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Report

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PULSED FIELD LOSS CALCULATION
FOR A CRYOGENICALLY STABILISED COMPOSITE CONDUCTOR

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ABSTRACT

Pulsed field losses in a possible conductor for a large Tokamak experiment are calculated. The cryogenically stabilised conductor carries 5000 A d.c. in a 7.5 T background field. Hysteresis and eddy current loss are calculated for a 0.5 T pulse oriented a) parallel to the conductor length and b) perpendicular to the conductor narrow edge. The dependence of pulsed field losses on the following parameters is discussed: a) pulse amplitude, b) background field, c) rise and decay time constant of the pulsed field, and d) superconducting filament diameter.

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CONTENTS

1. INTRODUCTION

The majority of conceptual design studies of fusion reactors(1) based upon present day technological capabilities utilise superconducting coils and incorporate a cyclic sequence of ignition, burn period and shutdown. The superconducting coils which provide the containment fields will consequently be subjected to pulsed fields of similar duty cycle. In a reactor the poloidal field due to the plasma current plus the azimuthal component of a stabilising field will be approximately parallel to the toroidal field windings. The radial component of the stabilising and divertor field will be perpendicular to these windings.

This report investigates pulsed field losses in a cryogenically stable conductor(*) carrying 5000A d.c. in a 7.5 T background field, for two possible orientations of the pulsed field; these are a) parallel to conductor length, and b) perpendicular to the narrow face of the conductor. The pulsed field has an amplitude of .5T, rising and decaying with a 1s time constant.

(*) The choice and design of a cryogenically stable conductor for Tokamak toroidal fields is discussed in a recent paper.(2)

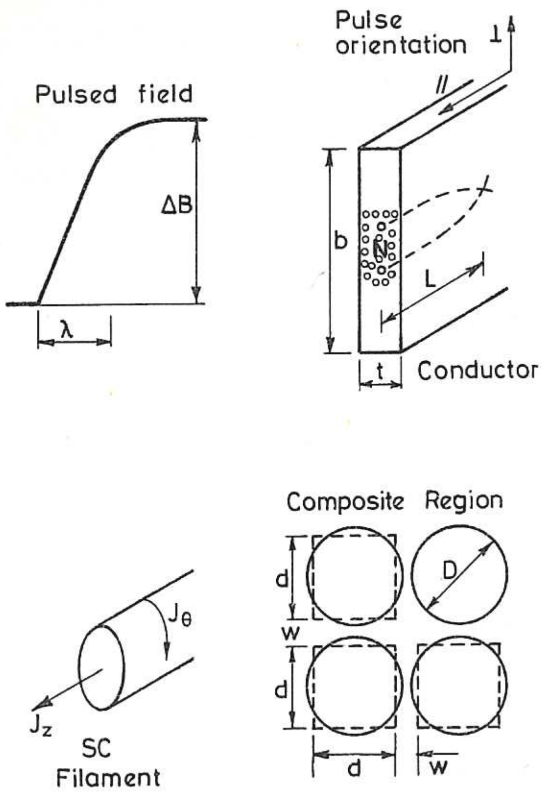
The conductor consists of 297 Nb-Ti filaments, 0.2mm diameter, embedded in a copper matrix 3mm thick and 40 mm wide (Fig. 1). The filaments are grouped in a central region approximately 8mm wide, which will be referred to as the composite region. The copper-to-superconductor ratio in the composite region is 1.5, while for the whole conductor it is 12. During the manufacturing stage a twist will be introduced in the composite region to reduce filament coupling.(3) The minimum practical twist pitch is approximately 5 times the composite region equivalent diameter, where

$$D_{eq} = \left[\frac{4}{\pi} \text{x composite region cross-section area}\right]^{\frac{1}{2}} \quad (1)$$

$\therefore$ twist pitch $\approx 27 \times 10^{-3}$m.

2. FILAMENT COUPLING

Type-II superconductors exhibit hysteresis losses when subjected to changing fields or currents(4) because flux lines are caused to move against the pinning forces in the superconductor.



