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Report

REVERSED FIELD PINCH REACTOR STUDY
I. Physical Principles

J D LAWSON

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REVERSED FIELD PINCH REACTOR STUDY

I. Physical Principles

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A B S T R A C T

This is the first of a series of reports(1-3) describing the evolution of a preliminary conceptual design for a fusion reactor based on the Reversed Field Pinch. This enterprise requires an understanding of the behaviour of the plasma during the various phases of operation: current rise, heating, burning, and run-down. The problem of stability, and in particular the practical realization of the conducting shell, need special attention. In this report these topics are examined in the light of present understanding obtained from experiment and theory. Quantitative estimates of the relation between parameters suitable for incorporation in the reactor design are obtained wherever possible, and attention is drawn to those areas where knowledge is poor and further progress awaits either further assessment, or, more often, greater experimental experience.

The work described in this report forms part of a Reversed Field Pinch reactor study undertaken by the following members, whose contributions to the study are gratefully acknowledged:-

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CONTENTS

1. INTRODUCTION

The first exploratory studies for a reactor based on the Reversed Field Pinch concept were made in the late 'sixties and reported at BNES Conference at Culham in 1969.[4,5] The main potential advantages and problems with this concept were identified at that time. Between then and 1975 some further assessments were made,[6-8] and the HBTX2 (now RFX) experiment proposed.[9] Since 1975 a more detailed study has been carried out, with the aims of ascertaining the physical conditions to be expected in the reactor, the technical requirements to produce them, and the optimum parameters for economic operation, chosen to exploit most profitably its special features. The first of these topics is dealt with here; the others are the subject of companion papers.[1-3] Studies based on somewhat different physical assumptions have also been made at Los Alamos.[10-11]

Basically the RFP is a toroidal current carrying plasma, confined by a combination of self and applied magnetic fields and in this respect it resembles a Tokamak. It differs from the Tokamak in that the toroidal magnetic field is much lower, and further, that the direction of this field reverses between the toroidal axis and the plasma edge. The configuration is stabilized by magnetic shear and the presence of a conducting wall, and the value of the safety factor q is much less than unity. Since the toroidal field is much less than that in a Tokamak, and confinement is associated with the poloidal field, the value of β obtainable might be expected to be higher. A penalty to be paid for this advantage is that the plasma configuration is likely to be less stable. Although it can, in principle, be stabilized by a 'perfectly conducting' shell, such a shell cannot be simulated on the time scale required for a reactor, and additional stabilization will be required.

These points will be discussed in more detail in subsequent sections, together with other requirements, such as that of setting up and heating the plasma. Scaling laws will be considered, and broad comparisons with the Tokamak made.

One fact that emerges is that despite a considerable amount of experimentation on smaller machines, and of theoretical work on various models, considerable uncertainty remains about the behaviour of the plasma under conditions necessary for a reactor. In these circumstances, the study can indicate certain criteria that must be met before a reactor can be considered feasible.

2. THE CHARACTERISTICS OF AN RFP PROFILE

The essential properties of this configuration have been outlined in a recent review by Bodin.[12] This contains an extensive list of references, and a historical introduction. Here we abstract information required for the reactor study.

The cornerstone of the concept is the observation on ZETA that the initially turbulent setting up phase was followed by a quiescent period exhibiting the reversed field configuration.[13,14] It may be shown theoretically that such configurations can be MHD stable, and experimentally that they can be set up by suitable field programming.[12] It was further shown theoretically by Taylor that if the plasma is surrounded by a 'flux conserving shell', such distributions develop naturally as the concomitant of a 'minimum energy state' when small but finite dissipation is present.[15,16]

We continue by describing the configuration in the 'quiescent' condition, after the current has been established, and introduce notation which will be used later. Quasi-cylindrical co-ordinates r, θ and ϕ will be used, but most of the theoretical work has been done in cylindrical geometry (R, the major radius, $\to\infty$).

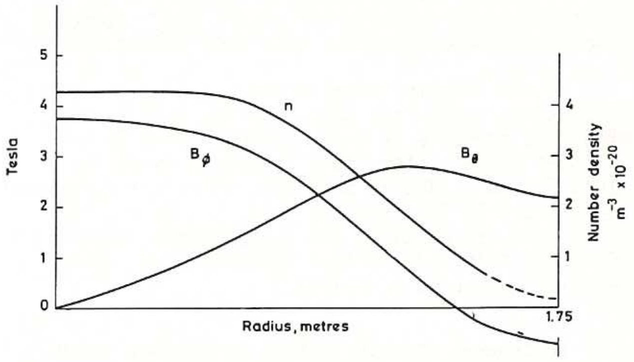

Fig.1 Profile used in reactor calculations.

The essential characteristics of the field and pressure profiles are shown in Figure 1. These represent the actual profiles used in the reactor study, their particular properties and origin will be described later. The density n(r) and temperature T(r) are not separately specified, though a convenient assumption in reactor studies is that T is constant so that $n \propto p$. The current distribution can be found once the fields are known, or vice versa. The current at the plasma edge was negative in ZETA, but this is not a necessary condition for a stable profile.

Two important parameters which characterize the field distribution are F and θ, defined as:

$$F = \frac{B_\phi(a)}{\overline{B}_\phi}, \quad \theta = \frac{B_\theta(a)}{\overline{B}_\phi} \qquad (1)$$

where

$$\overline{B}_\phi = (\textstyle\int_0^a 2\pi r\, B_\phi dr)/\pi a^2$$

and a is the radius of the first wall. For a 'reversed field', F is negative. For an ideal 'Taylor' configuration, undergoing field reversal with flux conservation, $\overline{B}_\phi = B_\phi\ (t = o)$. The quantitiy θ is often referred to as the 'pinch parameter', and a high value denotes a configuration with axial current concentrated near the axis.

Further parameters associated with such a distribution are $\beta(r)$ and β_θ. The first may be defined as the local value of the ratio of particle pressure to magnetic pressure; the value on axis, $\beta(o)$, is often quoted. The second is defined in terms of the total current,

$$\beta_\theta = \frac{8\pi(N_e+N_i)kT}{\mu_o I^2} = \frac{16\pi^2\int_o^a prdr}{\mu_o I^2} \qquad (2)$$

For an 'ideal' discharge in which the pressure at the walls is zero, and $B_\phi = o$, the poloidal β, β_θ would be unity. The ratio $\beta_\theta/\beta(o)$ depends very much on the shape of profile, and can exceed 3 in the region of practical interest. It is larger for profiles in which the ratio of the radius at which $B_\phi = o$ to the radius of the wall is small. (In Tokamaks, very much larger values of $\beta_\theta/\beta(o)$ are to be expected.)

3. MHD STABILITY

The quantities F, θ, β(r) and β_θ characterizing (but not completely defining) the plasma profile were introduced in Section 2. We now enquire what constraints are placed on these quantities by the requirements of MHD stability. First we note that stability requirements cannot be expressed as functions of these variables alone, but in general involve the actual profile in a complicated way.

In one limiting example there are simple rules; this is the force-free configuration ($\underline{j} \times \underline{B} = o$) with zero pressure ($\beta = o$), corresponding to the 'relaxed state' after field reversal, where the field B_θ and B_ϕ depend on radius as Bessel functions $J_1(\mu r)$ and $J_0(\mu r)$ respectively; $\mu = 2\theta$ and $1.2 < \theta < 1.55$ for a stable and axisymmetric state.[15] (Since in this model F is a function of θ, this determines a range of F also.)

In general however, when β is finite, there are no simple rules, nor is there a simple prescription for finding profiles with an acceptable degree of stability. Indeed, when all resistive effects are rigorously included, no completely stable profiles are found when $\beta > o$.[17] Even when profiles stable against major instabilities are found, it is not possible to relate them unambiguously to the setting up process (to be discussed later).

In addition to the uncertainty concerning profiles, it is not possible to estimate at all precisely what the achievable value of β is likely to be. Stability calculations relevant to particular models have been made, but it is difficult to estimate how well these will describe the behaviour of actual reactor plasma, especially near the wall. It should be recognised also that views on this topic are liable to change as understanding and experience grows. In the reactor study a profile stable against ideal MHD (not including resistive effects) given by Robinson,[18] was used, with wall radius chosen at such a position that θ = 2, (Figure 1). This is not a fully consistent choice since the pressure and current density are not zero at the walls; nevertheless, in view of the limits quoted for θ in the Bessel function model, it was felt that θ should not be too large). The value of β on axis for this profile is 0.31, and β_θ is about 0.45. Nevertheless, for caution, a lower value of 0.35 has been assumed.

