



UKAEA RESEARCH GROUP

Report

SPARK-GENERATED STEAM BUBBLES

(Experiment and Theory)

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Abstract

Sparks in water were produced by discharging a charged condenser through a spark gap using an ignitron circuit. Steam bubbles with a maximum radius $\simeq 1$ cm were obtained. A fast cine-camera was used to observe the bubble and a pressure transducer was used to measure pressure variations in the water. A simple computer program similar to that described by Judd^{(1), (2)} has been developed to calculate the bubble dynamics.

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Notation

C	-	Specific heat of water ($\text{J kg}^{-1} \text{K}^{-1}$)
C_p	-	Specific heat of steam at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)
E_T	-	Energy delivered by spark (J)
E_P	-	Potential energy of bubble against external pressure (J)
E_K	-	Kinetic energy in water surrounding bubble (J)
E_{\max}	-	Maximum value of E_T (J)
G	-	Gas constant for steam ($\text{J kg}^{-1} \text{K}^{-1}$)
J	-	Mass flux of condensing or evaporating steam ($\text{kg m}^{-2} \text{s}^{-1}$)
K	-	Thermal conductivity of water ($\text{W m}^{-1} \text{K}^{-1}$)
L	-	Latent heat of evaporation of water (J/kg)
M	-	Mass of steam evaporated (kg)
p	-	Pressure of steam in bubble (N/m^2)
p_e	-	External (atmospheric) pressure (N/m^2)
p_v	-	Vapour pressure of steam (N/m^2)
Q	-	Heat content in heated bubble wall (J)
r	-	Radial distance from bubble centre (m)
R	-	Bubble radius (m)
R_{\max}	-	Maximum bubble radius (m)
t	-	Time (s)
t_B	-	Bounce time for bubble (s)
t_c	-	Collapse time for bubble (s)
T	-	Temperature (K)
T_W	-	Initial water temperature (K)
T_s	-	Temperature at bubble wall surface (K)
T_g	-	Temperature of steam in bubble (K)
T_c	-	Temperature for thermal equilibrium for a given pressure (K)

1. Introduction

Vapour bubble growth and collapse within the sub-assembly of a sodium-cooled fast reactor has been treated, using a computer model by Brook (3). Nucleation was initiated within the superheated coolant which may occur locally in the proximity of a local blockage of the coolant subchannels, and the dynamics of the bubble were calculated as it expanded and collapsed across subcooled fuel pins. The pressure in the bubble was determined by means of a mass balance, and it was shown that the pressure tends towards the level at which evaporation and condensation are matched. A range of condensation coefficients was considered, as there are grounds for believing that even small amounts of non-condensables can seriously reduce this parameter. Although the interphase mass transfer coefficients are known for pure vapour, the effect of non-condensables has not been measured under these transient conditions in the presence of sweeping vapour flow across the fuel pins, and it is difficult to assess. A further practical complication is the break-up of the spherical bubble surface as the bubble collapses.

In an attempt to form an understanding of some of these effects, experiments have been made with spark-produced bubbles in water.

This bubble was photographed using a high-speed cine-camera and a pressure transducer was used to measure the time variation of pressure in the water. A computer program has been developed to explain these measurements following the method described by Judd⁽¹⁾⁽²⁾. Unfortunately this calculation can only be used for certain limited bubble conditions, which do not apply strictly to the conditions of the experiment. It is hoped that the method of calculation recently described by Loader⁽⁴⁾ will enable calculations to be made which apply strictly to the experimental conditions. It has been found to be experimentally possible to introduce a significant amount of incondensable gas into the bubble by using water saturated with carbon dioxide, but no systematic measurements have been made. The experiments made so far have been exploratory, and a number of unexpected effects have been noticed which it is proposed to explain in this report.

