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Report

REVERSED FIELD PINCH REACTOR STUDY
II. Choice of Parameters

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Abingdon Oxfordshire

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A B S T R A C T

This report considers the choice of parameters for a Reversed Field Pinch reactor with a net electrical output of 600 MW. The design assumes pulsed operation without refuelling during the burn, ignition by ohmic heating alone and the use of resistive magnetic field windings. Burn calculations are described, showing that operation at low plasma temperature with controlled energy loss is required to obtain adequate energy multiplication, and indicating the restrictions necessary to obtain ignition by ohmic heating. A choice of free parameters is made to minimise the capital cost of a complete generating unit including reactor, generators and energy storage system, and the sensitivity of the cost to the initial assumptions is analysed. The optimisation leads to a reactor design of 1.75 m first wall radius, with an energy multiplication factor Q of 6.9.

The work described in this report forms part of a Reversed Field Pinch reactor study undertaken by the following members, whose contributions to the study are gratefully acknowledged:-

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1. INTRODUCTION

The Reversed Field Pinch is one of several plasma confinement systems which use magnetic fields in a toroidal configuration and which are thought to be suitable as the basis of a fusion reactor. Although the Reversed Field Pinch has been studied for many years, especially in the United Kingdom, its potential as a reactor has not been assessed in detail. Consequently a reactor study has been undertaken with the objective of developing a conceptual reactor design which will exploit most profitably the characteristics of the Reversed Field Pinch configuration. The initial stages of this study and preliminary conclusions are described in a series of reports, of which this is the second, covering the physical requirements,[1] the choice of parameters, the principles of the mechanical arrangement,[2] and the associated energy storage and transfer system.[3]

This report deals with the choice of parameters for a Reversed Field Pinch reactor. Although a wide range of parameters is conceivable within the physical constraints, and there will be no unique reactor design, it is necessary to make a specific choice if engineering studies are to proceed in detail. The resulting conceptual design should show the characteristics which are typical of this reactor and allow conclusions to be reached concerning the system as a whole and its various components, subject to uncertainties due to the present limited understanding of the plasma physics.

Whilst the choice of most parameters must be made within well defined constraints the choice of others, such as the net output power of the reactor, are to a greater extent arbitrary, and since these exert a strong influence on the design as a whole the reasons for these choices are dealt with in detail in Section 2. The time dependent plasma parameters during the heating and burn phases of the operating cycle are considered in Section 3, and lead to further constraints on the reactor design. The choice of free parameters is determined by the desire to obtain the lowest reactor costs or highest efficiency and these criteria are applied in Section 4, to obtain the most attractive design. A sensitivity analysis of the results is performed to show which of the constraints and choices are most important. In Section 5 parameters are chosen which will form the basis of further engineering studies.

Comparing the chosen parameters with those proposed in previous studies of Reversed Field Pinch reactors shows that the minor radius is smaller than the value of 3 m used in several Culham studies,[4,5] and closer to the value of 2 m in a recent study at Los Alamos.[6]

2. INITIAL DECISIONS

Two general guidelines have been adopted in defining the parameters for the reactor study. These are:-

i) That the reactor should use to best advantage its distinctive features or strong points.

ii) That, where no other criterion is available, the choice should lead to characteristics which are different from those found in existing design studies.

The first of these guidelines leads to several choices which are dealt with below, and is designed to show this particular confinement system in its best light. The second guideline derives from the general proposition that one aim of conceptual reactor studies is to explore the whole range of alternatives possible in fusion systems, and seeks to avoid the natural tendency for such studies to take a common form at too early a stage in their development. Whilst there will be a temptation to assess this particular design as 'better' or 'worse' than others, it should more correctly be seen simply as a step in the process of exploring more fully the form and feasibility of a fusion reactor.

The characteristics of the Reversed Field Pinch and progress in the experimental and theoretical understanding of its behaviour have been reviewed elsewhere.[1] The most important plasma physics aspects which are relevant in the present parameter survey are:-

i) The high value of the ratio of plasma pressure to magnetic pressure, β, obtainable, resulting in lower levels of magnetic field being required than in present Tokamak reactor designs.

ii) The need for a flux conserving shell close to the plasma to allow the toroidal magnetic field profile to be established and to maintain plasma stability.

iii) The possibility of ignition by ohmic heating alone, due to the high current density in the plasma and to the current configuration.

iv) The transient nature of the required plasma and magnetic field profiles, which decay at a rate consistent with classical plasma resistivity.

v) The unrestricted choice of toroidal aspect ratio.

An early decision in this study was to consider a pulsed unrefuelled reactor. This choice was made partly in the expectation that the decay of plasma and magnetic field profiles would only allow the configuration to be maintained for a few burn times, and partly because no means of refuelling the reactor has yet been shown to be possible in principle. The first reason appears less certain now than when the decision was taken, and there is theoretical and experimental evidence to suggest that a mechanism exists to sustain the required profiles.[7,8] The second reason is still valid. The choice of unrefuelled operation, which leads to cycle times of 20 to 50 seconds, will be seen to have substantial repercussions on the choice of parameters and the acceptability of the final reactor design.

A consequence of the high value of β obtainable, together with the decision to use pulsed operation, is that the ohmic dissipation in resistive (i.e. not superconducting) magnetic field windings is not intolerable. Whereas in a Tokamak reactor it only appears practical to use superconducting windings to provide the toroidal magnetic field, it is possible to consider a Reversed Field Pinch reactor without superconducting windings. It was therefore decided to use normal copper windings in this study. This choice is justified by the second guideline above, and by the technical difficulty of building superconducting windings for pulsed operation with short current rise-times.

Another consequence of the decision to consider a pulsed reactor is that the use of auxiliary heating for ignition, such as neutral injection or radio frequency heating, is unattractive. The short burn time implies that the plasma must be ignited quickly, so that very high heating powers are required. The capital cost of the equipment required to achieve this would probably be excessive. The low efficiency of the auxiliary heating methods would also be a more serious disadvantage in pulsed systems, compared with quasi-steady-state reactors in which they are operating for only a negligible fraction of the operating cycle. Since a possible advantage of the Reversed Field Pinch reactor is that ignition may be obtained by ohmic heating alone, this has been assumed in the present study.

A final decision, taken at the beginning of the study, concerned the unit size of the reactor. The majority of Tokamak reactor designs have a net electrical output in the range 1000 to 2500 MW(e) due to the requirements for a large minor radius for adequate plasma confinement at reasonable levels of magnetic field and economic wall power loading. Whilst such sizes may be a reasonable extrapolation of current trends in power station construction, it is open to question whether such trends will continue in the future and whether fusion stations of this size are desirable. It will certainly be necessary to construct much smaller units when fusion reactors are first introduced and before their reliability and acceptability has been demonstrated. For this study, therefore, a smaller unit size was proposed and the parameters are presented for a reactor with 600 MW(e) net output.

3. PLASMA BURN CALCULATIONS

Two important questions relating to the plasma behaviour are the conditions under which the maximum energy is released in an unrefuelled burn and under which ignition by ohmic heating might be achieved. These questions have been studied using a simple point model of the plasma which includes ohmic and alpha-particle heating, bremsstrahlung and synchrotron radiation cooling, and the loss of energy or particles as characterised by a confinement time. Only D-T reactions were included and electron and ion temperatures were assumed to be identical. Following Yeung, Long and Newton,[9] the plasma resistivity was adjusted by a correction factor $F(\theta,\beta_\theta)$ to allow for the effects of the helical distribution of current associated with actual plasma profiles. In the results quoted the alpha-particles are assumed to be instantaneously thermalised, but more extensive calculations, including a more accurate representation of the alpha-particle energy spectrum, gave similar results.

3.1 Burn cycles

Various forms of unrefuelled plasma burn have been examined, and their effectiveness expressed as an energy multiplication factor Q_b, which is the ratio of the useful thermal energy extracted from the reactor divided by the net electrical energy required to heat and confine plasma. An alternative parameter for comparison is the fractional recirculating energy ε_b, which is the proportion of the electrical power generated which must be recirculated to maintain the cycle, and must be substantially less than unity in a power producing system. These parameters are related since $\varepsilon_b = 1/Q_b\eta$, where η is the efficiency of conversion of thermal to electrical power which is taken to be 40% in this study. Consequently Q_b must be greater than 2.5 for net power production. In the burn calculations the effects of losses in the system external to the reactor such as energy storage and transfer losses are ignored, with the result that the calculated values of Q_b and ε_b are optimistic.

The alternative burn cycles have been evaluated in a reactor of 1.75 m first wall radius and with the constraint that the mean power loading of the first wall averaged over the full cycle, $\bar{P}_w$, is fixed at 2.2 MW/m^2. This wall loading includes the nuclear power released in the plasma and blanket, taken to be 20 MeV per reaction, and the plasma and magnetic energy deposited on the wall at start-up and at the end of the burn. Unless otherwise stated, the fractional burn up is assumed to be 30% and the cycle time includes a constant off-time between burns to allow for evacuation of the reactor and introduction of new fuel. Other parameters are fixed at the levels chosen for the parameter surveys discussed later.

The simplest form of burn is that in which the plasma temperature is allowed to increase without control, with the plasma current chosen such that the limiting value of poloidal plasma pressure ratio, β_θ, is only reached at the end of the burn. The variations of plasma temperature, T, and wall loading, P_w, during such a burn are illustrated in Figure 1. For a plasma with equal proportions of deuterium and tritium, Q_b is 1.7.

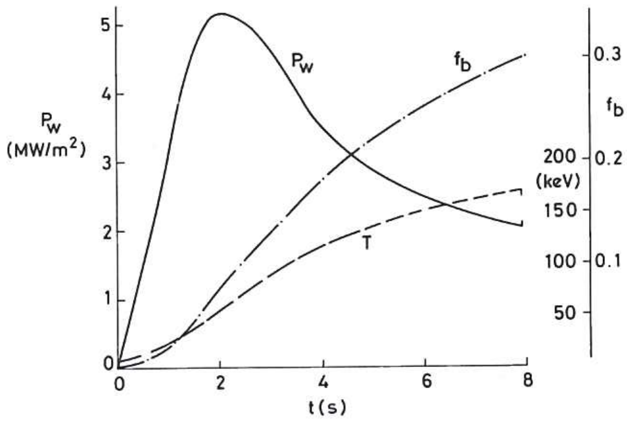

Fig.1 Plasma burn at constant density with unlimited temperature. Mean wall loading averaged over whole cycle is 2.2MW/m^2 with plasma radius of 1.75m.

Several factors influence the energy multiplication in a simple burn, such as the ratio of deuterium to tritium in the plasma,[10] the level of radiation losses, or the off-time between burns. None of these can be varied, however, to the extent that net power production is achieved. A further possibility is to assume a mechanism for limiting β_θ during the burn, as would happen if an enhanced particle loss occurred when the critical value of β_θ was exceeded. This mechanism has been shown to lead to a significant energy gain in a refuelled reactor,[9] but in the unrefuelled system considered here only increases Q_b to 2.4.

The low values of energy multiplication obtained in the examples above are mainly due to the high plasma temperatures attained, since β_θ increases linearly with temperature but the reaction rate passes through a maximum at a temperature of around 75 keV. An essential requirement for an efficient burn cycle, therefore, is a mechanism to control the plasma temperature. The variations of β_θ and P_w during a burn which assumes such a mechanism to maintain a constant temperature of 10 keV, are shown in Figure 2. The energy multiplication is 8.8 (ε_b = 0.28). These values are lower than those obtained by Yeung,[11] mainly because of the lower value of β_θ assumed and the inclusion of the magnetic energy lost in establishing the reversed field profile.

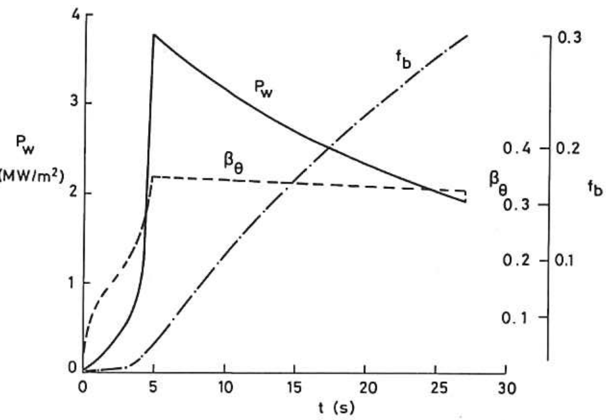

Fig.2 Plasma burn at constant density with temperature limited to 10keV. Mean wall loading averaged over whole cycle is 2.2MW/m² with plasma radius of 1.75m.

