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Report

THE USE OF LIQUID LITHIUM AS COOLANT IN A TOROIDAL FUSION REACTOR

PART I

Calculation of Pumping Power

J C R HUNT
R HANCOX

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THE USE OF LIQUID LITHIUM AS COOLANT IN A TOROIDAL FUSION REACTOR

PART I

Calculation of Pumping Power

J.C.R. Hunt*
R. Hancox

A B S T R A C T

An attractively simple concept for the nuclear blanket of a fusion reactor fuelled with a deuterium-tritium mixture would be the use of liquid lithium as both a tritium breeding and heat transfer medium. The main disadvantage of this idea is the pumping power necessary to circulate the lithium in the presence of the high magnetic field required for plasma containment.

The magneto-hydrodynamic power loss is calculated for the flow regimes relevant to a toroidal fusion reactor. The calculation is exact for uniform magnetic field and flow conditions, and is compared with published experimental work.

A preliminary estimate is made of the pumping power required in a reactor, but a further discussion of the application of the equations is left to Part II of this report.

*Trinity College, Cambridge

UKAEA Research Group
Culham Laboratory
Abingdon, Berks.

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C O N T E N T S

SYMBOLS AND DEFINITIONS

Vector quantities

$\underline{H}$	Magnetic field
$\underline{B}$	Magnetic flux density
$\underline{E}$	Electric field
$\underline{J}$	Current density
$\underline{v}$	Fluid velocity

(These quantities may have components e.g. v_x, v_y, v_z in the x, y, or z directions as defined in Figure 1, or mean values $\bar{v}$)

Scalar quantities

a, b	Dimensions of duct (2a, 2b are the height and width of a rectangular duct, a is the radius of a circular duct)
A	Cross-sectional area of duct
ℓ	Length of duct
t	Thickness of duct wall
δ	Thickness of the fluid boundary layer
Q	Volume flow of fluid
p	Pressure in fluid
ρ	Density of fluid
c	Specific heat of fluid
σ	Electrical conductivity of fluid
$\varkappa$	Thermal conductivity of fluid
h	Heat transfer co-efficient
η	Viscosity of fluid
u	Magnetic permeability of fluid
σ_w	Electrical conductivity of duct wall

Non-dimensional quantities

M	Hartmann number (Electro-magnetic stress/viscous stress)$^{\frac{1}{2}}$	$= aB(\sigma/\eta)^{\frac{1}{2}}$
R_e	Reynolds number (Inertial stress/viscous stress)	$= a\bar{v}\rho/\eta$
R_m	Magnetic Reynolds number (Induced magnetic field/imposed magnetic field)	$= a\bar{v}\sigma\mu$
N_u	Nusselt number	$= ah/\varkappa$
P_r	Prandtl number	$= \eta c/\varkappa$
C_f	Friction factor	$= \dfrac{p/\ell}{\rho v^2/2a}$
ϕ	Conductance ratio (Conductance of duct wall/conductance of fluid)	$= \sigma_w t/\sigma a$

MKS units are used in all equations.

1. INTRODUCTION

In a steady state fusion reactor, as at present visualized, a high temperature reacting plasma will be confined by a magnetic field. Several magnetic field configurations are possible, but the most promising are closed configurations with toroidal geometry from which the plasma can only leak slowly by diffusion across field lines. The first generation of such reactors will probably operate with a deuterium-tritium fuel mixture, with the majority of the reaction energy appearing in 14 MeV neutrons. Since tritium is not a naturally occurring isotope, it will be necessary to breed it in a lithium bearing nuclear blanket placed between the plasma and the magnetic winding, in which $^{6}Li(n,T)$ and $^{7}Li(n,n'T)$ reactions will take place. This blanket will also be used to absorb most of the energy in the neutrons, converting it to heat for use in generating electricity by means of a conventional heat engine such as a steam turbine.

From this brief description it is seen that an essential component of a deuterium-tritium fuelled fusion reactor is a lithium bearing blanket from which heat must be recovered at a sufficiently high temperature for the efficient generation of electrical power. Whilst in principle a range of lithium bearing molten salts could be used, such as $2LiF.BeF_2$ (Flibe), these only give a marginal tritium breeding ratio due to the parasitic capture of neutrons, whereas the use of lithium metal itself would allow a significantly higher breeding ratio. By analogy with a liquid metal cooled fast breeder reactor, the lithium could also be considered as the coolant circulating through the nuclear blanket, giving outlet temperatures up to 650°C. Thus, conceptually, the blanket could be a very simple structure with the lithium in it serving all the functions of neutron moderator, tritium breeding, and heat transfer medium.

The major disadvantage of such a scheme is that, due to the need to place the nuclear blanket inside the winding which generates the confining magnetic field, the lithium must be pumped across the magnetic field. Thus the pumping power may be unacceptably high. This problem is considered in detail in the following sections - beginning with the discussion of the physical phenomena which occur when a conducting fluid is pumped across a magnetic field, and leading to the application of pumping power calculations to possible reactor proposals.

2. PHYSICAL PRINCIPLES OF MHD DUCT FLOW

When a fluid with high electrical conductivity is pumped along a duct, the presence of a magnetic field can produce considerable changes in the overall properties of the flow and in the velocity distribution across the duct. For engineering design of the cooling circuit information is also needed about the way in which the magnetic field changes the pressure drop along the duct and the heat transfer from the duct walls to the fluid.

Before dealing with these effects in detail, however, it may be helpful to describe the nature of the flow in the absence of a magnetic field.

Consider the flow along a duct as the mean velocity, $\bar{v}$, is increased. When the velocity is low enough the flow is quite steady, any disturbances being damped by viscous forces, and the flow can be regarded as thin layers of fluid sliding over each other at different speeds; hence its description as 'laminar' flow. The velocity distribution is determined by viscous stresses between each layer of fluid, the layers in the centre moving faster than at the walls where the velocity must be zero. When the velocity is increased beyond a critical value, small disturbances to the flow are amplified and distorted by inertial effects until the velocity becomes random throughout the duct. Such a flow is called 'turbulent' and is not only different from laminar flow in that it contains random fluctuations but also in that the velocity distribution is changed: the parabolic velocity profile of laminar flow being replaced by an approximately constant velocity in the centre of the duct and narrow boundary layers close to the walls where the velocity decreases to zero. This difference is caused by the turbulent eddies transferring momentum from the centre of the duct to the walls, and vice versa, much more effectively than the molecular process of viscosity. By analogy these eddies also transfer heat more effectively than the molecular process of heat conduction.

In the absence of magnetic field effects the dividing line between laminar and turbulent flow is specified by the non-dimensional Reynolds number, R_e, which gives the order of magnitude of the ratio of inertial to viscous stresses in the fluid. For a typical duct a Reynolds number above 1000 gives turbulent flow. The practical significance of the velocity distribution in the duct and the difference between the two flow regimes can now be appreciated, given that the pressure drop is proportional to the viscous shear stresses at the wall, which depend on the local velocity gradient, and that the heat transfer is proportional to the temperature gradient at the wall. For, since turbulence increases the velocity and temperature gradients at the wall, it follows that the pressue drop and heat transfer are greater in a turbulent flow than a laminar flow at the same flow rate. It also follows that if the pressure drop and the heat transfer are to be understood or calculated, we must examine how the magnetic field affects the velocity distribution and whether, if the flow is turbulent, the velocity fluctuations are damped or amplified by the magnetic field.

