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Report

DIELECTRONIC RECOMBINATION IN LASER GENERATED PLASMAS

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DIELECTRONIC RECOMBINATION IN LASER GENERATED PLASMAS

by

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ABSTRACT

Dielectronic recombination coefficients have been computed for hydrogenic ions from He II to Fe XVI over a range of conditions typical of laser generated plasma. The results are displayed in a set of graphs together with the corresponding collisional radiative recombination coefficients. A comparison of these results indicates plasma conditions where dielectronic recombination is a significant process.

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Contents

1. INTRODUCTION

Calculations of dielectronic recombination coefficients have already been presented in the literature (for example Burgess 1964, Ansari et al 1970). The previous treatments are relevant to plasma electron densities less than 10^{16} cm^{-3}. Burgess and Summers (1969) have pointed out that densities greater than this figure introduce significant effects which must therefore be implicit in any treatment.

The aim of this paper is to investigate the role of dielectronic recombination in laser-generated plasma, where electron densities range from ~ 10^{17} cm^{-3} to 10^{23} cm^{-3}, and electron temperatures range from ~ 10 eV to 10 keV. Burgess (1965) has mentioned that the main comtributions to the dielectronic recombination coefficients come from quite a wide range of the principal quantum number of the recombining electron, and so the total dielectronic recombination coefficient, in high density laser-generated plasma, will be reduced due to the reduction of the ionisation limit. The following sections consider the relevance of the various processes occurring at electron densities between 10^{11} cm^{-3} and 10^{21} cm^{-3}, which can affect the recombination coefficient.

Although it is not possible to define a uniquely accurate value for the dielectronic recombination coefficient under these conditions, to a first approximation, working values of dielectronic recombination coefficients have been computed. Hydrogen-like ions have been considered, for ease of computation and to gain insight into the physical processes involved.

2. THEORY

2.1. Dielectronic Recombination

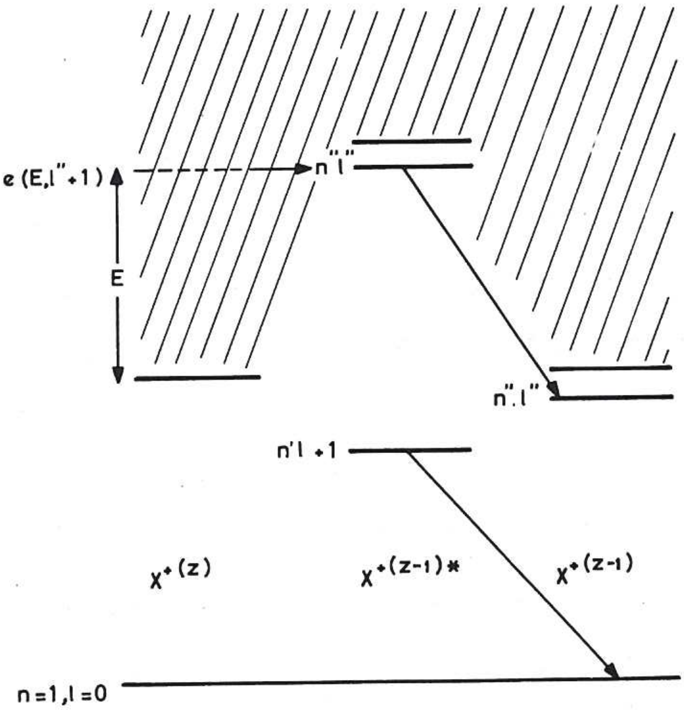

Fig. 1 Term Diagram for Dielectronic Recombination.

As can be seen from the term diagram in figure 1, dielectronic recombination is the consequence of inverse auto-ionisation (resonance capture), immediately followed by a stabilising decay, as detailed in the following equations; where an ion of species X, charge +z and valence electron state $n\ell$ recombines with an incident electron of energy E and angular momentum $\ell'' + 1$ resulting in an ion of charge (z - 1) and valence electron state $n''\ell''$:

$$X^{+(z)}(n\ \ell) + e^{-}\ (E\ell'' + 1) \rightarrow X^{+(z-1)}(n'\ell + 1;\ n''\ell'') \qquad \ldots(1a)$$

$$X^{+(z-1)}\ (n'\ell + 1;\ n''\ \ell'') \rightarrow X^{+(z-1)}(n\ \ell;\ n''\ell'') + h\nu. \qquad \ldots(1b)$$

The dielectronic recombination coefficient, α_d, for incident electrons of energies $E \rightarrow E + dE$, is expressed by the following formula, Burgess (1964):

$$\alpha_d = \frac{A(n'\ell+1;n''\ell'' \rightarrow n\ell;\ n''\ell'')C_1(kT_e)^{-\frac{3}{2}}\ \exp(-E/kT_e)EQ(n\ell;E\ell''+1 \rightarrow n'\ell+1;n''\ell'')dE}{A(n'\ell+1;n''\ell'' \rightarrow n\ell;n''\ell'') + C_2\omega(n\ell)EQ(n\ell;E\ell''+1 \rightarrow n'\ell+1;n''\ell'')dE/\omega(n'\ell+1;n''\ell'')} \quad ..(2)$$

where $C_1 = 4\ (2\pi m_e)^{-\frac{1}{2}}$, $C_2 = 16\pi m_e h^{-3}$, $A\ (j \rightarrow i)$ is the Einstein spontaneous decay rate for level j to level i, (process 1b), kT_e is the electron temperature, $Q(i \rightarrow j)$ is the collisional excitation partial cross-section from level i to level j, (process 1a), dE is the width of the electron energy spectrum, and $\omega(i)$ is the statistical weight of level i. The total dielectronic recombination coefficient is given by:

$$\alpha_d(\text{tot}) = \sum_{n'n''\ell''} \alpha_d\ (n'\ell+1;n''\ell''). \quad ..(3)$$

For a detailed derivation of this formula see Appendix A.

2.2. Supression of Dielectronic Recombination by Ionisation of Upper Quantum Levels

Collisional ionisation processes, as described by the following equations, suppress dielectronic recombination involving the higher n' and n'' levels

$$X^{+(z-1)}(n'\ell + 1;\ n''\ell'') + e^- \rightarrow X^{+(z)}(n'\ell + 1) + e^- + e^- \quad ..(4a)$$

$$X^{+(z-1)}(n'\ell + 1;\ n''\ell'') + e^- \rightarrow X^{+(z)}(n''\ \ell'') + e^- + e^-. \quad ..(4b)$$

The maximum principal quantum levels which remain appreciably populated are denoted by n'_t and n''_t. Lower lying energy levels can be strongly ionised when collisional excitation occurs as an intermediate process, so the level n_t, the thermal limit, is defined such that excitation followed by ionisation of the levels above n_t is faster than their radiative decay (see, for example, Jordan (1969)):

$$n_t = 1.26 \times 10^2 Z_n^{14/17}\ N_e^{-2/17} \left[\frac{kT_e}{\chi_i}\right]^{1/17} \exp\left[\frac{4\ \chi_i}{17 n_t^3\ kT_e}\right] \quad ..(5)$$

where N_e is the electron density in cm^{-3}, Z_n is the nuclear charge seen by the electron, χ_i is the ionisation potential of the electron from the ion ground state, and kT_e is the electron temperature.

Other processes suppressing recombination to the upper quantum levels are merging of these levels due to the Inglis Teller effect and photoionisation (see Appendix B). Both these were found to be less significant than the thermal limit process.

Formula (5) was derived on the assumption of hydrogenic levels so it is applicable to the $X^{+(z)}$ ion, and for $n'' > 3$ it was found to be applicable to the $X^{+(z-1)}$ ion. Thus, using (5), n_t' was calculated for $X^{+(z)}$ and n_t'' for $X^{+(z-1)}$. These were taken as the maximum principal quantum levels contributing to the dielectronic recombination coefficient.