The plasma temperature control mechanism assumed above implies energy loss without any loss of particles. In practice both energy and particle losses are likely to be present and therefore further burn calculations have been made including energy and particle confinement times τ_e and τ_p which are assumed to vary in such a way that a constant ratio τ_e/τ_p is maintained. In all cases the plasma is heated to a temperature of 10 keV, after which the temperature is controlled to maintain the critical value of β_θ during the burn. (The value of Q_b obtained in this way with infinite particle confinement is slightly higher than in the constant temperature burn.) The fractional burn-up was also adjusted to optimise the energy multiplication, maintaining the constraint of a fixed first wall power loading averaged over the full cycle. The dependence of the energy multiplication on the ratio τ_p/τ_e is shown in Figure 3, together with the fractional burn-up for optimum multiplication and the plasma temperature at the end of the burn. A substantial reduction in Q_b is already apparent with $\tau_p/\tau_e \sim 10$, and ratios nearer 100 would be preferred. The corresponding particle confinement times required are such that the product $n\tau_p$ would be 4 x 10^{21} and 6 x 10^{22} $m^{-3}s$.

The control processes embodied in the above calculation do not correspond to any known mechanism. Enhanced radiation losses would only be appropriate if the required impurity could be injected after the heating phase,[10] and suffer from the disadvantage that they lead to a thermally unstable situation at this low temperature. An alternative means of stabilizing the plasma temperature, used in the Los Alamos reference Theta Pinch reactor,[12] is the expansion of the magnetic field during the burn to give adiabatic cooling and efficient energy recovery.[6] This method is not considered appropriate to a Reversed Field Pinch reactor since plasma stability depends on the close proximity of the plasma to a stabilizing wall so that no space is available for the expansion. There remains the possibility that instabilities associated with operation at the critical β_θ limit will enhance thermal losses from the plasma in such a way as to produce a natural limit to the temperature at a convenient value.

It is concluded that for adequate energy multiplication in a pulsed, unrefuelled, system the burn must take place at a relatively low plasma temperature. The mechanism by which the plasma temperature might be controlled is at present unknown, but it must primarily affect the loss of energy rather than particles and the ratio between particle and energy confinement times must be at least 10.

3.2 Heating times

An important advantage claimed for the Reversed Field Pinch configuration relative to the Tokamak is that the higher toroidal current allows the plasma to be heated to ignition by ohmic heating alone. If this advantage is to be maintained it places restrictions on the parameters allowed for the reactor. These restrictions can be evaluated with the same point model used for the plasma burn calculations described in the previous section.

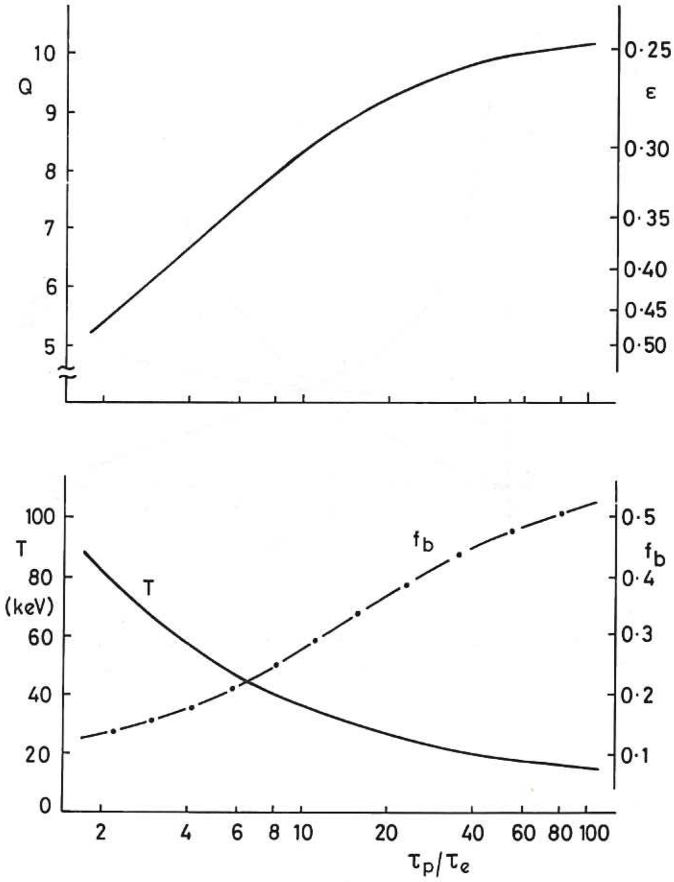

Fig.3 Energy multiplication, Q, and recirculating power fraction, ε, in a plasma burn at constant β_θ with given ratio of particle confinement time, τ_p, to energy confinement time, τ_e. The lower curves show the fractional burn-up, f_b, for optimum multiplication and the plasma temperature, T, at the end of the burn.

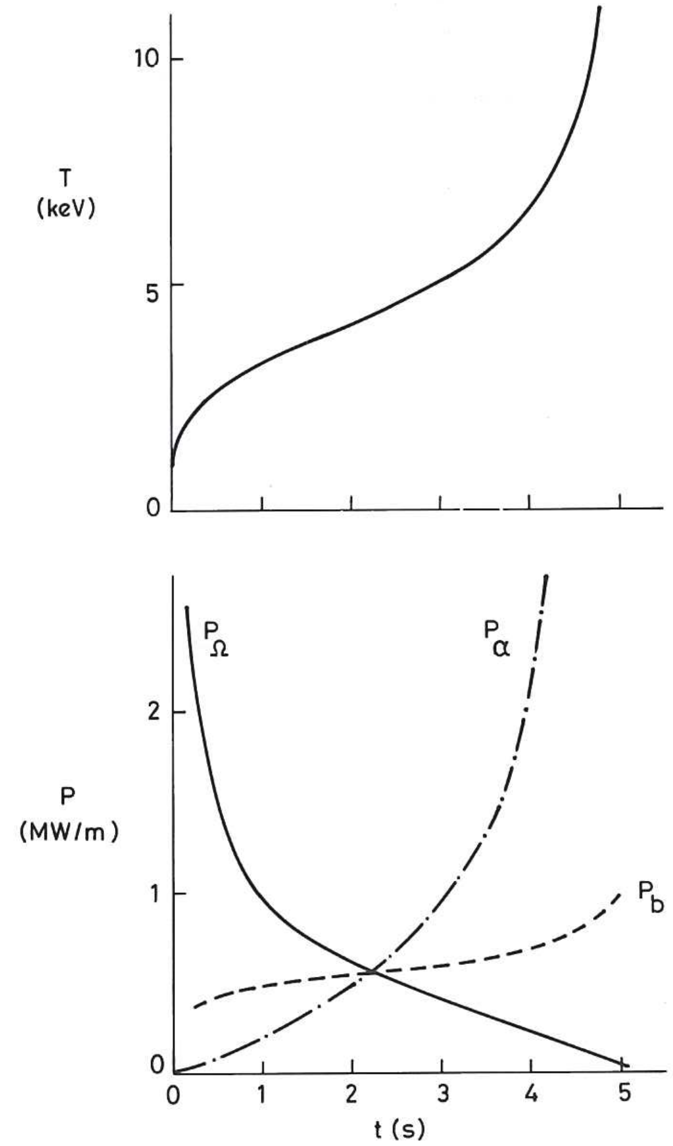

Fig.4 Heating phase of a plasma burn. The upper curve shows the plasma temperature, and the lower curves the power inputs from ohmic heating, P_Ω, alpha heating, P_α, and the radiation loss due to bremsstrahlung, P_b.

The increase of plasma temperature during the heating phase is illustrated in Figure 4, which also shows the variations of power input to the plasma from ohmic heating and nuclear reactions and the loss due to bremsstrahlung. In the following calculations all thermal conduction and particle losses have been ignored. The times required to reach a plasma temperature of 10 keV, τ_h, are shown in Figure 5 for reactors with several different first wall radii and mean neutron wall loadings, $\bar{P}_n$ (defined by the product of the primary neutron current per unit wall area, the neutron energy of 14.1 MeV and the ratio of burn to cycle time). If the energy transfer processes are written in terms of the toroidal plasma current, I, it is found that these heating times can be expressed more simply in the form:

$$\tau_h = f(I).a^2$$

where a is the first wall radius, and that the function f(I) goes to infinity at a current which depends only on the conditions chosen for the plasma burn. For the conditions chosen in Sections 4.1 and 4.2 (T_b = 10 keV, β_θ = 0.35) this current is 30 MA, although this value is sensitive to the value assumed for the correction factor $F(\theta,\beta_\theta)$ and the plasma profile. This upper limit on the current can also be expressed as an upper limit on the line density N of 4.3 x 10^{21} m^{-1} or a maximum value of the product $a^3 \hat{P}_n$ where $\hat{P}_n$ is the neutron wall loading when the burn temperature is attained. This last form is equivalent to the requirement given by Lawson,[1] and for a mean neutron wall loading of 1.5 MW/m^2 corresponds to a first wall radius of 3.5 m.

From the point of view of efficient operation it is important that the heating time, τ_h, should be short compared with the burn time, τ_b, otherwise power is wasted maintaining the field configuration without a corresponding gain in nuclear output. Since the burn time scales with a^2, the ratio τ_h/τ_b is also a function of the plasma current as shown in Figure 6. This ratio should generally be well below unity, so that the limitation on plasma current is in practice more restricted than suggested above, and in an optimised system should not exceed 20 to 25 MA. At a mean wall power loading of 2 MW/m^2 the corresponding maximum first wall radius lies in the range 2.0 to 2.5 m.

The effect of plasma contamination by impurities has also been studied. Assuming fully stripped oxygen as the impurity the effect of increasing impurity levels, as measured by the effective Z of the plasma ($Z_{eff} = [\Sigma_j n_j Z_j^2]/n_e$), is illustrated in Figure 7. For radii up to 2 m and mean first wall loadings up to 2 MW/m^2, a Z_{eff} of up to 1.5 appears acceptable.

In all these calculations it has been assumed that the plasma is heated at its working density. Schemes for easier ignition at low density have been postulated but are not considered here since they require the injection of fuel after ignition and this option has been excluded in the present study.

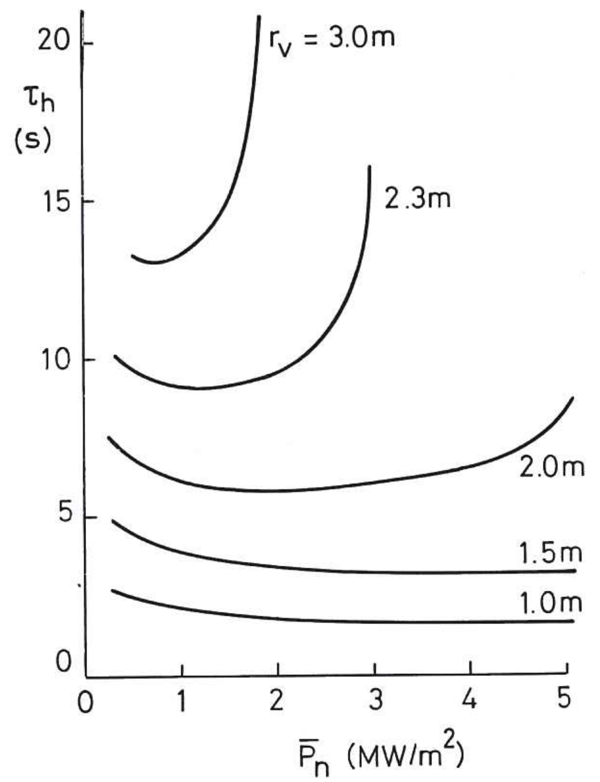

Fig.5 Heating times, τ_h, to reach a plasma temperature of 10keV in reactors of different plasma radii, r_v, and mean neutron wall loading, $\bar{P}_n$. In all cases the plasma pressure ratio β_θ is 0.35 at a temperature of 10keV.

