$$

Flute-like Modes

The inequality (9) reduces the dispersion equation (8a) considerably and the unstable modes then arise only at very specific harmonics of Ω_i. That is, the unstable harmonics $m\Omega_i$ are those defined by the condition $A(m\Omega_i) \simeq 1$, [see Fig. 2], and their growth rate γ_1 is given by

$$\gamma_1 \simeq \sqrt{\frac{\varepsilon_m}{m\Omega_i\ A'(m\Omega_i)}} \tag{11}$$

where $A'(\omega) \equiv \frac{\partial A}{\partial \omega}$. Using (11) the inequality (9) reduces to the more convenient form

$$|\varepsilon_m|^3 \gg \left| \frac{\delta_m^2 A'(m\Omega_i)}{m\Omega_i} \right| . \tag{12}$$

Clearly equation (11) defines the growth rates of the pure flute modes, $\frac{k_\parallel}{k_\perp} \equiv 0$, i.e. through the condition (12). However a stringent requirement for instability of these modes [at the given harmonic $\omega = m\,\Omega_i$] is that $E_m + \varepsilon_m < 0$ in the dispersion equation (7a). This requirement is very clear from Fig. 2 and also has a simple interpretation in terms of positive and negative energy waves. Thus, in principle at least, there exists a critical beam density defined by $E_m > |\varepsilon_m|$, that is :-

$$\frac{e^{-\lambda_i} I_m(\lambda_i)}{\lambda_i} > \left[\frac{\omega_B}{\omega_{p_i} \; m^{4/3}} \right]^2 \tag{13}$$

below which the pure flute modes become stable. It is shown in section (V) that the stability condition (13) is of no practical consequence for realistic beam densities [see (31)].

<u>Non Flute-like Modes</u>

The inequality (10) reduces (8a) considerably and the fastest growing waves occur at harmonics m again defined by $A(m\Omega_i) \simeq 1$, at which point the growth rate γ_2 is

$$\gamma_2 \simeq \frac{1}{m\Omega_i} \left\{ \frac{m\Omega_i \; \delta_m}{A'(m\Omega_i)} \right\}^{1/3} \tag{14}$$

the inequality (10) now, of course, reduces to

$$|\varepsilon_m|^3 \ll \left| \frac{\delta_m^{\,2} \; A'(m\Omega_i)}{m\Omega_i} \right| . \tag{15}$$

Thus given that the modes are non flute-like, i.e. condition (10), the peak growth rate is defined by (14) and occurs at the harmonics defined by $A \simeq 1$. However, under condition (10), all harmonics n for which $A(n\Omega_i) > 1$ are unstable; the requirement $A \geqslant 1$ for instability is very clear from Fig. 3. Consequently given that

$$\varepsilon_n^{\,2} \lesssim \delta_n \quad \text{and} \quad A(n\Omega_i) \gg 1 \tag{16a}$$

the growth rate γ reduces to

$$\gamma \simeq \frac{1}{n\Omega_i} \sqrt{\frac{\delta_n}{A(n\Omega_i)}} . \tag{16b}$$

The growth rates (16b) are smaller than (14) but can appear at lower harmonic numbers and have thus been included for completeness.

<u>Beam-Wave Resonance Condition</u>

Since all growth rates vanish in the limit $\varepsilon_m = \delta_m = 0$ the magnitude of the Bessel function $J_m(b)$ is important [see (7b)]. Thus $J_m(b)$ transforms from an exponentially small to a rapidly oscillating function[5] as b increases in magnitude through the value m [$m \gg 1$ is characteristic of the analysis]. That is, the growth rates maximise when

$$m \simeq b \equiv \frac{k_{\perp} V_o}{\Omega_i} \tag{17}$$

in which case

$$J_m(b)\Big]_{m=b} \simeq \left[\frac{2^{1/3}}{3^{2/3}\Gamma(2/3)}\right]\frac{1}{m^{1/3}} \; . \tag{18}$$

The condition (17) reduces ε_m and δ_m to the form

$$\varepsilon_m \simeq -\omega_B^2 \quad \text{and} \quad \delta_m \simeq \frac{\omega_B^2 \Omega_i^2}{m^{2/3}}\left(m\frac{k_\parallel}{k_\perp}\right)^2 \; . \tag{19}$$

[More precisely, it can be verified that the peak value of $-\frac{d}{db}\left[J_m^2(b)\right]$ is $\simeq \frac{0.6}{m}$ and occurs at $b \simeq m + 1.6\left(\frac{m}{2}\right)^{1/3}$].

The unstable frequencies $\omega \simeq m\Omega_i$ are defined by $A(m\Omega_i) \simeq 1$, while the condition (17) $\frac{\omega}{k_\perp} \simeq V_o$ determines the unstable wave number $k_\perp$, thus

$$\lambda_i \equiv \frac{(k_\perp \rho_i)^2}{2} \simeq \frac{m^2}{2}\cdot\frac{T_i}{W_H} \tag{20}$$

where W_H = directed beam energy.

Clearly $\frac{\omega}{k_\perp} \simeq V_o$ implies a resonance between the injected beam ions and a natural mode of the background plasma, i.e. an integral number m of unstable wavelengths fit inside one gyroradii of the injected beam ions.

Detailed Results of the Growth Rate Calculations

Since the unstable harmonics m are defined by $A(m\Omega_i) \simeq 1$ there are two unstable frequency regions for the dispersion equation (8), one below and one above the electron cyclotron frequency Ω_e .

The unstable 'low' frequency harmonics, $\omega < \Omega_e$, are defined by :

$$m \simeq \frac{\sqrt{\frac{m_i}{m_e}\left[\Lambda + \frac{m_i}{m_e}\left(\frac{k_\parallel}{k_\perp}\right)^2\right]^{1/2}}}{\left[1 + \Omega_e^2/\omega_{p_e}^2\right]^{1/2}} \tag{21}$$

where $\Lambda \simeq 1$ for $k_\perp^{-1} > \rho_i$ and $\Lambda = O\left[\frac{1}{\sqrt{\lambda_i}}\right]$ for $k_\perp^{-1} < \rho_i$.