B	7.5 T
I_t	5000 A
ΔB	0.5 T
λ	1 s
b	40×10^{-3}m
t	3×10^{-3}m
4L	27×10^{-3}m
D	$.2 \times 10^{-3}$m
N	297 filaments
Overall Cu : SC	12:1
Comp. region Cu : SC	1.5 : 1
d	$.18 \times 10^{-3}$m
w	$.1 \times 10^{-3}$m
J_z(7.5T,4.2K)	9.5×10^{8} A.m^{-2}
J_θ(0.5T,4.2K)	5.0×10^{9} A.m^{-2}
ρ(7.5T,4.2K)	4.4×10^{-10}Ω.m
$\bar{\rho}$(7.5T,4.2K)	16.0×10^{-10}Ω.m

Fig. 1. Pulsed field losses calculated for a cryogenically stabilised conductor with an overall copper to superconductor ratio of 12 : 1, transport current I_t = 5000 A, background field B = 7.5 T, pulse amplitude ΔB = .5T, rise/decay time constant of pulsed field λ = 1s.

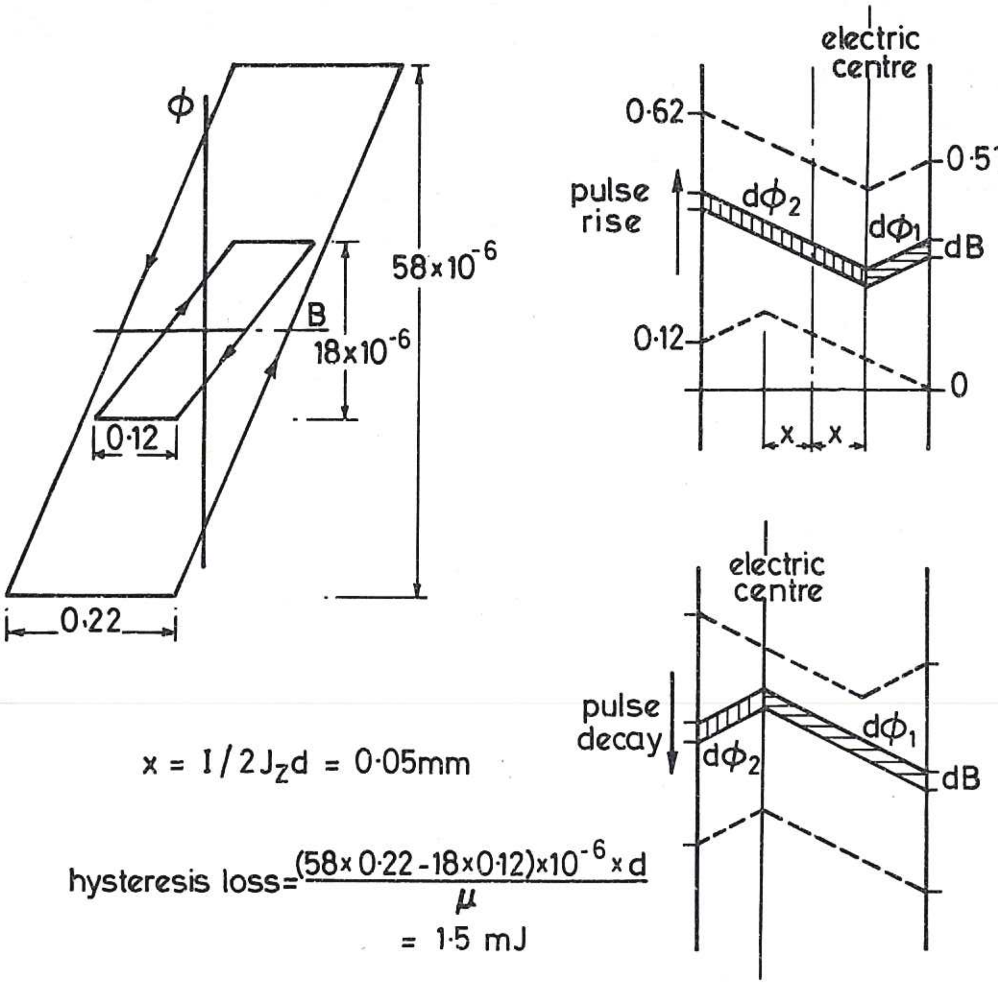


Fig. 2. Magnetic field through the cross-section of a superconducting filament carrying a d.c. transport current, and the corresponding hysteresis loops.