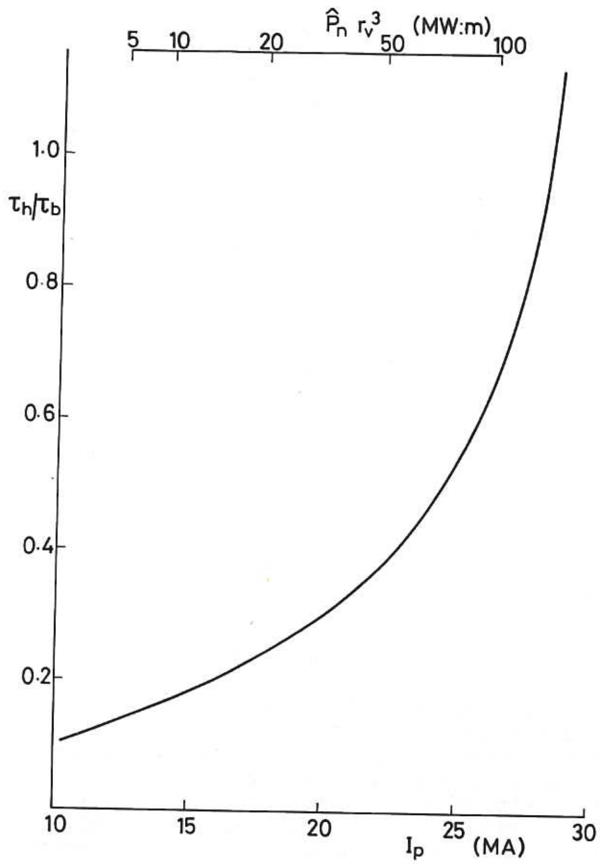

Fig.6 Ratio of heating time, τ_h, to burn time, τ_b, in reactors of different plasma radii, r_v and peak wall loading, $\hat{P}_n$. In all cases the plasma pressure ratio β_θ is 0.35 at the burn temperature of 10keV.

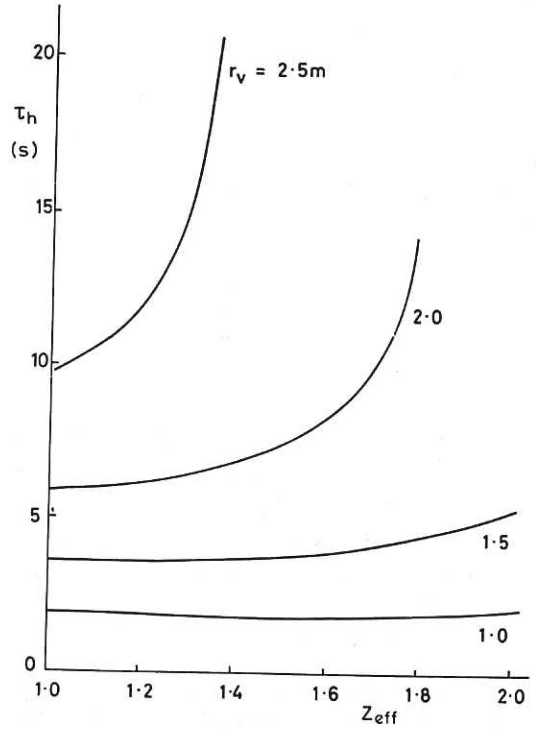

Fig.7 Effect on the heating time, τ_h, of plasma impurities expressed as an effective Z at various plasma radii, r_v. The mean wall loading is constant at $2 MW/m^2$.

In conclusion, in an unrefuelled reactor without auxiliary heating there is a maximum first wall radius above which the ohmic heating is insufficient to overcome radiation losses. The value of this limit is dependent on the evaluation of effects due to current density and plasma density profiles, but is thought in practice to be about 2.0 to 2.5 m.

4. PARAMETER SURVEY

The ultimate aim of a reactor design study will be to obtain the lowest possible electricity generating costs, combined with required flexibility of operation and adequate safety. During the early stages of a reactor study, however, cost factors and their scaling are not well defined and can only be used as a rough guide to the choice of parameters. For several fusion reactor systems it has been found that an alternative basis for choice is the energy multiplication factor Q of the whole system, since the cost of both the reactor and most of the conventional equipment depends on the gross power generated. A simple argument, detailed in Appendix I, suggests that a minimum acceptable level of Q is in the range 15 to 20, corresponding to a fractional recirculating power of 17 to 12.5% at 40% efficiency of conversion of thermal to electrical power. In the following discussion both the energy multiplication factor and the station capital cost, C, will be quoted, but where increasing Q leads to increasing C, the cost will generally be considered the better indication.

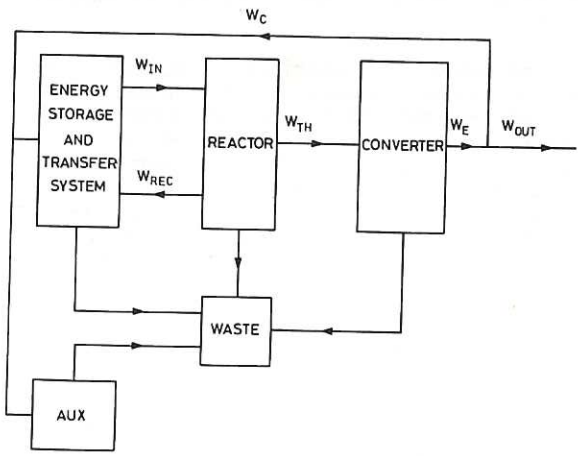

Fig.8 Simplified energy circulation diagram.

To quantify Q and C an energy flow diagram has been constructed which includes all the energy transfers during a complete cycle of operation. The full diagram and energy balance equations are given in Appendix II, and a simplified version of the diagram is given in Figure 8. The energy, W_{IN}, supplied to the reactor provides magnetic confinement of the plasma, overcomes losses during the setting-up of plasma current, and maintains coil currents against resistive losses. Some of this energy, W_{REC}, is recovered directly at the end of the pulse. The reactor multiplies the net input energy via thermonuclear reactions, and the heat produced, W_{TH}, is used in a conventional boiler-generator thermal conversion system. The gross electrical output of the system, W_E, must be partly recirculated, W_C, to overcome losses and provide the net energy input to the reactor. What remains, W_{OUT}, represents the net electrical output to the grid. Other losses occurring in the plant which are not recovered into the thermal cycle are lost to the environment. The system Q is defined as W_{TH}/W_C and will be lower than Q_b ($= W_{TH}/[W_{IN} - W_{REC}]$) used in the previous section.

The basis on which the capital costs have been estimated is the same as that used previously for the Culham Mk I and Mk II design.[13] The major reactor components such as the blanket, shield, windings, etc are costed according to their area or volume and the heat transport and generating equipment according to its thermal or electrical power rating. The quoted costs are for a complete generating unit including reactor, boiler, turbine, generator, auxiliaries, and building, but excluding customers on-costs or interest during construction, and assume quantity production for a series of commercial power stations. Replacement sections of breeding blanket are included, discounted to the date of construction and used to replace sections which have reached a total radiation dose of 4×10^{26} neutrons/m^2. This leads to blanket replacement about every five years throughout the 25 year reactor life. The cost data is based on 1976 values and has previously been used to estimate the capital costs of other tokamak reactor designs. and showed good agreement[14] with the published cost of the Princeton reactor.[15] Where possible the data has also been checked with figures used in a study of the cost optimisation of the JET experiment.[16]

The constraints described in Sections 2 and 3 have led to the consideration of a pulsed, non-refuelled, reactor of 600 MW(e) net power output in which the plasma burn is controlled at a constant temperature. The further constraints imposed by plasma physics and engineering requirements will be considered, after which the remaining free parameters will be chosen. The sensitivity of the final design to these choices will then be assessed.

4.1 Assumptions and Constraints

Various possible profiles of plasma pressure and magnetic field have been discussed by Robinson[17] and by Yeung.[11] Whilst these have been tested for MHD stability they are not entirely satisfactory in that they do not provide a vacuum region between the plasma and flux conserving boundary, which will be essential in a reactor to allow for the space occupied by the first wall and its cooling channels.[2] For the purposes of this reactor study a profile proposed by Robinson has been used, with the position of the flux conserving wall adjusted to give a pinch parameter, θ, of 2 and a reversal parameter, F, of -0.75. The limiting central plasma pressure for which this profile is expected to be stable corresponds to a β of 0.24 or a β_θ of 0.35 and these values are assumed in the study. In order to calculate the fractional burn-up of fuel the plasma temperature is assumed to be constant across the profile and the density to fall to zero at the wall.

Of the engineering constraints assumed, the most important is the maximum allowable first wall power loading. Tokamak reactor studies show that high wall loadings lead to the most economic designs,[14] but that the limit will probably be set by radiation damage and the frequency with which the wall will have to be replaced. These Tokamak studies, however, implicitly or explicitly assume a divertor to reduce the direct interaction of the plasma with the first wall, whereas in the Reversed Field Pinch considered here no such divertor is included. The assumed mechanism for the stabilization of plasma temperature implies that the reactor energy imparted to the α-particles (3.5 MeV/reaction) must be lost to the first wall. Furthermore, the short burn time in the Reversed Field Pinch may lead to failure due to fatigue. Thus lower wall loadings than the 3.0 to 6.0 MW/m^2 assumed in Tokamak reactors must be used. In this study a mean neutron wall loading of 1.5 MW/m^2 (based on 14.1 MeV/reaction) is assumed, which corresponds to a mean wall power loading of 2.2 MW/m^2.

Further engineering constraints include the physical dimensions of the nuclear blanket and shield, the resistivity of the magnetic field windings, the allowable flux density in the core, and the extent to which space must be made available between components for access. The blanket and shield should be as thin as possible to minimise the circulating magnetic energy,[18] and a value of 1.6 m is considered possible in a practical structure which includes a passive flux conserving shell close to the first wall, active stabilizing windings at the back of the blanket, and space for heat transport ducting. A resistivity of 2.25×10^{-8} Ωm for the magnetic field windings, which allows operation at 100^{o}C, and a copper packing factor of 60% in the windings, with a 50% winding space factor, are assumed. A maximum flux density of 1.4 Tesla, corresponding to a flux density swing of 2.8 Tesla, is allowed in the iron cored designs, with access space between the core structure and field windings of 1.0 m. For the air cored designs a maximum flux density of 4.0 Tesla is permitted.

Parameters which will affect the circulating energy are the efficiency with which energy is transferred to and from the energy store and the proportion of kinetic and magnetic energy recovered directly from the reactor. Based on previous studies,[10,12] a switching efficiency of 95% for each transfer to and from the store is assumed. The magnetic energy outside the flux conserving shell is assumed to be fully recoverable, but the magnetic and kinetic plasma energy inside the shell will only be partly recovered and it is assumed that 40% is directly returned to the store.

The cycle time is the sum of the times required to induce the plasma current, heat the plasma to its operating temperature, burn the fuel to a specified fractional burn-up, reduce the current to zero, and an off-time between burns. The heating and burn times are taken from calculations similar to those described in Section 3, and imply ignition by ohmic heating alone. The plasma current is assumed to have rise and decay times of 0.5 seconds. The off-time is fixed at 7 seconds, which allows at least 5 seconds for evacuating burnt fuel from the reactor vessel and refilling with new fuel. Preliminary calculations indicate that this will be sufficient with reasonable allowances for pumping port space.

4.2 Survey of free parameters

A list of parameters remaining fixed in the optimisation is shown in Table 1. The parameters which still remain free include the plasma temperature during the burn, the required fractional burn-up, and the physical dimensions of the reactor including the minor and major radius and the thickness of the windings. A choice also exists between an iron cored or an air cored system and the study has been carried through for both of these options. In the examples given below the reactor dimensions have been taken as unspecified variables within the constraints imposed by the parameters in Table 1.