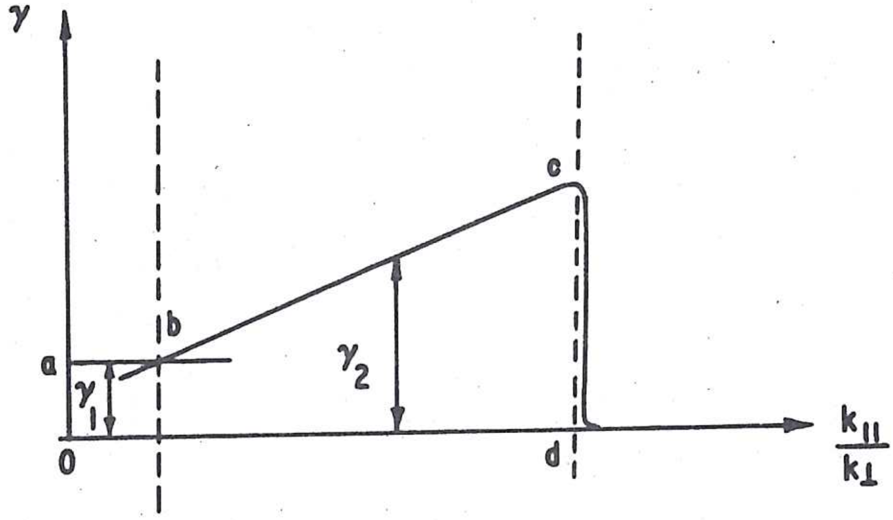
Similarly the high frequency harmonics, $\omega > \Omega_e$, are defined by:

$$m \simeq \left[\left(\frac{m_i}{m_e}\right)^2 + \frac{\omega_{p_e}^2}{\Omega_i^2}\left[1 + \left(\frac{k_\parallel}{k_\perp}\right)^2\right]\right]^{1/2} . \tag{22}$$

Under typical reactor conditions, $\omega_{p_e} > \Omega_e$, the modes (21) are essentially electron cyclotron modes[6], $\sqrt{\Omega_i\Omega_e} \lesssim \omega < \Omega_e$, while (22) are electron plasma waves, $\omega \sim \omega_{p_e}$.

The growth rates γ plotted as a function of $\frac{k_\parallel}{k_\perp}$ have the qualitative form sketched below.

Fig. 1



$\frac{k_\parallel}{k_\perp} = \alpha_o$ $\qquad$ $\frac{k_\parallel}{k_\perp} \simeq \frac{1}{2.5}\sqrt{\frac{W_H}{T_e}\frac{m_e}{m_i}}$

The line $\frac{k_\parallel}{k_\perp} = \alpha_o$ separates the flute like region ab, where $\gamma = \gamma_1$, from the non flute like region bc, where $\gamma = \gamma_2$. Modes for which $\frac{k_\parallel}{k_\perp} > \frac{1}{2.5}\sqrt{\frac{W_H}{T_e}\,\frac{m_e}{m_i}}$ are 'stabilized' by electron Landau damping [see section (V)].

Thus for the 'low' frequency modes (21)

$$\gamma_1 \simeq \left\{\frac{\omega_B \Omega_e}{\omega_{p_e}}\right\} \frac{1}{\left[1 + \frac{\Omega_e^2}{\omega_{p_e}^2}\right]^{1/2}}\,, \tag{23}$$

$$\gamma_2 \simeq \Omega_i \left\{\frac{\omega_B}{\omega_{p_i}} \cdot \frac{k_\parallel}{k_\perp} \cdot \frac{1}{\left[1 + \frac{\Omega_e^2}{\omega_{p_e}^2}\right]^{1/2}} \cdot \left(\frac{m_i}{m_e}\right)^{1/2}\right\}^{2/3} \cdot m^{1/9}\,, \tag{24}$$

and the line α_o, of Fig. 1, is :-

$$\alpha_o = \left[\frac{\omega_B}{\omega_{p_i}} \left(\frac{m_i}{m_e}\right)^{1/2} \frac{1}{\left[1 + \frac{\Omega_e^2}{\omega_{p_e}^2}\right]^{1/2}}\right]^{1/2} \frac{1}{m^{1/6}}\,.$$

Similarly for the 'high' frequency modes (22)

$$\gamma_1 \simeq \omega_B \frac{\omega_{p_e}}{\omega}\,, \tag{25}$$

$$\gamma_2 \simeq \left\{\frac{\omega_B\, \omega_{p_e}}{\omega^{1/2}} \cdot \frac{k_\parallel}{k_\perp}\right\}^{2/3} \cdot \frac{1}{m^{2/9}}\,, \tag{26}$$

and

$$\alpha_o = \frac{(\omega_B\, \omega_{p_e})^{1/2}}{\Omega_i} \quad \frac{1}{m^{2/3}}\,.$$

[Since $\lambda_e \simeq \frac{m^2}{2}\left[\frac{T_e}{W_H}\frac{m_e}{m_i}\right]$, the condition $\lambda_e < 1$ can break down for the electron plasma waves (22) especially at high electron temperatures; however it can be verified that this simply introduces the factor $\sqrt{\lambda_e}$ into the equations and does not significantly alter the main conclusions.]

Clearly, for a monoenergetic beam, the fastest growing modes are non flute like, their growth rates are represented by the line bc in Fig. 1. For example, taking $\omega_{pe} > \Omega_e$, $T_i \sim T_e \sim 10^2$eV and $W_H \sim 10^6$eV, the line dc occurs at $\frac{k_\parallel}{k_\perp} \sim 1$, in which case the fastest growing electron cyclotron and electron plasma waves have growth rates

$$\gamma_2 \sim \Omega_i\left(\frac{\omega_B}{\omega_{p_i}}\right)^{2/3} \cdot \left(\frac{m_i}{m_e}\right)^{1/3} \quad \text{and} \quad \gamma_2 \sim \left(\omega_B^{\,2}\,\omega_{p_e}^{\,1/3}\,\Omega_i^{\,2/3}\right)^{1/3}$$

respectively.

Given that $\omega_{p_e} \gtrsim \Omega_e$, all the above modes arise at very high harmonics, i.e. $m \gtrsim \sqrt{\frac{m_i}{m_e}}$. However weaker growth rate modes arise at all harmonics n for which $A(n\Omega_i) > 1$ [see Fig. 3], thus for $\frac{k_\parallel}{k_\perp} \gg \sqrt{\frac{m_e}{m_i}}$ and $n < \sqrt{\frac{m_i}{m_e}}$ [i.e. $\omega < \sqrt{\Omega_i\Omega_e}$] we expect to find unstable plasma waves, $\omega \simeq \omega_{p_e}\frac{k_\parallel}{k_\perp}$, which have growth rates γ of the order

$$\gamma \simeq \left[\frac{\omega_B\,\Omega_i}{\omega_{p_e}}\right] n^{2/3}$$

[where we have used (16a) and (16b)].

V. PLASMA HEATING

The time scale for collisional heating of the background plasma by the injected beam can be calculated from the well known test particle model(7). However, since the beam behaves in a collective fashion and drives electrostatic modes of the background plasma unstable, we investigate the overall consequence to background plasma [ion] heating of either electron Landau or ion cyclotron damping of these beam driven modes.

Collisional Heating

Applying the test particle model(7) to estimate the time scales $\tau(i_B \to e_p)$ and $\tau(i_B \to i_p)$ by which the ion beam (i_B) energy decreases due to long range Coulomb collisions with the background plasma electrons (e_p) and ions (i_p) respectively, we find

$$T_1 \equiv \frac{\tau(i_B \to e_p)}{\tau(i_B \to i_p)} \simeq \frac{T_e}{W_H} \cdot \frac{1}{\Psi\left(\sqrt{\frac{W_H}{T_e}\frac{m_e}{m_i}}\right)} \quad . \tag{27}$$

The function $\Psi(u)$ is plotted in Fig. 4 together with its asymptotic forms, clearly $T_1 < 1$ for electron temperatures

$T_e < W_H\left(\frac{m_e}{m_i}\right)^{1/3}\left(\frac{2}{3\sqrt{\pi}}\right)^{2/3}$ $\{ > 10$ keV given $W_H \sim 10^6$ eV and $m_i = 2m_{Proton} \}$.