More recent work by Robinson,[19] in which the effect of the resistive tearing mode instability is taken into account, suggests that the maximum value of β(o) is about 0.12 if there is to be a vacuum region at the edge of the plasma, and that the radial extent of the vacuum region cannot in any case exceed 4% of the plasma radius. He recommends a value of θ of 2.5-3, and this implies a value of β_θ of order 0.2-0.3. The profile is similar to the Bessel function model at small values of r, but departs considerably from it at the plasma edge. In view of the uncertainty in the profile, the form originally assumed has been retained, with β_θ as a free parameter. In this profile, the current density has a maximum value at an intermediate radius rather than on the axis. Although this feature is perhaps unrealistic, the characteristics of the reactor as understood at present are not sensitive to the profile shape; they depend more strongly on the value of β_θ.

So far, we have implicitly assumed that the plasma is within a conducting shell which conserves flux in the φ-direction, having been set up originally via the relaxation mechanism to the 'minimum energy state' described in Section 2. Subsequently the shell provides MHD stability against kink modes with displacement $\exp i(m\theta \pm n\phi)$. Those most likely to be serious have m = 1 and n from 0 to 2R/a.[20]

A practicable shell falls short of the ideal in two respects; first, the resistivity is not zero, and second, the shell must be subdivided to facilitate construction and maintenance. The first of these defects, which is the more fundamental, implies growth rates not greater than the L/R time constant, which is less than a second for achievable shell thicknesses.[21] Since this is short compared with the burn time of many seconds, active feedback is necessary. The problems of the shell, including criteria for divisibility, and the feedback system are examined in Section 9 and Appendix 5.

4. REACTOR REQUIREMENTS DURING THE BURN PHASE

The essential part of any reactor cycle is the 'burn' phase in which thermonuclear reactions occur and power is produced. We consider this now, and return later to the setting up and run-down phases.

Two modes of operation may be envisaged; in a refuelled system fuel is supplied continuously, and reaction products plus some unburned fuel leave at such a rate as to leave a steady state in the reactor. In a pulsed system, on the other hand, reaction products are contained until a significant fraction of the fuel (about 1/3), is burned; all are then extracted as exhaust.

Until recently it has been assumed that in the RFP the field configuration decays resistively in a time only a few times longer than the time to burn 30% of the fuel, and therefore the considerable complications associated with quasi-continuous operation are not at present worth introducing. (These include fuel injection and the provision of some balancing particle loss mechanism, and the inclusion in the design of a divertor.) All the analysis has therefore been done on the assumption of pulsed operation with no divertor or refuelling. Recent computational results by Sykes and Wesson,[22] and experimental results on HBTXI and FRSX[23] however, suggest that the plasma may continuously regenerate the required configuration by indulging in relaxation oscillations of small amplitude. If the losses associated with this process are adequate but not excessive, one can speculate on the possibility of quasi-continuous volt-second limited operation as proposed in some Tokamak designs. We do not consider this possibility further, but return to the study of a pulsed system.

Detailed arguments which show how the parameters are chosen are given in the companion paper.[1] A table therefore of typical parameters useful for illustrative purposes is given in Appendix 1. The optimum operating temperature is about 10 keV, and this is assumed constant during the burn. How this is maintained constant is completely unspecified and, in common with many other systems, no convincing mechanism has been proposed. Little progress can be expected in this direction until the overall plasma behaviour, wall effects, etc are better understood. One hope is that the system may be self-adjusting in the following way. As the burn proceeds, the temperature rises, so that β increases until the stability limit is reached. After this point the loss increases very rapidly with temperature. Small fluctuations then set in, which keep the system just on the verge of stability. (A model of this type with turbulent core and stable sheath, but relevant to a non-reacting plasma, has been developed by Christiansen and Roberts.[24]) It has been suggested that such fluctuations might allow loss to occur by lowering the confinement time of the fast α-particles.

In the absence of fluctuations, it has been assumed that the decay time of the reversed field to an unstable configuration is small compared with the diffusion decay time $\tau_D \approx 0.1\ \mu\sigma a^2$; for a = 1.75 m and T = 10 keV, $\tau_D \approx$ 160 secs as compared with the corresponding burn time of 20-30 s. In the quiescent phase of ZETA, however, a negative value of applied electric field was found to be essential. If this turns out to be necessary in a reactor in order to produce a reverse current in the outer region of the discharge, then the decay time will be reduced.

In the present reactor study the temperature both of ions and electrons has always been taken as 10 keV at all points in the plasma and impurities have been neglected. In practice, the edge temperature will certainly be lower and even small traces of impurity will be important.[1]

5. SETTING UP THE PLASMA - GENERAL REMARKS

A number of ways in which a burning plasma might be set up can be imagined, but which of them are capable of successful development is far from clear. In the present study we have considered slow build up in an essentially flux conserving shell, as in ZETA, whereas in the USA the emphasis has been on rapid compression and heating. These approaches are compared in a review by Baker, Davidson and DiMarco.[25]

Two overlapping stages in the process can be distinguished. First, the current is raised to the required value and second, the plasma is heated to ignition, which is initiated when the α-particle heating rate exceeds that of ohmic heating. The density can perhaps be varied during the setting up phase by feeding in additional gas in a controlled manner. Rapid mixing of the injected gas and existing plasma is desirable, but it is not yet clear how this can be achieved.

The requirements of a reactor impose several constraints both on the current rise and heating phases. We consider first the current rise. Very rapid setting up is limited by the cost and technical problems of rapid power transfer; a slow rise on the other hand implies large energy losses which severely impair the energy balance of the reactor, and lead to low Q.* Whether it will be better to aim for flux conservation within the shell or by independent programming of B_ϕ to provide 'aided self-reversal' (as has been demonstrated in small scale experiments on HBTX I),[5] is not yet known. A rise time in the range of 0.2-1 second is expected on present information. Arguments leading to this choice are given in the next section.

The complex question of heating, and whether ignition can be achieved by ohmic heating alone, is discussed in Section 7. There is no need to achieve rapid heating, provided that the heating time remains short compared with the burn time, and the energy confinement during the ignition phase is adequate.

It has been convenient to consider current rise and heating as two separate phases, though some overlap will occur. The total sequence of events might be:

1) Rising current is initiated; at the same time the vertical and toroidal fields, B_z and B_ϕ, are controlled in a suitable manner.

2) Initially, when $B_\phi \gg B_\theta$, the plasma resembles that in a Tokamak during start-up; for a favourable distribution of j(r) it will be stable.

3) When B_θ increases to give q(a) $\lesssim$ 3 the discharge becomes unstable, losing energy to the walls.

4) When B_θ, hence θ, is further increased, relaxation to a reversed field configuration occurs.

5) The configuration continues to change as θ increases, but since it is now relatively stable and losses are reduced, heating commences. External control of B_ϕ continues if necessary.

6) The current ceases to rise, but ohmic heating continues.

7) After the temperature reaches a few kilovolts α-particle heating dominates, and the temperature rises more rapidly.

8) At the burn temperature τ_E decreases (associated with β-limit?) and burn continues.

The run down at the end of the burn will be considered in Section 8. Some of the problems associated with this picture are described in more detail in the next two sections.

6. THE CURRENT RISE AND ASSOCIATED LOSSES

The principal parameter to be determined is the rise time of the current. This is likely to be a compromise between high capital costs arising from technical difficulties if the rate of rise is too large, and excessive loss if it is too slow.

Technical difficulties include the problems of energy storage and transfer, and switching (particularly if a programmed B_ϕ field is also required), eddy currents in the blanket structure and elsewhere, and large inductive fields which cause breakdown. The total volt-seconds associated with setting up a current of 15 MA in a reactor with a = 1.75 m is of order 1000.