The effect of a magnetic field on the flow of a conducting fluid may now be considered. In the cooling circuits of a fusion reactor the orientation of the magnetic field, $\bar{B}$, relative to the duct will probably vary between being perpendicular and parallel to the ducts. By considering the two extreme situations the properties of the flow at all intermediate orientations can be estimated. First consider the case where the magnetic field is at right angles to the duct, illustrated in Figure 1. The induction of electric currents is best understood by examining the change in flux linking a fluid circuit. The circuit ABCD links no flux, but since the velocity v_z is greater in the centre of the duct than at the walls the displaced circuit A'B'C'D' now links flux and a current is generated around the circuit. An

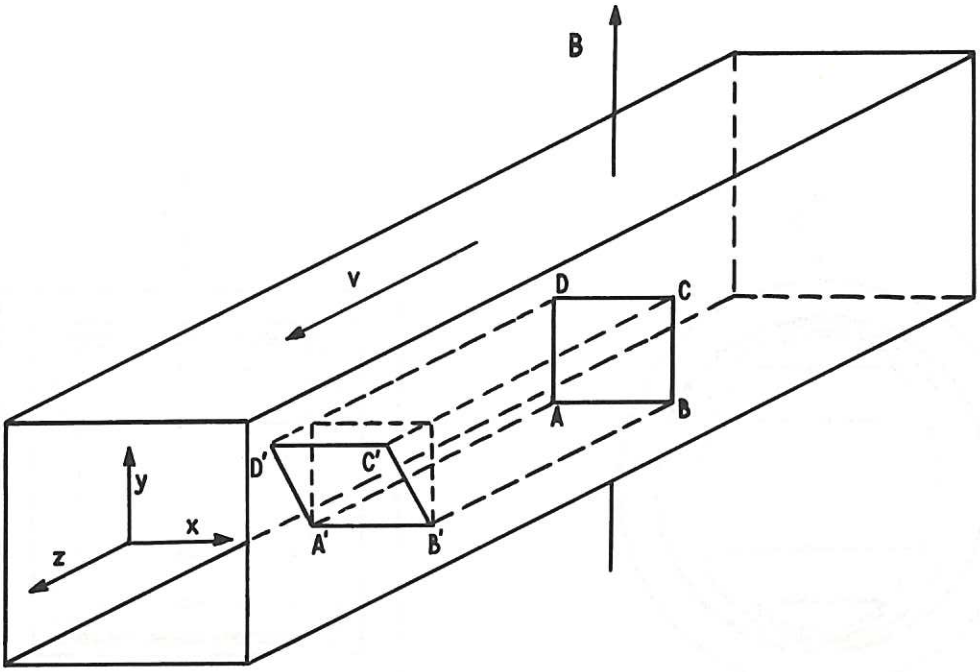

Fig.1 The movement of a circuit drawn in the fluid as it travels along a duct, showing how a transverse magnetic field induces a current round the circuit.

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alternative physical explanation can be given in terms of the $\underline{v} \times \underline{B}$ induced electric field where $\underline{v}$ is the velocity and $\underline{B}$ the local magnetic flux density. Some typical current paths for flow in circular and rectangular ducts when the magnetic field is large are shown in Figure 2. If the walls of the ducts are electrical insulators the currents generated in the core return through the boundary layers on the side walls, as in Figure 2(a). If the walls are electrically conducting, then the currents return through the walls, as in Figure 2(b), especially if the conductivity of the walls is high compared with the fluid and the boundary layers are thin compared with the thickness of the walls. Note that electric currents induce their own magnetic field, which is mainly directed along the duct because the currents flow in loops. The ratio of this induced field to the imposed magnetic field is known as the Magnetic Reynolds Number, R_m, and in most practical situations it is small.

These electric currents, with density $\underline{J}$, affect the velocity distribution by means of variations across the duct of the electro-magnetic body force, $\underline{J} \times \underline{B}$. If this force did not vary across the duct then it would simply be equivalent to an increased pressure gradient. For a duct with non-conducting walls the current distribution is such that this force acts to brake the fluid in the core of the flow but to speed up the flow near the walls. If the walls are highly conducting, the normalised velocity profile takes on a similar form with narrow boundary layers at the walls. The reason is that on those walls which are not parallel to the magnetic field, the component of current density parallel to the wall is much less than that in the core and therefore there is a large change in the $\underline{J} \times \underline{B}$ force between the core and the boundary layer, as in the duct with non-conducting walls. Figure 3 illustrates these changes in velocity across

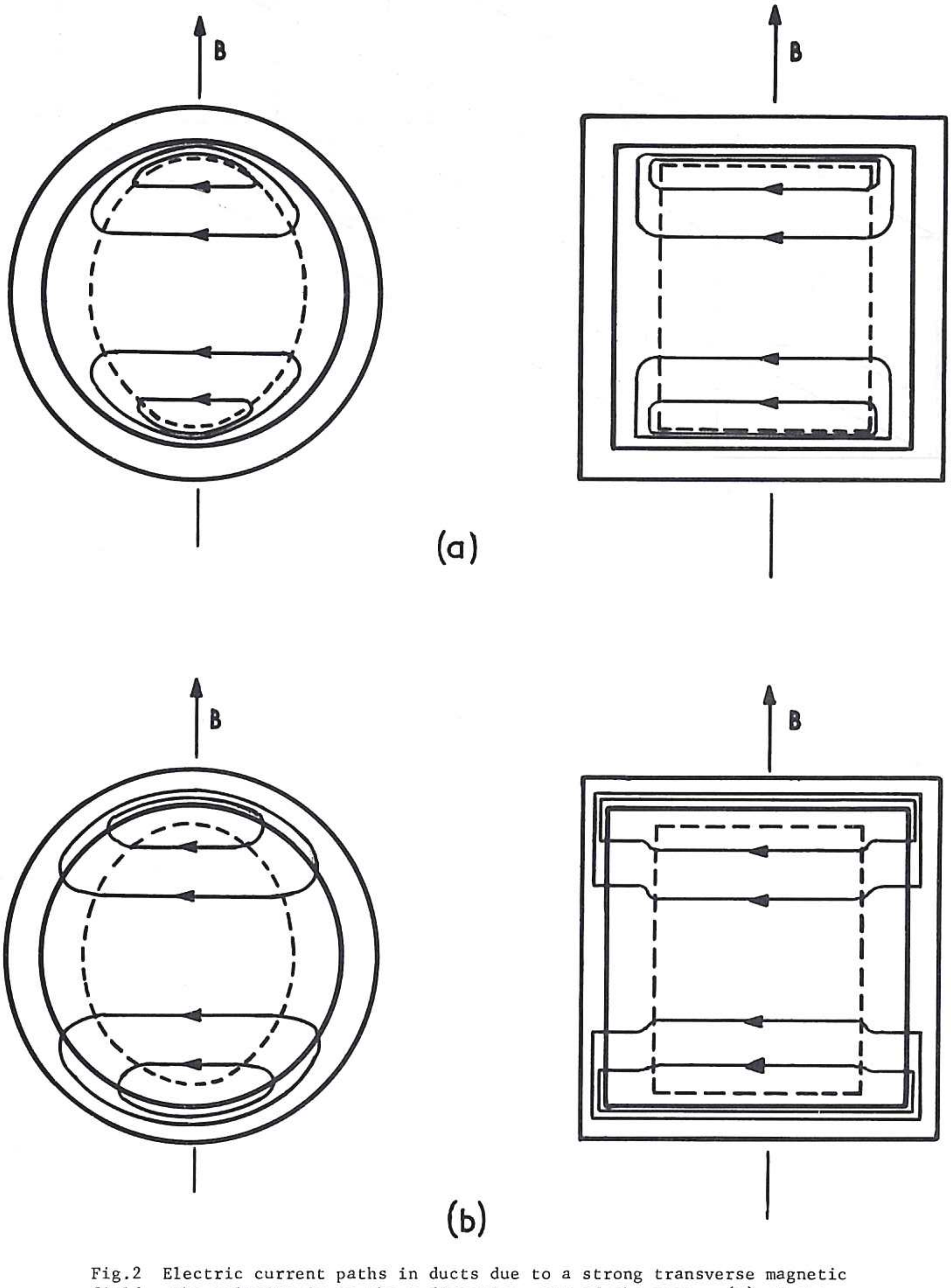

Fig.2 Electric current paths in ducts due to a strong transverse magnetic field. The velocity is in the z direction, out of the page. (a) Ducts with non-conducting walls. (b) Ducts with highly conducting walls.