Similarly

$$T_2 \equiv \frac{\tau(i_B \to e_p)}{\tau_{eq}(e_p \to i_p)} \simeq \frac{2}{3\sqrt{\pi}}\sqrt{\frac{W_H}{T_e}\frac{m_e}{m_i}} \cdot \frac{1}{\Psi\left(\sqrt{\frac{W_H}{T_e}\frac{m_e}{m_i}}\right)} \tag{28}$$

where $\tau_{eq}(e_p \rightarrow i_p)$ is the Spitzer equipartition time for the background plasma electrons to relax to thermal equilibrium with the background ions. Clearly

$$T_2 = \frac{4}{3\sqrt{\pi}} \cdot \left[\frac{W_H}{T_e} \frac{m_e}{m_i} \right]^{3/2} > 1 \quad \text{for} \quad T_e \ll \frac{m_e}{m_i} W_H \tag{29a}$$

$$T_2 \simeq 1 \quad \text{for} \quad T_e \gg \frac{m_e}{m_i} W_H \, . \tag{29b}$$

Since $\tau_{eq}(e_p \rightarrow i_p) \leq \tau(i_B \rightarrow e_p) < \tau(i_B \rightarrow i_p)$ we expect the ion beam to lose energy primarily to the background electrons which in turn relax to thermal equilibrium with the background ions on a faster [or similar] time scale. Consequently from purely collisional considerations, the background ions should heat up on the time scale $\tau(i_B \rightarrow e_p)$ while $T_i \sim T_e$ throughout the heating process.

Ion Cyclotron Damping

In principle modes for which $k_\parallel > 0$ can be 'stabilized' by electron Landau or ion cyclotron damping; that is if $A(\omega)$ takes on the complex form $A_R(\omega) + A_I(\omega)$ the corresponding damping decrement γ_D is $\gamma_D \simeq - \dfrac{A_I(\omega_R)}{A'_R(\omega_R)}$ for $\omega = \omega_R + i\gamma_D$. Thus damping should stabilize those modes for which γ_D dominates the beam driven instability.

It can be verified that the ion cyclotron damping decrement is given by [for $\omega_{p_e} \gtrsim \Omega_e$]

$$\gamma_{CD} \simeq m\Omega_i \left[\frac{m_i}{m_e} \frac{k_\perp}{k_\parallel} \sqrt{\frac{W_H}{T_i}} \right] \frac{\left[e^{-\lambda_i} I_m(\lambda_i) \right]}{\lambda_i} \quad \text{[electron cyclotron modes]} \tag{30a}$$

and

$$\gamma_{CD} \simeq \omega_{Pe} \left[\frac{\omega_{Pi}^2}{\Omega_i^2} \cdot \frac{k_\perp}{k_\parallel} \sqrt{\frac{W_H}{T_i}} \right] \frac{\left[e^{-\lambda_i} I_m(\lambda_i) \right]}{\lambda_i} . \tag{30b}$$

[electron plasma waves]

The above function $e^{-\lambda_i} I_m(\lambda_i)$ has a peak value of $\frac{1}{4m}$ (approx.) at $\sqrt{\lambda_i} \simeq m$, that is, when $T_i \simeq W_H$ [using (20)], a condition which is clearly never satisfied in practice. However[5] for $m < \lambda_i < m^2$

$$e^{-\lambda_i} I_m(\lambda_i) \simeq \frac{e^{-\frac{m^2}{2\lambda_i}}}{\sqrt{2\pi \lambda_i}} \propto e^{-\frac{W_H}{T_i}} \tag{31}$$

consequently ion cyclotron damping is never an effective mechanism by which the background plasma [ions] are heated.

The exponentially small asymptotic form (31) also implies that the stability criteria (13), for flute mode stabilization, is of no practical consequence for these beam driven unstable modes.

Electron Landau Damping

Electron Landau damping is included in the analysis by replacing the electron term $\frac{\omega_{Pe}^2}{\omega^2} \left(\frac{k_\parallel}{k_\perp} \right)^2$ in equation (8) by $\frac{\omega_{Pe}^2}{\Omega_e^2 \lambda_e} \left[Z'\left(\frac{\omega}{k_\parallel V_{T_e}} \right) \right]$. The form of $R_e[Z'(x)]$ is well known[4,8] in particular the previous expansion is valid provided $x > 2.5$, while for $x < 2.5$ its value increases rapidly [until $x \simeq 0.9$]. Consequently we expect very strong electron Landau damping when $\frac{\omega}{k_\parallel V_{T_e}} \lesssim 2.5$ i.e.

$$\frac{k_\parallel}{k_\perp} \gtrsim \frac{1}{2.5}\sqrt{\frac{W_H}{T_e}\frac{m_e}{m_i}} \qquad (32)$$

For example at $\frac{\omega}{k_\parallel V_{T_e}} \simeq 2.5$ the damping decrement γ_L is [taking $\omega_{pe} > \Omega_e$]

$$\gamma_L \simeq m\Omega_i \left\{\frac{2.5\sqrt{\pi}}{\lambda_e \exp[6]}\right\} \qquad \text{[electron cyclotron modes]}$$

$$\gamma_L \simeq \omega_{pe} \left\{\left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{2.5\sqrt{\pi}}{\lambda_e \exp[6]}\right\} \qquad \text{[electron plasma waves]} .$$

Thus taking $T_e \sim 100$ eV and $W_H \sim 10^6$eV we have $\frac{k_\parallel}{k_\perp} \sim 1$ for marginal electron Landau damping; in which case the time scale for energy exchange between the beam and the background electrons may be $\sim \frac{1}{\text{growth rate}} \simeq \frac{1}{\{\omega_B{}^2 \omega_{p_e}{}^{1/3} \Omega_i{}^{2/3}\}^{1/3}}$.

The "Landau" heated electrons will presumably relax to thermal equilibrium with the background ions, through collisions, at the rate defined by τ_{eq} $(e_p \rightarrow i_p)$, and since $\frac{1}{\text{growth rate}} \ll \tau(i_B \rightarrow e_p) \geqslant \tau_{eq}(e_p \rightarrow i_p)$ we can expect that $\tau\,(i_B \rightarrow i_p) \simeq \tau_{eq}(e_p \rightarrow i_p)$. However (29a) and (29b) imply that electron Landau damping will also lead to faster ion heating at low electron temperatures $T_e < \frac{m_e}{m_i} W_H$, while at high values $T_e > \frac{m_e}{m_i} W_H$, the rate of ion heating is unchanged. Unfortunately, since $\tau_{eq} \propto T_e{}^{3/2}$, the overall time scale in a typical reactor for background ion heating, i.e. from 100eV to 10keV, is not much reduced. This latter point is most clearly illustrated in Fig. 5.