In addition to the volt-seconds associated with the inductance, there will be a 'volt-second consumption' associated with dissipation arising both from ohmic heating and loss of energy to the walls,

$$\int E\, dt \doteq LI + \int RI\, dt \qquad (3)$$

where L is defined such that $\frac{1}{2} LI^2$ is the total change of stored energy, including that associated with re-distributing B_ϕ, and R is an 'effective' resistance. The energy associated with this dissipation is defined by the second term on the r.h.s. of the energy equation:

$$\int EI\, dt = \tfrac{1}{2} LI^2 + \int I^2 R\, dt \qquad (4)$$

Part of the energy accounted for by the second term goes into ohmic heating of the plasma, but a larger part represents energy lost to the walls by plasma contact and radiation during the current rise. It is interesting to note that there must necessarily be energy lost to the walls during the heating stage; if this did not occur the temperature would rise too fast during the early stages when the resistance is high, and the β-limit, proportional to NT/I^2 (equation 2), would be exceeded.[26]

*The quantity Q is used differently by different authors. Here we define it as the ratio of the total thermal energy produced per cycle both in the fusion process and by nuclear processes in the blanket, divided by the total electrical energy dissipated per cycle in the reactor components and structure. If η_T is the efficiency with which the reactor output is converted to electricity then the fraction of the electrical output which is fed back is $1/\eta_T Q$. For further discussion see companion paper.[1]

Energy dissipated during the current rise is substantial, and has an important influence on the value of Q which can be achieved. Unfortunately, since the setting up process is so ill understood, we have not been able to produce a credible estimate of what the energy loss might be. It is expected that it will be a function of the rise time τ_R, and for ZETA, over a limited range of parameters a variation of the form:

$$\int I^2 R dt / \tfrac{1}{2} LI^2 = K_0 + K_1 \tau_R \qquad (5)$$

has been reported.[27]. Over the range of times in the ZETA experiments, the quantity represented by the l.h.s. of equation (5) was found to vary from 0.4-0.7, where the energy represented by $\frac{1}{2}LI^2$ includes also that between the plasma and the windings. If only the energy within the plasma is included, the figure quoted is increased by a factor of order 1.3. If we assume that a dependence of the same form as equation (5) applies also in the reactor, the question arises of how the co-efficient K_1 scales both with radius and current. Scaling with radius as a^{-2} is probably a reasonable conjecture, but there are no clear experimental or theoretical grounds for determining the scaling with I. Assuming $K \propto I^{-n}$, the experimental evidence from ZETA is not clear, but n is probably less than unity. The large range over which current scaling is required, of order 25:1, together with the fact that important components of the loss (such as associated with impurities) scale in a complicated way, led us to reject numerical estimates based on equation (5) as unrealistic. There is scope however for further study and clarification.

An alternative approach, assuming that the temperature depends on current and that the resistivity is classical, gives an expression of the form:

$$\int I^2 R dt / \tfrac{1}{2} L_p I^2 = f \frac{\tau_R}{\tau_p} \qquad (6)$$

where L_p refers to the magnetic energy within the plasma, f is a form factor of order unity which depends on how the temperature varies with current, and τ_p is the L/R time constant of the plasma after the current has been set up. Unforunately, it is unclear both what the temperature will be at the end of the current rise and how it varies during the rise. Thus meaningful estimates of energy loss still elude us.

It is interesting to note that for a solid conductor, with constant resistivity, it is not possible to set up a uniform current with dissipation of less than about twice the amount of energy stored as magnetic field within the conductor. For a filamented, transposed conductor on the other hand, (Litz wire), the dissipation can be arbitrarily small. Although it is not possible to say what the minimum loss associated with setting up the required current profile in a plasma is, turbulence can in principle perhaps, act somewhat as a 'dynamic' Litz wire, to provide rapid penetration of fields with reduced dissipation.

After much discussion, and examination of data from various large experiments, it was only possible to arrive at 'optimistic' and 'pessimistic' estimates of the energy ratio (equation 6) of 0.4 and 1, with the feeling that rise times of 0.2-1 seconds might be possible. The pessimistic value of energy makes the energy balance of the reactor marginal since parameter studies show that the Q of the reactor rapidly falls to uneconomic values as the energy ratio approaches and exceeds unity. Thus the acceptable rise time is determined by economic if not by physical considerations.

7. PLASMA HEATING; WILL OHMIC HEATING SUFFICE?

One of the advantages claimed for the RFP reactor is that ohmic heating to ignition is possible, so that the complications associated with beam injection or r.f. heating can be avoided. Compared with a Tokamak, an RFP requires higher current to produce given plasma conditions, since β_θ is lower; (the ratio of currents is the ratio of the values of $\beta_\theta^{\frac{1}{2}}$ and this can be quite large, ~ 3). Furthermore, because there is not the same limit associated with B_ϕ an RFP can be made with smaller minor radius than a Tokamak, so making ohmic heating more efficacious. (Both these advantages unfortunately have associated disadvantages, Appendix 4). The higher efficiencies associated with smaller radius can be seen but comparing ohmic heating with the competing radiation loss:

$$\frac{P_\Omega}{P_r} \propto \frac{j^2}{n^2} f(T) \qquad (7)$$

This comparison is of course only valid if other loss processes, such as charge exchange and thermal conduction to the wall are negligible. For a basis of comparison for reactors of different minor radius we take the total mean energy flux at the wall as constant. This needs to be high for an efficient reactor, but is limited for technical reasons to a mean value of 1-2 MW/m^2. In Appendix 2 it is shown that with 'constant wall loading scaling', n^2a is constant, and $j^2 \propto a^{-5/2}$ so that $P_\Omega/P_b \propto a^{-3/2}$. It is to be expected that conduction losses increase with decreasing a, so that the advantage of small minor radius may be reduced.

We now derive a necessary but not sufficient criterion that ignition should be possible by ohmic heating alone, using a simplified model with a uniform plasma column. Accordingly, we include only bremsstrahlung loss which is the irreducible minimum. This must be less than the sum of the ohmic heating, falling as $T^{-3/2}$ and α-particle heating, which rises rapidly with T. The heating curve, $P_\Omega + P_\alpha$ vs. T, has a minimum at about T = 4keV, and the criterion for ignition is that at this minimum $P_\Omega + P_\alpha > P_b$. At this temperature P_α (T) can be represented with reasonable accuracy by a T^4 dependence. For a uniform plasma of radius αa, where α is chosen to give the same value of $\langle n^2 \rangle$ and N, the criterion is derived in Appendix 3 as:

$$P_n \, a^3 \, Z_{eff}^{8/7} < \frac{0.04}{K} \left(\frac{fT}{\alpha^2 \beta_\theta} \right)^2 \qquad (8)$$

where P_n is the mean neutron wall loading in MW/m^2, T the burn temperature in kilovolts, β_θ the value during the burn, and f a form factor discussed below. The factor K is the normalizing constant for wall loading:

$$K = 10^{-40} \langle n^2 \rangle \, a/P_n \qquad (9)$$

where K is typically 6-9 m/MW for the range of reactor designs studied.

Equation (8) indicates the desirability both of small minor radius and high burn temperature, and shows that conditions become more difficult as β_θ is increased. It represents of course a necessary but not sufficient condition, since conduction losses, particle losses, impurity radiation and charge exchange have been ignored. Using the figures for P_n, a, K, α, β_θ in Appendix 1, and taking Z_{eff} = 1 yields l.h.s. and r.h.s. respectively of 8 and $7f^2$. To see by what margin the criterion is satisfied requires an estimate of f. This is a form factor to correct for the fact that in the derivation of equation (8) a uniform plasma profile and Spitzer resistivity were assumed, and azimuthal currents neglected.

Unfortunately it is not possible to say what the form factor f should be, without embarking on a more sophisticated calculation. Such a calculation would need to take into account the following factors. First, because the density varies with radius, the ratio of ohmic heating power to bremsstrahlung loss varies across the plasma; unless therefore thermal conduction makes the temperature uniform, ignition will occur first at a particular point in the plasma. Second, owing to the finite range of the α-particles, their energy will not all be deposited where they are produced. Third, heating arising from θ components of current must be included. On balance, these effects might be expected to improve the chance of ignition. Rough calculations indicate a substantial contribution from θ components of current so that the quantity f will exceed unity and the criterion be well satisfied.

In the heating calculations employed in the reactor design a β-dependent value of f based on curves given by Yeung, Long and Newton[28] was used. This gave results consistent with a value of f = 2.4.

It is interesting to note that the criterion can be written in an equivalent but quite different looking form. Eliminating j from equations (7) and (8) of Appendix 3 yields

$$N < 6.5 \times 10^{19} \frac{f\,T}{\beta_\theta^2 Z_{eff}^{4/11}} \qquad (10)$$

The wall loading has not been introduced, and the relation is perhaps a more fundamental one.