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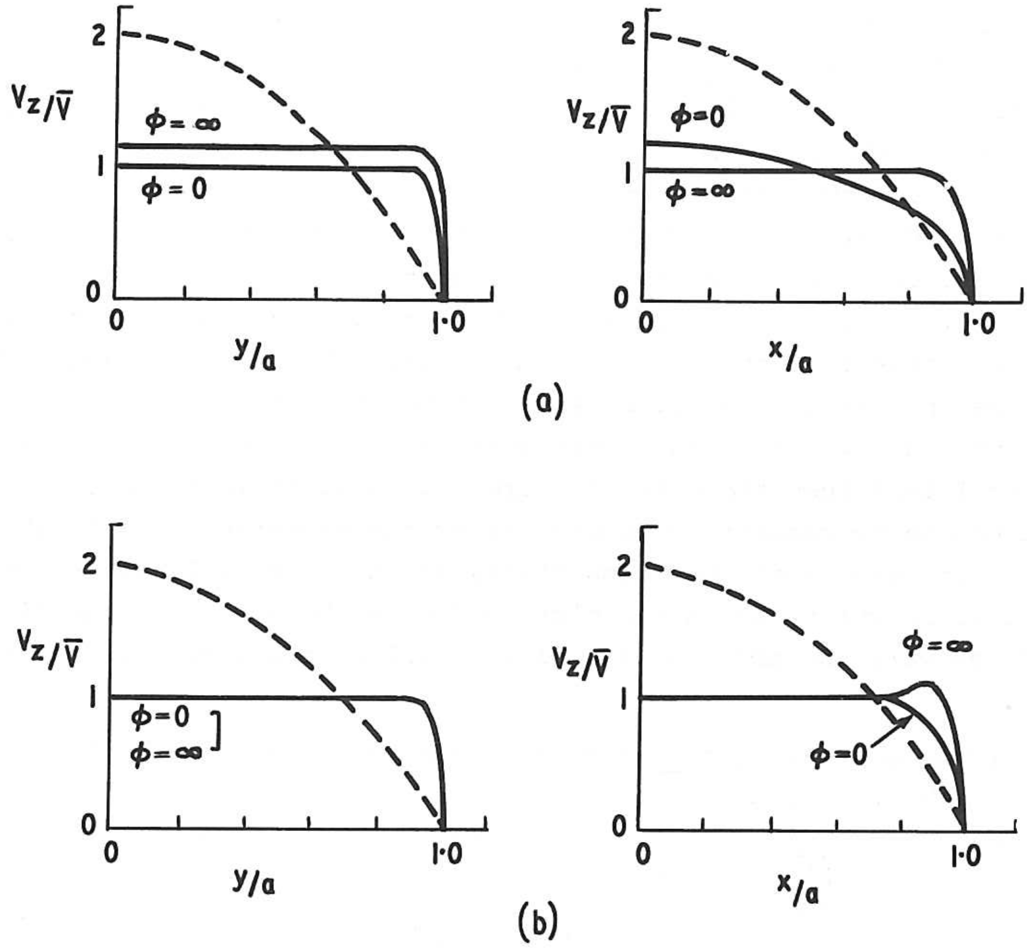

Fig.3 Velocity profiles in ducts with zero (---) and strong (___) transverse magnetic fields, and walls which are non-conducting ($\theta = 0$) and highly conducting ($\theta = \infty$). (a) Duct with circular cross-section. (b) Duct with rectangular cross-section.

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ducts in the directions parallel and perpendicular to the magnetic field, and also shows how the duct shape affects the velocity distribution. The velocity profile in the direction parallel to the magnetic field $v_z(y)/\bar{v}$ is similar for both the circular and square ducts, but the profile in the perpendicular direction $v_z(x)/\bar{v}$ is quite different for the two types of duct. If the conductivity of the various walls of a rectangular duct differ, then some remarkable velocity profiles can occur. In general the degree to which the velocity distribution is affected by the magnetic field is indicated by the value of the Hartmann number, M, which physically is the ratio of electromagnetic to viscous stresses.

It is clear that if the effect of a magnetic field is to reduce differences in the time averaged velocity, then it will have a similar effect on fluctuating velocities. Since the essential feature of turbulence is the random distribution of eddies all moving with different velocities, then if these velocity differences are suppressed the whole turbulent flow is suppressed. However the turbulence is also affected in another way. Since the turbulence receives its energy from the transverse gradients in the average velocity, in the core where these gradients are reduced the turbulence is damped; in the boundary layers near the walls, although the gradients are increased, the layers are so thin that viscous stresses damp the turbulent eddies. It appears that these eddies in the boundary layers are more persistent than those in the core[1]. A sufficient condition for the turbulence to be completely suppressed by the magnetic field is that[2] $R_e/M < 130$. In some cases the conductivity of the duct walls varies round the perimeter, in which case a new picture emerges because the magnetic field produces very unstable boundary layers which positively promote turbulence[3].

The electromagnetic $\underline{J} \times \underline{B}$ forces on the fluid can affect the pressure gradient both directly and indirectly. If the walls of the duct are insulators so that all the currents form closed paths in the fluid then $\int_{-a}^{a} J_x \, dy = 0$, and $\iint_A J_x B_y \, dx dy = 0$, so there can be no net direct body force. However, it has been seen that the effect of $\underline{J} \times \underline{B}$ forces is to increase the shear stresses at the wall, and this indirectly increases the frictional forces and hence the pressure gradient. On the other hand if the incident flow is turbulent, $\underline{J} \times \underline{B}$ forces also reduce the turbulence, and thereby indirectly decrease the pressure gradient. These two conflicting effects are demonstrated by measurement of the pressure gradient as a function of magnetic field at constant values of the flow rate. The results of many experiments have been correlated by Branover and Tsinober[4] and are shown in Figure 4 in non-dimensional form as a friction factor, C_f, plotted against the ratio $2M/R_e$ at constant values of Reynolds number. These results show that if the flow is turbulent with zero magnetic field, then at low flow rates as an increasing magnetic field is applied the pressure gradient at first decreases slightly and then increases, but at higher flow rates the pressure gradient always increases as the magnetic field increases. If the initial flow is laminar, the pressure gradient always increases as the magnetic field increases.

For a duct with conducting walls, since the currents circulate through the walls $\iint J_x B_y \,dxdy$ is not zero in the fluid and there is a net contribution by $\underline{J} \times \underline{B}$ to the pressure gradient. Usually this effect dominates over the two indirect effects of increasing the wall shear stress and reducing the turbulence, so that even for turbulent flows as the magnetic field increases, the pressure gradient always increases[5].

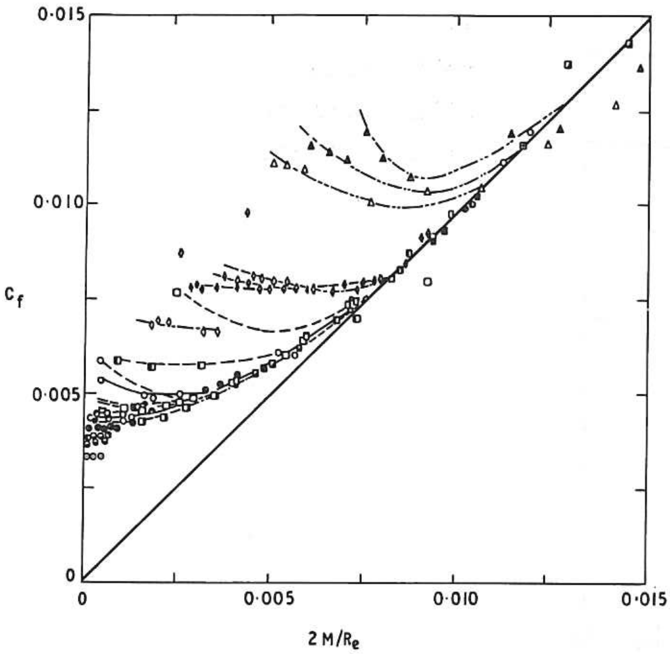

Fig.4 Experimental measurements of the friction factor C_f for turbulent flows with a transverse magnetic field in a rectangular duct with insulating walls at various values of Reynolds numbers ranging from 8.1×10^2 (△) to 1.0×10^5 (⊙). Branover and Tsinober[4].