TABLE 1

Parameters fixed during optimisation (Iron core figures in parentheses)	
Pinch parameter, θ	2.0
Field reversal parameter, F	- 0.75
Poloidal beta, β_θ	0.35
Mean neutron wall loading during cycle, $\bar{P}_n$ (MW/m^2)	1.5
Energy output per reaction, Q_R (MeV)	20.0
Thermal efficiency, η_{TH}	0.4
Unit size, P_{OUT} (MW$_e$)	600.0
Energy transfer efficiency, c	0.95
Kinetic energy recovery efficiency, a	0.4
Magnetic energy recovery efficiency, b	0.4
Off time, τ_{off} (secs)	7.0
Current rise time, τ_r (secs)	0.5
Maximum core flux density, B_{cmax} (T)	4.0 (1.4)
Blanket thickness, Δr_b (m)	1.6
Winding packing factor, f_w	0.3
Fraction of gross electrical output required for auxiliaries, d	0.05
Blanket neutron dose at replacement (m^{-2})	4.0 x 10^{26}
Space between core structure and coils (m)	0.0 (1.0)

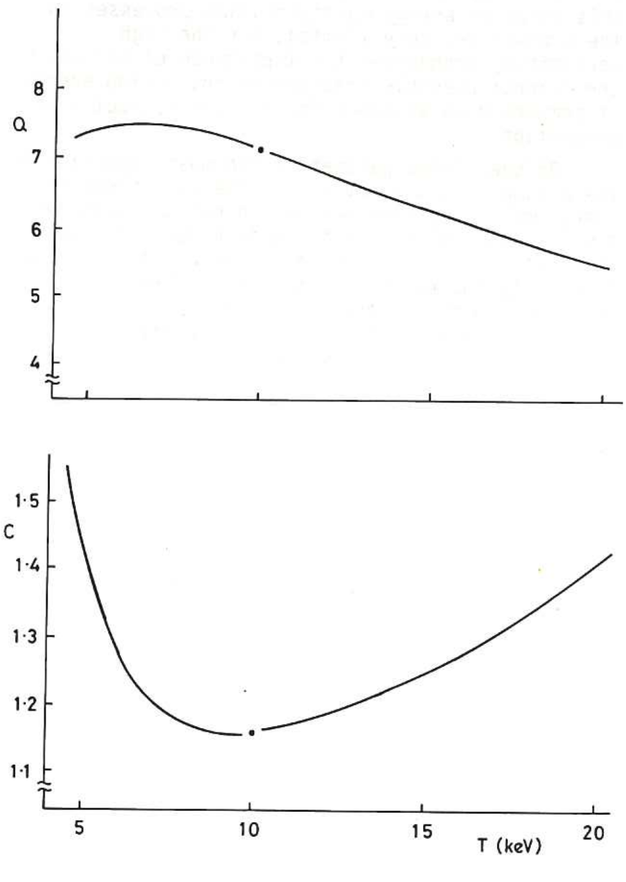

Fig.9 Energy multiplication factor, Q, and nominal capital cost per kilowatt, C, of a 600MW(e) unit with an air cored reactor, as a function of the plasma burn temperature.

The influence of plasma temperature on energy multiplication and capital cost is illustrated in Figure 9. As in most β limited systems the optimum temperature is around 10 keV, which is lower than the temperature giving the minimum value of the required product of density and confinement time $n\tau$ for a self-sustained reaction and much lower than the temperature at which thermally stable operation is obtained if energy losses are neo-classical.[19] An operating temperature of 10 keV has beeen chosen for this study.

The effect of the specified burn-up of fuel is shown in Figure 10. At low values the energy released is small compared with the energy expended in establishing the plasma, and at high values the reaction rate falls due to dilution of the fuel with reaction products. A broad optimum in the range 0.3 to 0.5 is obtained and the lower value is taken in this study in the expectation that impurity build-up in the plasma due to interactions with the reactor wall will in practice make higher values less attractive.

The choice of the current density in the windings during the burn is an example in which the requirements for maximum Q and minimum capital cost lead to differing conclusions. The energy multiplication factor can be significantly improved by the use of very low current densities, but the increased cost of the windings makes this

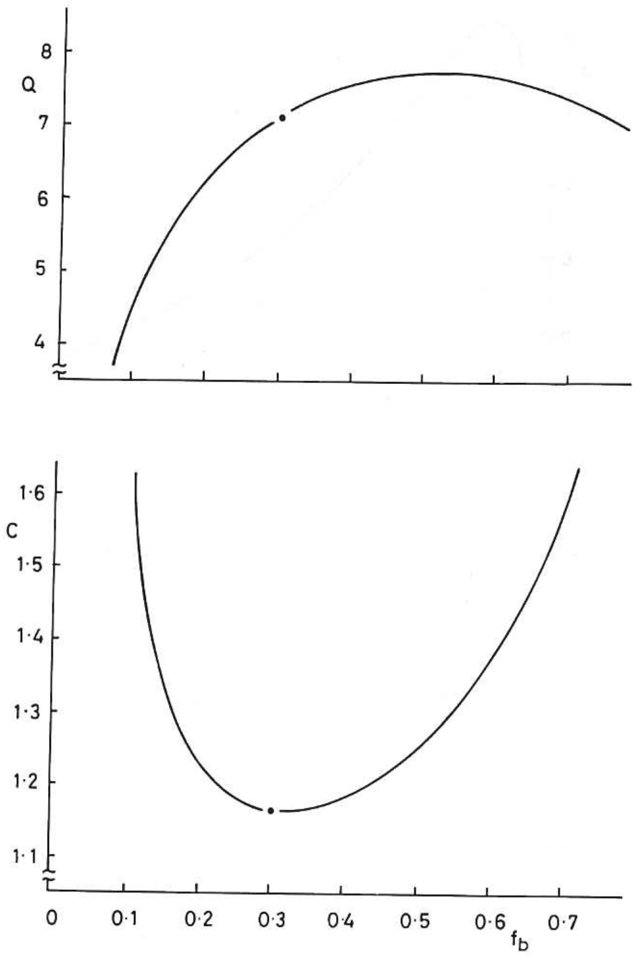

Fig.10 Energy multiplication factor, Q, and nominal capital cost per kilowatt, C, of a 600MW(e) unit with an air cored reactor, as a function of the fractional burn-up of fuel.

uneconomical, as illustrated in Figure 11. Moreover, at very low current densities the constraints on core flux density and net power output together with the increased winding thickness combine to force the first wall radius to low values and reduce the energy multiplication factor Q. The current densities in the poloidal field and toroidal field windings have also been varied independently, but the choice of a common value of 2.5 MA/m^2 results in near-optimum cost.

For an air cored reactor the choice of current density during the burn is complicated by the fact that the current in the poloidal field winding need not necessarily equal the plasma current. Reduced ohmic dissipation in the windings during the burn would be advantageous, but the cost savings might be counteracted by the increased cost of energy storage to provide the increased current and stored energy in the core during the biased phase before the rise of plasma current. The ratio of primary current to plasma current during the burn has been varied, and it is found that these effects roughly balance. For the present parameter survey, therefore, this ratio has been taken to be unity although it should be noted that a more detailed consideration of the energy storage and transfer system may lead to a different conclusion.

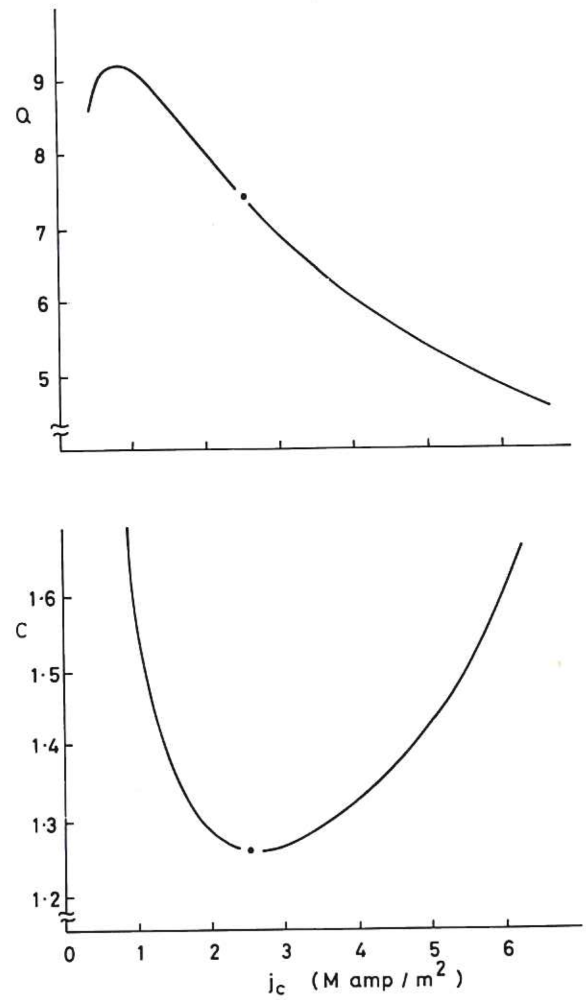

Fig.11 Energy multiplication factor, Q, and nominal capital cost per kilowatt, C, of a 600MW(e) unit with an iron cored reactor, as a function of the current density in the poloidal and toroidal field windings.

The parameters taken above lead to iron and air cored systems with minor radii of 1.70 and 2.15 m and aspect ratios of 9.0 and 5.7 respectively. The iron cored system has a higher cost, mainly because the cost of the iron transformer far outweighs the increased cost of the energy storage system for the air cored reactor. On the other hand, the iron cored system has a higher energy multiplication factor. In both cases the energy multiplication factor is lower than desirable and the capital cost high. Further parameters for the two alternatives are shown in Table 2.

The energy storage and transfer requirements for the optimum reactor parameters are shown in Table 3. Resistive losses in the windings account for nearly half the circulating energy and are twice as large as the net magnetic energy transfer. The size of energy store is determined by the maximum instantaneous energy that must be available for transfer during the pulse. This peak demand occurs during the premagnetisation phase for the air cored reactor and during the current rise for the iron cored reactor.

4.3 Sensitivity of Choices

The effect upon Q, and C, of small variations of any parameter, P, have been calculated and are listed in Table 4 in rough order of importance with respect to the sensitivity of cost. The sensitivities are defined as $(\Delta Q/Q)/(\Delta P/P)$ and $(\Delta C/C)/(\Delta P/P)$. It should be noted that where the signs of the two sensitivities are the same it is not possible to improve both Q and C simultaneously.

The most sensitive assumption is seen to be the useful energy released per fusion reaction, which was taken to be 20 MeV on the basis of previous studies. The possibilities of increasing this value by energy multiplication processes in the blanket are very limited, but the high sensitivity emphasises the importance of collecting the highest possible fraction of the fusion energy at temperatures suitable for efficient electricity generation.

Of the plasma parameters the most important is the plasma pressure ratio β_θ. The variations of energy multiplication factor and capital cost over a wider range of β_θ are shown in Figure 12, from which it is seen that values below about 0.25 lead to rapidly increasing costs. It is therefore important to future experiments to establish the practically achievable level of β in stable profiles which are usable under reactor conditions.