The heating time can be found by integration of the expression for energy gain in terms of the known heating and loss processes. This has been done by Hancox considering Ohmic heating, bremsstrahlung (with Zeff = 1) and α-heating.[1]

$$\frac{dT}{dt} = (P_\Omega + P_\alpha - P_b)/3kN \qquad (11)$$

Insertion of the expression for the P's (Appendix 3, equations 2-4) yields an equation of form:

$$\frac{dT}{dt} = f(I,T)/a^2 \qquad (12)$$

giving a heating time τ_h/a^2 which for given β and T is a function of I only, or (by using the Bennett relation) of N only. In the current range 10-25 MA the ratio τ_h/a^2 lies between 1.5 and 2 s/m² with a minimum at 20 MA. For the reactor parameters the heating time is 6 seconds. Too large a value of the ratio of heating time to burn time results in a loss of efficiency. The ratio τ_b/τ_h for constant wall loading varies roughly as $a^{-3/2}$, again emphasizing the desirability of small a.

In all this analysis it has been assumed that the energy confinement time is long. The required value varies during the heating period, and needs to be greatest when the heating rate divided by the temperature is a minimum. This occurs close to the ignition temperature at about 4 keV; if bremsstrahlung is the only loss τ_E is about 15 secs for the parameters used earlier in this section. Since the constant 0.22 on the r.h.s. of inequality (8) contains the bremsstrahlung loss coefficient C_b to the power -22/7, doubling the loss would prevent ignition; additional tolerable loss must therefore be smaller than the bremsstrahlung loss, leading to a total τ_E of order 10 seconds. Since τ_E is unknown, and extrapolation from ZETA and present Tokamak data uncertain, it is not possible at present to assume that ohmic heating to ignition is assured; it all depends on loss mechanisms determining τ_E during the heating process.

8. CURRENT RUN-DOWN

At the end of the burn it is necessary to reduce the current to zero, cool the gas, and pump it away. At the present time it is not clear how this may best be done; indeed the process is poorly understood in present experiments.

Two essential requirements may be identified; first, because of the tight energy balance in the reactor as a whole, it is desirable that as much as possible of the magnetic energy within the plasma, and also the kinetic energy associated with the particles, should be recovered with high efficiency as electrical energy. Second, the run-down must be such that energy stored in the plasma is deposited uniformly on the walls, and not concentrated in small regions to form hot-spots.

Concerning the second of these requirements, we see no reason at present why uniform deposition of energy within a factor of say 1.5 to 2 (to allow for the asymmetry of toroidal geometry) should not be possible. With regard to the first requirement, for efficient recovery of the kinetic and magnetic energy within the plasma, we enquire first how the run-down might be achieved.

At the end of the burn, the current must be smoothly reduced to zero, and the plasma cooled, de-ionized and pumped away. Since the method by which the temperature is controlled during the burn is not known, it is clearly not possible to describe the run-down process in detail. To initiate it the current can be reduced by applying a reversed E_ϕ field, and at the same time the B_ϕ field can be decreased. Just what the plasma will then do is difficult to predict. Varying the external fields will initially induce currents in the outer regions of the plasma; this will upset the equilibrium, and during the re-adjustment the current and density profiles will change. During this period, the energy flux to the poloidal circuit will be $\frac{1}{2}\int E_\phi B_\theta ds$ over the plasma boundary. The energy flux $\frac{1}{2}\int E_\theta B_\phi ds$, associated with the change in B_ϕ, will be of such a sign as to transfer thermal energy from the expanding gas to the circuit which controls B_ϕ. Suppose, for example, that the plasma forms a column of uniform density and radius 2a/3, then evaluation of $\int p dv$ shows that 40% of the energy can in principle be recovered before the plasma strikes the wall, and the total recovery is 65%.

In the absence of more definite understanding, we make the assumption that 40% of the energy associated with both the magnetic and kinetic energy within the plasma can be recovered as electrical energy with 100% efficiency; the remainder appears as heat and is included in the wall loading.

It is interesting to note that in a solid conductor with constant conductivity, only about 1/3 of the magnetic energy within the conductor can be extracted, and this requires a finite time. There is no experimental demonstration of energy recovery with existing plasmas, and it might be argued that the figure chosen of 40% is optimistic.

After the current has disappeared, it is necessary to pump away the hot residual gas, and introduce new fuel. No fundamental difficulty is expected to arise during this process, a time of several seconds should be adequate. Provision of adequate space for pumping introduces engineering complications, and problems with the stabilizing shell, discussed in the next section.

9. STABILIZING SHELL AND FEEDBACK WINDINGS

The requirements for a stable plasma were discussed in Section 3. An important component, assumed there, is a perfectly conducting shell surrounding the plasma, with a 'vacuum' region of at most a few percent of the plasma volume between the plasma, limited by some form of liner, and the shell. Although schemes without a liner were considered earlier, the liner is now considered to be necessary to avoid electrical breakdown between gaps in the shell. At least one gap is needed along the short circumference to allow penetration of the E_ϕ field; similarly any fast external control of B_ϕ requires a gap also in the long circumference.

The basic difficulty with the shell has long been recognised. The time constant for flux penetration in all known material is short compared with the burn time so that a 'perfectly conducting shell' is an unobtainable abstraction. Before discussing what to do about this, we evaluate the time constant associated with finite conductivity.

There are several time constants associated with a toroidal shell, which relate to the penetration or decay of different current configuration. One of these, the time for a suddenly applied field to penetrate to the full thickness of the copper is 'short', of the order $\mu\sigma d^2$ where d is the thickness of the shell. Typical of the 'long' time constants, is the L/R decay time of θ-currents associated with trapped toroidal flux. Neglecting toroidicity, and assuming that the current flows uniformly through the thickness of the shell, it may readily be shown that:

$$\tau_\theta = \tfrac{1}{2}\mu\sigma a d \qquad (13)$$

It may likewise be shown, but not so simply,[29] that the time constant for penetration of a field perpendicular to the axis is also τ_θ.

Several factors need to be taken into account when assessing the shell thickness. It is at present not possible to state the optimum value from the point of view of the feedback scheme; we conjecture that a time constant of the same order as the current rise time is probably the best. If the shell is too thick, neutron absorption becomes excessive, so that breeding is impaired. Equally, if not more serious, however, is the loss of efficiency due to energy absorption, since the temperature must be maintained well below that of the first wall to achieve high conductivity. First estimates[2] indicate that, considering both electrical conductivity and neutron absorption, either copper or aluminium is the best material; further, a thickness of 4 cm represents an upper limit with 2 cm or less preferred. The temperature will probably be of order 300°C, corresponding to a conductivity $\mu\sigma$ of about 30 s/m² for copper. From equation (13), with a = 1.5 m and d = 0.03 m, we find τ_θ = 0.7 so this is much less than the burn time, and of the same order as the current rise time.

Before considering the effect of finite τ_θ, we describe the functions of a 'perfect' shell, with no gap in the short circumference. At t = o there is finite B_ϕ but no current. An increasing current is then generated by applying E_ϕ from the transformer; B_ϕ at the wall first decreases as the discharge begins to pinch, and finally reverses to produce the stable reversed-field distribution. The current axis is at a larger radius than the toroidal axis; the outward force exerted by the current ring may be considered as being balanced by currents induced in the walls. For a circular cross section, with suitable profile (Section 2), the system is in equilibrium, and stable against kink instabilities.

If now the shell has finite conductivity, the fields induced by the changing plasma current diffuse through it; flux in the φ-direction is no longer conserved, and the currents in the wall can no longer hold the plasma in equilibrium. Further, kink instabilities can grow with time constant of order τ_θ or less.[21]

All these undesirable effects can of course be prevented if the external fields are controlled so that they exactly match the internal fields. We consider these effects in turn. First, poloidal field matching can be obtained by suitable design of the transformer windings, to produce a flux surface at the outside of the shell of the same form as that which is produced inside by the current. For a very thin shell an arbitrarily good match can in principle be obtained, though practical constraints may limit the accuracy with which this can be achieved.

If the only function of the shell were to produce suitable boundary conditions for the required equilibrium, then it could be dispensed with, as in a Tokamak, and a programmed vertical magnetic field used instead. Some field trimming by means of feedback might nevertheless be helpful.

The two other functions of the shell, control of φ-flux and stability against kinks, cannot so easily be provided by other means. We consider these in turn. During field reversal we can distinguish two components of the wall current. The first of these, flowing in the θ-direction, is associated with the change in B_ϕ at the wall. It is independent of φ, and (apart from toroidal effects) independent of θ. If there is a slot in the long circumference of the torus, then the variation in B_ϕ at the wall can be controlled by suitable programming of the current in the toroidal windings. The φ-flux can be increased, decreased, or held constant as required. There is also however a 'random' fluctuating component of B_ϕ, which can vary with φ and θ, which exists during the field reversal process. The shell is necessary during this phase to provide the boundary condition required for the emergence of the 'quiescent' reversed field state. The final function of the shell is to provide stability against gross kink modes during the quiescent phase, as discussed in Section 3.