2.3. Effects of High Electron and Radiation Densities on the Stabilisation Rate

At low densities stabilisation is effected by spontaneous decay, but at the high densities which are typical in laser-generated plasma, stabilisation can be enhanced by collisional de-excitation, while reversal of stabilisation by collisional excitation remains negligible.

$$X^{+(z-1)}(n'\ell + 1;\ n''\ell'') + e^- \rightarrow X^{+(z-1)}(n\ell;\ n''\ell'') + e^- \qquad .. \ (6a)$$

$$X^{+(z-1)}(n'\ell + 1;\ n''\ell'') + e^- \rightarrow X^{+(z-1)}(n'\ell + 1;\ n\ell) + e^- . \qquad .. \ (6b)$$

Stabilisation by stimulated emission was found to be negligible, but re-absorption of radiation can reverse the stabilisation process, so this effect was included by means of the escape factor $g(\tau_o)$ see, for example, McWhirter (1965) and Holstein (1947). The optical depth τ_o was calculated assuming a typical scale length of 50 microns for plasma generated by 1.06 μm radiation, Donaldson et al (1973), and 400 microns for plasma generated by 10.6 μm radiation, Donaldson and Spalding (1976). The fractional abundances of ions in the plasma used in the calculations of $g(\tau_o)$ and stimulated emission rates, were taken from House (1964). The dielectronic recombination coefficients obtained, compared to within 10% with those calculated using abundances taken from Jordan (1969). The high density effects mentioned above are included in equation (2) by replacing:

$$A(j \rightarrow i) \text{ with } A(j \rightarrow i)g(\tau_o) + \alpha_{de-ex} \qquad .. (7)$$

α_{de-ex} is the collisional de-excitation rate for ions, see for example Van Regemorter (1962):

$$\alpha_{de-ex} = 20.60\ \lambda^3\ N_e(kT_e)^{-\frac{1}{2}}\ A(j \rightarrow i)P(\lambda,kT_e) \qquad .. (8)$$

where N_e is the electron density in cm^{-3}, λ is the wavelength of the transition $j \rightarrow i$ in cm, kT_e is the electron temperature in eV and the parameter $P(\lambda,kT_e) \sim 1.0$ is tabulated by Van Regemorter (1962).

Electron collisions can also induce statistical redistribution between degenerate $n''\ell''$ states, favouring the states of highest angular momenta, so that auto-ionisation, the inverse of reaction (1), is reduced, thus enhancing dielectronic recombination. For the plasma conditions considered $N_e > 10^{16}$ cm^{-3}, the results of Pengelly and Seaton (1964) indicate that a complete redistribution of ℓ'' states will occur, for $n'' > 4$. States with $n'' < 4$ were not considered sufficiently degenerate to undergo any statistical redistribution of ℓ'' states and as will be apparent from section (3), even a complete statistical redistribution would not appreciably affect the dielectronic recombination rates to levels where $n'' < 4$.

2.4. Computational Details

For the calculations of dielectronic recombination of a hydrogen-like ion, the resulting helium-like ion was visualised by a two un-coupled electron model. This can be justified by the quantum mechanical treatment Shore and Menzel (1968), Trefftz (1969). It is applicable for $n'' > n'$ and thus does not introduce a significant error in most cases considered, because the main contributions to the dielectronic recombination coefficient arise from $n' = 2$ and $6 \geq n'' \geq 3$. Inaccuracies only arise for very high electron densities, $N_e > 10^{20}$ cm^{-3}, for which $n''_t < 3$, so that coupling of the electrons is still strong. However, at these densities α_d is too small to be of significance. In this model $A(j \rightarrow i)$ is independent of the captured electron ie:

$$A(n'\ell + 1;\ n''\ell'' \rightarrow n\ell;\ n''\ell'') = A(n'\ell + 1 \rightarrow n\ell). \qquad ..\ (9)$$

Recombination involving decay of the core electron to the ground state is the only significant contribution to the value of $\alpha_d(tot)$, so that

$$A(n'\ell + 1 \rightarrow n\ell) = A(n'p \rightarrow 1s)\ . \qquad ..\ (10)$$

The incident electron kinetic energies, $E \rightarrow E + dE$ are specified by the initial and intermediate bound state energies, $E_{n\ell}$, and $E_{n'\ell+1;\ n''\ell''}$ i.e.

$$E = E_{n'\ell+1} - E_{n\ell} - \frac{Z^2}{(n'')^2} \qquad ..\ (11)$$

and

$$dE = \frac{2Z^2}{(n'')^3}\ . \qquad ..\ (12)$$

The spontaneous decay rates were obtained from tables of Wiese et al (1966).

The collisional excitation partial cross-section in equation (2) was evaluated for threshold excitation of the $1s \rightarrow n'p$ transition using:

$$Q(n\ell;E\ell''+1 \rightarrow n'\ell+1;n''\ell'') = \frac{8\pi}{\sqrt{3}}\frac{1}{E}\frac{f(n'\ell+1 \rightarrow n\ell)g_{\ell''+1}(E_{th})\,\pi a_o^2}{(E_{n'\ell+1} - E_{n\ell})} \quad ..(13)$$

where E is the incident electron kinetic energy in Rydbergs, $E_{n'\ell+1}$ is the intermediate bound state energy, $E_{n\ell}$ is the initial bound state energy, a_o is the first Bohr radius, $f(i \rightarrow j)$ is the emission oscillator strength for the transition $i \rightarrow j$ and $g_{\ell''+1}\,(E_{th})$ is the partial free-bound Gaunt factor evaluated for an incident electron with angular momentum $\ell''+1$ and threshold energy $E_{th} = E_{n'\ell+1} - E_{n\ell}$.

For low electron density $g_{\ell''+1}\,(E_{th})$ was calculated using the Coulomb-Born approximation, Burgess et al (1969). For high electron densities, as mentioned in section 2.3, complete redistribution of ℓ'' states was assumed, for $n'' > 4$, so that it was necessary to replace $g_{\ell''+1}\,(E_{th})$ by:

$$g_{\ell''+1}\,(E_{th}) = \frac{(2\ell''+1)}{(n'')^2}\sum_{\ell''=0}^{n''} g_{\ell''+1}\,(E_{th}). \quad ..(14)$$

This assumption considerably simplifies the calculations.

A FORTRAN routine was written to calculate the values of di-electronic recombination coefficients of hydrogen-like ions from He II to Fe XXVI at electron densities ranging from 10^{16} cm^{-3} to 10^{21} cm^{-3}, and at the low density limit, 10^{11} cm^{-3}.

For the purposes of computation some simplifications are possible when the first term in the denominator of equation (2) is greater than the second ie:

$$\frac{(Z+1)^6}{Z^2} > \frac{9.4 \times 10^6}{(n'')^3}\,\frac{g_{\ell''+1}(E_{th})}{(2\ell''+1)}\,. \quad ..(15)$$

When condition (15), is satisfied, Donaldson and Peacock (1975), equation (2) reduces to a simplified form.

$$\alpha_d = C_1(kT_e)^{-\frac{3}{2}} \exp(-E/kT_e)\ EQ(n\ell;E\ell''+1 \rightarrow n'\ell+1;\ n''\ell'')dE. \quad ..(16)$$

Equation (3) reduces to:

$$\alpha_d(\text{tot}) = \sum_{n'n''} C_1(kT_e)^{-\frac{3}{2}} \exp(-E/kT_e)\ EQ(\text{tot})\ dE. \quad ..(17)$$

Q(tot) can be obtained from equation (13) by putting $g_{\ell''+1} = \sum_{\ell''+1} g_{\ell''+1} = 0.2$.

The sum over n'' can be avoided if E is taken as the energy of the transition $n\ell \rightarrow n'\ell+1$. E cancels with the value in Q(tot); the only other value of E appears in the exponential, and only a small error is produced in α_d.

For $Z \geqslant 17$ condition (15) is satisfied for all values of n'', and α_d can be easily evaluated using equations (13) and (16) to better than 50% accuracy, and to better than 15% accuracy for $Z > 25$. Note that α_d is then independent of a redistribution in ℓ'' values and independent of $g(\tau_0)$.

At low densities, for $Z \geqslant 17$, a large number of n'' terms must be summed, and thus equation (3) can be further simplified to:

$$\alpha_d(\text{tot}) = C_1(kT_e)^{-\frac{3}{2}}\ 2Z^2\ \xi(3) \sum_{n'} \exp(-E/kT_e)\ EQ(\text{tot}) \quad ..(18)$$

where $\xi(3)$ is a Reimann zeta function $= \sum_{n=1}^{\infty} 1/n^3 = 1.2021$. Usually not more than four values of n' need be summed. Formulae (17) and (18) should also be useful for non-hydrogenic ions.