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When the magnetic field is parallel to the axis of the duct and provided the flow is steady and has no swirl component, then the velocity is parallel to the magnetic field and no electric fields are induced. Alternatively it can be seen that any circuit drawn in the fluid, such as ABCD in Figure 1, links a constant quantity of flux as it moves down the duct. Thus a parallel magnetic field has no effect on the pressure gradient or heat transfer of a steady flow. However if the flow is turbulent, then there exist motions across the magnetic field and in general circuits such as ABCD will link variable quantities of flux. As might be expected, it is found experimentally that the magnetic field damps the turbulence. But because the magnetic field does not distort the time averaged velocity profile which provides the turbulent energy it is found that to suppress the turbulence in a given flow a considerably larger parallel magnetic field is required than perpendicular field[6]. As the turbulence is suppressed, the exchange of momentum between the core and the boundary layers is reduced and the shear stress at the wall is reduced. Consequently the effect of a parallel magnetic field on the pressure gradient in a turbulent flow is always to reduce it.

The effect of a perpendicular magnetic field on the heat transfer to a duct depends on whether, after the field is applied, the flow is laminar or turbulent. If the flow is laminar in the absence of the magnetic field then, as we have seen, the effect of a magnetic field is to maintain the flow laminar and to reduce the thickness of the boundary layers. This must lead to an increase in heat transfer which is of the order of 50%, as shown in Figure 5. However, if a flow which is initially turbulent enters a magnetic field sufficiently strong for its turbulence to be suppressed, then the heat can no longer be transferred from the duct walls to the fluid by the turbulent eddies, and the heat transfer will be considerably reduced from its initial value. If the flow is turbulent even in the presence of a

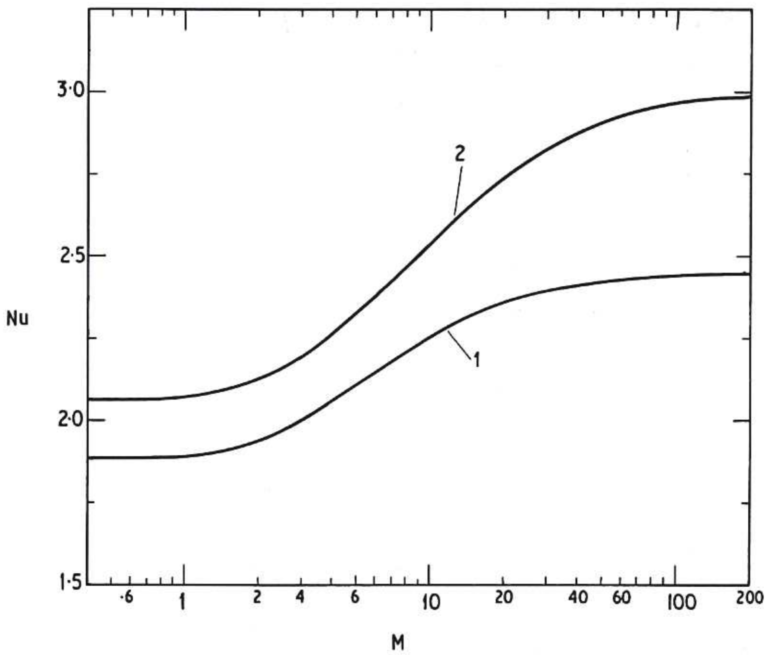

Fig.5 Experimental measurements showing the influence of a transverse magnetic field on the heat transfer to a laminar flow along a flat rectangular duct. Branover and Tsinober[4]. (1) Constant temperature difference. (2) Constant heat flux.

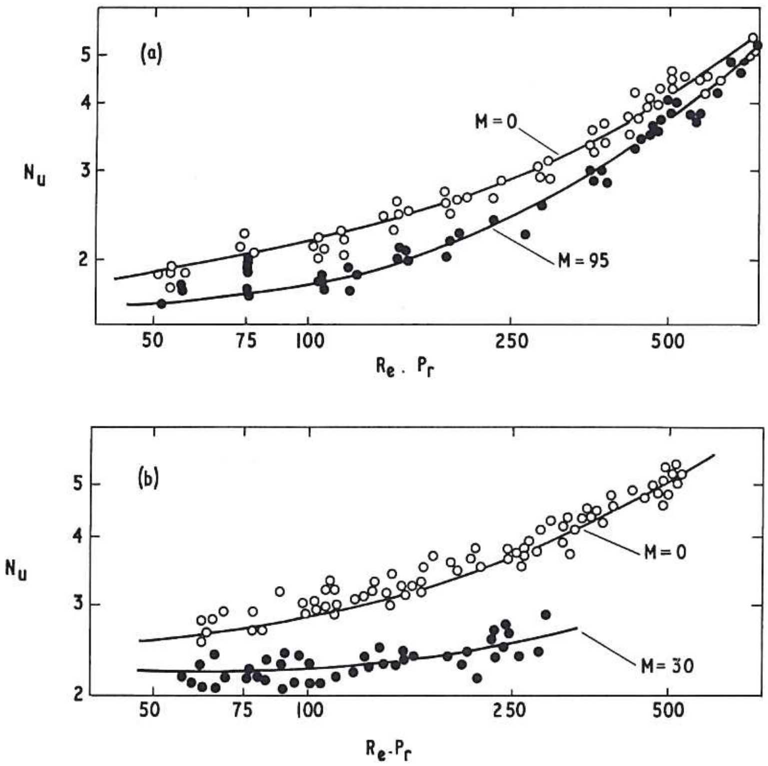

Fig.6 Experimental measurements showing the influence of a magnetic field on heat transfer to turbulent flows. Branover and Tsinober[4]. (a) Parallel field (circular duct) (b) Perpendicular field (rectangular duct).

magnetic field, then the magnetic field must be relatively weak. But there will still be some damping of turbulence and consequently a reduction in the heat transfer. A parallel magnetic field will not affect the heat transfer in a laminar flow but in a turbulent flow will damp the eddies and thus reduce the heat transfer. These effects are shown in the experimental results of Branover and Tsinober[4] shown in Figure 6.

We have so far mentioned only viscous, pressure, electromagnetic, and turbulent stresses acting on the mean flow, because these are the only forces acting in a fully developed flow in a long straight duct with a constant magnetic field. However in a reactor the ducts will have curved sections, which may have cross sectional area changes, and will certainly pass through non-uniform magnetic fields. Furthermore the flow will not be fully developed at many places. In these circumstances the velocity will be changing along the duct and inertial effects will occur, so that another important question is how these inertial forces and electromagnetic forces interact. This is a much more difficult problem than the first, but fortunately the effects are small in the fusion reactor situation on account of the parameters involved, as will be made clear later.

A recent review has been published by Hoffman and Carlson[7] of techniques for calculating pressure losses in fusion reactor cooling circuits. To some extent this present report covers similar ground but, on the whole, it concentrates on practical flows where the magnetic field is large and where simple analytical results are possible. It does not attempt to provide empirical correlations of formulae over the whole range of Hartmann number. In addition extensive reference has been made to work done in the USSR which was largely ignored by Hoffman and Carlson.