2. Description of the Experiment

The underwater spark was produced in a gap ≈ 0.5 mm between two sharply-pointed zirconium welding rods for electrodes. Condensers totalling $16\mu\text{F}$ and charged to 3.0 kV were switched onto this gap using an ignitron (Bk396) circuit. The bubble was photographed using a 'Hycam' 16 mm high speed motion picture camera. This camera used a rotating prism and continuously moving film and was used at a

3. Assumptions Made in the Calculation of Bubble Dynamics

The approximations and assumptions made in the computer calculation are as follows:

(1) The approximation first made by Judd⁽¹⁾, (2) was used. Write:

T_W - Initial temperature of water (K),

T_S - Temperature of water at bubble surface (K),

T - Temperature at a point radius r from the bubble centre (K),

K - Thermal conductivity of water ($W m^{-1} K^{-1}$),

C - Specific heat of water ($J kg^{-1} K^{-1}$),

ρ - Density of water (kg/m^3),

R - Bubble radius (m).

The heat conduction equation is not solved in the region near the bubble surface, instead the temperature is assumed to be given by the equation

$$T - T_W = (T_S - T_W) \frac{R}{r} \exp \left\{ - \frac{(r - R)}{\lambda} \right\},$$

where λ is a time-varying skin depth for the heated region of the bubble surface. The following quantities can be expressed as functions of λ .

Heat flux at the bubble surface

$$\phi = K \left[\frac{\partial T}{\partial r} \right]_{r=R}$$

$$\phi = -K (T_S - T_W) \left(\frac{1}{\lambda} + \frac{1}{R} \right) \quad (1)$$

(W/m^2)

Total heat content of the heated bubble surface

$$Q = \rho C \int_R^\infty (T - T_W) 4 \pi r^2 dr$$

$$Q = 4 \pi \rho C (T_S - T_W) \lambda R (R + \lambda) \quad (2)$$

(2) The electrical power delivered to the spark (w), which heats the water, vaporises the water and then heats the steam, is supplied over the time interval $0 < t < \tau$ and is of the form

$$w = w_{\max} \sin^2 \left(\frac{\pi t}{\tau} \right) \quad (3)$$

where τ has been taken as $10 \mu s$.

(5) The mass flux of steam j ($\text{kg m}^{-2} \text{s}^{-1}$) condensing at the bubble wall is given by the formula referred to by Collier (5),

$$j = \left(\frac{2\sigma}{2 - \sigma} \right) \frac{1}{(2\pi G)^{\frac{1}{2}}} \left\{ \frac{p}{T_g^{\frac{1}{2}}} - \frac{p_v}{T_s^{\frac{1}{2}}} \right\} \quad (9)$$

where σ is a constant, which for steam is 0.04.

4. Method of Computation of Bubble Dynamics

Writing:

p_e - External (atmospheric) pressure (N/m^2),

The equation of bubble motion first derived by Rayleigh is

$$R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 = \frac{p - p_e}{\rho} \quad (10)$$

The solution of equation (10) may be regarded as the simultaneous solution of the equations

$$\frac{dR}{dt} = Z \quad (11)$$

$$\frac{dZ}{dt} = \frac{p - p_e}{\rho R} - \frac{3}{2} \frac{1}{R} Z^2 \quad (12)$$

For the bubble there are also the equations

$$\frac{dQ}{dt} = 4\pi R^2 \phi \quad (13)$$

$$\frac{dM}{dt} = -4\pi R^2 j \quad (14)$$

The equations (11), (12), (13) and (14) were solved using the second order Runge-Kutta method as described by Lance (6). Taking a series of time intervals Δt_n such that

$$t_{n+1} = t_n + \Delta t_n,$$

and writing R_n for $\left[R \right]_{t=t_n}$. First a set of approximate values R^1 , Z^1 , Q^1 and M^1 for the quantities R_{n+1} , Z_{n+1} , Q_{n+1} and M_{n+1} were obtained from the equations:

$$R^1 = R_n + Z_n \Delta t_n,$$

From the conservation of energy,

$$E_T = Q + E_P + E_K + MU .$$

$$\therefore U = (E_T - Q - E_P - E_K) / M \quad (15)$$

Inverting equation (8) of the previous section to give ΔT in terms of U

$$\Delta T = 3.24675 \cdot 10^{-4} \left\{ - (4.94707 \cdot 10^6 - U) + \sqrt{(16.4575 \cdot 10^{12} - 6.23387 \cdot 10^6 U + U^2)} \right\} \quad (16)$$