For all values of Z the same formulae can be applied to sufficiently large values of n'' which satisfy condition (15). For example, for C VI at low densities, α_d can be calculated from n'' = 1 to n'' = 100 using equations (2) and (3) and from n'' = 100 to n'' = ∞ by use of equation (18), with an error in the answer for the worst case of better than 1% and the sum over n' not greater than n' = 8.

3. RESULTS AND CONCLUSIONS

The low density values obtained for He II were in excellent agreement with results calculated by Burgess (1964) using values of Q extrapolated below threshold. The values of C VI and N VII were in good agreement with those calculated by Ansari et al (1970), from the general formula for dielectronic recombination, Burgess (1965), but those for O VIIIwere a factor of 3 greater than Ansari's results.

The graphs in figures (2) show the results obtained for typical laser-generated plasma conditions of N_e and kT_e. The collisional-radiative recombination coefficients, α_{cr}, Bates et al (1961), are also plotted to illustrate the regime of dominance of dielectronic recombination. As can be seen it dominates at high temperatures where $kT_e > \chi_i$ and is in some cases a sufficiently important process to be included in calculations of equilibrium conditions.

It can be deduced from equation (18) that, as Z increases, α_d tends to an asymptotic decrease as α_d/Z^3 with kT_e scaling as Z^2kT_e, while α_{cr} increases as $Z\,\alpha_{cr}$ with kT_e scaling as Z^2kT_e, so that for large $Z(> 17)$, although the plasma processes tending to suppress dielectronic recombination are less significant, collisional radiative recombination dominates over dielectronic recombination.

The density and dimensions of the hot core of the laser-generated plasma considered range from 10^{21} cm^{-3}: 50 μm for Nd^{3+} laser radiation, to 10^{19} cm^{-3}: 400 μm for CO_2 laser radiation, so favourable conditions for dielectronic recombination are obtained for the range $8 < Z < 17$ in the low density plasma produced by CO_2 laser radiation at the longer wavelength.

Tables I(a) and (b) compare the magnitude of the most important plasma processes with the spontaneous emission rate, at densities of 10^{19} cm^{-3} and 10^{20} cm^{-3} for the 1s - 2p transition in CVI.

TABLE I(a)

COMPARISON OF PLASMA PROCESS FOR C VI 1s - 2p TRANSITION ($N_e = 10^{20}$ cm^{-3})

kT_e (eV)	Spontaneous Emission Rate	Collisional De-excitation Rate	Stimulated Emission Rate	Escape Factor $g(\tau_o)$ Scale Length 50 μm	n''_t
10	8.14 11	0.380 11	-	1.00	1
50	8.14 11	0.17 11	-	0.22	2
100	8.14 11	0.12 11	-	0.018	2
200	8.14 11	0.085 11	0.069 11	0.59	2
300	8.14 11	0.069 11	0.037 11	0.88	2
500	8.14 11	0.057 11	-	0.98	2
1000	8.14 11	0.059 11	-	1.00	2
2000	8.14 11	0.054 11	-	1.00	2

TABLE I(b)

COMPARISON OF PLASMA PROCESSES FOR C VI 1s - 2p TRANSITION ($N_e = 10^{19}$ cm^{-3})

kT_e (eV)	Spontaneous Emission Rate	Collisional De-excitation Rate	Stimulated Emission Rate	Escape Factor $g(\tau_o)$ Scale Length 50 μm	n''_t
10	8.14 11	0.038 11	-	1.00	2
50	8.14 11	0.017 11	-	0.84	3
100	8.14 11	0.012 11	-	0.32	3
200	8.14 11	-	-	0.95	3
300	8.14 11	-	-	0.99	3
500	8.14 11	-	-	1.00	3
1000	8.14 11	-	-	1.00	3
2000	8.14 11	-	-	1.00	3

It can be seen that when the density is sufficiently high for the collisional de-excitation rate to be significant, the thermal limit is so low that the dielectronic recombination coefficient is negligible, so that for $Z < 8$ the only effects which affect the accuracy significantly in an estimation of α_d are the reduction of the thermal limit for $N_e > 10^{11}$ cm^{-3} and a finite optical depth for $N_e > 10^{17}$ cm^{-3}. Table II shows the approximate reduction in α_d for C VI due to reduction of the thermal limit n_t'' in C V.

TABLE II

THE EFFECT OF THE REDUCTION IN THERMAL LIMIT n_t'' ON α_d

n_t''	α_2/α_1
2	0.08
3	0.22
4	0.40
5	0.55
6	0.65

α_1 is the low density value of α_d

α_2 is the value of α_d summed up to and inclusive of n_t''

As mentioned previously for $Z \geqslant 17$ equation (17) can be used, while for $Z < 17$ condition (15) is reversed for sufficiently low n_t' and n_t'', so that α_d reduces to the following form

$$\alpha_d = A(n'\ell+1;n''\ell'' \rightarrow n\ell;n''\ell'') \frac{C_1}{C_2}[\omega(n'\ell+1;\ n''\ell'')/\omega(n\ell)]\exp(-E/kT_e)\ (KT_e)^{-\frac{3}{2}}. \quad ..\ (19)$$

At the densities considered for $Z < 8$, n_t' and n_t'' are less than

~ 7 so that equation (19) applies. Thus α_d is not very sensitive to the value of Q. So at high electron densities and $Z < 8$ the approximation for Q is relatively unimportant, and even a complete redistribution of ℓ'' states has a small effect, as can be seen from Table III, where the percentage increase in α_d is shown summed for all levels up to and inclusive of n''_t for C V.

TABLE III

PERCENTAGE INCREASE IN α_d AS A FUNCTION OF THE TERMINAL LEVEL n''_t DUE TO A COMPLETE REDISTRIBUTION OF ℓ'' STATES IN C V

n''_t	Increase in α_d
2	2%
3	2%
4	12%
5	25%
6	32%

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APPENDIX A

Dielectronic Recombination

The dielectronic recombination term diagram is illustrated in figure 1. A free electron of energy E and angular momentum $\ell'' + 1$ can recombine with an ion $X^{+(z)}(n\ell)$ by the transfer of its kinetic energy into excitation energy of a core electron in the ion $X^{+(z-1)}$ and consequent capture into an autoionising state of the ion. The resulting ion $X^{+(z-1)*}$ is thus in a doubly excited state with a term scheme similar to that of the singly excited scheme shifted by an amount equal to the excitation potential of the excited core electron (see figure 1). This process is described by:

$$X^{+(z)}(n\ell) + e^{-}(E\ell'' + 1) \rightarrow X^{+(z-1)*} \quad (n'\ell + 1;\ n''\ell'') . \qquad .. \text{(A1)}$$

The inverse process is autoionisation, which will occur unless the ion $X^{+(z-1)*}$ stabilises by radiative decay of the excited core state to the ground state; described by:

$$X^{+(z-1)*} \quad (n'\ell + 1;\ n''\ell'') \rightarrow X^{+(z-1)} \quad (n\ell;\ n''\ell'') + h\nu . \qquad .. \text{(A2)}$$

The process of recombination by means of these two reactions is referred to as dielectronic recombination, and was first suggested by Massey and Bates (1942). An expression for the dielectronic recombination coefficient is derived below.