3. GENERAL PRESSURE DROP CALCULATIONS

3.1 Uniform Laminar Flows

First consider the flow in a duct with a transverse magnetic field when the following conditions are satisfied:

(a) the duct is straight,

(b) the duct dimensions remain constant along its length,

(c) the magnetic field does not change appreciably in a distance of the order of the duct size.

These conditions may not be untypical of most parts of the liquid metal circuit of a toroidal fusion reactor. In addition it will be found that in the reactor the non-dimensional parameters, Hartmann number and Reynolds number, which define the nature of the flow of the fluid have values $M >> 1$, $R_e \sim M$, $R_m \ll 1$.

Given these assumptions, it follows[2] that the flow will be laminar, since $R_e/M \ll 130$. It also follows[8] that the flow will be fully developed provided the duct is long enough that $\ell/a \gg R_e/M$. Both these conditions are satisfied by a lower value of Reynolds number or a higher value of the Hartmann number if the duct walls are highly conducting. Thus it can be assumed that inertial forces in the fluid are negligible, so that a balance has to exist between viscous, pressure, and electro-magnetic forces. The mathematical analysis for this situation is given in Appendix I. The salient points in the argument are shown below.

First the analysis shows that thin boundary layers are created on the walls of the duct with thickness $\delta \sim (a/(M \cos\theta))$, where θ is the angle between the normal to the wall and the magnetic field. Where the walls are parallel to the field there is a small region in the case of a round pipe of width $\delta \sim (aM^{-\frac{2}{3}})$, but if the duct is rectangular then on these parallel walls $\delta \sim (aM^{-\frac{1}{2}})$. A review by Hunt and Shercliff[3] contains a lengthy discussion of these various layers.

In the core electromagnetic forces balance pressure forces so that

$$\partial p/\partial z = \underline{J} \times \underline{B} \quad . \qquad (3.1)$$

Ohm's law gives the current density $J_x = \sigma(E_x - v_z B_y)$, so that

$$\partial p/\partial z = \sigma B_y(E_x - v_z B_y) \ . \quad (3.2)$$

If the conductivity of the walls is very high, $\phi \gg 1$, the electric field is zero, and

$$\partial p/\partial z = - \sigma v_z B_y^2 \ . \quad (3.3)$$

The mathematical analysis in Appendix I shows that this result is, in fact, exact. It follows directly that for a total volume flow rate through the duct, Q,

$$\partial p/\partial z = - Q\sigma B_y^2/A \ . \quad (3.4)$$

If the conductivity of the walls is small and finite so that $0 < \phi \ll 1$, but the magnetic field is large enough that $M\phi \gg 1$, then the currents still circulate through the walls because of the high resistance of the boundary layers. At the mid-plane (x = 0) the total current flowing through the walls is equal to the total current in the fluid, $tJ_w = - bJ_f$, so that the electric field is

$$E_x \sim \frac{J_w}{\sigma_w} \sim \frac{J_f b}{\sigma_w t} \sim (J/\phi\sigma)(b/a)$$

whence from (.32)

$$\partial p/\partial z = - \sigma v_z \phi B_y^2 (a/b) \ . \quad (3.5)$$

This order of magnitude result is confirmed by the exact solution of Chang and Lundgren[9], derived in Appendix I which for the particular cases of a circular or rectangular duct simplify to

$$\partial p/\partial z = - Q\sigma\phi B_y^2/A \ . \quad (3.6)$$

The exact solution allows us to consider the possibility of minimizing the pressure gradient by changing the shape of the duct. Variations by a factor 2 or 3 are possible, as indicated in Appendix I.

The analysis in the Appendix also shows that if the duct is completely non-conducting, so that $M\phi = 0$, then

$$\partial p/\partial z = - Q\sigma B_y^2/AM \quad (3.7)$$

so that the pressure gradient has decreased by a factor[8] of M from the case with fully conducting walls.

3.2 Laminar Flow Under Non-Uniform Conditions

There has been no theoretical or experimental work on the effects of non-uniformity of the magnetic field in conditions where the Hartmann number is great enough that inertial effects are negligible i.e. $M^2/R_e \gg 1$. There has been extensive work[8] on the problem when $M^2/R_e \ll 1$, and the results have been used by Hoffman and Carlson[7], but, as can be seen from the parameters likely to occur, this approximation is quite inappropriate to a toroidal fusion reactor. If, however, the magnetic field varies along the length of the duct then a simple examination of the equations indicates the pressure gradient $\partial p/\partial z$ changes by a factor $\epsilon_B \sim a(\partial B/\partial z)/B$, provided $\epsilon_B \ll 1$.

When the area of the duct changes, but the magnetic field is constant, then there has been some theoretical analysis[10,11] in the conditions when $M \gg 1$, $M^2/R_e \gg 1$ and $R_m \ll 1$. The analysis suggests that if the duct walls are conducting then for given pressure gradient the flow changes by a factor $\epsilon_A \sim a(\partial A/\partial z)/A$, but that if the duct walls are non-conducting then even if $\epsilon_A \ll 1$, the changes in the normalized velocity may be of the order of unity. Experiments by Branover et al[12] on a step change of cross-section in a non-conducting duct show some quite surprising but theoretically predictable effects, so that qualitatively at least these theories are reasonable reliable.

The non-uniformity of magnetic field and duct area are the two main limitations on the applicability of the theory to fusion reactor conditions. By analogy with conventional circuits it might have been thought that the effect of bends or junctions in the pipe upstream would be felt over much of the staight section of the pipe but this is not the case because, since $M^2/R_e \gg 1$, these effects all disappear within one or two pipe diameters. Also the actual pressure loss in the bend is only of the order of the pressure loss over one or two pipe diameters in a straight section.

Another limitation which could be serious is the effect of induced magnetic fields if induced electric currents pass from one duct to another. We have assumed that the duct is electrically isolated from any neighbouring ducts. If this is not so, for example if cooling ducts were to be placed in a bath of lithium, then further calculations would be needed.

Note that where the applied magnetic field has components perpendicular and parallel to the duct axis, if the flow is laminar the component parallel to the duct can be ignored. If the magnetic field is exactly parallel, then upstream or downstream influences could become important.

3.3 Experimental Verification of Laminar Duct Flow Theories

There have been many experiments in the last five years to test the MHD laminar flow theories used in Section 3.1. They have been reviewed by Hunt and Shercliff[3], and Branover and Tsinober[4]. For most types of duct there have now been measurements of the pressure gradient as a function of the Hartmann number and also velocity profile measurements.

For the flow through circular ducts Gardner and Lykoudis[1] among others have verified equation (3.6). Ihara et al[5] measured the pressure gradient in ducts made of non-conducting (glass) and two weakly conducting materials (stainless steel, $\phi = 0.034$; and carbon $\phi = 0.051$). The striking agreement between theory and experiment is shown in Figure 7. (The theoretical curve has been obtained by solving numerically the equations A.8, A.9; when $M \gg 1$ the numerical solutions agree with the asymptotic solution (3.5)). Branover and Tsinober[4] report similar measurements for a copper duct (ϕ = 12.5) with equally good agreement. Gnatyuk and Paramonova[13] have reported profile measurements which confirm the theoretical profile shown in Figure 3(a) very adequately.