TABLE 2

Parameters for optimum air and iron cored Reversed Field Pinch reactors	Air Core	Iron Core
Net electrical output (MW)	600	600
Gross thermal output (MW)	2300	2260
First wall radius (m)	2.16	1.70
Major radius (m)	12.3	15.4
Aspect ratio	5.7	9.0
First wall mean neutron loading (MW/m^2)	1.5	1.5
Core flux density (T)	4.0	1.4
Plasma line density ($m^{-1} \times 10^{-21}$)	2.7	1.9
Plasma pressure ratio, β_θ	0.35	0.35
Plasma current (MA)	22	18
Burn time (sec)	30	27
Cycle time (sec)	45	40
Energy multiplication factor, Q	7.1	7.4
Recirculating power fraction, ε	0.35	0.34
Estimated capital cost (£/kW(e))	1170	1260

TABLE 3

Energy storage and transfer requirements for alternative air and iron cored Reversed Field Pinch reactors	Air	Iron
Components of circulating energy:-		
Magnetic energy input (GJ)	16.5	7.5
(Start-up losses)	(1.2)	(1.3)
Thermal energy to plasma (GJ)	0.4	0.4
Energy transfer and switching losses (GJ)	2.5	1.2
Dissipation in windings (GJ)	6.6	5.8
Auxiliary systems (GJ)	2.1	1.8
Magnetic/Kinetic energy recovered directly (GJ)	- 13.3	- 4.6
TOTAL (GJ)	14.7	12.1
Components of stored energy contributing to store size:-		
Initial toroidal magnetic field (GJ)	1.6	-
Core bias magnetic field (GJ)	15.7	-
Transferred to reactor at start-up (GJ)	-	8.6
Dissipation in field windings (GJ)	6.6	5.8
TOTAL (GJ)	23.9	14.5
Thermal energy from blanket/cycle (GJ)	105.0	89.8
Electrical energy generated/cycle (GJ)	42.0	35.9
Net output energy/cycle (GJ)	27.3	23.9

TABLE 4

Sensitivity of capital cost per kilowatt, C, and energy multiplication factor, Q, to changes in the assumed parameters.

Parameter	Air Core C	Air Core Q	Iron Core C	Iron Core Q
Energy per reaction, Q_R	- 0.84	0.52	- 0.79	0.50
Plasma pressure ratio, β_θ	- 0.66	0.58	- 0.67	0.57
Plasma pinch parameter, θ	0.45	- 0.40	0.46	- 0.35
Mean wall loading $\bar{P}_n$	- 0.30	0.04	- 0.24	0.06
Resistivity of windings, ρ	0.27	- 0.40	0.26	- 0.45
Blanket and shield thickness, Δr_b	0.26	- 0.17	0.26	- 0.16
Winding packing factor, f_w	- 0.17	0.09	- 0.18	0.08
Net output power, P_{OUT}	- 0.18	0.21	- 0.22	0.24
Neutron dose at which blanket replaced	- 0.11	-	- 0.10	-
Switching losses (1-c)	0.09	- 0.16	0.04	- 0.09
Auxiliary power fraction, d	0.06	- 0.13	0.06	- 0.13
Plasma current rise time, τ_r	0.05	- 0.05	0.06	- 0.05
Ratio of primary to plasma current, x	- 0.04	- 0.11	-	-
Working space behind windings, Δr_s	-	-	0.04	- 0.03
Efficiency of energy recovery, a	- 0.03	0.08	- 0.03	0.09
Maximum core flux density, B_{cmax}	0.01	0.03	- 0.12	0.10
Off time, τ_{off}	0.01	0.04	0.02	0.05

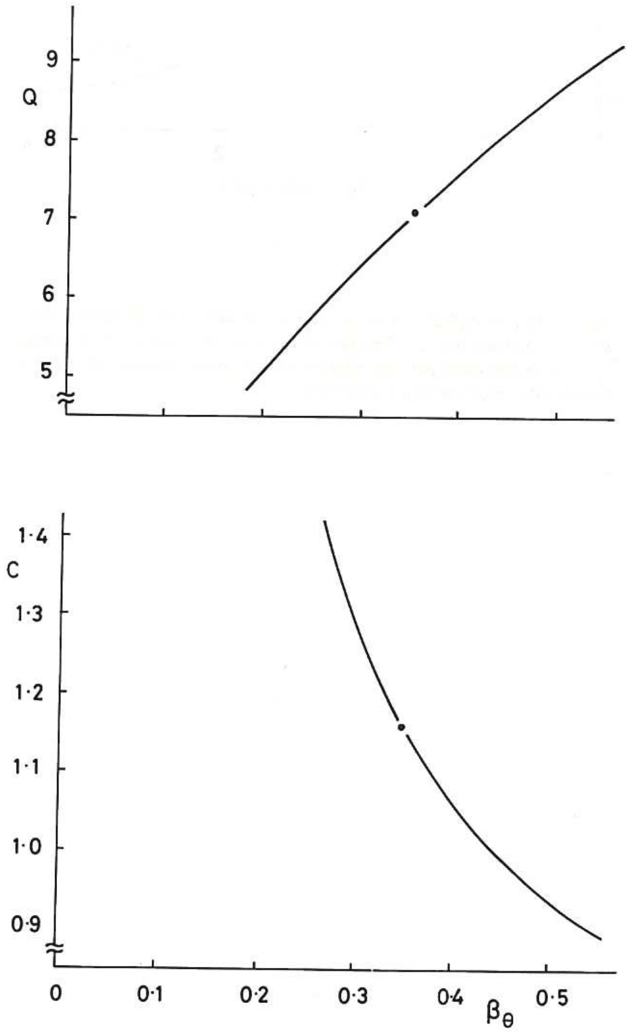

Fig.12 Energy multiplication factor, Q, and nominal capital cost per kilowatt, C, of a 600MW(e) unit with an air cored reactor, as a function of the plasma pressure ratio, β_θ.

Of the engineering parameters the most important are the first wall loading and the net power output of the reactor. The variation of capital cost with the mean neutron wall loading is illustrated in Figure 13, and shows that higher loadings would have been more economic if technically possible. In a 600 MW(e) reactor the economic advantage of higher wall loadings is limited by the constraint on magnetic flux density in the core, and at even higher loadings, by the minimum plasma radius acceptable for plasma confinement. In reactors of higher net output power, however, the possible economic advantage of higher wall loadings is considerable. Since the wall loading is limited by the energy flux to the surface of the first wall rather than the neutron flux passing into the bulk of the blanket, the design of this wall is of critical importance in a pulsed Reversed Field Pinch reactor.

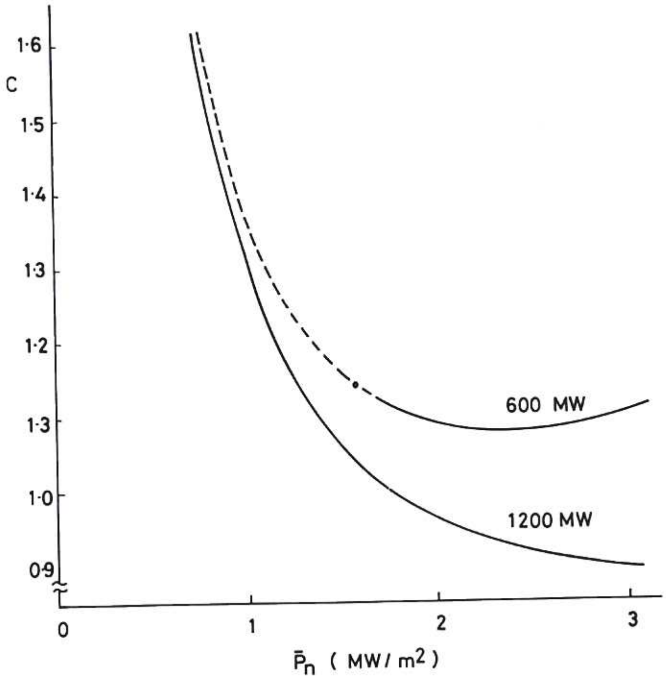

Fig.13 Nominal capital cost per kilowatt, C, of 600 and 1200MW(e) units with air cored reactors, as a function of the mean first wall neutron loading, $\overline{P}_n$. The dashed curve indicates that the lowest cost is obtained with a core flux density below the maximum value.

The variation of capital cost with total reactor power at a fixed mean neutron wall loading of 1.5 MW/m^2 is shown in Figure 14. At low powers the limitation of magnetic flux density in the core gives a steep dependence, but for large units the cost becomes almost independent of size. This latter effect is due to the strong dependence of plasma heating time on plasma radius, noted in Section 3.2, which forces the optimization to large aspect ratios. Whilst the numerical values of the heating time are not very reliable due to uncertainties in the ohmic heating correction factor, the overall effect is probably genuine. Thus the requirement to ignite the reactor by ohmic heating alone favours small reactors operating at high current density, which is consistent with the initial choice of a small unit size for this study. As noted in the previous paragraph, this limitation could be overcome if it were technically feasible to operate the reactor at considerably higher mean wall loadings.

5. CHOICE OF PARAMETERS

The parameter survey of the previous section has led to two alternative designs, with iron or air cores. For the purposes of continuing engineering studies it would be desirable to reduce these to a single set of physical dimensions and in so doing to take account of any further constraints which have not yet been included.

Preliminary engineering studies have drawn attention to one particular difficulty associated with the Reversed Field Pinch configuration, which

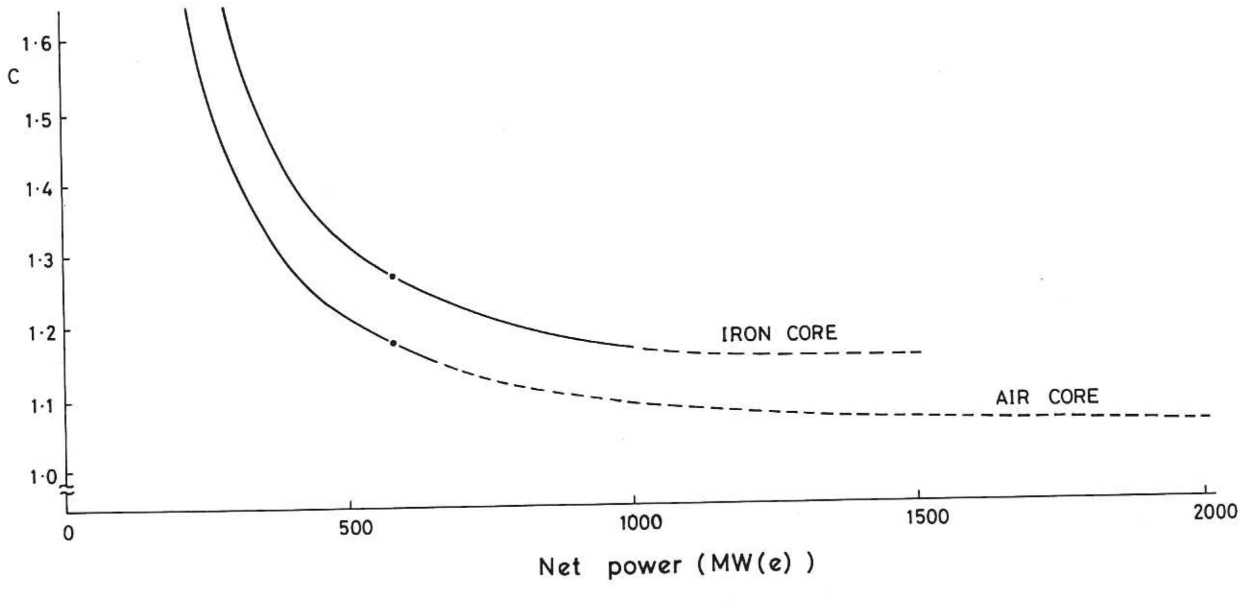

Fig.14 Nominal capital cost per kilowatt, C, of units with iron and air cored reactors, as a function of the net output power, P_{OUT}. The dashed curve indicates that the lowest cost is obtained with a core flux density below the maximum value.