Using the law of conservation of energy and observing the convention that the energy of an unbound state is positive, while that of a bound state is negative and where the energy of the state $X^{+(z)}(n\ell) = E_{n\ell}$, the incident electron energy $E_{\ell''+1}$ is defined, viz.

$$E_{\ell''+1} - E_{n\ell} = - E_{n''\ell''} - E_{n'\ell+1} . \qquad .. \text{(A3)}$$

Assume the outer excited electronic states to be hydrogen like, i.e.

$$E_{n''\ell''} = \frac{Z^2}{n''^2} . \qquad \text{.. (A4)}$$

So that for incident electron capture the electrons must have energies defined by:

$$E = E_{\ell''+1} = E(n\ell \rightarrow n'\ell+1) - \frac{Z^2}{n''^2} . \qquad \text{.. (A5)}$$

If the state $X^{+(z-1)*}(n'\ell+1;\ n''\ell'')$ is assumed stationary the number of electron losses due to autoionisation and radiative decay equals the number of electron captures:

$$\left[N^{+(z-1)*}(n'\ell+1;\ n''\ell'')\right]\left[A_r(n'\ell+1;\ n''\ell'' \rightarrow n\ell;\ n''\ell'') + A_a(n'\ell+1;\ n''\ell'' \rightarrow n\ell;\ E\ell''+1)\right] = \left[N_e N^{+(z)}(n\ell)\right] A_c(n\ell;E\ell''+1 \rightarrow n'\ell+1;n''\ell'') \qquad \text{..(A6)}$$

where $N^{+(z)}(n\ell)$ is the number of ions of charge $+(z)$ in state $(n\ell)$, A_r is the radiative rate coefficient, A_a is the autoionising rate coefficient, A_c is the electron capture rate coefficient or collisional excitation rate, and N_e is the number of electrons. Since stabilisation is necessary for recombination, the dielectronic recombination rate per unit volume equals the stabilisation rate of the $X^{+(z-1)*}$ species per unit volume:

$$\left[\alpha_d(n\ell;E\ell''+1)\right] N_e N^{+(z)}(n\ell) = \left[N^{+(z-1)*}(n'\ell+1;n''\ell'')\right] A_r(n'\ell+1;n''\ell''\rightarrow n\ell;n''\ell'') \qquad \text{..(A7)}$$

where α_d is the dielectronic recombination coefficient. Eliminating $N^{+(z-1)*}(n'\ell+1;\ n''\ell'')$ by combining equations A6 and A7:

$$\alpha_d N^{+(z)} N_e = \frac{N^{+(z)} N_e A_c A_r}{A_r + A_a} . \qquad \text{.. (A8)}$$

To express A_c in terms of A_a, L.T.E. is assumed between the processes of autoionisation and electron capture. This is valid in the laser - generated plasma, since most of the levels involved in dielectronic recombination are in L.T.E. with the continuum.

Hence
$$N^{+(z)} N_e A_c = A_a N^{+(z-1)}. \qquad ..(A9)$$

So that from the Saha equation (Griem 1964),

$$A_a = A_c \frac{2\,\omega(n\ell)}{\omega(n'\ell+1;\ n''\ell'')} \frac{(2\pi m_e kT_e)^{\frac{3}{2}}}{h^3} \exp(E/kT_e) \qquad ..(A10)$$

where $\omega(n\ell)$ is the statistical weight of the level $(n\ell)$ and all other symbols have their conventional meaning or have been defined previously.

$$\alpha_d = \frac{A_c A_r}{2 A_c (2\pi m_e kT_e)^{\frac{3}{2}} h^{-3} \exp(E/kT_e)\ \omega(n\ell)/\omega(n'\ell+1;\ n''\ell'') + A_r}. \qquad ..(A11)$$

The quantity A_c can be expressed in terms of a collision cross-section Q, with incident electrons in the velocity range $v \rightarrow v + dv$:

$$A_c = Q \left(\frac{2E}{m}\right)^{\frac{1}{2}} \frac{dn(E)}{n}$$

where $\frac{dn(E)}{n}$ is the fraction of electrons with energy in the range $E \rightarrow E + dE$. The electron energy distribution function dn(E) is assumed to be Maxwellian:

$$A_c = Q \left(\frac{2E}{m}\right)^{\frac{1}{2}} \frac{2}{\sqrt{\pi}} (kT_e)^{-\frac{3}{2}}\ E^{\frac{1}{2}} \exp(-E/kT_e)\ dE \quad cm^3\ sec^{-1}. \qquad ..(A12)$$

So the final expression for the dielectronic recombination coefficient is:

$$\alpha_d = \frac{A(n'\ell+1;n''\ell''\rightarrow n\ell;\ n''\ell)C_1(kT_e)^{-\frac{3}{2}} \exp(-E/kT_e)EQ(n\ell;E\ell''+1\rightarrow n'\ell+1;n''\ell'')dE}{A(n'\ell+1;n''\ell''\rightarrow n\ell;\ n''\ell'')+C_2\omega(n\ell)EQ(n\ell;E\ell''+1\rightarrow n'\ell+1;n''\ell'')dE/\omega(n'\ell+1;n''\ell'')}$$

$$..(A13)$$

where all quantities are in cgs units, $C_1 = 4(2\pi m_e)^{-\frac{1}{2}} = 5.29 \times 10^{13}$ cgs units and $C_2 = 16\pi mh^{-3} = 1.58 \times 10^{53}$ cgs units, and α_d is in $cm^3\ sec^{-1}$.

The total dielectronic recombination coefficient is obtained by summing over all possible levels, ie:

$$\alpha_d(\text{tot}) = \sum_{n'n''\ell''} \alpha_d(n'\ell+1;\ n''\ell''). \qquad ..(A14)$$

APPENDIX B

Photoionisation

Photoionisation rates were calculated using the hydrogenic formula derived in this appendix. The cross-section for photoionisation of a principal quantum level, n, in an atom or ion, by radiation of frequency ν, is given by $\sigma(n,\nu)$. See for example Marr (1968).

$$\sigma(n,\nu) = \frac{64\pi^4 e^{10} m_e Z^4}{3\sqrt{3}\, ch^6 \nu^3 \omega(n) n^3} \text{ cm}^2 \qquad ..(B1)$$

where e is the electronic charge, m_e is the electronic mass, c is the velocity of light, h is Planck's constant and $\omega(n)$ is the statistical weight of level n. For a level, n, the total photoionisation rate of an atom or ion, P_i, is given by

$$P_i = \int_{\nu_n}^{\infty} \sigma(n,\nu)\, q(\nu)\, d\nu \text{ sec}^{-1} \qquad ..(B2)$$

where ν_n is the frequency of radiation required for threshold ionisation of the level n, $q(\nu)d\nu$ is the number of photons in the frequency range $\nu \rightarrow \nu + d\nu$ emitted in the plasma per unit area per unit time. Assuming a Black Body radiation distribution:

$$q(\nu)d\nu = \frac{2\pi\nu^2}{c^2} \frac{d\nu}{[\exp(h\nu/kT_e) - 1]} \text{ cm}^{-2} \text{ sec}^{-1}. \qquad ..(B3)$$

Thus, from B1, B2, B3:

$$P_i = \frac{128\pi^5 e^{10} m_e Z^4}{3\sqrt{3}\, c^3 h^6 n^5} \int_{\nu_n}^{\infty} \frac{d\nu}{\nu[\exp(h\nu/kT_e) - 1]}. \qquad ..(B4)$$

If $h\nu_n > kT_e$ (true in some important cases) an approximate analytic form for the integral can be found:

$$P_1 = \frac{2.0 \times 10^9 Z^4}{n^5} E_1(\chi_n/kT_e) \text{ sec}^{-1} \qquad ..(B5)$$

where kT_e is the electron temperature, χ_n is the ionisation potential of the level n, and E_i is the exponential integral, values of which are tabulated, (see for example Weast and Selby (1967)).

APPENDIX C

Stimulated Emission

The stimulated emission rate, S_r, is given by:

$$S_r = B(j \rightarrow i)\ \rho(\nu)\ \mathrm{sec}^{-1} \qquad ..(C1)$$

where $B(j \rightarrow i)$ is the Einstein stimulated emission coefficient for the transition $j \rightarrow i$ and $\rho(\nu)\ d\nu$ is the stimulating radiation density in the frequency band $\nu \rightarrow \nu + d\nu$ corresponding to the transition $j \rightarrow i$.

$$B(j \rightarrow i) = \frac{c^3}{8\pi\ h\nu^3}\ A(j \rightarrow i)\ \mathrm{ergs}^{-1}\ \mathrm{cm}^3\ \mathrm{sec}^{-2} \qquad ..(C2)$$

where c is the velocity of light, h is Planck's constant, and $A(j \rightarrow i)$ is the Einstein spontaneous decay coefficient.