VI BEAM ENERGY SPREAD

The instabilities studied so far have been of a reactive kind, similar in nature to the two stream; the effect of thermal motion of the beam particles is to reduce the growth rates presumably by "dephasing" the bunching process in some way[9,10]. In fact the assumption that the beam is monoenergetic, i.e. $\Delta V_{T_B} = 0$, arises at two points in the simplification of (5b). Firstly it appears through the condition (v) i.e. $\dfrac{|\omega - m\Omega_i|}{k_{\parallel} \Delta V_{T_B}} \gg 1$, and secondly through the approximation

$$I \equiv \frac{1}{\sqrt{\pi}\, \Delta V_{T_B}} \int_0^{\infty} dv_{\perp}\, e^{-(v_{\perp} - V_o)^2/\Delta V_{T_B}^2} \cdot \frac{\partial}{\partial v_{\perp}} J_m^2\left(\frac{k_{\perp} v_{\perp}}{\Omega_i}\right) \sim - \frac{k_{\perp}}{m\Omega_i} . \tag{33}$$

Clearly condition (v) must always remain valid for the flute modes while the approximation (33) will remain valid provided the Bessel function J_m does not oscillate before the integrand in (33) becomes exponentially damped. This latter requirement reduces to the condition

$$\Delta V_{T_B} \ll \frac{V_o}{m^{2/3}} \tag{34}$$

which is clearly most likely to break down at high harmonic numbers. Conversely finite beam energy spread is defined by $\Delta V_{T_B} \gg \frac{V_o}{m^{2/3}}$ in which case (33) reduces to

$$I \simeq \frac{1}{\sqrt{\pi}\ \Delta V_{T_B}\ m^{2/3}} \tag{35}$$

and consequently the flute mode growth rates γ_1 are then reduced according to the prescription

$$\gamma \simeq \gamma_1 \left\{ \frac{V_o}{\Delta V_{T_B}\ m^{2/3}} \right\} . \tag{36}$$

The stabilizing effect of spread in the injected beam energy is even more marked for the non-flute modes since both (v) and (34) can break down independently. Firstly the two basic conditions (v) and (iv) are mutually consistent only provided that $\frac{\Delta V_{T_B}}{V_{T_i}} \ll 1$, consequently the condition (v) breaks down when

$$\Delta V_{T_B} \gtrsim V_{T_i} . \tag{37}$$

Formally one then reverses the inequality (v) and the instability, arising from the imaginary contribution of the Z function in (5b), is of the Rosenbluth-Post loss cone[11] type. That is, the dispersion equation (4) reduces to

$$1 + \left(\frac{k_\parallel}{k_\perp}\right)^2 \simeq A(\omega) + \frac{i\,\omega_B{}^2\,\omega}{[\Delta V_{T_B} V_o k_\perp^2 k_\parallel]} \left[\frac{1}{\sqrt{\pi}\Delta V_{T_B}} \int_o^\infty dv_\perp\, e^{-(v_\perp - V_o)^2/\Delta V_{T_B}^2} \cdot \frac{\partial}{\partial v_\perp} J_m^2\left(\frac{k_\perp v_\perp}{\Omega_i}\right) \right] \tag{38}$$

where $A(\omega)$ is unchanged [the real contributions from the beam are again typically of order $\frac{\omega_B{}^2}{\Omega_i{}^2 m^{4/3}} \ll 1$ and are neglected].

Equation (38) predicts instability from regions of velocity space where the beam distribution has positive slope when plotted against $v_\perp$, as for the loss cone modes[11], provided a wave exists with suitable phase velocity i.e. $\frac{\omega}{k_\perp} \simeq V_o - \Delta V_{T_B}$ [c.f. $\frac{k_\perp V_o}{m\Omega_i} \simeq 1$..(17)]. Clearly by comparison with the monoenergetic beam case these modes are very weakly unstable. For example the growth rate γ of the high frequency modes (22), which propagate across the magnetic field at an angle for which electron Landau damping is just becoming dominant [see Fig. 1] is now reduced to

$$\gamma \simeq \omega_B^2 \left[\frac{V_{T_e}}{\Delta V_{T_B}}\right] \left\{\frac{\omega_{pe}^2\,\omega}{[\omega^2 - \Omega_e^2]^2}\right\} \simeq \frac{\omega_B^2}{\omega_{p_e}} \left[\frac{V_{T_e}}{\Delta V_{T_B}}\right] \quad \text{for} \quad \omega_{p_e} > \Omega_e \tag{39}$$

[where, of course, $\Delta V_{T_B} \gtrsim V_{T_i}$]. The above growth rate (39) is further reduced by the factor $\frac{V_o}{\Delta V_{T_B}\, m^{2/3}}$ should the inequality (34) also become reversed.

Thus there is clearly an advantage from the overall stability point of view from injecting a beam with finite thermal spread, such that both

(a) the inequality (34) is reversed, the flute mode growth rates γ_1 are then reduced according to the prescription (36)

and

(b) the condition (37) is satisfied. Under the condition (37) the fastest growing modes become the flute modes, since the non flute mode instabilities are of a weak resonant particle type; [e.g. see (39)].

VII MODIFICATIONS IMPOSED BY FINITE GEOMETRY

The analysis so far has been confined to an infinite uniform plasma. We now study the effects of boundaries, nonuniformities and shear on the unstable modes. Clearly the group velocities are of primary importance in any system of finite size.

Group Velocities

For $\omega_{pe} \gtrless \Omega_e$ we have:-

$$\frac{\partial \omega}{\partial k_\perp} \simeq V_o \left(\frac{k_\parallel}{k_\perp}\right)^2 \left[1 - \frac{\omega_{pi}^2}{\omega^2}\frac{m_i}{m_e}\right] \frac{\Omega_e^2}{\omega_{pe}^2} \quad , \frac{\partial \omega}{\partial k_\parallel} \simeq - \left\{\frac{\partial \omega}{\partial k_\perp}\right\} \frac{k_\perp}{k_\parallel} \tag{40a}$$

[electron cyclotron modes]

and

$$\frac{\partial \omega}{\partial k_\perp} \simeq \frac{V_o \left(\frac{k_\parallel}{k_\perp}\right)^2 \frac{\Omega_e^2}{\omega^2}}{1 + \left(\frac{k_\parallel}{k_\perp}\right)^2} \quad , \frac{\partial \omega}{\partial k_\parallel} \simeq - \left\{\frac{\partial \omega}{\partial k_\perp}\right\} \frac{k_\perp}{k_\parallel} \quad . \tag{40b}$$

[electron plasma waves]

The non-flute modes thus propagate energy towards the boundary but it is easily verified [from (8b) and (8c)] that $k_\perp \to 0$ as $\omega_{pe} \to 0$, consequently we expect reflection rather than absorption from the boundary region.

Non Uniformity of the Ambient Magnetic Field

It is also quite easily shown that unstable modes which are extended any distance along a given field line are essentially stabilized [their growth rates being drastically reduced] by field line non-uniformity. This result stems from the fact that the resonance condition $\omega = m\Omega_i$ can only be satisfied locally in a non-uniform magnetic field. This implies that the fastest growing waves are essentially wave-packets highly localized along the field lines.