Several imperfections associated with a real shell need to be considered in detail. First, the finite resistivity; second, the effect of gaps required for assembly and maintenance, and third, the effect of holes required for pumping. We consider these in turn.

Since the shell conductivity is insufficient to prevent slowly growing kink modes, external feedback is required. Sensing elements must be placed near the plasma, inside the shell. The correcting coils however are behind the blanket. There is certainly no room for them inside the shell, and even immediately behind it the technical problems of insulation and high voltage in a harsh radiation environment appear to be excessive.

We now consider the requirements for the feedback windings. It is assumed that in the steady state the field external to the shell is matched to that inside the shell. This can be achieved by suitable design of poloidal windings, though matching throughout the current rise and heating phases, where the plasma configuration changes, may be difficult. For this reason, some feedback control of the vertical field, and perhaps a quadrupole field to provide some shaping, would be desirable. Such fields would have no structure in the ϕ-direction, and in the θ-direction would be of form $B_\phi = B_0 + B_1 \cos(\theta + \theta_0)$ in the absence of plasma (neglecting toroidicity).

The structure of the detection system and coil system required to stabilize gross kink instabilities effectively is difficult to predict; experimental experience is needed. Several approaches are possible; on the one hand one can detect local displacements and try to correct them locally, or alternatively, a particular mode structure can be anticipated, and when the mode is identified an optimum current distribution can be set up to correct it. The design of an effective system requires a good knowledge of the dynamic characteristics of the instabilities as well as their spatial characteristics and it is possible at this stage to make only a rough estimate of what will be needed.

The fields from the windings must penetrate the shell, and therefore only slowly growing modes can be dealt with by this system. If modes with a growth rate much faster than the shell time constant prove to be important, as suggested in ref (21), then this represents a fundamental difficulty.

The simplest model for gross kink modes represents the plasma as a flexible current loop. If a perturbation of form $\cos n\phi$ appears, then clearly a vertical field proportional to $\cos n\phi$, of such a sense as to produce $\underline{j} \times \underline{B}$ force towards the axis, should reduce it. A field of sinusoidal shape, and arbitrary phase, requires a complicated winding. A correcting field for which $\cos n\phi$ is the lowest harmonic, in which the phase is variable, can however be produced by an array of 4n pairs of rectangular coils, as sketched in Figure 2. To cope with all modes up to n to 2R/a, the total number of coil pairs must be divisible by 4 x the lowest common multiple of all n from 1 to 2R/a. For n = 18 this is an impracticably large number. At short wavelengths we can perhaps consider the kinking as a local phenomenon, so that judicious use of 4 x 2 R/a = 72 coil pairs should suffice. If the circumference is divided into 16 segments, this implies 4 coils per segment.*

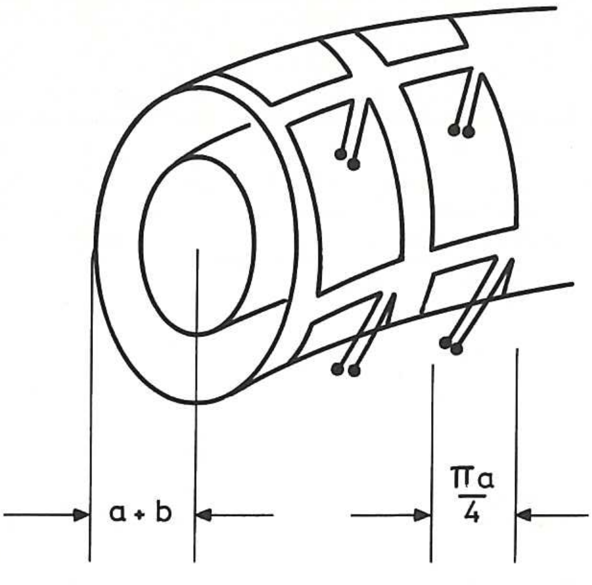

Fig.2 Arrangement of coils to provide feedback to control kink instability.

So far, we have considered only kinks in the horizontal direction. Variations of the form $\cos m\theta$ also can occur; among such possibilities only that with m = 1 is considered likely to be serious. This implies a displacement of the current, which, combined with finite n, gives rise to helical deformation. In order to provide a force towards the centre, a horizontal as well as vertical component of feedback field is required. A mode number m = 2 represents distortion to an elliptical cross section, which would need a quadrupole field for its suppression.

Preliminary thoughts about a feedback system consisting of a network of independent coils placed outside the blanket are outlined in Appendix 5. No specific fundamental problems have yet been identified, provided that the response does not need to be faster than that associated with the time constant of the shell, apart from difficulties associated with gaps, described below. Nevertheless, this is a complex topic, of which little is known, and considerable analysis and experimental work need to be done before the feasibility of the proposed system can be established.

We now consider the question of gaps in the shell. These have two effects; first, they distort the 'natural' current patterns which in an undivided shell would produce stabilization, and second, they produce local distortions where the external and internal fields do not match. One slot might be needed in the short circumference, to control B_ϕ if this should be desirable; this can be placed in the median plane either inside or outside. In the long circumference, however, it is desirable for ease of assembly and maintenance that the shell be divided into a number of sections.

The lower limit to the spacing which can be permitted between gaps determines the minimum size of section, if the awkward problem of making adequate joints between the sections of the shell is to be avoided. No detailed analysis of this problem has been made, but unless the length of the sections greatly exceeds its diameter its effectiveness will be reduced. As a working rule, pending a more complete investigation, a ratio of length to radius of not less than 4 is recommended.

The actual configuration of the perturbations introduced by the gaps, and their effect, are both difficult to estimate. Such perturbations can be significant if there is any mismatch between the external and internal fields; this may be the case during start-up when the fields are changing rapidly. The presence of gaps and access holes in the shell may adversely affect the feedback control; the correcting field will enter much more rapidly through the gaps than through the body of the shell, giving rise to a local time-dependent distortion. How serious this will be is not clear.

In conclusion, it may be stated that considerable uncertainties remain about the behaviour of the shell and its associated feedback system.

10. CONCLUSION

An attempt has been made in this report to discuss those features of the reversed field pinch which need to be understood if it is to form the basis of a fusion reactor. The objective is to provide basic information, qualitative and quantitative, for the design study outlined in the companion reports.[1-3]

We first summarize the position on detailed aspects of the physics which have been discussed.

Profile

In the presence of a perfectly conducting shell, profiles which are stable against the more important types of MHD instability exist, but the 'vacuum region' between the plasma edge and conducting wall can be at most a few percent of the minor radius a.

β-value

The maximum value of β_θ which can be achieved is not yet clear; the value of 0.35 assumed in the study is probably optimistic. (Note when quoting figures that β_θ and β on axis can differ by a large factor; at least 2).

Stabilization

Since the time constant of a practical shell is finite, additional stabilization by external feedback windings will be required. This gives rise to complications and problems not yet seriously assessed.

Temperature control

No definite mechanism for providing control at the appropriate temperature has been identified, though natural processes may exist.

Current rise

There is no detailed model of what happens during current rise, and it has not been possible to find a quantitative relation between energy loss and rise time. For a reactor, a value optimistic on present evidence is required.

Heating

Provided that the only loss mechanism during heating is bremsstrahlung and classical diffusion, ohmic heating to ignition will occur. There is little margin for additional loss however. 'Partial' confinement times defined as 3nkT/(loss rate) for losses other than bremsstrahlung must be of order tens of seconds.

*The present mechanical design is based on half this number, using an earlier estimate of R/a rather than 2R/a as the maximum value of n.

Current run-down

The assessment of the fraction of thermal or magnetic energy which can be recovered directly as electrical energy during the run-down phase is little more than a guess, and it is not possible to assert with confidence that this will be reasonably uniformly distributed on the walls.

General

We do not attempt here any absolute or relative assessment of the prospects for a reactor, this is a matter on which the reader may draw his own conclusion from the evidence of this paper and its companions. Since comparison with Tokamaks has been made previously, a brief and superficial comparison is nevertheless made in Appendix 4.

The broad conclusion which can be drawn is hardly surprising, it is merely that our understanding of the underlying physics is at present totally inadequate to produce a convincing reactor design. Many unknown features will need to turn out favourably for this to be possible. Although this 'conclusion' may appear negative, and could have been confidently foretold at the start, it is nevertheless of value to ask 'what do we actually know, what do we need to know, and how can this knowledge be acquired?' Careful perusal of these questions must surely provide guidelines for future strategy in this field.