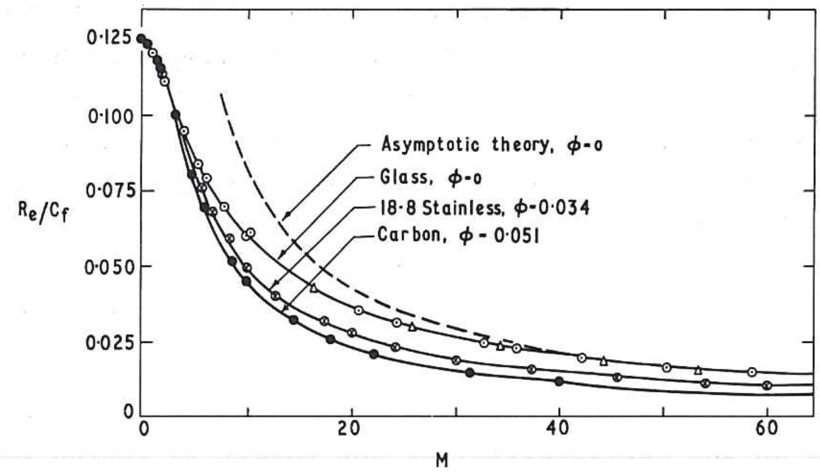

Fig.7 Theoretical and experimental results for the pressure drop in laminar flow along a circular duct in a transverse magnetic field. Ihara et al[5].

The theory for rectangular ducts has also been extensively checked. Branover et al[14] verified the theoretical results for the pressure gradient, equations (3.4) and (3.6), for flow in highly conducting and non-conducting ducts. Figures 8 and 9 show the velocity profile measurements in two types of duct, taken along the mid-plane (y = 0). The profiles show the remarkable effects of changing the conductivity of the ducts' walls and, more to the point for our purposes, show that our theoretical methods can satisfactorily predict these phenomena. The agreement is not exact because higher flow rates are required to measure the velocity than to measure the pressure gradient, so that the flows are partially turbulent.

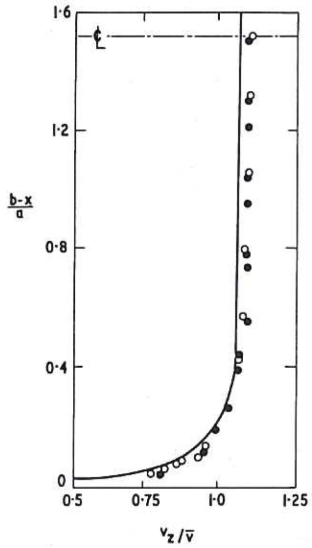

Fig.8 Theoretical and experimental velocity profiles for flow in a rectangular duct with non-conducting walls (b/a = 1.52) and transverse magnetic field. The points are experimental measurements by Branover et al[15] and the solid line is from calculations based on Shercliff[16]. Flow is probably just laminar.

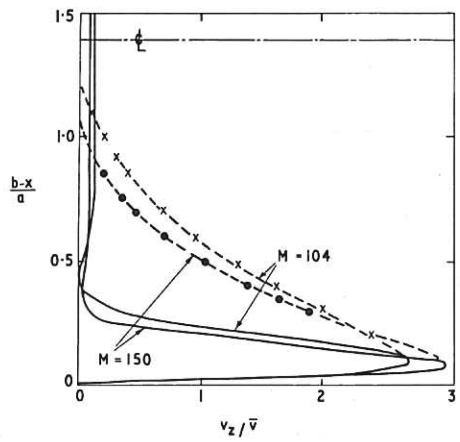

Fig.9 Theoretical and experimental velocity profiles for flow in a rectangular duct with transverse magnetic field. For the walls perpendicular to the magnetic field $\phi = \infty$, and for the walls parallel to the field ϕ = 18.5. The points are experimental measurements by Gelfgat et al[17]. Flow is turbulent in the boundary layers.

3.4 Turbulent Flow Experiments

For the case where the magnetic field is perpendicular to the duct, Branover[2] has reviewed all the existing experiments and found that for non-conducting ducts the critical Hartmann number, M_{crit} is given by

$$M_{crit} = R_e/k ,$$

where $130 \le k \le 215$ for all known cases. Subsequent investigations, such as those of Gardner and Lykoudis[1], gave results for k which lie within these limits. If the duct is conducting the experiments show a very much lower critical Hartmann number than for the equivalent non-conducting duct, but no general relationship has emerged. When the Hartmann number is less than the critical value, for a given value of Reynolds number the curves of pressure gradient against Hartmann number vary considerably between flows in different shapes of duct (compare, for example, the flows in rectangular and circular ducts examined by Brouillette and Lykoudis[18], and Gardner and Lykoudis[1]). However when the Hartmann number is small enough that $M^2/R_e < 1$, Branover and Tsinober[4] believe that all results can be expressed in the form

$$C_f/C_f\,(M = 0) = (1 + (\alpha\, M^2/R_e)),$$

where the coefficient α depends on the duct's shape and on the electrical conductivity and surface roughness of the walls.

When the magnetic field is parallel to the duct the experimental results are less well understood. For, as the Hartmann number increases for given Reynolds number the pressure gradient decreases until it reaches an asymptotic value. Unlike the case of a perpendicular field, this asymptotic value of pressure gradient is not equal to that of the equivalent laminar flow but is higher. In their experiments Levin and Chinenkov[19] found that, with $R_e = 48{,}000$, the pressure gradient reached its asymptotic value when $M = 1300$ and remained constant up to the maximum value of $M = 2{,}300$. The pressure gradient in this case was about a fifth of its turbulent value and three times as great as the equivalent laminar value. When $M^2/R_e < 1$ the experiments for circular pipes can be correlated to give the formula, quoted by Branover and Tsinober[4],

$$C_f/C_f(M = 0) = (1 - (37.7\, M^{1.65}/R_e^{1.45})).$$

For the purposes of predicting pressure drops in fusion reactor circuits where the Hartmann number is very large, the uncertainty in interpretation of the experimental results means that we can do little better than estimate the pressure gradient to lie between the turbulent and laminar values; closer to the latter than the former.

4. PUMPING POWER IN A REACTOR

In order to give a feel for the magnitude of the pressure gradient and pumping power in a reactor, the equations derived in Section 3 will be applied to the reactor design of Carruthers, Davenport and Mitchell[20]. A more detailed consideration of the problem will be left to Part II of this report.

The main parameters of the reactor are -

Power output	5000 MW(th)
Magnetic field	10 tesla
Plasma radius	1.25 metres
Inner radius of nuclear blanket	1.75 metres
Outer radius of nuclear blanket	3.00 metres
Outer radius of magnetic winding	3.50 metres
Major radius of torus	5.60 metres
Power entering nuclear blanket	13 MW/m^2

A possible scheme for cooling the nuclear blanket is indicated in Figure 10. The torus is divided into N sections along its major circumference. In each section liquid lithium is pumped into the blanket across the magnetic field through several feeder tubes, passes along ducts in the blanket parallel to the magnetic field and leaves by a second series of feeder tubes. The longitudinal ducts would, in practice, be distributed through the full radial depth of the blanket but, since most of the neutron and radiation energy is deposited near the inside surface of the blanket, it will be assumed for simplicity that all the ducts are at this inside surface. Assuming that the space required for the magnetic windings and the need for good neutron shielding limit the surface area available for the tubes to 10% of the total surface area of the blanket, and that the lithium temperature rise in passing through the blanket can be 300°C, we may derive the following parameters -

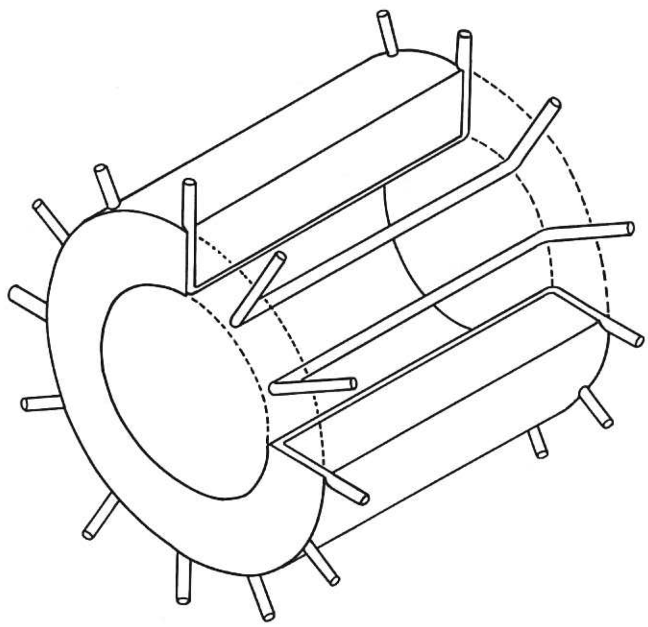

Fig.10 Scheme for cooling the nuclear blanket of a toroidal fusion reactor.