Thus we imagine the ambient field strength to vary along a field line as $\underline{B} = \underline{B}(s)$ where s is distance measured from some arbitrary point. First order W.K.B. analysis replaces the flute-like dispersion equation by its local approximation form

$$1 - A[\omega, s] = \frac{\varepsilon_m}{[\omega^2 - (m\Omega_i)^2]} \quad . \tag{41}$$

Given that the unstable mode is of finite extension L along the field line we have:-

$$[1 - A(\omega, s_o)]L \simeq \frac{1}{2}\int \frac{ds \ \varepsilon_m/(m\Omega_i)^2}{\{i\tilde{\gamma}_1/m\Omega_i - (s - s_o)B'(s_o)/B(s_o)\}} \tag{42}$$

[where the integration is along the length of the normal mode; $\omega \simeq m\Omega_i(s_o) + i\tilde{\gamma}_1$ and $B'(s_o) \equiv \frac{\partial B(s_o)}{\partial s_o}$]. If the extension L is such that $\frac{B'L}{B} \simeq 1 \gg \frac{\tilde{\gamma}_1}{m\Omega_i}$, then we can easily solve (42) for $\tilde{\gamma}_1$ to find

$$\tilde{\gamma}_1 \simeq \frac{\pi \gamma_1^2}{m\Omega_i} \quad . \tag{43}$$

Similarly the non-flute mode growth rates γ_2 are reduced as

$$\tilde{\gamma}_2 \simeq \frac{\pi \gamma_2^3}{(m\Omega_i)^2} \tag{44}$$

by non-uniformity of the ambient magnetic field.

Variation across the magnetic field is similarly effective and is quite likely to stabilize the lowest frequency modes $\left[\text{i.e. } m < \sqrt{\frac{W_H}{T_i}}\right]$ since these have wavelengths greater than the mean ion gyroradii of the ambient plasma ions i.e. $k_\perp^{-1} > \rho_i$ [see (20)]. Clearly these modes are most likely to 'sample' significant magnetic field variations in any system of practical interest. However the modes (22) are all short wavelength $k_\perp^{-1} \ll \rho_i$ and field variation is less effective against these modes.

Shear Stabilization

We study the effect of shear by analysing the slab model[12] in which the magnetic field is given by

$\underline{B} = B_o[\hat{z} + \frac{x}{L_s}\hat{y}]$ and the perturbation is of the form

$\phi = \phi_1(x)\, e^{i(\omega t + k_\perp y + k_\parallel z)}$, where L_s = shear length; the plasma density is assumed constant. In order[13, 14] to investigate the effect of shear it is essential to include the x dependence of $\phi_1(x)$. Without loss of generality we take $\underline{k} \cdot \underline{B} = 0$ at $x = 0$; the equation determining the potential $\phi_1(x)$ can then be written in operator form by replacing $k_\perp^2$ by $k_y^2 - \frac{\partial^2}{\partial x^2}$ and $k_\parallel$ by $\frac{x}{L_s} k_y$ wherever they appear[13] in equation (5a); the equation as a whole then operates on $\phi_1(x)$.

Using first order W.K.B. theory $\frac{\partial^2 \phi_1(x)}{\partial x^2}$ reduces to $k_x^2(x)$, the criteria for shear stabilization is then established by studyung the unstable x region. The most difficult modes to stabilize are the flute modes since they have zero group velocity $\frac{\partial\omega}{\partial k_x}\left[= \frac{\partial\omega}{\partial k_\perp}\frac{dk_\perp}{dx} = 0 \text{, see (40 a, b)}\right]$. Thus a localized wave packet, growing in the unstable region, will not convect into the surrounding stable region. Consequently the shear stabilization criteria is derived from the condition that a suitably localized unstable wave packet cannot be generated, i.e.

$$2\pi\, k_x^{-1} > \delta x \qquad (45)$$

where δx = width of the unstable region.

The unstable harmonics m are defined by the condition $A(m\Omega_i) \simeq 1$ [see Fig. 2]. The range of frequency $\delta\omega$, near to the value $\omega = m\Omega_i$, over which the instability arises is given by $\delta\omega \simeq \frac{\varepsilon_m}{(m\Omega_i)^2 A'(m\Omega_i)}$. In a sheared magnetic field the frequency spread $\delta\omega$ defines the width δx of the unstable region. Thus for the high frequency modes $\omega \simeq \sqrt{\Omega_e^2 + \omega_{p_e}^2\left[1 + \left(\frac{k_\parallel}{k_\perp}\right)^2\right]}$ ... (22) we have $\delta x \simeq L_s \frac{\sqrt{2\omega\,\delta\omega}}{\omega_{p_e}}$. Using (20) the stability requirement (45) reduces to

$$\frac{\rho_H}{L_s} > \frac{1}{2\pi}\left\{\frac{\omega_B\,[\omega^2 - \Omega_e^2]}{\Omega_i\,\omega_{p_e}^2}\right\} \simeq \frac{1}{2\pi}\,\frac{\omega_B}{\Omega_i} \quad \text{for } \omega_{p_e} > \Omega_e \qquad (46)$$

where $\rho_H = \rho_i\sqrt{\frac{W_H}{T_i}}$, the ion gyroradii of the injected particles. Clearly, for $\omega_{p_e} > \Omega_e$, the criteria (46) can be written more concisely as $\frac{2\pi\,\lambda_B}{L_s} > 1$, where λ_B = effective Debye length of the beam component.

Similarly the 'low' frequency, (21), shear criteria is

$$\frac{\rho_H}{L_s} > \frac{1}{2\pi}\left\{\frac{\omega_B}{\omega_{p_i}}\ \frac{\omega}{\Omega_e}\right\}\sqrt{\frac{m_i}{m_e}} \quad \lesssim \frac{1}{2\pi}\ \frac{\omega_B}{\omega_{p_i}} \ . \tag{47}$$

The non flute modes are more easily stabilized by shear since they are convective in nature [see (40)] ; their stability criteria are estimated from the requirement that a wave packet does not exponentiate more than a given number of times n_o before it convects into the stable region. That is

$$\int_{x_1}^{x_2} dx \ \frac{\gamma_2(x)}{U_g(x)} < n_o \tag{48}$$

where the unstable region $\delta x = x_2 - x_1$.

The shear requirement (48) is in general much less stringent than (45). For example, taking $\omega_{p_e} > \Omega_e$, $T_e \sim 10^2 \mathrm{eV}$ and $W_H \sim 10^6 \mathrm{eV}$ the shear requirement is very roughly of the order

$$\frac{\rho_H}{L_s} > \frac{1}{n_o}\left(\frac{\omega_B}{\Omega_e}\right)^2 G \tag{49}$$

where

$$G = \frac{\gamma_2}{m^{2/3}\ \Omega_i} \quad \text{and} \quad \gamma_2 \sim \left[\frac{\omega_B^2\ \omega_{p_e}}{m^{2/3}}\right]^{1/3} .$$

VIII SUMMARY

We have studied the electrostatic instabilities associated with the injection of a tenuous [i.e. $\omega_B \lesssim \Omega_i$], monoenergetic [$\Delta V_{T_B} = 0$], neutral, but subsequently ionized, beam* into a background plasma. Particular emphasis has been placed on the reactor requirements: beam energy $W_H \sim 10^6$eV, $100\text{eV} \lesssim T_i \sim T_e \lesssim 10$keV, and $\omega_{p_e} \gtrsim \Omega_e$. The main conclusions are summarized below:-

Growth Rates and Frequencies

High growth rate instabilities arise in two frequency ranges which, for $\omega_{p_e} > \Omega_e$, are $\sqrt{\Omega_i\Omega_e} \lesssim \omega < \Omega_e$ [electron cyclotron waves] and $\omega \sim \omega_{p_e}$ [electron plasma waves]. The growth rates, wavelengths, etc. are calculated in section (IV). For a monoenergetic beam, in a uniform plasma, the fastest growing modes are non flute-like and the electron plasma waves tend to be marginally faster. Thus, taking $T_e \sim 10^2$eV, $\omega_B \sim 10^8$Hz, $\Omega_i \sim 5.10^8$Hz, $\omega_{p_i} \sim 10^{10}$Hz, then at $\omega \sim \omega_{p_e}$, γ_2 (peak value) $\sim 10^8$Hz [see (26)]; while at $\omega \lesssim \Omega_e$, γ_2 (peak value) $\sim 10^7$Hz [see (24)].