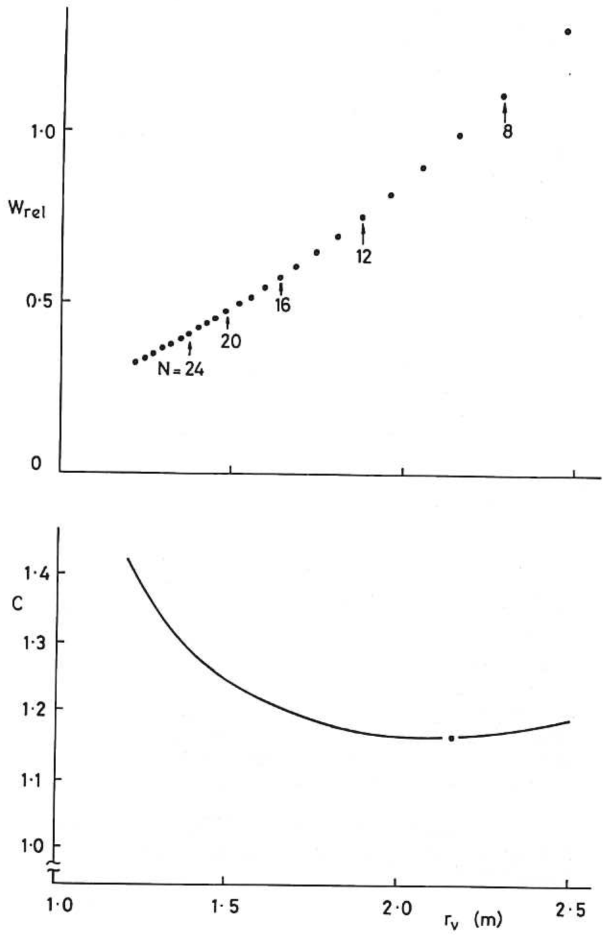

Fig.15 Relative weights, W_{rel}, of sectors of an air cored reactor divided into N sectors each with a length equal to four times their first wall radius, r_v. The lower curve shows the nominal capital cost per kilowatt of a 600MW(e) unit based on such a reactor.

is the difficulty of reactor maintenance in a system including a conducting shell close to the first wall. In order to be effective this shell must have a length several times greater than its radius and a suggested minimum ratio of length to radius is 4. This length of shell determines the minimum length of the blanket segments which can be removed for maintenance, unless it is envisaged that the thick shell can be cut and rejoined in situ. The result of this limitation is that in low aspect ratio systems the blanket can only be divided into a few segments for maintenance, each of which has a relatively large size and weight. In the air cored system, therefore, it may be advantageous to work at a lower flux density in the core than the value determined by stress limitations and at a correspondingly smaller minor radius, in order to increase the number of segments and reduce their weight. This effect is illustrated in Figure 15, which shows that the cost penalty is relatively small but the reduction in segment size quite significant. As a result the parameters for the air cored system have been adjusted to be close to those obtained for the iron cored system and a common set of physical dimensions adopted.

A preliminary consideration of the electromagnetic system of the reactor,[3] carried out in parallel with this study, has shown that the cost of an air cored reactor can be significantly lower

TABLE 5

Parameters for a 600 MW(e) air cored Reversed Field Pinch reactor	
First wall radius (m)	1.75
Major radius (m)	15.5
Aspect ratio	8.9
First wall neutron loading (MW/m^2)	1.5
Core flux density (T)	2.1
Plasma line density ($m^{-1} \times 10^{21}$)	2.0
Plasma pressure ratio, β_θ	0.35
Plasma current (MA)	19
Burn time (sec)	28
Cycle time (sec)	40
Thermal energy from blanket/cycle (GJ)	95.3
Electrical energy generated/cycle (GJ)	38.1
Net output energy/cycle (GJ)	24.2
Circulating energy (GJ)	13.9
Energy storage capacity (GJ)	21.1
Energy multiplication factor, Q	6.9
Recirculating power fraction, ε	0.36
Estimated Capital cost (£/kW(e))	1190

than the cost of an iron cored reactor. The cost saving is mainly due to the absence of the iron transformer core, which with a weight of 150,000 tonnes would be the most expensive component in the iron cored system. The corresponding increase in the cost of energy storage can be minimised by choosing to operate with a symmetrical current swing in the primary winding of the air cored transformer, in which case the power dissipated in the magnetic field windings will be constant throughout the operating cycle and can be supplied directly from the reactor output. It has also been shown that the stray fields from the air cored transformer can be more cheaply screened by means of an active winding than by the iron shield assumed in this study. Taking these factors into account, the air cored reactor has been chosen as the basis of further engineering studies. Its parameters are shown in Table 5.

6. CONCLUSIONS

A set of parameters has been derived for a 600 MW(e) Reversed Field Pinch reactor. To emphasise the particular features of this confinement geometry the design assumes pulsed operation without refuelling during the burn, ignition by ohmic heating, and the use of resistive (i.e. not superconducting) magnetic field windings.

Ignition by ohmic heating alone appears possible, on the basis of the point model used in Section 3 which includes a correction factor to allow for the helical distribution of currents associated with actual magnetic field profiles. The requirement of ignition by ohmic heating introduces severe limitations on the reactor parameters, however, and in particular a first wall radius less than 2 to 2.5 m is required if the heating time is to be kept short compared with the burn time.

The only plasma burn cycle which gives adequate energy multiplication is one with a low plasma temperature ($\sim$ 10 keV), but at the present time no suitable mechanism is known which will provide the necessary controlled energy loss from the plasma. The required mechanism must maintain a particle confinement time which is between one or two orders of magnitude greater than the energy confinement time, so that the product of plasma density and particle confinement time, $n\tau_p$, is greater than 10^{22} m^{-3}sec.

The need for a control mechanism during the plasma burn which enhances the energy loss from the plasma implies a high energy flux to the reactor first wall, so that operation at a relatively low mean wall loading is necessary. Combined with the limitations on first wall radius due to the ignition requirement, this leads to a low net output power for a reactor with optimum parameters. Whilst larger reactors are possible, no improvement is obtained in capital costs for larger sizes.

The most important assumptions in the study, as indicated by the sensitivity analysis, were the plasma pressure ratio β_θ and the plasma profile as characterised by the pinch parameter θ. The assumed value of β_θ was 0.35, corresponding to a central value of β_0 of 0.24, and values much below this give rapidly increasing capital costs.

A preliminary consideration of engineering requirements has led to the choice of an air cored system with a first wall radius of 1.75 m. A large aspect ratio is required to permit the removal of sections of the flux conserving shell during maintenance without the difficulty of remote cutting and welding of electrical conductors.

The chosen reactor parameters give an overall energy multiplication factor, Q, of 6.9, which at the assumed efficiency of conversion of thermal energy to electricity of 40% corresponds to a recirculating power fraction of 36%.

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APPENDIX I: Minimum Energy Multiplication required in reactor

The energy multiplication factor Q of a fusion reactor may be defined as the useful thermal energy released in the reactor divided by the net electrical energy required to establish and maintain the plasma. In principle any plasma confinement system which amplifies energy can be used as a power producing reactor, provided that the energy multiplication factor Q exceeds the reciprocal of the product of the efficiencies of the series coupled systems which transform the energy into the required forms. In general these systems include a turbo-alternator whose efficiency is about 40% so that the minimum necessary value of Q is about 2.5. If direct conversion systems are used with efficiencies around 80% the value of Q need only be about 1.25.

If the power source is to be the basis of a competitive commercial power station, further constraints are placed on the minimum acceptable value of Q. The evaluation of the necessary Q is difficult, however, because the argument must be an economic one and the factors involved are not well defined. In order to make some progress despite these difficulties a very simplified argument can be developed to obtain a first estimate of the minimum value of Q which is required.

Consider a simple system in which a reactor is supplied with energy from a store/injector of efficiency η_i, and delivers thermal energy to a turbo-alternator of efficiency η_t. Two cases can be considered; the ideal one in which the Q of the reactor is infinite (i.e. no net electrical energy input is required), and a second system with a reactor which has a finite value of Q. The cost of the reactor in the first system is a fraction R of the total station cost, and is replaced in the second system with a reactor whose cost is a factor F times the cost of the first reactor. Assuming no other changes are necessary, the relative capital costs of the two systems in terms of power output to the grid are:

$$C_{rel} = C2/_{C1} = \{1-R(1-F)\}/\{1-1/Q\eta_i\eta_t\}$$

In general η_i may be taken as ~ 0.8, $\eta_t \sim 0.4$, $R \sim 0.2$, so that C_{rel} may be evaluated as a function of F and Q as in Figure AI.1. Obviously no value of Q satisfies the requirement $C_{rel} < 1$ if $F > 1$.

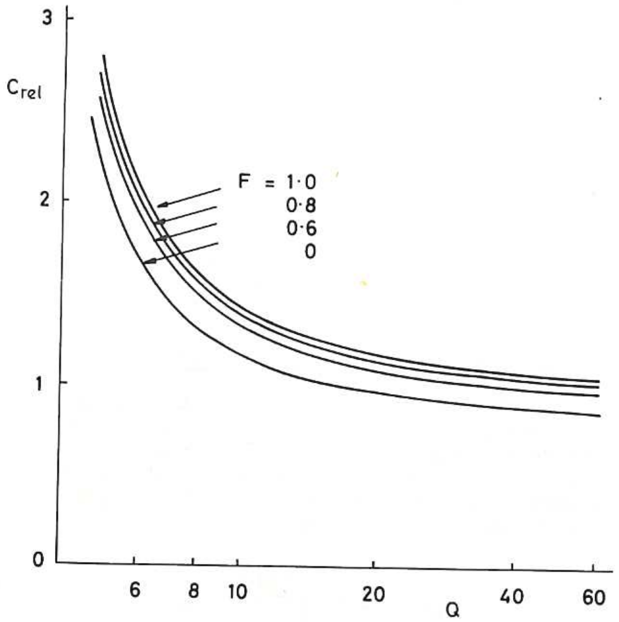

Fig.AI.1 Relative capital cost, C_{rel}, of a power station with a reactor of energy multiplication factor Q, to a station with a reactor of infinite Q.

The argument may be applied to a possible fusion reactor which replaces a fission reactor having a very large Q. In this case the fuel cost of the fission system must be assessed as an equivalent capital cost, and if the fuel costs of the fusion reactor are neglected this leads to a required value of $C_{rel} \sim 1.1$. It is seen that with infinite values of Q, cost savings are possible if the cost of the fusion reactor does not exceed the cost of the fission reactor by more than a factor 1.5. However, all estimates so far suggest that fusion reactors will be significantly more expensive than fission reactors.

If the cost of a fusion reactor could be brought down to the cost of a fission reactor, an economically acceptable value of Q would be $\sim$ 35. If the environmental advantages of fusion were judged to be worth a further 10% increase in station costs, the required value of Q would be reduced to $\sim$ 20. If the efficiency of the energy store/injector is included in the Q of the reactor, as is the case in calculations for the Reversed Field Pinch reactor, the required value of Q is reduced to $\sim$ 15. Only if there were grounds for believing that the fusion reactor capital costs might be less than fission reactor capital costs would lower values of Q be economically acceptable. This conclusion would be modified, however, by the use of direct conversion systems or additional energy multiplication systems such as fissile blankets.

It is concluded that, in the absence of detailed reactor cost comparisons, the lowest economically acceptable value of Q for a fusion reactor is probably in the range 15 to 20.

APPENDIX II.1: Circulating Energy Calculations

This Appendix describes the mathematical model used for the calculation of energy flows in the Reversed Field Pinch reactor, based on the energy flow diagram, Figure AII.1. The model is linear, including no toroidal effects on the field, current or density profiles of the plasma, and all parameters are calculated per unit length of the plasma major circumference ($2\pi R_m$). The reactor components - coils, blanket, shield and plasma region - are modelled by a series of concentric cylindrical shells of variable thickness. The analysis is carried out in SI units.

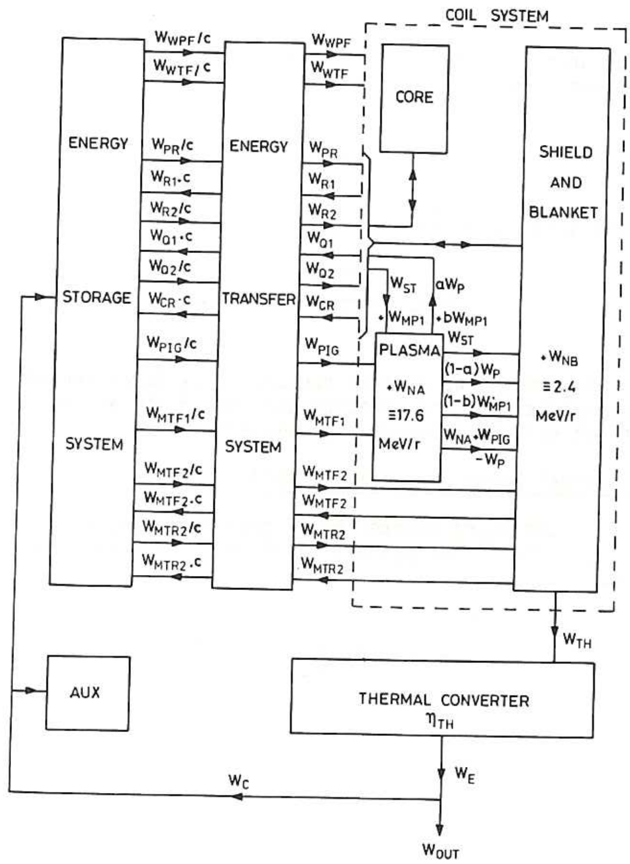

Fig.AII.1 Energy circulation diagram.