Heat removed in each section	5000/N	MW
Length of each section	35/N	metres
Area of feeder tubes	19.3/N	m^2
Volume flow of lithium	8.4/N	m^3/sec
Mass flow of lithium	4000/N	kg/sec
Velocity of lithium flow	0.43	m/sec

The physical properties of the materials concerned, in the temperature range 300 to 600°C, are taken to be

Density of lithium	475	kg/m^3
Specific heat of lithium	4200	joules/kg°C
Viscosity of lithium	4.5×10^{-4}	kg/m.sec
Electrical conductivity of lithium	2.2×10^{6}	mho/m
Electrical conductivity of duct walls	5.0×10^{6}	mho/m

It is further assumed that the feeder tubes have a radius of 0.05 m and a ratio of wall thickness to radius of 0.05, so that the non-dimensional parameters which determine the flow conditions are -

Hartmann number	M	35000
Reynolds number	R_e	28000
Magnetic Reynolds number	R_m	0.07
Conductance ratio	ϕ	0.1

These numbers, as expected, satisfy the conditions for fully developed laminar flow.

From the equations of section 3 we can now obtain

Pressure gradient in feeder tubes	9.6	MN/m^3
Total pressure drop	27	MN/m^2 (4000 psi)
Pumping power per section	225/N	MW
Total pumping power	225	MW
Ratio of pumping power to reactor thermal output	4.5%	

Thus we find that the pumping power is of the order of 10% of the electrical power output of the reactor, which is unacceptably high. It is interesting to note that the pressure drop in the absence of the magnetic field would be six orders of magnitude smaller.

5. CONCLUSIONS

The calculation of the pressure gradient due to the movement of a conducting fluid across a magnetic field has been discussed in detail for those conditions relevant to a toroidal fusion reactor. It has been shown, both theoretically and by comparison with published experimental results, that this pressure gradient can be calculated with an acceptable accuracy in uniform conditions, and that the influence of non-uniformities can be estimated.

The initial application of these equations to an outline reactor design suggests that the pumping power required to remove the heat from the nuclear blanket by circulating liquid lithium is unacceptably high. This question and the problem of stresses in the duct walls will be considered in more detail in Part II of this report.

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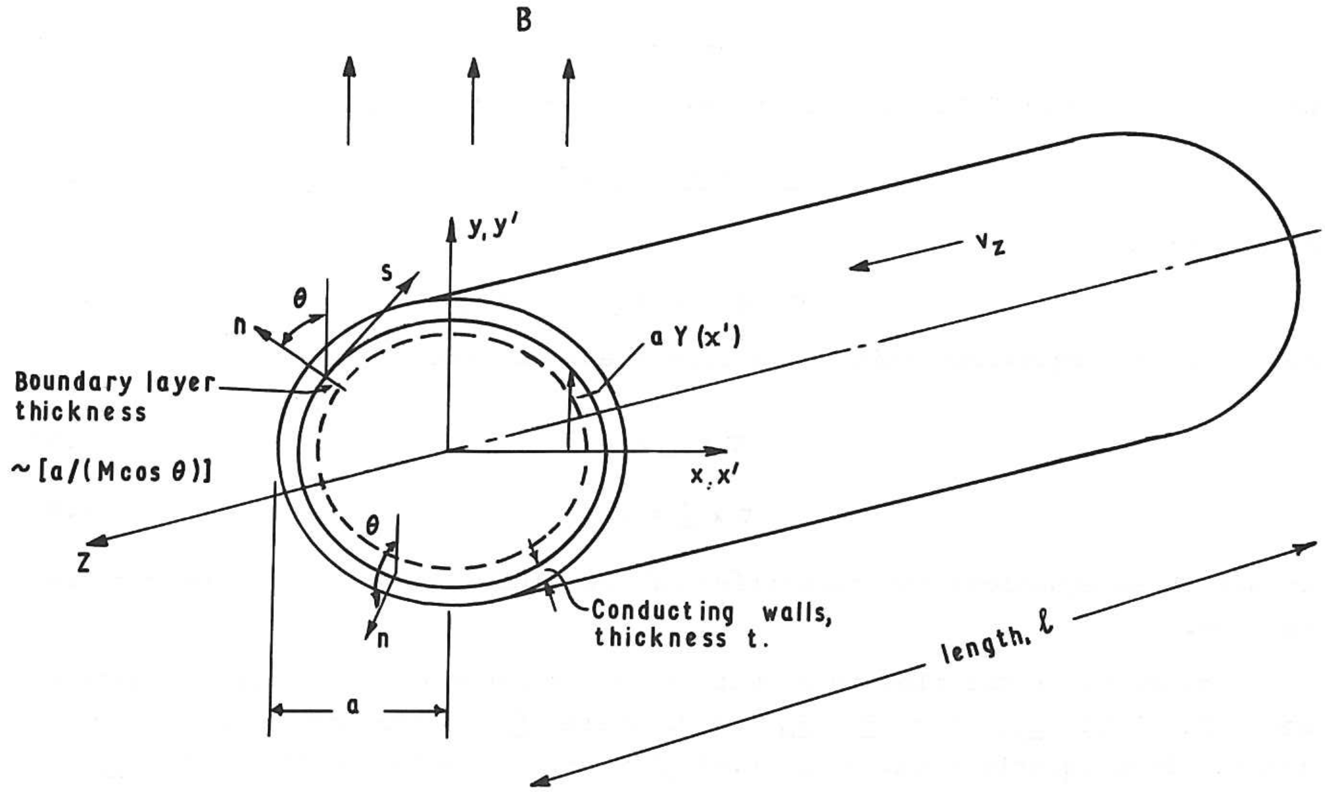

Fig.11 Cross-section of a typical duct, with notation used in Appendix I.

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APPENDIX I

MATHEMATICAL DERIVATION OF MAGNETO-HYDRODYNAMIC DUCT FLOW FORMULAE

The basic equations for incompressible, steady MHD flows are[21] the Navier Stokes equations governing the momentum of the fluid

$$\rho(\underline{v} \cdot \nabla)\underline{v} = - \nabla p + \underline{J} \times \underline{B} + \eta \nabla^2 \underline{v} \;, \qquad (A.1)$$

the conservation of mass or continuity equation

$$\nabla \cdot \underline{v} = 0 \;, \qquad (A.2)$$

the combined "Ohm's" law and electromagnetic induction equation

$$\underline{J} = \sigma(\underline{E} + \underline{v} \times \underline{B}) \;, \qquad (A.3)$$

Faraday's law

$$\nabla \quad \underline{E} = - \partial \underline{B}/\partial t = 0 \;, \qquad (A.4)$$

and Maxwell's equations (with no displacement current)

$$\nabla \cdot \underline{B} = 0 \;, \qquad (A.5)$$

$$\nabla \times \underline{B} = \mu \underline{J} \,. \qquad (A.6)$$

In all these equations the properties of the fluid ρ, η, σ are assumed to be uniform.