Plasma Heating

Plasma heating, which arises through collisional relaxation of the injected beam, is briefly summarized in

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* Injection normal to the ambient magnetic field was studied, but in practice a beam velocity component given along the field lines may introduce additional instabilities of the two stream type[2].

section (V) [see Fig. 5].

Ion cyclotron damping is not an important mechanism for background ion heating since the number of ions in resonance is very small, i.e. the ion cyclotron damping decrement is proportional to $\exp - \left[\frac{W_H}{T_i} \right]$. However modes for which $\frac{k_{\parallel}}{k_{\perp}} \gtrsim \frac{1}{2.5} \sqrt{\frac{W_H}{T_e} \frac{m_e}{m_i}}$ are strongly electron Landau damped, consequently the time scale for electron heating may not be classical. That is, in order of magnitude at least, the relevant time scale may be as short as the inverse of the beam driven growth rate. The relevant time scale for ion heating then becomes the Spitzer equipartition time, but because reactor ignition occurs at $T_e > \frac{m_e}{m_i} W_H$ the integrated time for background ion heating is only slightly reduced [see Fig. 5].

Stabilization Criteria

Beam Energy Spread

There is an advantage from the overall stability point of view from injecting the beam with a finite energy spectrum $[\Delta V_{T_B} \neq 0]$ such that both :-

$$\Delta V_{T_B} \gtrsim V_{T_i} \quad \text{(a)} \quad \text{and} \quad \Delta V_{T_B} \gg \frac{V_o}{m^{2/3}} \quad \text{(b)} .$$

Thus under condition (a) the non flute mode instabilities are reduced to a weak resonant particle type [e.g. see (39)] and the flute modes should then dominate. Condition (b) implies that all growth rates [including the flute modes] are reduced by the factor $\dfrac{V_o}{\Delta V_{T_B}\ m^{2/3}}$.

Finite Geometry Effects

The non flute modes are convective but reflect from the plasma boundary where $\omega_{p_e} = 0$. The effect of non-uniformity of the ambient magnetic field is to cause the unstable modes to grow as localized wave packets since these will not sample large field variations. The shear requirement

$$\frac{2\pi\ \lambda_B}{L_s} > 1 ,$$

where λ_B = effective Debye length of the beam component, is sufficient to stabilize all modes although the non flute modes are much more easily stabilized [e.g. see (49)].

Threshold Densities

A finite beam density threshold does exist, in principle, for the flute mode stabilization but is of no consequence in any real system [i.e. see (13) and (31)] but clearly all unstable modes vanish for very tenuous background plasmas where $\omega_{p_e} < \Omega_i$ [a condition never satisfied in any reactor system].

IX CONCLUSIONS

It is concluded that the high growth rate instabilities which are associated with the injected beam may lead to more rapid heating of the background electrons and ions than expected on classical grounds. However the rate of ion heating is only increased at 'low' electron temperatures

$T_e < \frac{m_e}{m_i} W_H$; consequently it appears that, although useful during the initial heating stage, the instabilities are only marginally useful in the overall heating of a reactor to its ignition temperature [see Fig. 5].

The non flute modes are faster growing at least for a monoenergetic beam injected into a uniform plasma; however in any real system the flute modes are likely to be more dangerous since they are non-convective, require greater shear for their suppression and are not much affected by finite thermal spread in the injected beam. The requirement for stability of all modes is

$$\frac{2\pi \lambda_B}{L_s} > 1 .$$

We have applied the above results to reactor systems, and also to the next generation of toroidal experiments such as ORMAK[15] [see appendix]. Thus complete stabilization of all electrostatic modes can scarcely be achieved in a typical reactor since the shear requirement is $L_s \lesssim 100$cm and stabilization of all modes in ORMAK is not possible since the shear requirement is $L_s \lesssim 10$cm. Enhanced plasma [ion] heating, which can arise from the beam driven modes, is not important at any stage of the heating process in the ORMAK experiment.

The suppression of electrostatic instabilities may not be vital from the containment point of view since rather crude estimates of the confinement time $\tau_c[\sim(\text{plasma radius})^2/D_\perp]$ are $\sim$ 1 sec. for both the reactor and for ORMAK [see appendix].

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APPENDIX

Application to Proposed Experimental Devices

Examples can be given of immediate interest to toroidal experiments, such as the Oak Ridge tokamak ORMAK. In this experiment[15] it is proposed to heat the toroidal plasma in a field $B \sim 30kG$, by means of neutral beams of 8 amperes equivalent, whose directed energy W_H is $\sim 25keV$. If the period of injection is several milliseconds and the plasma volume ~ 1 metre^3, the beam component density is at least $\sim 10^{10}/cm^3$ hence $\omega_B \sim 10^8\ sec^{-1}$. Using Tokamak T3-A as a guide[16], then, $10eV \lesssim T_e \lesssim$ few keV and the background number density $\lesssim 10^{14}/cm^3$. It is easily verified that with the above parameters all unstable modes [i.e. electron cyclotron and electron plasma waves] have growth rates γ within the range

$$5.10^7 sec^{-1} \lesssim \gamma \lesssim 5.10^8 sec^{-1} .$$

Collisional theory alone predicts that the ion beam loses energy preferentially to the background ions only if $T_e > W_H\left(\frac{m_e}{m_i}\right)^{1/3}\left(\frac{2}{3\sqrt{\pi}}\right)^{2/3} \simeq 1$ keV [see (27)]. However since electron Landau damping should lead to very rapid electron heating the relevant time scale for plasma [ion] heating is the Spitzer equipartition time $\tau_{eq}\ (e_p \rightarrow i_p)$. However $\tau_{eq}\ (e_p \rightarrow i_p) \simeq \tau(i_B \rightarrow e_p)$ since $T_e \gg \frac{m_e}{m_i} W_H \simeq 6$ eV throughout [see (29)].

The shear requirement, for stabilization of all modes, in the ORMAK experiment is $L_s \lesssim 10$ cm [see (46)] and is clearly not attainable [c.f. $L_s \lesssim 100$ cm for the toroidal reactor discussed within the text]. Stabilization of all modes implies that the electrons are heated preferentially if $T_e < 1$ keV while the ions again heat at the rate τ_{eq} [$\simeq \tau(i_B \rightarrow e_p)$ for $T_e > 6$ eV]. Consequently the instabilities appear to be purely detrimental in such a device.