From equations (4), (6) and (7)

$$T_g = \Delta T + \frac{4889.98}{\log (2.08477 \cdot 10^{11} \frac{R^3}{GMT_g})} \quad (17)$$

With all the quantities known but T_g , equation (17) was solved by a method of trial and error to give T_g .

Write:

L - Latent heat of evaporation of steam (J/kg)

C_p - Specific heat at constant pressure of steam (J kg⁻¹ K⁻¹)

then

$$\phi = J \left\{ L + C_p (T_g - T_s) \right\} .$$

Using this equation together with equations (1), (5) and (9)

$$\frac{K(T_s - T_w) \left(\frac{1}{R} + \frac{1}{\lambda} \right)}{L + C_p (T_g - T_s)} - 0.0835102 \frac{MT_g^{\frac{1}{2}}}{R^3} + 3.77252 \cdot 10^7 \frac{\exp \left(- \frac{4889.98}{T_s} \right)}{T_s^{\frac{1}{2}}} = 0 \quad (18)$$

Inverting equation (2)

$$\lambda = \frac{1}{2} \left[\sqrt{ \left\{ R^2 + \frac{Q}{\pi C(T_s - T_w) R} \right\} - R } \right] \quad (19)$$

Equations (15), (16), and (17) were solved to give successive values of U , ΔT and T_g . A method of trial and error was then used to solve equations (18) and

in the term E_p , the bubble potential energy, between the first and second maximum radii should be equal to the increase in Q the thermal energy in the bubble wall. Numerical calculation shows that the observed decrease in E_p , which is about a factor of 8, is much larger than can be accounted for by an increase in Q . Some energy was radiated as a sound wave when the bubble was small, and an estimate of this could be made from the cavitation produced by the sound wave shown in Fig 5. Work on underwater explosions⁽⁸⁾ suggests that about 30% of the maximum bubble potential energy is radiated as a sound wave at the bubble minimum radius. Thus, to account for the considerable reduction in the potential energy of the bubble on passing through the first minimum radius, some additional mode of energy dissipation must exist. This could be the turbulence produced when a bubble which is not a perfect sphere contracts to a small volume.

In the numerical computation the initial values of the bubble radius R_0 , the bubble radial velocity $\left[\frac{dR}{dt} \right]$, the mass of steam evaporated M_0 and the skin

depth of the heated bubble surface λ_0 were assumed. The total electrical energy to be delivered E_{\max} was also assumed. The step by step calculation described in section 4 was then carried out.

According to Kling⁽⁹⁾ electrical breakdown is preceded by local heating of the electrolyte and the formation of vapour. Kling gives a value of 20,000 K for the steam in the arc on breakdown. This value is consistent with the measurements of Lockte-Holtgreven.⁽¹⁰⁾ By starting the calculation at time t_0 which was a fraction of the time duration of the heating pulse τ it was arranged that the initial steam temperature was low. By adjusting the value of M_0 the required value of 20 000 K could be obtained for the maximum steam temperature. Although E_{\max} might have been measured calorimetrically an assumed value was used for this calculation.