Now consider the flow in a duct in the presence of a uniform transverse magnetic field $\underline{B}_o$. Let $\underline{B} = \underline{B}_o + \mu \underline{H}$, where $\underline{H}$ is the induced magnetic field. Then equations (A.3) to (A.6) give the following equation for $\underline{H}$,

$$u[(\underline{v} \cdot \nabla)\underline{H} - (\underline{H} \cdot \nabla)\underline{v}] = \frac{1}{\sigma} \nabla^2 \underline{H} + (\underline{B}_o \cdot \nabla)\underline{v} \;,$$

whence, if $R_m = \bar{v} a \mu \sigma \ll 1$,

$$(\underline{B}_o \cdot \nabla)\underline{v} + \frac{1}{\sigma} \nabla^2 \underline{H} = 0 \,. \qquad (A.7)$$

Therefore $|\mu \underline{H}| \ll \underline{B}_o$, a result we assumed in Section 3.1. In the absence of a magnetic field a basic assumption of the theory of laminar duct flow is that, if the duct is long enough, eventually the flow reaches a steady state, i.e. $\partial \underline{v}/\partial z = 0$. It can then be proved that $v_x = v_y = 0$, so that the flow has only one component v_z. Given a similar assumption about the flow eventually becoming fully developed, it can again be proved that no secondary flow can exist[22], so that $\underline{v} = (0, 0, v_z)$. Then equations (A.1), (A.6) and (A.7), when normalised in terms of $(- \partial p/\partial z)$, σ, a, and η become:

$$- 1 = M \, \partial h/\partial y' + \nabla^2 v \;, \qquad (A.8)$$

$$0 = M \, \partial v/\partial y' + \nabla^2 h \;, \qquad (A.9)$$

where

$$v = \frac{v_z}{(a^2/\eta)(-\partial p/\partial z)} \qquad h = \frac{H_z}{a^2(\sigma/\eta)^{\frac{1}{2}}(-\partial p/\partial z)} \quad ,$$

$$\nabla^2 = \frac{\partial^2}{(\partial x')^2} + \frac{\partial^2}{(\partial y')^2} \quad ,$$

$$x' = x/a \qquad y' = y/a \quad .$$

We shall need to consider the distribution of current in conducting walls, where these exist. In the walls, it follows from equations (A.3) to (A.6) that

$$\underline{J} = \sigma_w \underline{E} \qquad \text{where} \qquad \nabla \times \underline{E} = 0 \tag{A.10}$$

and

$$\underline{J} = \nabla \times \underline{H} \qquad \text{where} \qquad \nabla \cdot \underline{H} = 0 \quad . \tag{A.11}$$

Hence we find

$$\nabla^2 H = 0 \ . \tag{A.12}$$

The boundary conditions on the fluid-wall interface are

$$E_{s_f} = E_{s_w} \qquad J_{n_f} = J_{n_w} \qquad v = 0 \ , \tag{A.13}$$

and on the outer face of the duct walls $(n = t)$

$$J_{n_w} = 0 \ , \tag{A.14}$$

where the local co-ordinates s, n are shown in Figure 11. If the thickness of the wall $t \ll a$, it follows that (A.12) becomes

$$\partial^2 h/\partial n^2 = 0 \quad .$$

Thence equations (A.13) and (A.14) lead to the following conditions on v and h in the fluid at $n = 0$,

$$\partial h/\partial n + h/\phi = 0 \tag{A.15a}$$

$$v = 0 \ . \tag{A.15b}$$

(Since h is constant at $n = t$, we take this constant to be zero).

Thus our mathematical problem has been reduced to finding the solution to equations (A.8) and (A.9), subject to the boundary conditions (A.15). Since, in the situation we need to consider, $M \gg 1$, it transpires that solutions are possible for a general shape of duct (other than a rectangle) with the perimeter given by

$$y' = \pm Y(x') \ .$$

The solution that follows was derived by Chang and Lundgren [9].

The first essential step in the analysis is to recognise that when $M \gg 1$ the flow can be divided into two regions, (i) 'core' region occupying most of the duct away from the walls, where gradients are small, so that (A.8), (A.9) become

$$-1 = M\partial h/\partial y' \qquad M\partial v/\partial y' = 0 \quad , \tag{A.16}$$

(ii) the boundary layer region on the walls where gradients normal to the wall are large, so that in terms of the co-ordinate normal to the wall, n, (A.8), (A.9) become

$$M'\partial h/\partial n + \partial^2 v/\partial n^2 = -1 \qquad M'\partial v/\partial n + \partial^2 h/\partial n^2 = 0 \tag{A.17}$$

where $M = M\cos\theta$. Note $M' < 0$ on the lower wall.

The second essential step is to consider the combined variables $Z_1 = (v + h)$ and $Z_2 = (v - h)$. It is useful to introduce the value of h on the upper wall, $h_w(x')$, which is, at this stage an unknown function. Then by symmetry it follows that $h = -h_w(x')$ on the lower wall. By inspection of (A.17) it is easily seen that Z_1 can only have a boundary layer structure on $y' = -Y(x')$ and Z_2 have one on $y' = Y(x')$. Then it follows that the solutions to (A.16) and (A.17), subject to (A.15b) are: in the core

$$\left.\begin{aligned} Z_1 &= Y(x') - y')/M + h_w(x') \\ Z_2 &= (y' + Y(x'))/M + h_w(x') , \end{aligned}\right\} \tag{A.18}$$

in the boundary layers at $y' = Y(x')$

$$\begin{aligned} Z_1 &= h_w(x') - n\cos\theta/M \\ Z_2 &= 2Y(x')/M + h_w(x') + n\cos\theta/M - 2[Y(x')/M + h_w(x')]e^{(M'n)} \end{aligned} \tag{A.19}$$

and at $y' = -Y(x')$

$$\begin{aligned} Z_2 &= h_w(x') - n\cos\theta/M, \\ Z_1 &= 2Y(x')/M + h_w(x') + n\cos\theta/M - 2[Y(x')/M + h_w(x')]e^{(M'n)} . \end{aligned} \tag{A.20}$$

Substituting (A.19) into (A.15a) we find that to satisfy the boundary condition on h, h_w must satisfy

$$-\cos\theta/M + (Y(x')/M + h_w(x'))\, M\cos\theta + h_w/\phi = 0$$

whence

$$h_w = \frac{\phi\cos\theta\,.[1 - MY(x')]}{M(1 + M\phi\cos\theta)} . \tag{A.21}$$

Therefore, in the core,

$$v = \frac{Y(x') + \phi/\sqrt{1 + Y'(x')^2}}{M\left[1 + M\phi/\sqrt{1 + Y'(x')^2}\right]} . \tag{A.22}$$

If $\phi \ll 1$ and $M\phi \gg 1$, then reverting to physical variables the core velocity is

$$v_z = \frac{(-\partial p/\partial z)\sqrt{Y(x') \; 1 + Y'(x')^2}}{\sigma \, B_o^2 \, \phi} \quad . \tag{A.23}$$

If $\phi = 0$, then

$$v_z = \frac{(-\partial p/\partial z) \, a^2 \, Y(x')}{\eta \, M} \quad . \tag{A.24}$$

Note that the asymptotic method used for the limit $M \to \infty$ in this MHD duct flow analysis and other analyses has been rigorously justified by Eckhaus and de Jager[(23)].

It is interesting to investigate the possibility of maximizing v_z by making $Y(x') = \pm\,(1 - |x'|)/(b/a)$ then the relevant parameter which demonstrates the effect of the duct's shape, is

$$v = \frac{Q}{A} = \frac{-\partial p/\partial z}{\sigma \, B_o^2 \, \phi} \left(\tfrac{2}{3}\right)(b/a)(1 + b^2/a^2)^{\frac{1}{2}} \tag{A.25}$$

so that if $b/a = 2$ the mean velocity is increased by a factor of 3. If, instead of a lozenge, an elliptical cross-section is used with $b/a = 2$ the increase would only be a factor 2.

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