A good estimate of the cross field diffusion would require a rigorous non-linear treatment. However if we assume that a turbulent spectrum of waves develops we can use the conventional random walk expression for $D_\perp$, this reduces to $\frac{1}{2}\frac{\langle E_\perp\rangle^2}{B^2}\Delta t$, where $\frac{\langle E_\perp\rangle}{B}$ is the drift velocity due to the r.m.s. turbulent perpendicular electric field, Δt is the fluctuation time of the field seen by the particle.

The resonant particle diffusion of the 'Landau heated' background electrons is almost certainly dominant (since Δt is maximum for this particular component). We take $\Delta t \sim \frac{B}{k_\perp\langle E_\perp\rangle}$ the time for a resonant electron to $\underline{E} \wedge \underline{B}$ drift a perpendicular wavelength (alternative characteristic times can be considered, such as[17] $\Delta t \sim$ (linear growth rate)$^{-1}$, however simple numerical estimates verify that $\frac{B}{k_\perp\langle E_\perp\rangle}$ is the smaller).

Assuming an equilibrium situation between the energy input from the beam and the electron Landau damping of the unstable modes we find $D_\perp \sim \frac{1}{k_\perp B}\sqrt{\frac{4\pi\, W_H\, n_B}{\tau_{inj}\,\gamma_D}}$ where τ_{inj} = period of beam injection, γ_D = electron Landau damping decrement. Thus $D_\perp \sim 10^3$ cm^2/sec in ORMAK i.e. containment time ~ 1 sec [while $D_\perp \sim 10^4$ cm^2/sec in the above typical reactor].

Non-linear effects are discussed in more detail in a further report(2).

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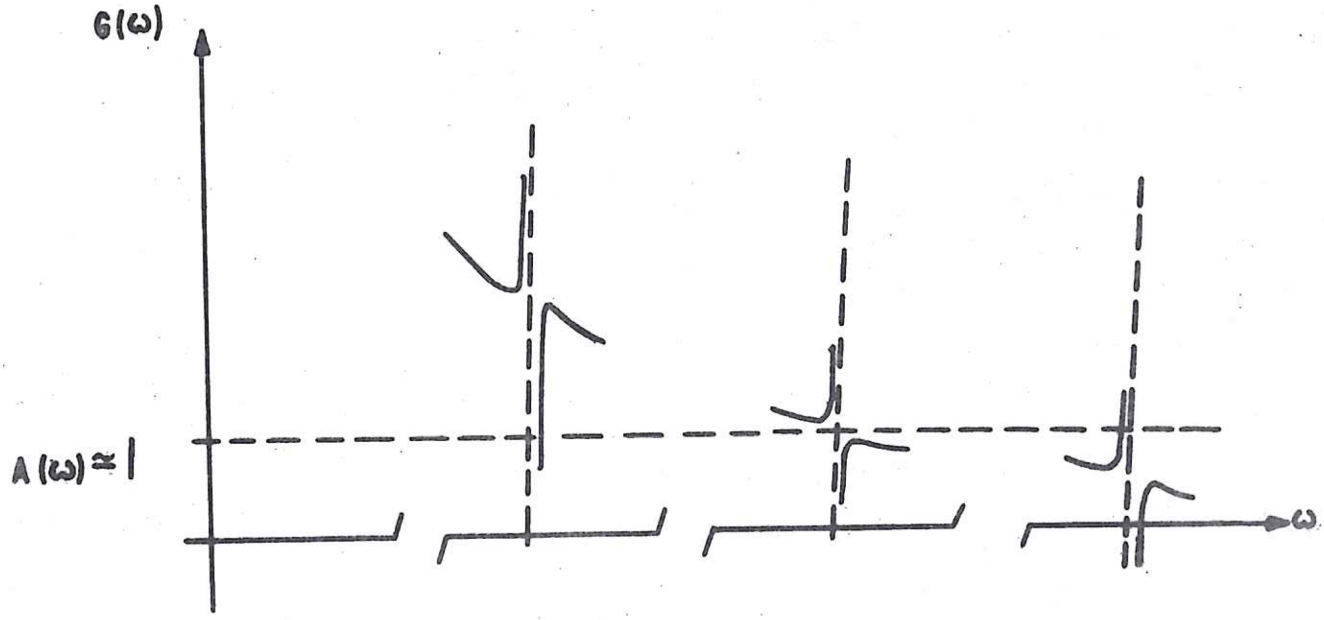
<u>Fig. 2</u>

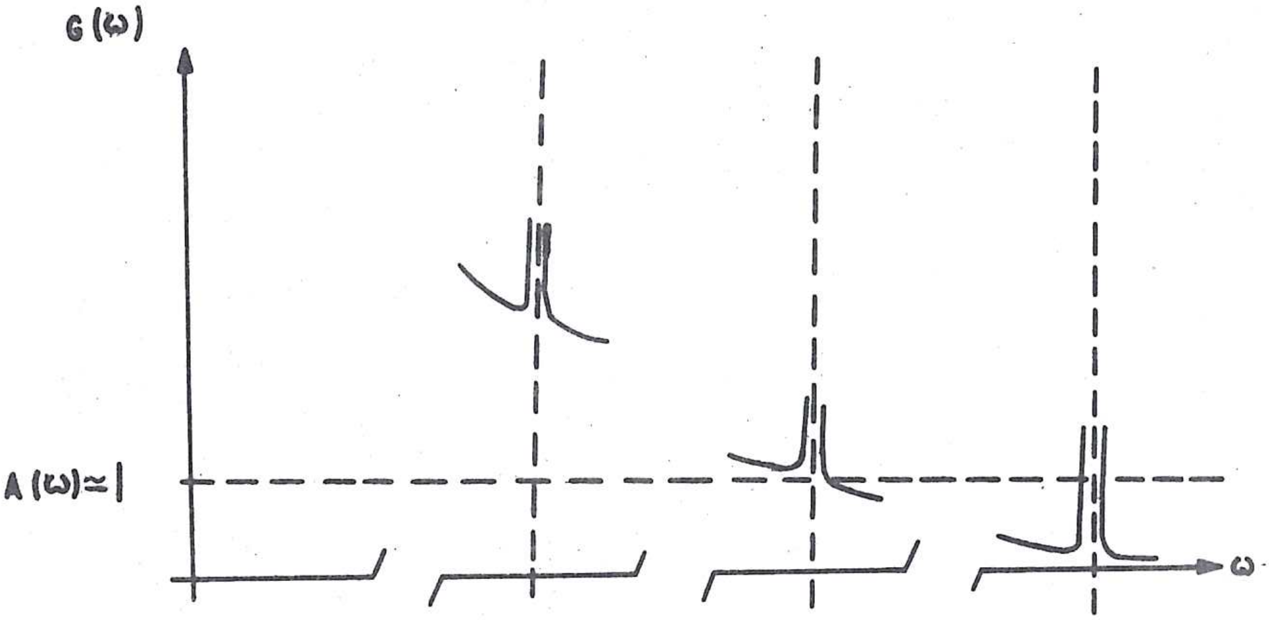
$$\left[\text{Form of } 1 + \left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 = A(\omega) + \sum \frac{\alpha_n}{[\omega^2 - (n\Omega_i)^2]} = G(\omega),\ \alpha_n \text{ negative}\right]$$