4.2 Eddy current loss

The eddy current loss is given by an expression identical to equation (13) in which τ_1 is now calculated for a diffusion process across a quarter twist pitch.

$\therefore \quad \tau_1 = .04\ s,\ V_1 = 13.86 \times 10^{-6} m^3$

τ_2 and V_2 are the same as thosed used to evaluate the result of equation (13).

$\therefore \quad W_{e\perp} = .08\ J.m^{-1}$ per field rise or decay

$W_{e\perp} = .16\ J.m^{-1}$ per pulse.

5. SUMMARY OF CALCULATED LOSSES

Pulsed field orientation	Hysteresis loss. $J.m^{-1}$	Eddy current loss $J.m^{-1}$
	0·2	0·056
	0·72	0·16

6. VARIATION OF LOSSES WITH PULSED FIELD AMPLITUDE IN THE RANGE .1 $<\Delta B<$ 1T (FIG.3)

Eddy current loss varies as $(\Delta B)^2$ for both field orientations.

Hysteresis loss in the parallel field, as given by equation (9), varies approximately as $(\Delta B)^3$. This approximate cube dependence persists until the penetration profile reaches the filament centre, i.e. equation (9) is valid in the range:

$$0<\Delta B< \mu J_\theta D = 1.26\ T \qquad (18)$$

In fact the hysteresis loss deviates from the $(\Delta B)^3$ relationship because a) as ΔB decreases from 1 to .1T, the term $(1-\Delta B/2\mu J_\theta D)$ in equation (9) increases from 0.6 to 0.98, and b) J_θ increases as ΔB decreases[6] reaching a value $10 \times 10^9 A.m^{-2}$ at $\Delta B \simeq .1T$.

In the perpendicular case equation (16) is valid for complete flux penetration, i.e. in the range:

$$\Delta B > \mu J_z\left(\frac{d}{d+w}\right) d \simeq .14\ T \qquad (19)$$

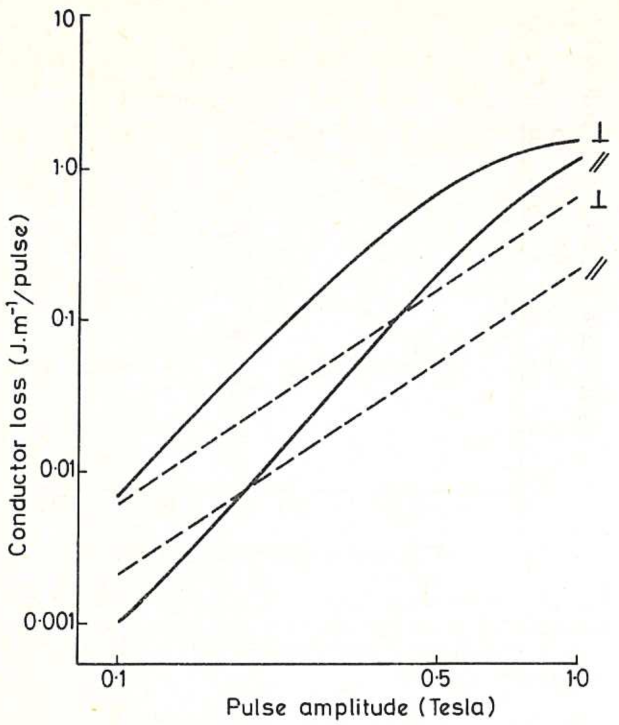


Fig. 3. Pulsed field losses in the conductor as a function of pulse amplitude in a background field of 7.5 T and a pulse rise/decay time constant of 1 s.
(----) eddy current loss, (——) hysteresis loss.