11. ACKNOWLEDGEMENT

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APPENDIX 1: Reactor Parameters

This list contains some of the more important parameters assumed or derived in companion reports[1-3] appropriate to a reactor of output 600 MW(e). Parameters are grouped according to type, and sections of the report where they are defined or discussed are shown in brackets.

1. Dimensional parameters

Major radius, R	15.5 m
Minor radius, a	1.75 m
Thickness of blanket and shield	1.6 m

2. Plasma parameters

Peak current	19 MA
β_θ (§ 2)	0.35
F (§ 2)	-0.75
θ (§ 2)	2.0
Temperature (§ 4)	10 keV
Fractional burn-up	0.3
Density on axis, n	4.3×10^{20} m^{-3}
$\langle n^2 \rangle^{1/2}$ averaged over cross section	2.5×10^{20} m^{-3}
Line density, N	2×10^{21} m^{-1}

3. Characteristic times

Burn time, τ_b (§ 7)	28 s
Time for current rise (§ 6)	0.5 s
Heating time, τ_H (§ 7)	4.6 s
Off time (§ 8)	8 s
Time constant of shell (§ 9)	0.7 s

4. Characteristic energies and powers

Mean wall loading (neutrons , P_n (§ 7)	1.5 MW/m^2
Mean total wall loading - (neutrons, radiation and bombardment), P_w (§ 7)	2.2 MW/m^2
Magnetic energy within plasma volume: (§ 6)	
at beginning of current rise	4 MJ/m
at peak current	33 MJ/m
Total magnetic energy stored, (excluding core): (§ 6)	
at beginning of current rise	18 MJ/m
at peak current	75 MJ/m
Thermal energy in plasma (§ 6)	9 MJ/m

5. Magnetic and electric fields

Volt-seconds to establish plasma (§ 6)	800 V.s
Poloidal magnetic field at wall	2.2 T
Toroidal magnetic field at wall	-0.77 T
Toroidal magnetic field on axis	3.8 T
Initial toroidal field on axis	1.0 T

6. Efficiencies associated with plasma behaviour

Ratio of energy lost in setting up plasma to magnetic energy stored within it (§ 6)	0.47
Fraction of kinetic and magnetic energy within plasma at end of burn recovered as electrical energy (§ 8)	0.4

7. Overall system parameters

Electrical power output	600 MW(e)
Energy multiplication, Q (§ 5)	6.9
Thermal to electrical conversion efficiency, η_T	0.4
Recirculating power fraction, $\varepsilon = 1/\eta_T Q$, (§ 5)	0.36

APPENDIX 2: Scaling laws for constant wall loading

The ability of the first wall to withstand the energy flux falling on it is an important constraint in fusion reactor design. The energy associated with particle bombardment and radiation is dissipated in the wall; that associated with neutrons mostly passes through, but radiation damage is an important factor. A design for the most economic operation invariably leads to as high a wall loading as can be tolerated; the mean wall loading, averaged over the cycle, is restricted to values of order 1-2 MW/m^2. It is interesting therefore to see how the various parameters scale with minor radius for constant wall loading, assuming fixed plasma conditions (temperature, β_θ and profile).

In order to obtain the scaling laws, we note first that the fusion power is proportional to n^2a^2R, the wall area is proportional to aR, so that for given wall loading

$$n^2a = \text{constant}$$

For given total power from the reactor, we also have

$$n^2a^2R = \text{constant}$$

From the Bennett relation, $\beta_\theta \mu_0 I^2/4\pi = 4NkT$, with β_θ and T constant

$$I^2/N \propto I^2/na^2 = \text{constant}$$

Other quantities may readily be found, using the appropriate physical formulae. These are shown in the Table.

Quantity	Power of a
Line density of ions N	3/2
Density of ions n	-1/2
Current, I	3/4
Magnetic field, B	-1/4
Major radius, R	-1
Volume, V	1
No of particles $2\pi NR$	1/2
Current density, j	-5/4
Ohmic heating power, j^2V	-3/2
Plasma energy, W_p	1/2
Magnetic energy within plasma, W_m	1/2
Bremsstrahlung loss	0
Larmor radius/minor radius	-3/4
Heating time	2
Field diffusion time	2
Classical diffusion time	2
Bohm diffusion time	7/4
Alfvén transit time	1
Burn time	1/2

It is interesting to display the scaling graphically, and Figure 3 shows a plot of various parameters with I and a as axes. The constants, such as n^2a/P_W where P_W is the time averaged wall loading, are taken from the parameter list. The ohmic heating limit, from equation (8), is shown; since β_θ and T are fixed N is a function of I. The quantity ξ, equal to the ratio of drift to thermal velocity, is also shown; this should be kept as small as possible; and less than $(m_e/m_i)^{\frac{1}{2}} \sim 1/60$ to avoid instability.

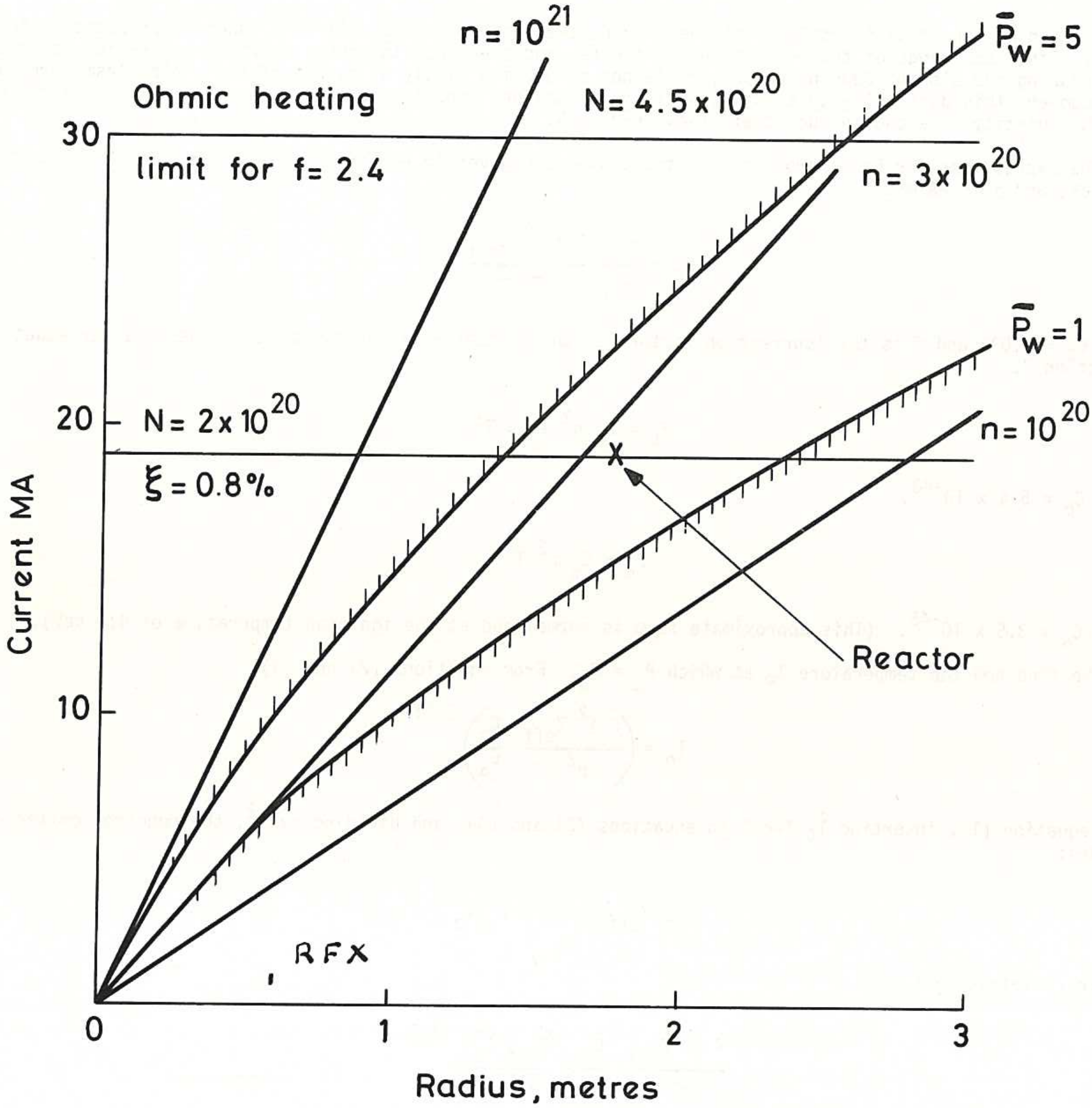

Fig.3 Various parameters plotted on current vs radius diagram for T = 10 keV and $\beta_\theta = 0.35$. $\bar{P}_w$ represents total wall loading in MW per m^2, and the point corresponding to the reactor design in Ref.1 is shown. Ohmic heating to ignition is only possible below the limit shown.