The main limitation of this calculation was introduced by the approximation (1) of section 3, which was first used by Judd. When the bubble surface temperature T_s is greater than the water temperature T_w equation (19) gives a value for λ , but as $T_s \rightarrow T_w$, $\lambda \rightarrow \infty$. Further if T_s falls slightly below T_w there is no real value of λ . Thus the calculation must be restricted to cases where T_s is always greater than T_w . The condition T_s greater than T_w implies from equation (1) that there is a net flow of heat to the bubble surface, and hence that there is a net condensation of steam at the bubble surface. In order to ensure that the condition $T_s > T_w$ holds for the entire calculation it is necessary to assume an artificially low water temperature T_w and an artificially high value for the initial mass of steam M_0 .

TABLE 1

Internal Energy of Steam (MJ/kg)

Pressure (N/m ²)	Temperature (°C)							
	100.0	200.0	300.0	400.0	500.0	600.0	700.0	800.0
1.0x10 ⁵	2.19 2.51	2.61 2.66	2.96 2.81	3.55 2.97	3.80 3.13	4.04 3.30	4.04 3.48	4.27 3.66
1.0x10 ⁶		2.26 2.62	2.69 2.79	3.03 2.96	3.33 3.12	3.60 3.30	3.85 3.48	4.09 3.66
1.0x10 ⁷				2.60 2.83	2.96 3.05	3.27 3.24	3.54 3.43	3.80 3.63
1.0x10 ⁸						2.55 2.59	2.91 2.97	3.23 3.28

Value calculated from approximate formula/Value from steam tables

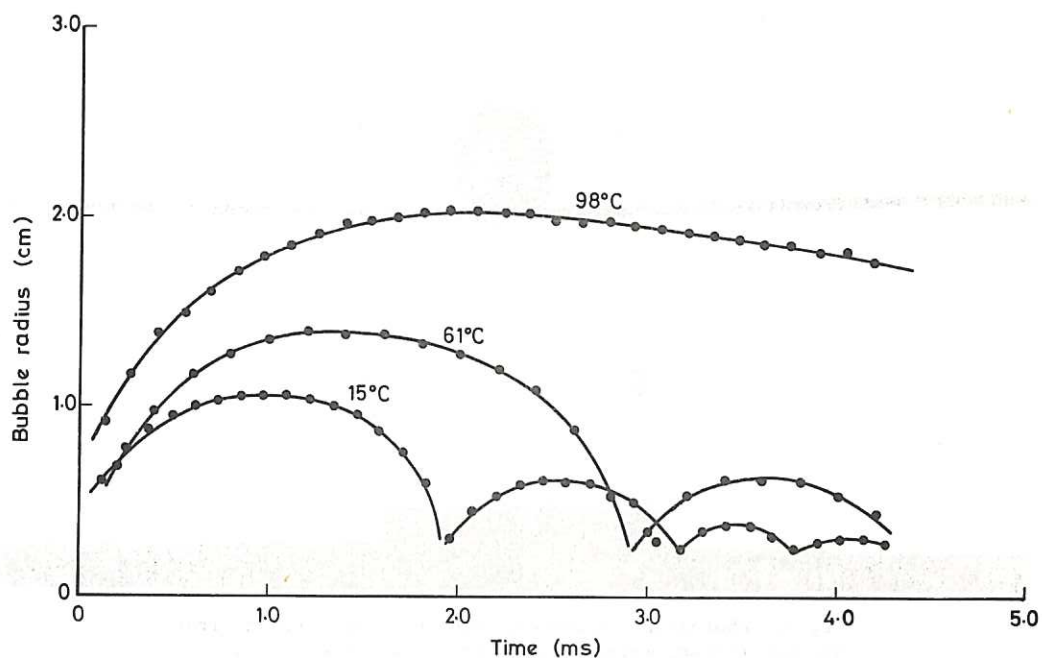


Fig 1. Measured bubble radius plotted against time for water temperatures of 15°C, 61°C and 98°C.