The radiation density $\rho(\nu)\Delta\nu$ was calculated by adopting the assumption that the coronal model describes the laser-generated plasma to a first approximation, Donaldson et al (1973), so that:

$$\rho(\nu)\Delta\nu = \frac{N_e N_z X(i \rightarrow j) D\ h\nu}{c}\ \mathrm{ergs\ cm}^{-3} \qquad ..(C3)$$

where N_e is the electron density, N_z is the number density of ions in the state of ionisation z in the plasma, obtained from Jordan (1969), D is a typical scale length in the plasma $\sim$ 50 microns for plasma created by 1.06 μm radiation and 400 microns for plasma created by 10.6 μm radiation, $\Delta\nu$ is the Doppler width of the transition and $X(i \rightarrow j)$ is the excitation rate of the transition $i \rightarrow j$, McWhirter (1965)

$$X(i \rightarrow j) = \frac{8.50 \times 10^{-4}\ \bar{g}\ f(i \rightarrow j)\exp(-1.16 \times 10^4 \chi(i,j)/kT_e)}{T_e^{\frac{1}{2}}\ \chi(i,j)}\ \mathrm{cm}^3\ \mathrm{sec}^{-1} \qquad ..(C4)$$

where $\bar{g}$ is the effective Gaunt factor for the excitation process, taken as 0.2, $f(i \rightarrow j)$ is the absorption oscillator strength, $\chi(i,j)$ is the energy of the transition in eV, and T_e is the electron temperature in ^oK.

Combining equations C1 and C3 the stimulated emission rate is obtained:

$$S_r = \frac{N_e N_z\ X(i \rightarrow j)\ Dh\nu B\ (j \rightarrow i)}{\Delta\ \nu\ c}\ \sec^{-1}. \qquad ..\ (C5)$$

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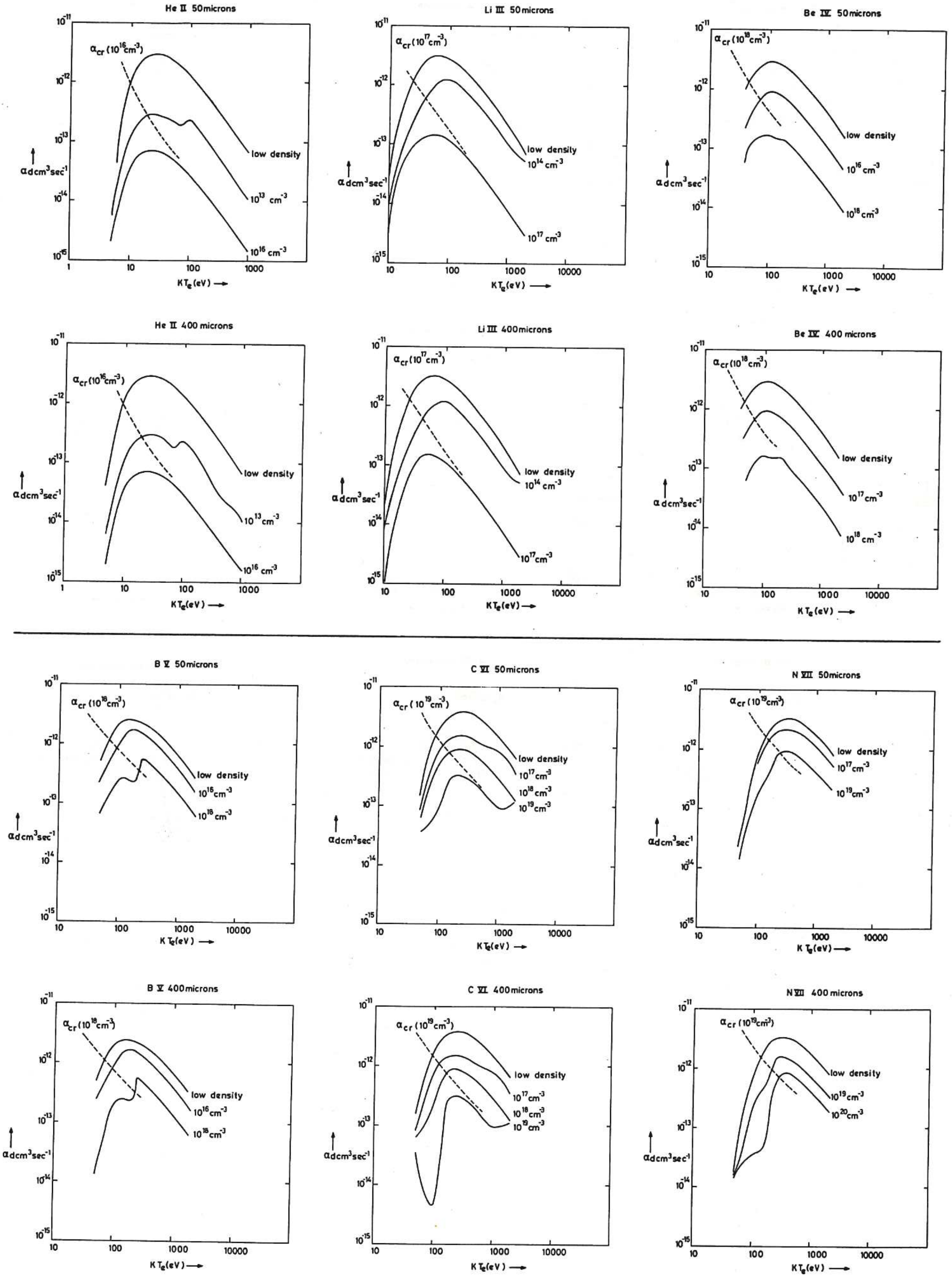
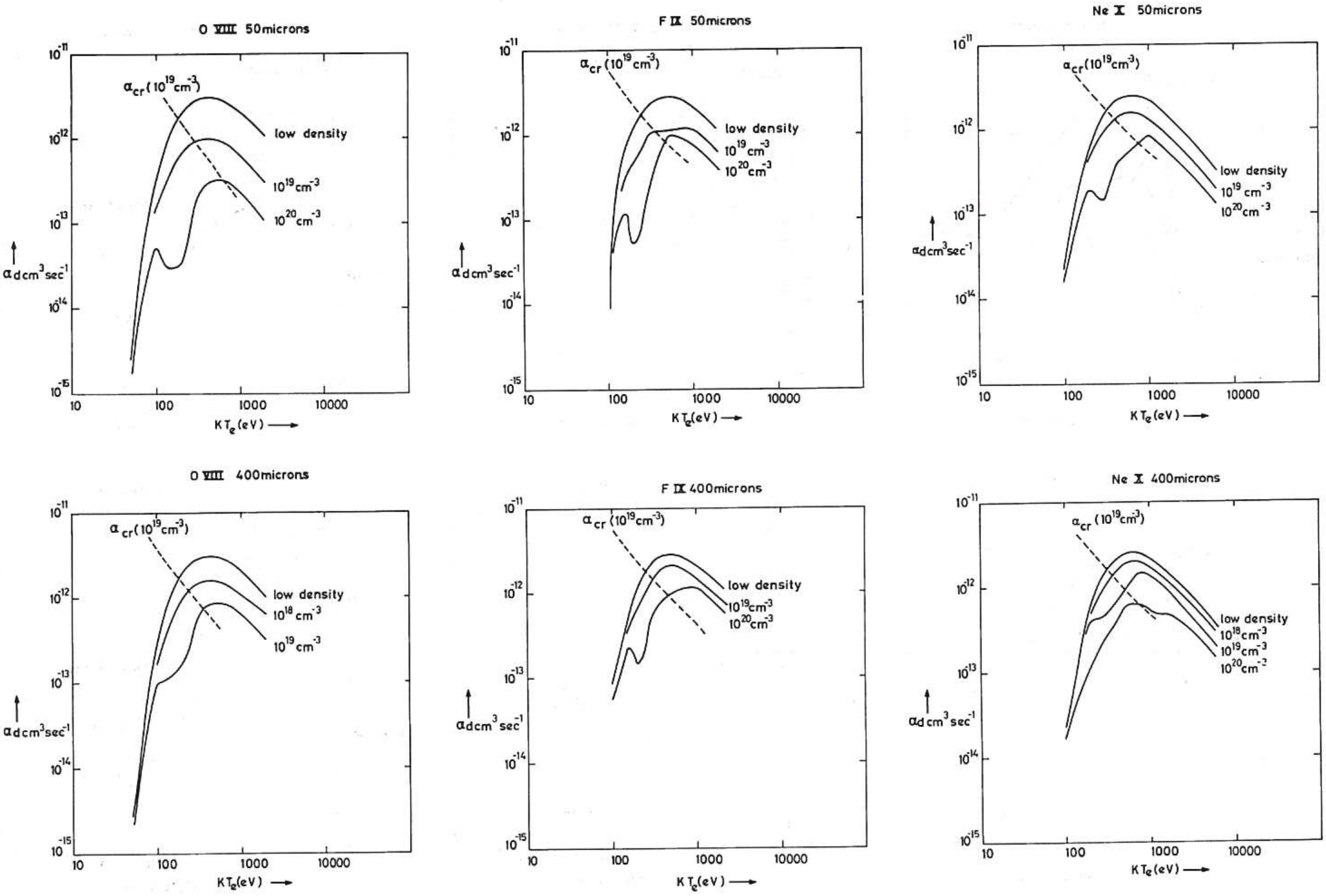
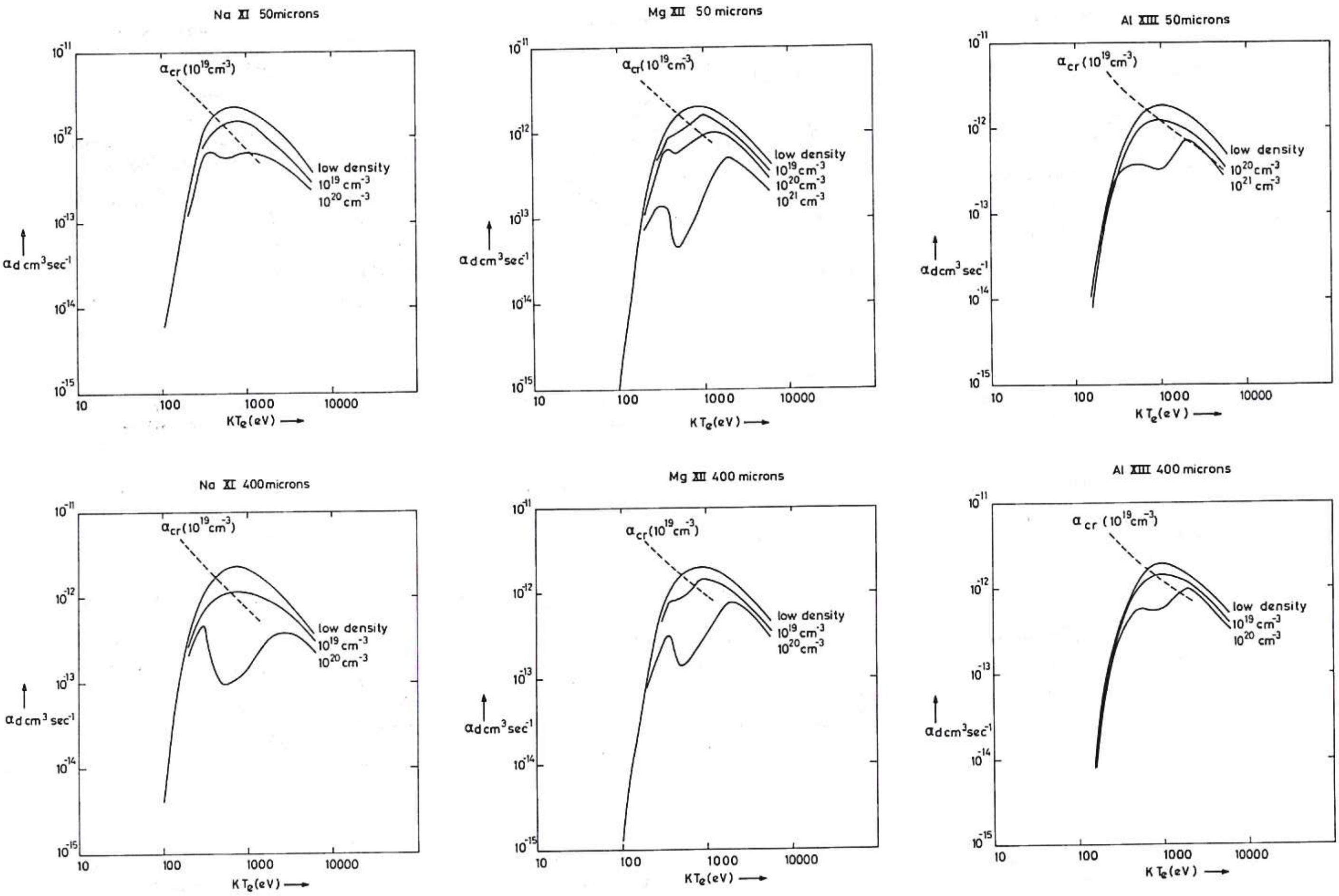