A careful study of the Figure AII.1 with the assistance of the glossary of symbols given in Part II.2 of this Appendix leads to the conclusion that the majority of the energy exchanges involve magnetic energy transfer to and from the toroidal field (TF) and poloidal field (PF). These will therefore be dealt with first. In the following it will be assumed that the TF and PF profiles out to the relevant coils are known quantities, and that the flux conserving shell is coincident with the first wall.

Toroidal Field

Let $B_\phi(r)$ denote the reversed TF profile during the burn phase and $\overline{B}_\phi$ the constant TF before the pinch profile is set up. Then the toroidal field energies that must exist before and after set up are given by:

$$W_{MTF1} = \frac{\overline{B}_\phi^{\,2}\pi r_V^{\,2}}{2\mu_o} \tag{1}$$

$$W_{MTF2} = \frac{\overline{B}_\phi^{\,2}\pi(r_{TF}^{\,2}-r_V^{\,2})}{2\mu_o} \tag{2}$$

$$W_{MTR1} = \frac{\pi}{\mu_o}\int_0^{r_V} B_\phi^{\,2}(r)rdr \tag{3}$$

$$W_{MTR2} = \frac{\pi}{\mu_o}\int_{r_V}^{r_{TF}} B_\phi^{\,2}(r)rdr \tag{4}$$

per unit circumferential length of reactor. Using Amperes law, the TF coil current is given by:

$$I_{TFF} = \frac{\overline{B}_\phi}{\mu_o} \tag{5}$$

before set up and by:

$$I_{TFR} = \frac{B_\phi(r_V)}{\mu_o} = F\ I_{TFF} \tag{6}$$

during the burn, so that toroidal flux is conserved by maintaining the reversed toroidal field at the first wall (F = reversal parameter).

Poloidal Field inside First Wall

Let $B_\theta(r)$ denote the PF profile during the burn. The PF energy, after set up, inside the first wall is:

$$W_{MP\theta} = \frac{\pi}{\mu_o}\int_0^{r_V} B_\theta^{\,2}(r)rdr \tag{7}$$

and from this the specific geometric plasma inductance:

$$\ell_g = \frac{2W_{MP\theta}}{\hat{I}_p^{\,2}} \tag{8}$$

may be defined, where $\hat{I}_p$ is the peak value of plasma current, calculated from the modified Bennett relation, $\beta_\theta \hat{I}_p^{\,2} = 16\pi NkT/\mu_o$, which is attained and held for the period of the burn. Since toroidal flux is conserved by control of the TF coil currents, the additional toroidal flux required by $B_\phi(r)$ over that provided by $\overline{B}_\phi$ must be provided by energy input via the PF coil current. Therefore the energy that must be transferred via the PF coil system to the region inside the first wall must be at least:

$$W_{MP1} = W_{MP\theta} + W_{MTR1} - W_{MTF1} \tag{9}$$

from which the specific effective plasma inductance:

$$\ell_e = \frac{2W_{MP1}}{\hat{I}_p^{\,2}} \tag{10}$$

may be specified. The energy transfer from poloidal to toroidal field is not ideal, since the toroidal field penetrates the plasma dissipatively, and so the fraction:

$$W_{ST} = (K_e - 1)W_{MP1} \tag{11}$$

is assumed to be lost in start-up. As start-up dissipation is not clearly understood at present, and the burn model used includes only alpha particle and ohmic dissipation as the mechanisms of plasma heating to constant temperature, the equivalent of this energy is assumed to be lost to the first wall. In scaling this loss from the results of Zeta,[20] the equation:

$$K_e = K_{eo} + F_N\ K_{elz}\left(\frac{a_z}{a_r}\right)^2\ \frac{I_z}{I_r}\tau_r \tag{12}$$

has been used, where a refers to first wall radius, I to plasma current, z to Zeta results, r to reactor parameters, $K_{eo} = 1.3$ and $K_{elz} = 162\ s^{-1}$, and $F_N = 1.0$.

Poloidal Field Coil Current Evolution

In the above analysis of poloidal fields, only those energies internal to the first wall have been considered since the first wall is assumed to be flux conserving on the timescale of the plasma current rise. In order to define the energies outside the first wall, let:

$$\ell_{pe} = \frac{\mu_o}{2\pi}\,\ell n\left(\frac{r_{PF}}{r_V}\right) + \ell_e \tag{13}$$

$$\ell_m = \frac{\mu}{2\pi}\left[\ell n\left(\frac{8R_m}{r_{PF}}\right) - 2\right] \tag{14}$$

where ℓ_{pe} and ℓ_m represent the specific internal and external PF coil inductances during the burn. To calculate the PF energy transfers involved both in the air and iron cored reactors it is necessary to begin by considering the PF coil current evolution. It will be assumed that the PF current during the burn is some multiple, x, of the peak plasma current ($x \gtrsim 1$ for iron core). The plasma conductivity at its burn temperature is assumed to be sufficiently high that the change in flux required to maintain constant plasma current during the burn is negligible. Using classical resistivity, these "resistive" volt seconds represent about 5% of the total required flux swing in the cases studied, which is well within the bounds of uncertainty of the scaling law of equation (12).

The core flux swing during start-up is assumed to be:

$$\Delta\phi_c = \gamma 2\pi R_m \ell_{pe}\ \hat{I}_p \tag{15}$$

where

$$\gamma\ell_{pe} = \frac{\mu_o}{2\pi}\,\ell n\left(\frac{r_{PF}}{r_V}\right) + K_e\ \ell_e \tag{16}$$

The relationship between γ and K_e will in general depend on the time evolution of the plasma current and applied voltage, but the above result is assumed to apply here. At the start of the rise phase, no plasma and therefore no plasma current exists, so the core flux is given by:

$$\phi_C\ (0) = 2\pi R_m \ell_m I_{PF}(0) \qquad (17)$$

At the end of the rise time, τ_r, plasma current is at its peak so the external flux in the core is:

$$\phi_C\ (\tau_r) = 2\pi R_m \ell_m \hat{I}_p\ (x-1) \qquad (18)$$

since the PF current has components of plasma image current ($\hat{I}_p$) and magnetising current ($\hat{I}_p[x-1]$). The definition of PF coil current during the burn and equation (15) result in:

$$I_{PF}\ (\tau_r) = x\hat{I}_p \qquad (19)$$

$$I_{PF}\ (0) = \hat{I}_p \left[x-1-\gamma\frac{\ell_{pe}}{\ell_m} \right] \qquad (20)$$

and the latter current must be set up during the premagnetisation phase, of duration τ_{PR}.

During the plasma quench phase (duration τ_q) at the end of the burn (duration τ_b), the plasma current is driven to zero and the plasma allowed to expand against the magnetic field, returning energy to it. Some fraction of thermal and magnetic energy/flux remains locked in the plasma and is screened from the PF circuit by surface currents in the plasma. The flux that can be recovered by changing the PF coil current during the quench is denoted by:

$$\Delta\phi = 2\pi R_m \ell_{per} \hat{I}_p \qquad (21)$$

where

$$\ell_{per} = \frac{\mu_o}{2\pi} \ell n\left(\frac{r_{PF}}{r_V}\right) + \alpha\ \ell_e \qquad (22)$$

and

$$\alpha = \frac{b\ W_{MP1} + a\ W_P}{W_{MP1}} \qquad (23)$$

Here W_P is the plasma thermal energy at burn temperature. The energy that can be recovered directly from the plasma is given by the numerator of equation (23) and there remains:

$$W_{INT} = (1-a)W_P + (1-b)W_{M1} \qquad (24)$$

available for thermal recovery by heat transfer to the wall in a controlled exhaust phase. At the end of the burn the core flux is given by equation (18) because of the assumption about resistive volt seconds requirements. The flux available at the end of the quench phase inside the PF coils must be transferred into the core for recovery and thus:

$$\phi(\tau_r+\tau_b+\tau_q) = 2\pi R_m \ell_m I_{PF}(\tau_r+\tau_b+\tau_q) \qquad (25)$$

Consequently, using equation (21):

$$I_{PF}(\tau_r+\tau_b+\tau_q) = \hat{I}_p \left[x-1-\frac{\ell_{per}}{\ell_m} \right] \qquad (26)$$

At the end of the deionisation phase (duration τ_D), assumed negligible in the optimisation) when no plasma exists, the PF coil current is reduced to zero in time τ_{CR}.

Thus, in summary, the PF coil current throughout the cycle may be described by:

$$I_{PF}(-\tau_{PR}) = 0$$

$$I_{PF}(0) = \hat{I}_p\{x-1-\rho_S\}$$

$$I_{PF}(\tau_r) = x\hat{I}_p = I_{PF}(\tau_r+\tau_b) \qquad (27)$$

$$I_{PF}(\tau_r+\tau_b+\tau_q) = \hat{I}_p\{x-1-\rho_L\} = I_{PF}(\tau_r+\tau_b+\tau_q+\tau_D)$$

$$I_{PF}(\tau_r+\tau_b+\tau_q+\tau_D+\tau_{CR}) = 0$$

where

$$\rho_S = \frac{\gamma\ell_{pe}}{\ell_m} \qquad \rho_L = \frac{\ell_{per}}{\ell_m} \qquad (28)$$

Poloidal Field Energy Transfers

The total energy in the poloidal field system at any time is given by the sum of magnetic energy in the core and that inside the PF coil. That is:

$$W_{PF}(t) = \frac{1}{2}\ell_m\{I_{PF}(t)-I_p(t)\}^2 + \frac{1}{2}\ell_m\rho I_p^2(t) \qquad (29)$$

where ρ takes one of the forms specified in equation (28). To calculate the energy that must be exchanged with the PF system it is useful to consider the maximum and minimum values of W_{PF} during the cycle, since the energy exchange does not depend on the exact time evolution of the currents between the values quoted in equation (27). These are:

$$W_{PF}\ (-\tau_{PR}) = 0$$

$$W_{PF}(0) = \frac{1}{2}\ell_m\hat{I}_p^2\{x-1-\rho_S\}^2$$

$$W_{PF}(\tau_{rmin}) = \frac{1}{2}\ell_m\hat{I}_p^2\frac{\{x-1-\rho_S\}^2}{1+\rho_S}$$

$$W_{PF}(\tau_r) = \frac{1}{2}\ell_m\hat{I}_p^2\ \{(x-1)^2+\rho_S\} \qquad (30)$$

$$W_{PF}(\tau_r+\tau_b) = \frac{1}{2}\ell_m\hat{I}_p^2\{(x-1)^2+\rho_L\}$$

$$W_{PF}(\tau_r+\tau_b+\tau_{qmin}) = \frac{1}{2}\ell_m\hat{I}_p^2\frac{\{x-1-\rho_L\}^2}{1+\rho_L}$$

$$W_{PF}(\tau_r+\tau_b+\tau_q) = \frac{1}{2}\ell_m\hat{I}_p^2\{x-1-\rho_L\}^2$$

$$W_{PF}(\tau_r+\tau_b+\tau_q+\tau_D+\tau_{CR}) = 0$$

where τ_{rmin} and τ_{qmin} are the minimum PF energies during τ_r and τ_q respectively, occurring when $I_{PF} = 0$. The differences between successive maxima and minima represent direct energy exchanges with the poloidal field, except in the case of $W_{PF}\ (\tau_r+\tau_b) - W_{PF}\ (\tau_r)$ which represents energy lost to the thermal cycle. The six energy exchanges with the PF occur:

(i) during premagnetisation:

$$W_{PR} = +\frac{1}{2}\ell_m\hat{I}_p^2\{x-1-\rho_S\}^2$$

(ii) during rise time:

$$W_{R1} = -\frac{1}{2}\ell_m \hat{I}_p^{\,2}\{x-1-\rho_S\}^2 \qquad (31)$$

$$W_{R2} = +\frac{1}{2}\ell_m \hat{I}_p^{\,2} x^2 \frac{\rho_S}{(1+\rho_S)}$$

(iii) during quench:

$$W_{Q1} = -\frac{1}{2}\ell_m \hat{I}_p^{\,2} x^2 \frac{\rho_S}{(1+\rho_S)}$$

$$W_{Q2} = +\frac{1}{2}\ell_m \hat{I}_p^{\,2}\{x-1-\rho_L\}^2 \frac{\rho_S}{(1+\rho_L)}$$

(iv) during PF current run-down:

$$W_{CR} = -\frac{1}{2}\ell_m \hat{I}_p^{\,2}\{x-1-\rho_L\}^2$$

where a minus sign indicates energy recovery from the PF system. The net energy transfer to the poloidal field can be shown from the above equations to be:

$$W_{MP} = W_{MP1}+W_{ST}-aW_p-bW_{MP1} \qquad (32)$$

Resistive Losses in Coils

Since the PF and TF coil systems are not superconducting, resistive losses will occur in the coils during the cycle and these are given, for the TF coils, by:

$$W_{WTF} = \frac{B_\phi^{\,2}(r_V)R_{TF}}{\mu_o^{\,2}}\left[\tau_b+\tau_q+\tau_D+\tau_{CR}+\right. \qquad (33)$$

$$\left.+\frac{1}{F^2}\left\{\frac{\tau_{PR}}{2}+\tau_r\left[1+\frac{(1-F)^2}{2}-\frac{4(1-F)}{\pi}\right]\right\}\right]$$

where the resistance:

$$R_{TF} = \frac{\eta 2\pi r_{TF}}{\Delta_{TF} f_{TF}} \qquad (34)$$

for η = resistivity, Δ_{TF} = radial extent of TF coil and f_{TF} is the copper packing fraction; for the PF coils:

$$W_{WPF} = \hat{I}_p^{\,2}R_{PF}\left\{\frac{\tau_{PR}}{2}(x-1-\rho_S)^2 + \right.$$

$$+\tau_r\left[\frac{(1+\rho_S)^2}{2} + (x-1-\rho_S)^2 + \frac{4(x-1-\rho_S)(1+\rho_S)}{\pi}\right] + \qquad (35)$$

$$+ x^2(\tau_b+\tau_q) + \tau_q\left[(1+\rho_L)^2\left(\frac{3}{2}-\frac{4}{\pi}\right) - 2x(1+\rho_L)\left(1-\frac{2}{\pi}\right)\right] +$$

$$\left.+\left(\tau_D+\frac{\tau_{CR}}{2}\right)(x-1-\rho_L)^2\right\}$$

where the resistance is:

$$R_{PF} = \frac{\eta}{2\pi r_{PF}\Delta_{PF} f_{PF}} \qquad (36)$$

with similar definitions to the above. The currents have been assumed to vary sinusoidally in time in the derivation of equations (33) and (35) but this is not an important element in the calculations.

Plasma Thermal and Thermonuclear Energy

The plasma thermal energy is calculated from the density profile, using:

$$W_p = \frac{6kT}{r_V^{\,2}}\int_o^{r_V} n_e(r)rdr \qquad (37)$$

where electrons and ions are assumed to have the same density and temperature. Temperature is assumed constant across the profile. Ignition of the plasma is assumed to occur at 4 keV so the energy required to heat the plasma to ignition is:

$$W_{PIG} = \frac{4}{T} W_p \qquad (38)$$

(for T in keV). The thermonuclear output of the burn is assumed to be:

$$W_N = \frac{1}{2} f_B Q_R \int_o^{r_V} n_i(r)rdr \qquad (39)$$

since 1 neutron is produced for every 2 fuel ions consumed, where f_B is the fractional burn up and Q_R the energy resulting from each reaction. The energy consists of 14.1 MeV per neutron plus 3.5 MeV per alpha particle plus 2.4 MeV due to blanket multiplication reactions of the incident neutron. The thermonuclear output of the plasma region alone is referred to by W_{NA} in Figure AII.1, and that of the blanket by W_{NB}.

System Energy Balance

The thermal output of the reactor via the blanket and shield system, viz:

$$W_{TH} = W_N+(1-b)W_{MP1}+W_{ST}+W_{PIG}-aW_p \qquad (40)$$

is converted with thermal efficiency η_{TH} to give the gross reactor electrical output:

$$W_E = \eta_{TH} W_{TH} \qquad (41)$$

Since each direct energy transfer to and from the energy store during the cycle is assumed to occur with transfer efficiency c, the circulating energy that must be supplied out of the gross electrical output can be stated as:

$$W_C = (W_{PR}+W_{R2}+W_{Q2}+W_{PIG}+W_{MTF1}+W_{MTF2}+W_{MTR2}+W_{WTF}+W_{WPF})/c-$$

$$-(W_{R1}+W_{Q1}+W_{CR}+W_{MTF2}+W_{MTR2})c+W_A \qquad (42)$$

and thus the net electrical reactor output is:

$$W_{OUT} = W_E - W_C \qquad (43)$$

from which

$$Q = \frac{W_{TH}}{W_C} \qquad \varepsilon = \frac{W_C}{W_E} \qquad (44)$$

The energy lost in switching the direct energy transfers is given by:

$$W_S = (W_{PR}+W_{R2}+W_{Q2}+W_{PIG}+W_{MTF1}+W_{MTF2}+W_{MTR2}+W_{WPF}+W_{WTF})\times$$

$$\left(\frac{1}{c}-1\right)+(W_{R1}+W_{Q1}+W_{CR}+W_{MTF2}+W_{MTR2})(1-c) \qquad (45)$$

and the total energy lost from the reactor, by:

$$W_{LOST} = W_S+W_{WPF}+W_{WTF}+W_A+(1-\eta_{TH})W_{TH} \qquad (46)$$

All the energies involved in this equation are assumed to be lost as low grade heat to the environment. Finally, the size of energy store required is:

$$W_{STO} = \text{Maximum } \{[(W_{MTF1}+W_{MTF2}+W_{PR}+W_{WTF}+W_{WPF})/c], [(W_{PIG}+W_{MTR2}+W_{R2}+W_{WTF}+W_{WPF})/c]\} \qquad (47)$$

Summary

This Appendix has described the exact means by which the results contained in this report have been obtained and has been included to permit a full understanding of the Reversed Field Pinch reactor system optimisation. The poloidal and toroidal magnetic energies required for the pinch inside the flux conserving shell have been described. The variation in poloidal field coil current to provide the required plasma current evolution has been described in a general way, applying to air and iron cored reactors. This approach ignores the requirements for vertical fields for plasma equilibrium, and represents an intermediate stage in the development of a model of the PF and TF coil operation described in its latest form in reference 3. Finally, these energies have been linked together with the aid of a flow diagram and a set of energy balance equations, to provide a means of evaluating the system energy multiplication factor Q, and to act as a basis for costing studies.

APPENDIX II.2: Glossary of Symbols

$B_\phi(r)$	Reversed TF radial profile during burn.
$\overline{B}_\phi$	Forward constant TF before start-up.
W_{MTF1}	Energy of TF inside flux conserving shell before start-up.
r_v	Flux conserving shell and first wall radius.
W_{MTF2}	Energy of TF outside first wall and inside r_{TF} before start-up.
r_{TF}	Active radius of TF coil, i.e. radius at which a current sheet may be placed to give the required field (of order inside radius + 1/3 thickness of coil).
W_{MTR1}	Energy of TF inside flux conserving shell during burn.
W_{MTR2}	Energy of TF between flux conserving shell and r_{TF} during burn.
I_{TFF}	TF coil current required to produce $\overline{B}_\phi$.
I_{TFR}	TF coil current required to produce $B_\phi(r_v)$.
F	Field reversal parameter (definition: equation 6).
$B_\theta(r)$	PF radial profile during burn.
$W_{MP\theta}$	Energy of PF inside flux conserving shell during burn.
ℓ_g	Specific (i.e. per unit circumferential length) geometric plasma inductance out to the flux conserving shell.
$\hat{I}_p$	Plasma current at end of current rise.
β_θ	Poloidal beta, or current plasma pressure ratio.
N	Plasma ion or electron line density.
k	Conversion constant = 1.6021×10^{-16} J/keV.
T	Plasma ion or electron temperature in keV.
W_{MP1}	Minimum energy input via PF coil required to establish fields inside first wall during start-up.
ℓ_e	Specific effective plasma inductance out to flux conserving shell.
W_{ST}	Energy lost in start-up by inefficient PF to TF energy transfer.
K_e	Loss factor describing extra PF energy supplied during plasma current rise.
τ_r	Plasma current rise time.
ℓ_{pe}	Specific plasma inductance out to r_{PF}.
r_{PF}	Active radius of PF coil - see r_{TF}.
ℓ_m	Specific magnetising inductance of PF coil.
R_m	Plasma major radius.
μ	Core permeability.
$\Delta\phi_c$	Core flux swing.
γ	Volt second multiplier.
$\phi_c(t)$	Core flux at time t.
x	Ratio of PF coil current to plasma current during burn.
τ_{PR}	Core premagnetisation (bias) time.
τ_q	Plasma current quench time.
τ_b	Burn time.
$\Delta\phi$	Flux available for recovery at end of burn.
ℓ_{per}	Specific plasma "inductance" recoverable inside r_{PF}.
b	Fraction of magnetic energy inside flux conserving shell that is recoverable at end of burn.
a	Fraction of plasma thermal energy recoverable by expansion at end of burn.
α	Above fractions expressed as fraction of magnetic energy recoverable at end of burn.
W_{INT}	Internal plasma energy, a mixture of magnetic and thermal, not recoverable directly at end of burn.
τ_D	Deionisation time.
τ_{CR}	PF coil current run-down time.
$I_{PF}(t)$	PF coil current at time t.
ρ_S	Ratio of start-up to magnetising inductance.

ρ_L	Ratio of recoverable to magnetising inductance.
$W_{PF}(t)$	Total energy in PF system at time t.
$I_p(t)$	Plasma current at time t.
τ_{min}	Time during τ_r at which W_{PF} is at a minimum.
τ_{qmin}	Time during τ_q at which W_{PF} is at a minimum.
W_{PR}	Energy input to PF during premagnetisation.
W_{R1}	Energy extracted from PF during rise time.
W_{R2}	Energy input to PF during rise time.
W_{Q1}	Energy extracted from PF during quench.
W_{Q2}	Energy input to PF during quench.
W_{CR}	Energy extracted from PF during PF current run-down.
W_{MP}	Net magnetic energy transfer to PF system.
W_{WTF}	Energy dissipated in TF coils.
W_{WPF}	Energy dissipated in PF coils.
R_{TF}	Resistance, per unit circumferential length, of TF coils.
R_{PF}	Resistance, per unit circumferential length, of PF coils.
η	Resistivity of windings.
$\Delta_{PF,TF}$	Radial extent of PF, TF coils.
$f_{PF,TF}$	Copper packing factor of extent $\Delta_{PF,TF}$.
W_p	Plasma kinetic energy during burn.
W_{PIG}	Energy required to heat plasma to ignition.
W_N	Thermonuclear output of burn including particles and blanket reactions (W_{NA} + W_{NB}).
f_B	Fractional burn-up.
$n_e(r)$	Electron number density radial distribution.
$n_i(r)$	Fuel ion number density radial distribution.
Q_R	Energy per reaction (J).
W_{TH}	Reactor thermal output.
W_E	Reactor gross electrical output.
η_{TH}	Thermal conversion efficiency.
c	Energy transfer efficiency.
W_C	Circulating energy.
W_A	Energy supply to auxiliaries (= d.W_E).
Q	Energy multiplication of reactor system.
W_S	Energy lost in switching.
W_{LOST}	Energy from reactor system lost as low grade waste heat.
W_{STO}	Energy storage capacity.

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