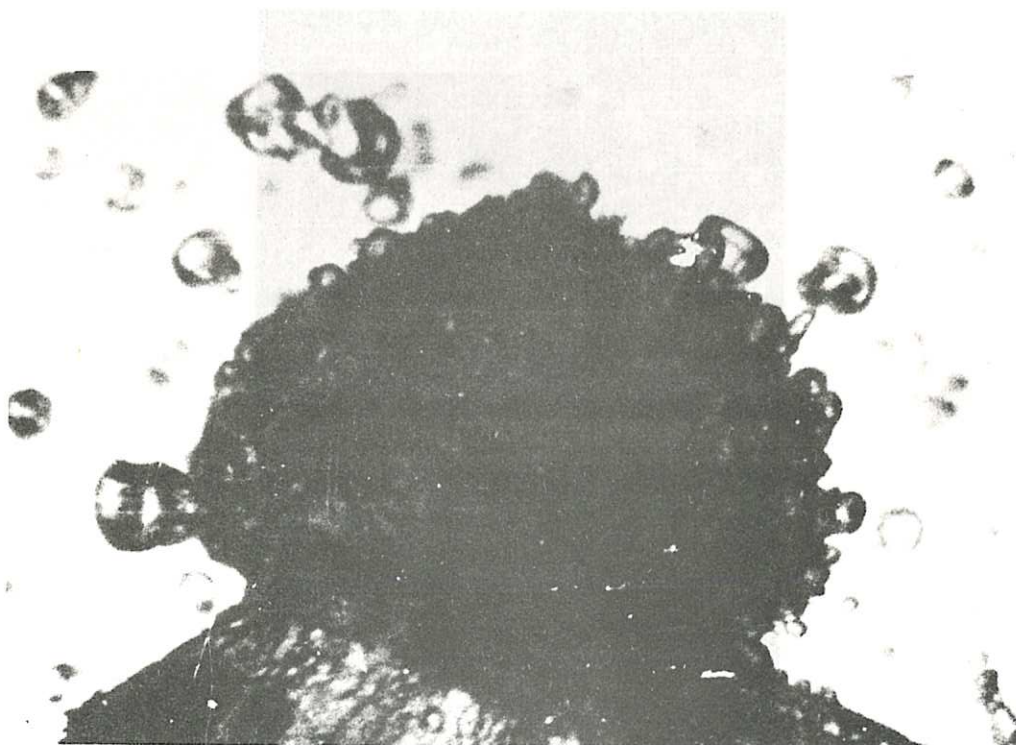


Fig 2. Photograph of collapsed bubble, water temperature 98°C, time 6.7 ms.