Fig. 2 Dielectronic Recombination Coefficients and Collisional Radiative Recombination Coefficients for the Hydrogenic Ions He II through to Fe XXVI.


O VIII 50microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
F IX 50microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Ne X 50microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
O VIII 400microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{18}cm^{-3}$
$10^{19}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
F IX 400microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Ne X 400microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{18}cm^{-3}$
$10^{19}cm^{-3}$
$10^{20}cm^{-2}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$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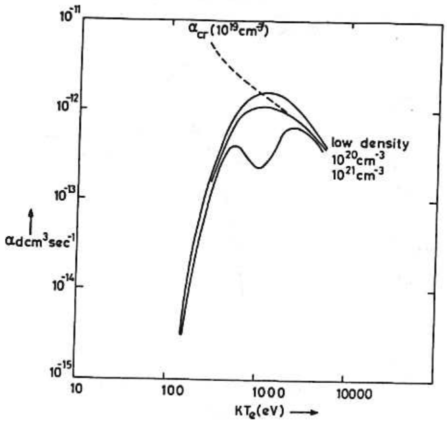
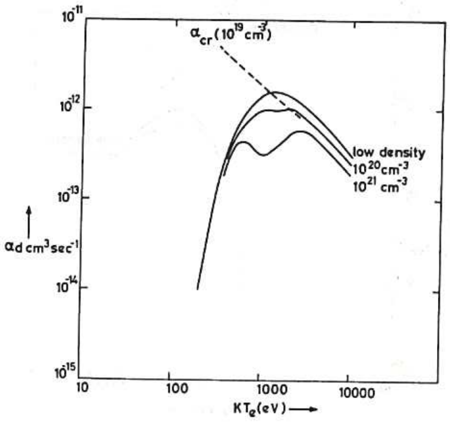
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Mg XII 50 microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$10^{21}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Al XIII 50microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{20}cm^{-3}$
$10^{21}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Na XI 400microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Mg XII 400 microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$
Al XIII 400 microns
$\alpha_{cr}(10^{19}cm^{-3})$
low density
$10^{19}cm^{-3}$
$10^{20}cm^{-3}$
$\alpha d\,cm^3sec^{-1}$
$KT_e(eV)$

Si XIV 50 microns



P XV 50 microns



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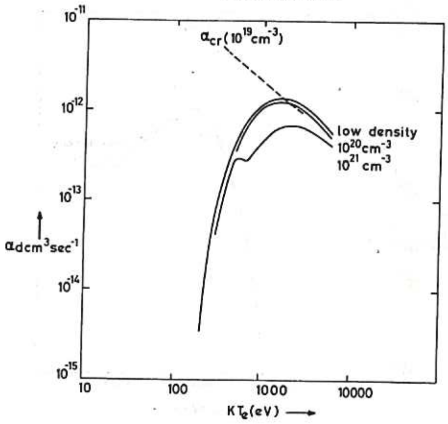
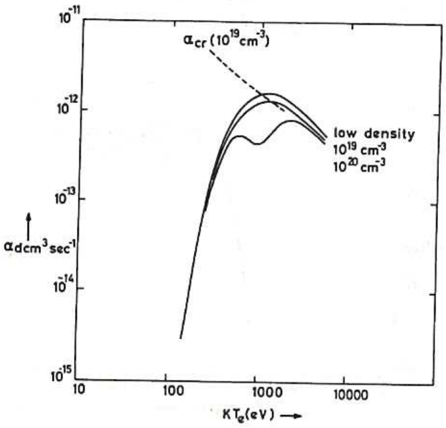