In this range the hysteresis loss will vary at a rate greater than that predicted by a "$Q_{\perp} \propto \Delta B$" relationship because a) the term $(1-2\mu J_z d/3\Delta B)$ decreases as ΔB decreases, and b) filament coupling decreases linearly with ΔB as can be seen from equation (4). For example:

at $\Delta B = 1T$ $\quad Q_{\perp} \simeq 2.25 Q_{o\perp}$

and at $\Delta B = .1T$ $\quad Q_{\perp} \simeq 1.26 Q_{o\perp}$

In the range $\Delta B < .14$ T the loss varies approximately as $(\Delta B)^3$, this is described in the following section, equation (23).

7. VARIATION OF LOSSES WITH BACKGROUND FIELD IN THE RANGE 0<B<7.5T (FIG. 4)

Eddy current loss in the resistive matrix increases as the background field decreases. This can be seen by substituting the expression for τ from equation (11) into equation (10):

$$W_e = \frac{(\Delta B)^2 V t^2}{2\lambda\pi^2\rho}$$

i.e. $W_e \propto \rho^{-1}(B)$ (20)

The magnetoresistance exhibited by copper is such that there will be approximately a five fold decrease in ρ as B decreases from 7.5T to zero.

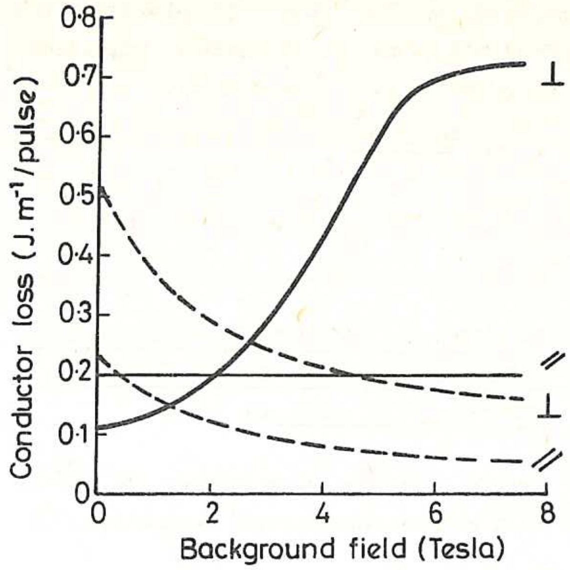


Fig. 4. Pulsed field losses in the conductor as a function of the background field for a .5 T pulse with a rise/decay time constant of 1 s.
(----) eddy current loss, (——) hysteresis loss.

The effect of varying the background field on hysteresis loss will depend mainly on the variation of the critical current density.

For the parallel pulsed field case the equilibrium of Lorentz forces and pinning forces gives the following equation:[10]

$$J_\theta B_z - J_z B_\theta = \text{constant} \qquad (21)$$

where B_z is the axial field and would represent the pulsed field, and B_θ is the field created by a transport current, i.e. the background field. Assuming $J_z B_\theta$ is a constant, then according to equation (21) J_θ is unaffected by the variation of the background field; the loss as expressed by equation (9) is therefore also unaffected. $v_{/\!/}$ in equation (4) increases to .00005 at B=0 due to the decrease of $\bar{\rho}$ but the conclusion that $Q_{/\!/} \simeq Q_{o/\!/}$ is still valid. The hysteresis loss in the parallel pulsed field case is, to a first approximation, unaffected by the variation of the background field.

In the perpendicular case the effect of reducing the background field from 7.5T to zero is to increase J_z from 9.5×10^8 to about 9.0×10^9 A.m^{-2}. As a result $v_{\perp}$ decreases from .16 to .05 and f_{dc} from 1.5 to 1.1. As stated previously equation (16) is valid for the case of complete penetration, i.e. in the range:

$$J_z < \frac{\Delta B}{\mu(\frac{d}{d+w})d} = 3.4 \times 10^9 \text{A.m}^{-2}$$

or B> 4 T for Nb-Ti (22)

As the background field is reduced below this limit flux penetration into the filaments is incomplete and the hysteresis loss[9] is given by:

$$W_{h\perp} = \frac{4d(\Delta B/2)^3}{3\mu^2 J_z} \qquad (23)$$

which indicates that $Q_\perp$ varies approximately as J_z^{-1} or B.