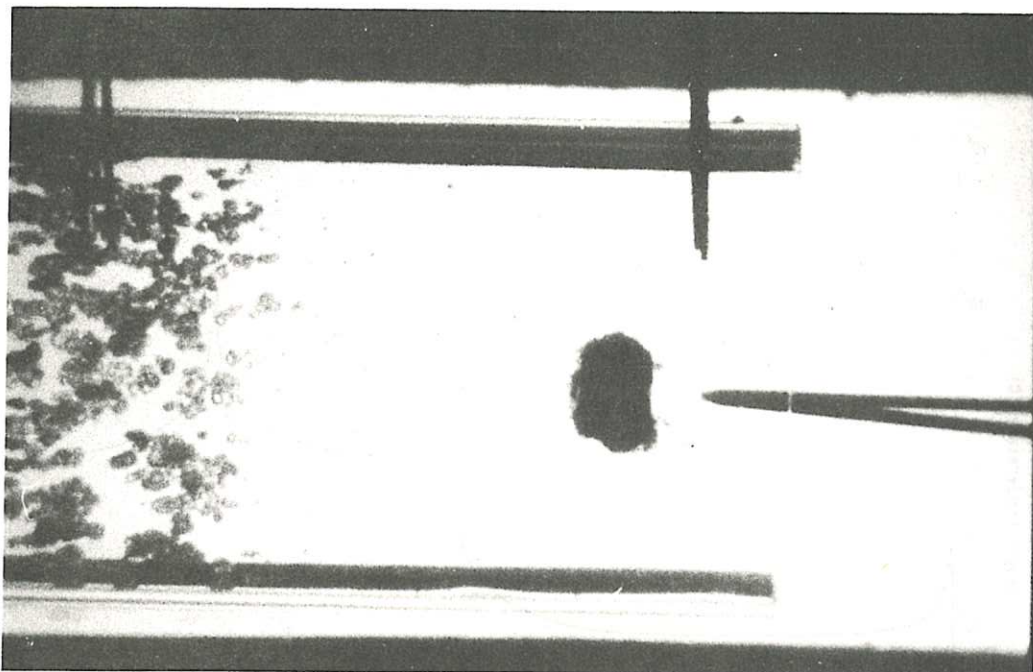


Fig 5. Photograph of the cavitation produced at the end of a tube 4 cm dia with the bubble expanding after the first minimum radius.

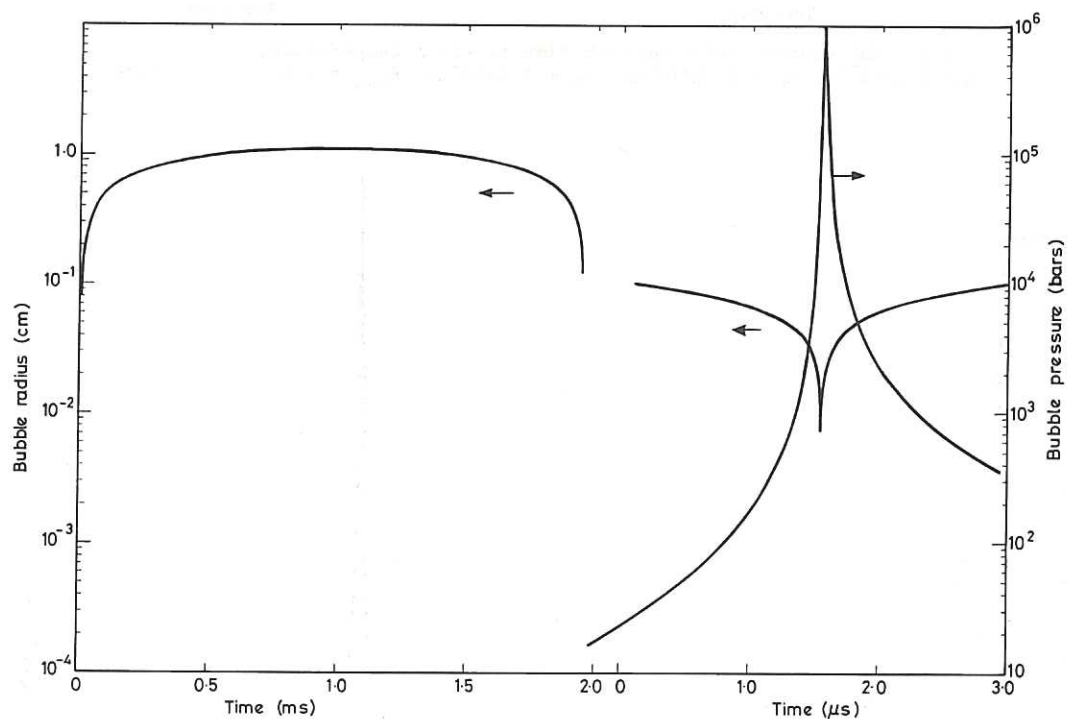


Fig 6. Calculated variation with time of the bubble radius and bubble pressure, $R_0 = 8.0 \times 10^{-4} \text{ m}$, $M_0 = 8.0 \times 10^{-8} \text{ kg}$, $\lambda_0 = 1.0 \times 10^{-5} \text{ m}$, $E_{\text{max}} = 1.0 \text{ J}$, $T_w = -10^\circ \text{C}$.