Si XIV 400 microns



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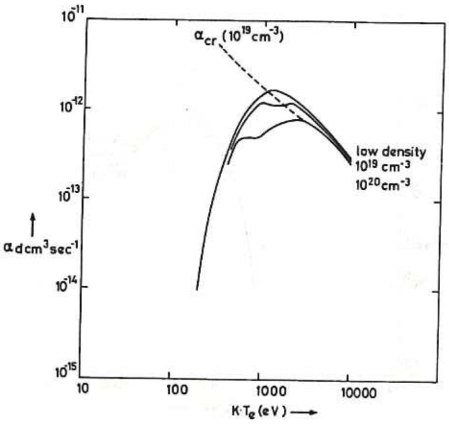
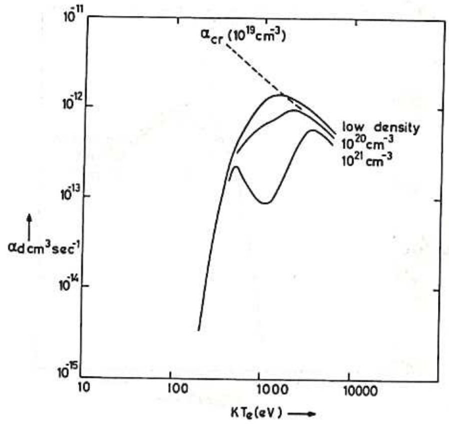


S XVI 400 microns



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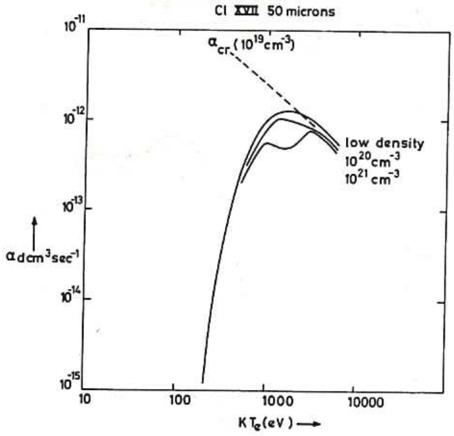

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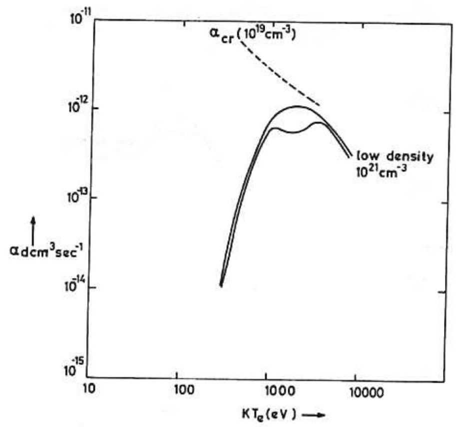

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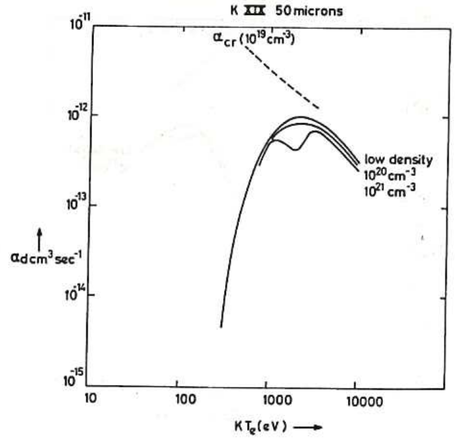

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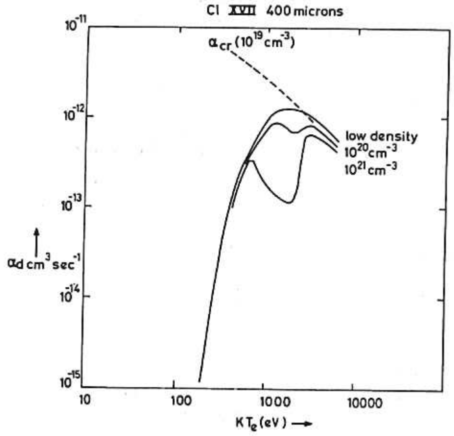

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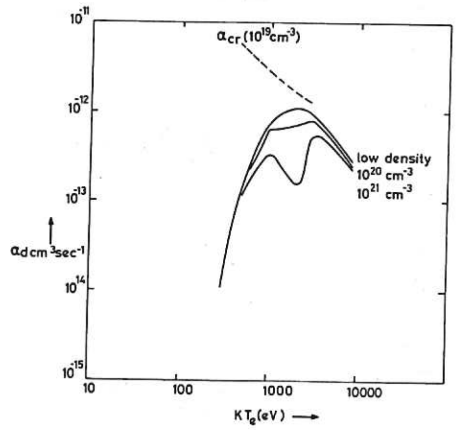

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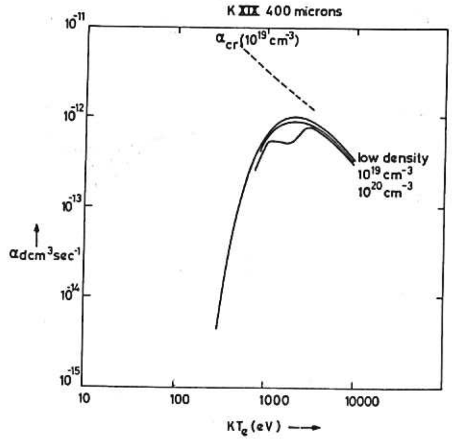