8. VARIATION OF LOSSES WITH THE RISE AND DECAY TIME CONSTANTS OF THE PULSED FIELD IN THE RANGE $1<\lambda<10$ s. (FIG. 5)

Eddy current loss is proportional to λ^{-1} for both field orientations as given by equation (13).

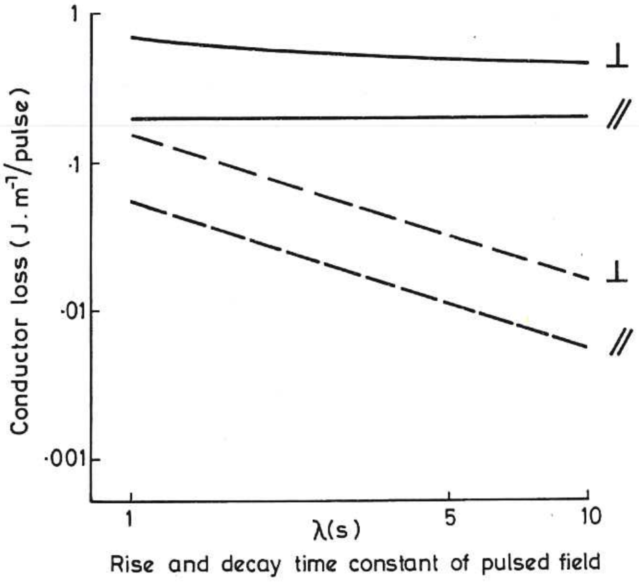


Fig. 5. Pulsed field losses in the conductor as a function of the pulse rise/decay time constant for a .5 T pulse in a background field of 7.5 T. (----) eddy current loss, (——) hysteresis loss.

A decrease in the rate of rise or decay of the pulsed field reduces the extent of filament coupling, equation (4). For the parallel field case the filaments are virtually uncoupled at λ = 1 s, equation (7a), and hence the conclusion that $Q / Q_0 \simeq 1$ is also valid for values of $\lambda>$ 1 s. $v_\perp$ on the other hand decreases from .16 at λ = 1 s to .016 at λ = 10s. As a result Q / Q_0 in the perpendicular field case is reduced from 1.63, equation (7b), to 1.06.

9. VARIATION OF HYSTERESIS LOSS WITH FILAMENT DIAMETER IN THE RANGE .02<D< .2mm (FIG.6)

For the parallel pulsed field case complete penetration occurs for a diameter less than $D_{/\!/} = \Delta B/\mu J_\theta$ = .08mm. In the range .08<D< .2mm, the loss per filament is given by equation (9).

$$\therefore \quad W_{h/\!/} \propto D$$

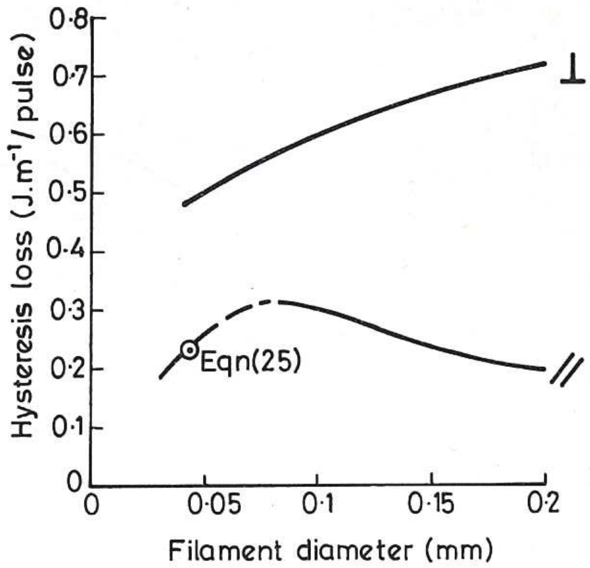


Fig. 6. Hysteresis loss in the conductor as a function of filament diameter for a .5 T pulse with a rise/decay time constant of 1 s, in a 7.5 T background field.