APPENDIX 3: Criterion for ohmic heating

We derive now a rough criterion which must be satisfied for ignition by ohmic heating alone, assuming the only energy loss from the plasma to be by bremsstrahlung. The correct way of performing the calculation would be to find the condition that the bremsstrahlung power loss, P_b, is always less than the sum of the heating power, P_Ω, plus α-particle heating power, P_α. Owing to the fact that P_Ω and P_α are respectively decreasing and increasing rapidly with temperature at the point where they are equal (at T_o), and P_b is slowly varying, the following simpler criterion becomes a good approximation:

$$P_b\ (T_o) < P_\Omega\ (T_o) + P_\alpha\ (T_o) = 2P_\alpha\ (T_o) \tag{1}$$

Since P_b and P_α depend differently on the density n, the temperature T_o will be a function of position in the plasma. Since the range of the α-particles is finite, and they deposit energy in different regions of the plasma during the slowing down processes, P_α is not a function simply of n, but of the whole plasma profile. We circumvent this difficulty by assuming a plasma of uniform density, occupying the volume within a radius αa. The quantity α is chosen such that $\pi(\alpha a)^2\langle n^2\rangle^{\frac{1}{2}} = N$.

The expressions for basic processes in the plasma are given in equations (2-4). Units are MW/m^2, with T in keV, and j in MA/m^2.

$$P_\Omega = \frac{j^2}{\sigma} = \frac{C_\Omega f\ j^2\ Z_{eff}}{T^{3/2}} \tag{2}$$

where $C_\Omega = 0.033$ and f is the 'correction factor' to the Spitzer resistivity value discussed after equation 8 in Section 7.

$$P_b = C_b\ n^2\ Z_{eff}\ T^{\frac{1}{2}} \tag{3}$$

where $C_b = 5.4 \times 10^{-43}$.

$$P_\alpha = C_\alpha\ n^2\ T^4 \tag{4}$$

where $C_\alpha = 3.5 \times 10^{-45}$. (This approximate form is normalized at the ignition temperature of 4.2 keV).

We find now the temperature T_o at which $P_\Omega = P_\alpha$. From equations (2) and (4):

$$T_o = \left(\frac{f\ j^2\ Z_{eff}}{n^2}\ \frac{C_\Omega}{C_\alpha}\right)^{2/11} \tag{5}$$

From equation (1), inserting T_o for T in equations (2) and (4), and dividing by n^2, the required criterion becomes:

$$C_b\ Z_{eff}\ T_o^{\frac{1}{2}} < 2\ C_\alpha\ T_o^4 \tag{6}$$

whence, eliminating T_o,

$$\frac{f\ j^2\ Z_{eff}}{n^2}\ \frac{C_\Omega}{C_\alpha} > \left(\frac{Z_{eff}\ C_b}{2\ C_\alpha}\right)^{11/7} \tag{7}$$

To obtain the criterion in the required form we first eliminate j using the Bennett relation:

$$\frac{\mu_o}{4\pi}\ \beta_\theta\ j^2\ \left(\pi\alpha^2 a^2\right) = 4 \times 10^{-9}\ n\ q\ T \tag{8}$$

or

$$j^2 = 2 \times 10^{-21}\left(\frac{nT}{\beta_\theta \alpha^2 a^2}\right) \tag{9}$$

We require the result in terms of the neutron wall loading:

$$KP_n = 10^{-40} \langle n^2 \rangle a \tag{10}$$

where K is a constant to be found later. Substituting j^2/n^2 from equations (9) and (10) into equation (7) yields the required result:

$$\frac{2 \times 10^{21} \; f \; T \; Z_{eff}}{\beta_\theta \alpha^2 (KP_n \; a^3)^{\frac{1}{2}}} > \left(\frac{Z_{eff} \; C_b}{2 \; C_\alpha}\right)^{11/7} \left(\frac{C_\alpha}{C_\Omega}\right) \tag{11}$$

Rearrangement, and insertion of the numerical values for C_b, C_Ω and C_α yields:

$$P_n a^3 \; Z_{eff}^{\;8/7} < \frac{0.04}{K} \left(\frac{fT}{\alpha^2 \beta_\theta}\right)^2 \tag{12}$$

The factor K depends on the 'form factor' associated with the duty cycle of the reactor, but is essentially constant over the range of parameters studied. Using the values of P_w, n and a in Appendix 1, it is found that α = 0.83 and K = 7.2.

APPENDIX 4: Comparison with Tokamaks

It is natural to wish to compare the Tokamak and RFP as candidates for a reactor, and in earlier papers this has been done. Both have advantages and disadvantages, but the relative weight to be given to these is a matter of judgement. We have not made a Tokamak study in the same style as the present RFP study, and hence the comparison is rather simple-minded.

The following advantages have been claimed for the RFP:

1) For machines of corresponding size and output, the RFP has a much lower B_ϕ field. This implies much less stored energy, more manageable B_ϕ forces, and the possibility of dispensing with superconducting magnets.

2) Since the current for the same plasma conditions is larger than in a Tokamak, ohmic heating is more efficient. The heating ratio, proportional to I^2, is, apart from a factor due to differences in current distribution, in the ratio of the values of β_θ for the two devices. This may be as much as 10.

3) Since it is not necessary to have a tight aspect ratio to ensure stability in the RFP, a free choice of aspect ratio is possible. This eases a number of engineering problems.

In the reactor design study an attempt has been made to utilize the first advantage, that the field strength is not a limitation, by making a physically smaller device, with lower output and at less total cost than a Tokamak reactor. The smaller size also aids ohmic heating as discussed in Section 7.

The disadvantages associated with the RFP appear to be as follows:-

1) The larger current for given plasma conditions, together with the necessity of circulating the energy associated with the B_ϕ field, makes it very difficult to achieve a favourable energy balance in a pulsed reactor. Especially worrying is the possible large loss at start-up, which we do not know how to calculate.

2) The need for a shell and feedback implies considerable complications, both mechanical and electrical. It is far from clear whether the tentative ideas outlined in Section 9 indicate a route to a successful system.

3) Extension to quasi-continuous operation, even granting that the continuous type relaxation process works, will be difficult if a divertor is found to be necessary in addition to a shell.

These considerations are of course not new. The study does, however, enable the nature of the difficulties to be seen more clearly.

APPENDIX 5: Requirements for feedback coils

In Section 9 the coil configuration required to provide feedback control was discussed, using very simple and straightforward assumptions. Here we make an order of magnitude calculation of the currents and voltages required in the coils, assuming that the system does not need to respond in a time short compared with the time constant of the shell.

We assume the torus to be fitted with a uniform array of 256 saddle coils, 4 around the minor circumference and 64 around the major, as shown in Figure 2. They are situated at a radius (a + b) from the circular axis, b being the blanket thickness.

It will be seen that by suitably exciting pairs of coils AA' and BB', a resultant correcting field at any angle θ to the axis can be obtained for each of the 64 discrete segments in the ϕ-direction. Since there is no preferred axis for displacement, the coil axes can be vertical and horizontal or at 45^o. The coils provide restoring forces for kink displacements, and can trim the vertical field. They probably cannot deal effectively with m = 2 distortion, implying ellipticity in the minor cross section of the plasma.

We now make a naive calculation of the currents required in the feedback coils to suppress a local kink. Suppose that the plasma column at some point has drifted a distance Δa, then at the plasma edge the feedback coils are required to provide a field which will restore the field to the value it had when the plasma was centred. The field at the plasma edge is:

$$\frac{4\pi B_e}{\mu_o} = \frac{2I_p}{a} \qquad (1)$$

and hence the error field associated with the drift is:

$$\frac{4\pi \Delta B}{\mu_o} = \frac{2I_p \Delta a}{a^2} \qquad (2)$$

We now calculate the current in two pairs of coils to produce a field of which the uniform (dipole) component is B_s. Setting this equal to ΔB gives the feedback current required.