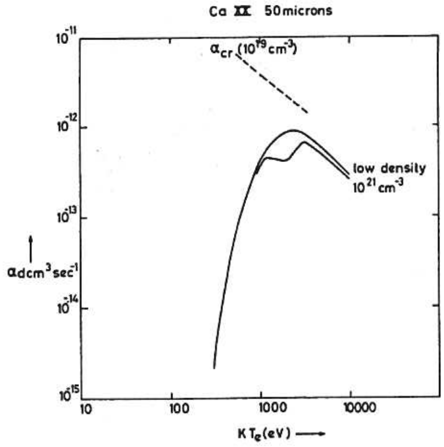
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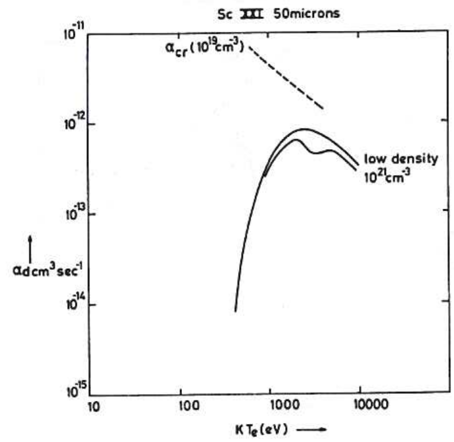
low density
10^{21} cm^{-3}
α d cm^3 sec^{-1}
KT_e(eV)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low density
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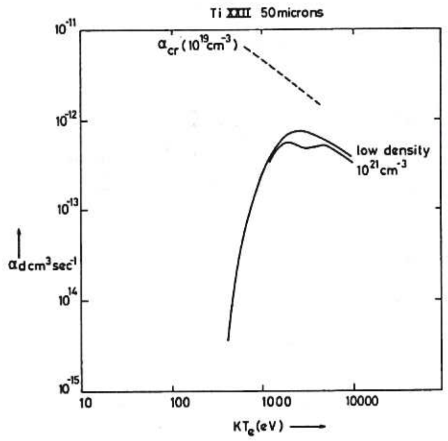
α d cm^3 sec^{-1}
KT_e(eV)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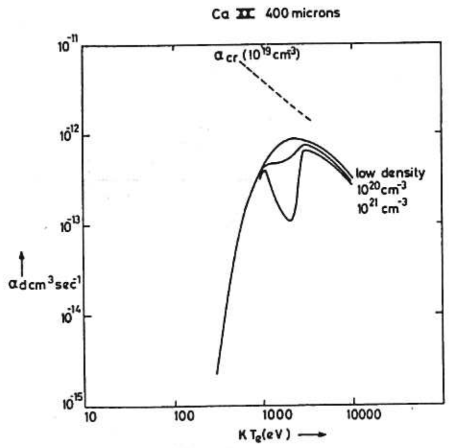
low density
10^{21} cm^{-3}
α d cm^3 sec^{-1}
KT_e(eV)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low density
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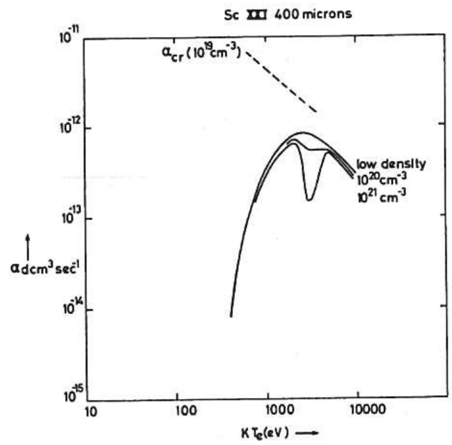
α d cm^3 sec^{-1}
KT_e(eV)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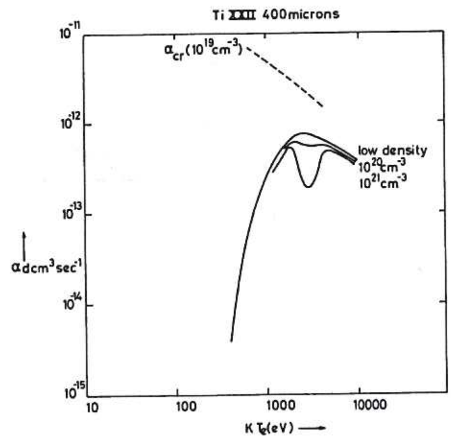
α d cm^3 sec^{-1}
KT_e(eV)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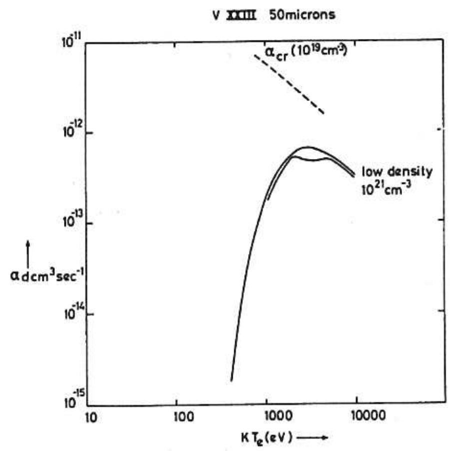
α d cm^3 sec^{-1}
KT_e(eV)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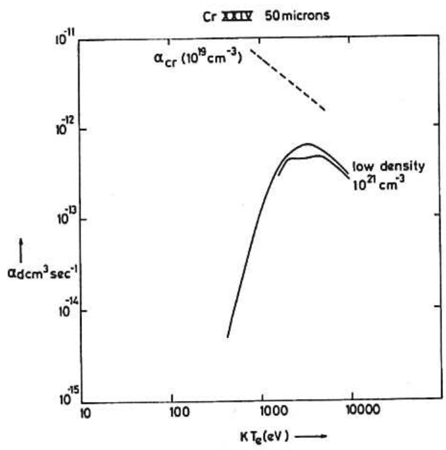
KT_e(eV)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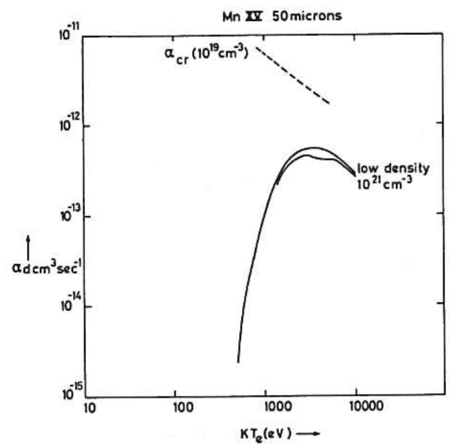
α d cm^3 sec^{-1}
KT_e(eV)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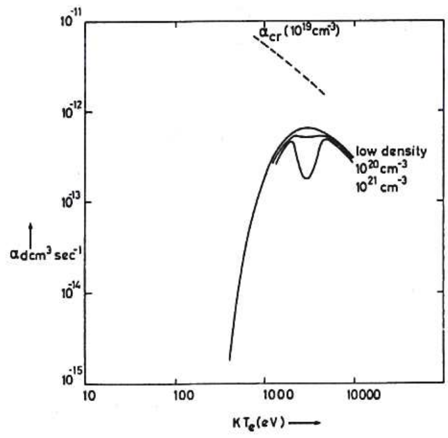
KT_e(eV)


V XXIII 400 microns
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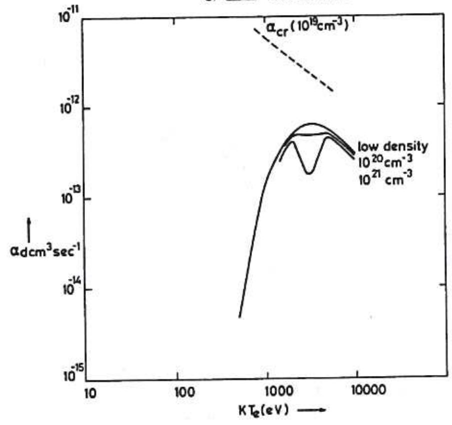
low density
10^{20} cm^{-3}
10^{21} cm^{-3}
α d cm^3 sec^{-1}
KT_e(eV)


Cr XXIV 400 microns
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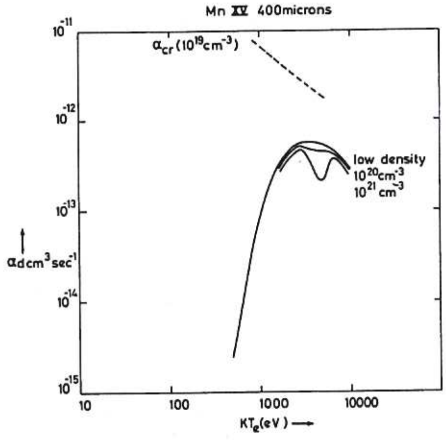
α d cm^3 sec^{-1}
KT_e(eV)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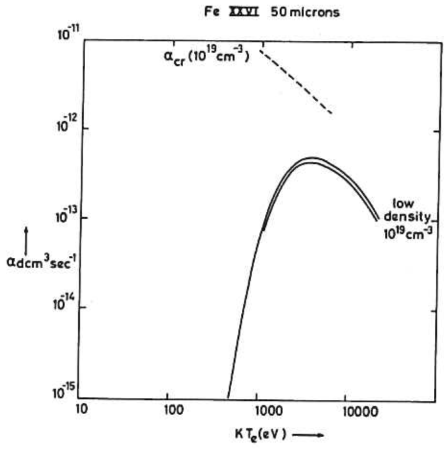
α d cm^3 sec^{-1}
KT_e(eV)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α_d cm^{3}sec^{-1}
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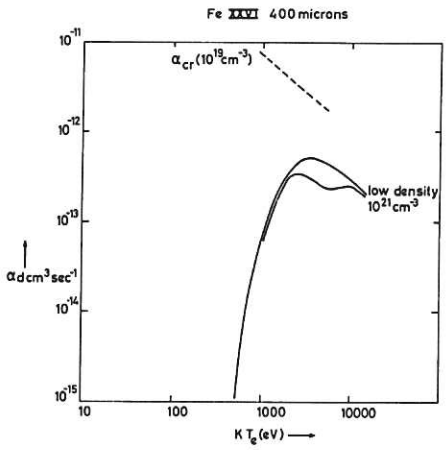
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10
100
1000
10000
K T_e(eV)


Fe XXVI 400 microns
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low density
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α_d cm^{3}sec^{-1}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10
100
1000
10000
K T_e(eV)

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