Unstable modes arise at $A(\omega) \simeq 1$, provided α_n is negative

<u>Fig. 3</u>

$$\left[\text{Form of } 1 + \left(\frac{k_{\parallel}}{k_{\perp}}\right)^2 = A(\omega) + \sum \frac{\delta_n}{[\omega^2 - (n\Omega_i)^2]^2} = G(\omega),\ \delta_n \text{ positive}\right]$$



All harmonics for which $A(\omega) \simeq 1$ are unstable

Fig. 4

Plot of Ψ(u) and its two asymptotic forms

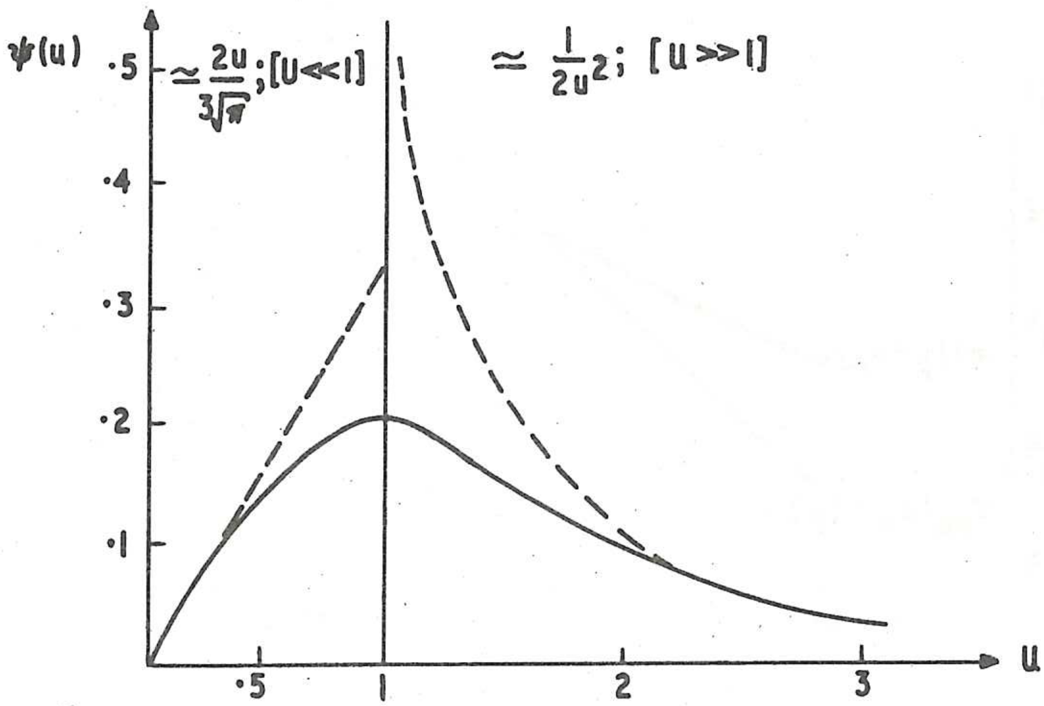


Fig. 5

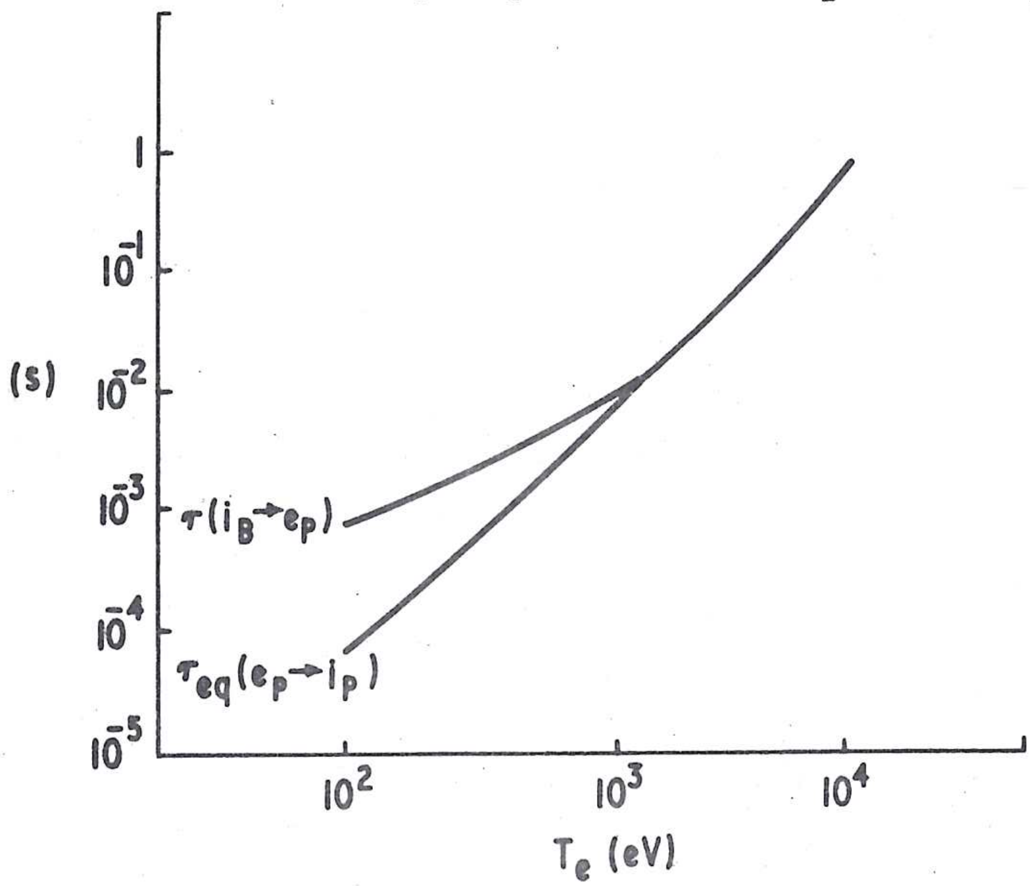
Plot of $\tau(i_B \rightarrow e_p)$ and $\tau_{eq}(e_p \rightarrow i_p)$ against T_e

[We take the typical reactor parameters:-

Directed beam energy $W_H \sim 10^6$ eV,

Background plasma density $\simeq 10^{14}$ cm^{-3} and $\log \Lambda \simeq 15$,

where $\Lambda = \dfrac{\text{Debye Shielding Distance}}{\text{Impact parameter for } \pi/2 \text{ scattering}}$]



Notice $\tau_{eq}(e_p \rightarrow i_p) < \tau(i_B \rightarrow e_p)$ only for $T_e < \frac{m_e}{m_i} W_H \sim 10^3$ eV. Thus rapid electron heating, which may arise from electron Landau damping of the unstable modes, will result in more rapid ion heating only in the initial phase $T_e \sim 10^2$ eV. However the time for ion heating from 10^2 to 10^4 eV is clearly not much reduced.

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