At the axis only the dipole component of field is present, and we can readily find this by vectorial addition of the fields from the four conductors, each carrying I_s:

$$\frac{4\pi B_s}{\mu_o} = 4 . \frac{1}{\sqrt{2}} \cdot \frac{2I_s}{a+b} \qquad (3)$$

In this calculation we have assumed cylindrical geometry with conductors parallel to the axis of the cylinder. This will be appropriate if all the saddle coils carry the same current, so that there is no contribution from the conductors in the θ direction; (currents in adjacent conductors are equal and opposite).

From equations (2) and (3), setting $B_s = \Delta B$:

$$\frac{I_s}{I_p} = \frac{(a+b)\ \Delta a}{2\sqrt{2}\ a^2} \qquad (4)$$

For a = 1.75 m, Δa = 0.02, b = 0.75 m, we find $I_s/I_p \approx 6 \times 10^{-3}$, so that when I_p = 19 MA, $I_s \approx 10^5$A. For a current density of 10^7 A/m^2 this represents a conductor cross section of 0.01 m^2. The saddle coils are rectangular with sides $2\pi R/64$ and $\frac{1}{2}\pi(a+b)$. As a very crude approximation we assume the inductance to be equal to that of a square coil with the same perimeter, p.

$$L_c \approx \mu_o\ (p/4)\ \ln\ (p/4r_c) \qquad (5)$$

where r_c is the conductor radius. Inserting values of p = 11 m and r_c = 0.06 m, we find $L_c \approx 13\ \mu$H.

In order to make an estimate of the reactive power, we need to know how rapidly the current in the windings needs to be changed. In order to allow for the penetration time of the feedback system through the shell this time should be much less than the shell time constant; we quite arbitrarily choose τ_f = 0.1 s, so that an emf of $LI_{max}/0.1$ volts is required. Numerically this is 13 V, representing a peak reactive power of order $V_{max}\ I_{max}$, equal to 1.3 MW/coil. There are 256 coils; assuming somewhat arbitrarily a 'duty factor' which represents an average reactive power in any coil equal to a quarter of the peak, this represents a total of less than 100 MVA, which is not unreasonable.

This is a very crude estimate; it is possible for example that, because of the time delay of the shell, or the existence of modes with growth time faster than the time constant of the shell, much higher currents than those quoted may be needed. On the other hand, our understanding is so vague that the problem may be less severe than the above figures suggest.

Ohmic loss in the feedback windings and shell have not been included in the overall energy balance in the reactor. The loss in the coils is of order $\frac{1}{2}j^2V/\sigma$, where V is the volume of the coils. The volume of a single coil is 0.01×11 m^3. Inserting values of $j = 10^7$ A/m^2 and $\sigma = 5 \times 10^7$ mho/m, and assuming that all of the 256 coils are carrying half the full current, this represents a total power of 14 MW. This figure represents a few percent of the reactor output, and is not embarrassing; the very tentative nature of the estimate must however be emphasised.

REFERENCES

1.* Hancox R and Spears W, Reversed Field Pinch Reactor Study: II Choice of Parameters, Culham Report CLM-R172, 1977.

2.* Hollis A A and Mitchell J T D, Reversed Field Pinch Reactor Study: III Preliminary Engineering Design, Culham Report CLM-R173, 1977.

3.* Bobbio S, Gasparini F, Martone R, Rostagni G, Reversed Field Pinch Reactor Study: IV Electromagnetic System, University of Padua Report UPee/7705, 1977.

4.* Bodin H A B, James T E and Newton A A, A Pulsed Fusion Reactor Based on a Toroidal Pinch, Proc BNES Conf on Nuclear Fusion Reactors, Culham, p255, 1969.

5. Butt E P, Feasibility Studies of Pulsed Toroidal Reactors, Ibid p268.

6. Bodin H A B, James T E, Newton A A and Rostagni G, Further Considerations of the Toroidal Diffuse Pinch Reactor. Proc IAEA Workshop, Culham 1974, p83.

7. Carruthers R, Some Considerations of the Toroidal Diffuse Pinch, Pulsed Fusion Reactors, Proc Erice Summer School 1974, Pergamon Press 1975, p277.

8. Bodin H A B, Butt E P, Carruthers R, James T E, Newton A A and Rostagni G, The Pulsed Reversed Field Pinch Reactor, Proc 5th IAEA Conference on Plasma Physics, Tokyo, p361, 1975.

9. James T E and Bodin H A B, Conceptual Design Problems in Future Reversed Field Experiments, Proc 9th Symposium on Fusion Technology, Garmisch-Partenkirchen 1976, p251.

10. Baker D A, Di Marco J N, Forman P R and Phillips J A, Preliminary Engineering Calculations for a Toroidal Reverse-Field Pinch Reactor, Los Alamos Report LA-UR-2338, 1975.

11. Hagenson R L, Preliminary Physics and Engineering Considerations for a Fusion Power Plant Based on Reversed-Field Z-pinch (RFZP) Confinement, Los Alamos Report LA-UR-76-2190, 1976.

12. Bodin H A B, Reversed Field Pinches, Proc 3rd Topical Conference on Pulsed High Beta Plasmas, Culham 1975, p39.

13. Butt E P, Cole H C, Dellis A N, Gibson A, Rusbridge M and Wort D, Conditions for Improved Stability in ZETA, Proc IAEA Conf on Plasma Physics, Culham 2, 751, 1966.

14. Robinson D C and King R E, Factors Influencing the Period of Improved Stability in ZETA, Proc 3rd IAEA Conf on Plasma Physics, Novosibirsk, 1, 263, 1969.

15. Taylor J B, Relaxation of Toroidal Plasma and Generation of Reverse Magnetic Fields, Phys Rev Letters 33, 1139, 1974.

16. Taylor J B, Relaxation of Toroidal Discharges, Proc 3rd Topical Conf on Pulsed High Beta Plasmas, Culham 1975, p39.

17. Coppi B, Greene J M and Johnson L J, Resistive Instabilities in a Diffuse Linear Pinch, Nuclear Fusion 6, 101, 1966.

18. Robinson D C, Tearing Mode Stable Diffuse Pinch Configurations. Culham Report CLM-P503, (1977).

19. Robinson D C, High β Diffuse Pinch Configurations, Plasma Physics 13, 439, 1971.

20. Robinson D C, Private Communication.

21. Furzer A S and Robinson D C, Effects of Equilibrium Flow, Radiation, and Resistive Walls on MHD and Tearing Modes in a Pinch, 7th European Conference on Controlled Fusion, Lausanne 1975, p114.

22. Sykes A and Wesson J A, Field Reversal in Pinches, 8th European Conference on Controlled Fusion and Plasma Physics, Prague 1977.

23. Bunting C A, Gowers C W, Ogawa K, Robinson D C, and Watts M R C, The Production and Sustainment of a Reversed Field Pinch, 8th European Conference on Controlled Fusion and Plasma Physics, Prague 1977.

24. Christiansen J P and Roberts K V, Evolution of the Reversed Field Pinch, Culham Report CLM-P484, 1977.

25. Baker D A, Davidson R C and Di Marco J N, Overview of the LASL Z-pinch Program, Proc High Beta Workshop, Los Alamos 1975, ERDA 76/108, 1976, p528.

26. Baker D A and Phillips J A, Pressure-balance Limitations in Z-pinches with Diffusion Heating, Phys Rev Lett 32, 202, 1974.

27. Butt E P and Newton A A, New Results from ZETA, Proc 3rd Topical Conference on Pulsed High Beta Plasma, Culham 1975, p425.

28. Yeung B C, Long J W and Newton A A, Reactor Burning Calculations for a Model Reversed Field Pinch, Ibid p575.

29. Aleskin V F and Romanov S S, Penetration of Quasistationary Magnetic Fields into a Conducting Chamber, 1 Uniform Magnetic Field, Sov Phys Tech Phys 18, 734, (1973).

*NOTE: A summary report of Refs 1-4 was presented at the IAEA Workshop on Fusion Reactor Design, Madison, Wisconsin, October 1977. This is reproduced as a Culham Report; A 600 MW(e) Reversed Field Pinch Reactor Study, Hancox R, Bagatin M, Baker L J, Bobbio S, Crawley H J, Davenport P A, Gasparini F, Hollis A A, Lawson J D, Martone R, Mitchell J T D, Newton A A, Reynolds P, Rostagni G, Shercliff J A and Spears W R. CLM-P501, (1977).

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