The overall number of filaments varies approximately as D^{-2} for a fixed transport current,

$$\therefore \quad Q_{/\!/} \propto D^{-1} \qquad (24)$$

The actual rate of change of $Q_{/\!/}$ will be less than that predicted by equation (24) because of the $(1-\Delta B/2\mu J_\theta D)$ term in equation (9). An expression for the hysteresis loss in the range D<.08mm can be obtained, in principle, through an identical procedure to that used in the appendix. The formulation is lengthy and has been attempted only at $D = \Delta B/2\mu J_\theta$ = .04mm. The loss in this particular case is:

$$W_{h/\!/} = \frac{(\Delta B/2)^4}{8\mu^3 J_\theta^{\,2}} = .031 \text{ mJ.m}^{-1}$$

The number of filaments $= 297(\frac{.2}{.04})^2 = 7425$

$$\therefore \quad Q_{/\!/} \,(D = .04\text{mm}) \simeq .23 \text{ J.m}^{-1} \qquad (25)$$

For the perpendicular pulsed field case complete penetration already occurs for D = .2mm, as shown by equation (15). The loss per filament as given by equation (16) shows that:

$$W_{h\perp} \propto d^3 \text{ or equally } W_{h\perp} \propto D^3$$

but the total number of filaments varies as D^{-2}

$$\therefore \quad Q_{o\perp} \propto D \qquad (26)$$

The actual rate of decrease of hysteresis loss with filament diameter will be less than that anticipated from equation (26) because a) the $(1-2\mu J_z d/3\Delta B)$ term increases as d decreases, and b) filament coupling increases as d decreases, equation (4).

10. ACKNOWLEDGEMENT

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12. APPENDIX

Hysteresis loss in a cylindrical filament due to a small axial external pulsed field.

The loss can be calculated by evaluating the Poynting's vector for a complete cycle.[9]

$$W = \frac{1}{\mu} \int_{\text{cycle}} \tilde{E} \times \tilde{B} \, dt \qquad (1)$$

since $E = \frac{d\phi}{dt}$, where ϕ is the magnetic flux in the filament, and since $\tilde{E}$ and $\tilde{B}$ are perpendicular

$$\therefore \quad W = \frac{1}{\mu} \int B d\phi \qquad (2)$$

which is the area of the magnetisation loop.

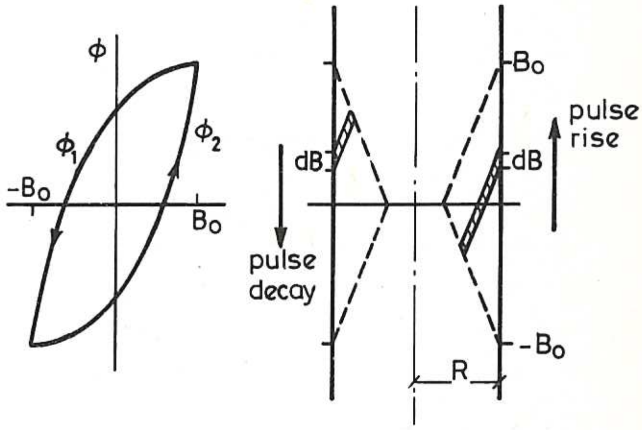

Fig. A1. Axial magnetic field through a cylindrical filament and the resulting hysteresis loop.

The expression for ϕ can be easily calculated with the aid of Fig. A1, where the 'critical-state' model[11] is used to calculate the flux penetration pattern.

$$\phi_1 = \phi_o - \frac{\pi(B_o-B)^2}{2\mu J_\theta} \left[R - \frac{B_o-B}{6\mu J_\theta} \right] \qquad (3)$$

$$\phi_2 = -\phi_o + \frac{\pi(B_o+B)^2}{2\mu J_\theta} \left[R - \frac{B_o+B}{6\mu J_\theta} \right] \qquad (4)$$

$$\phi_o = \frac{\pi B_o^2}{\mu J_\theta} \left[R - \frac{B_o}{3\mu J_\theta} \right] \qquad (5)$$

substituting in equation (2):

$$W = \frac{4\pi R B_o^3}{3\mu^2 J_\theta} \left[1 - \frac{B_o}{2\mu J_\theta R} \right] \qquad (6)$$

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