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Report

CUPID

A RAY-TRACING PROGRAM FOR CONTINUOUSLY VARYING REFRACTING MEDIA

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A RAY-TRACING PROGRAM FOR CONTINUOUSLY VARYING REFRACTING MEDIA

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ABSTRACT

A semi-analytic ray-tracing routine is described that allows refraction through continuously varying refracting media to be studied. The program was developed to investigate scattering from laser produced plasmas, but could be adapted for many other situations.

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During a series of experiments[1] involving CO_2 laser generated plasmas using the APPLE and TROJAN facilities at Culham, it was observed that for interactions with plane massive targets, tilted from normal illumination, the scattered radiation was (a) diffuse and (b) peaked at an angle greater than specular. An intuitive explanation of this effect was given by assuming that the geometry of the expanding plasma was non-planar and that this non-planar expansion profile could refract the beam in a manner which would account for the experimental observations. To test this assumption it is necessary to follow the trajectory of a light ray as it refracts through an inhomogeneous plasma. This can be done analytically only in two special cases, ie in planar and spherical geometry.[2] To simulate more general geometries a numerical ray-tracing routine is required. This report describes a ray-tracing routine that gives the path of a ray in a media with a continuous refractive index within the accuracy of geometrical optics.

We restrict ourselves to cases for which the refractive index and its gradient are analytic functions of the co-ordinate variables.

It appears from the literature that most of the existing ray-tracing routines use a shooting method, whereby the plasma is considered to be layered, and the ray is traced by applying Snell's law at each interface between two layers of constant refractive index. This type of method usually requires lengthy computation for reasonable accuracy. For this reason we decided to try an alternative approach and treat the problem analytically as far as possible.

If $\underline{r}$ (l) is the position vector of a point on a ray (considered as a function of the length of arc l of the ray), given an initial position vector $\underline{r}$ (lo), we can calculate the position vector $\underline{r}$ (l) of a point in its neighbourhood by using a Taylor expansion approximation:

$$\underline{r}\,(l) = \underline{r}\,(lo) + (l-lo)\left[\frac{d\underline{r}}{dl}\right]_{l=lo} + \frac{(l-lo)^2}{2!}\left[\frac{d^2\underline{r}}{dl^2}\right]_{l=lo}$$

From differential geometry[3] we know that:

$$\left[\frac{d\underline{r}}{dl}\right]_{l=lo} = \underline{l}\ (lo) \text{ and } \left[\frac{d^2\ \underline{r}}{dl^2}\right]_{l=lo} = \left[\frac{d\underline{l}}{dl}\right]_{l=lo}$$

$$\text{Thus, } \underline{r}\ (l) = \underline{r}\ (lo) + \Delta l\ \underline{l}\ (lo) + \frac{(\Delta l)^2}{2}\left[\frac{d\underline{l}}{dl}\right]_{l=lo} \qquad \ldots(1)$$

where $\underline{l}$ (lo) is a unit vector tangential to the ray. Within the validity of geometrical optics[(4)]

$$\frac{d\underline{l}}{dl} = \frac{1}{n}\left[\underline{\nabla n} - \underline{l}\ (\underline{l}\ .\ \underline{\nabla n})\right] \qquad \ldots(2)$$

where n is the refractive index. Therefore, given $\underline{r}$ (lo) and $\underline{l}$ (lo), and calculating ($d\underline{l}$ / dl) at l = lo, we can obtain the position vector of the next point on the ray (for sufficiently small Δl).

Once we know $\underline{r}$ (l), we must calculate the unit tangent vector at this point to proceed. To do this we again use a Taylor expansion:

$$\underline{l}\ (l) = \underline{l}\ (lo) + \Delta l \left[\frac{d\underline{l}}{dl}\right]_{l=lo} \qquad \ldots(3)$$

and we have all we need to go to the next point, repeating the procedure. Thus, given the refractive index and its gradient, at each point on the ray; and the co-ordinates and tangent vector of an initial point on the ray, we can use equations (1), (2) and (3) to calculate the co-ordinate points of the ray path.

If n and $\underline{\nabla n}$ are analytic functions of the co-ordinate variables, we can use the computer code shown in the appendices to trace a ray. In the particular listing given we consider a 3-D plasma whose density is given by:

$$N\ (x,\ y,\ z) = N_{crit} \exp\left[-\frac{1}{L}\left\{\sqrt{x^2 + \frac{y^2 + z^2}{E^2}} - a_{crit}\right\}\right]$$

where L is a specified density scale length along the x-axis, a_{crit} is the distance (measured along the x-axis) from the critical surface to the origin, and E is the eccentricity of the ellipsoidal co-axial surfaces of constant density. The density profile is exponentially decreasing. The refractive index is assumed to be:

$$n\ (x,\ y,\ z)\ =\ \sqrt{1 - \frac{N\ (x,\ y,\ z)}{N_{crit}}}$$

An elliptical plasma profile (in the x, y plane) is used as this permits planar geometry ($E \gg 1$) or spherical geometry ($E = 1$) to be modelled easily. The extent of the critical surface in the y direction is determined by the width of the incident beam. Other important features of the program are indicated by the comment lines. The output is in the form of (x, y, z) co-ordinate points for each ray which can be displayed using the GHOST graphical output system[(5)]. Examples of the output are shown in Figs.2,3 and 4.

Due to the axial symmetry of the case considered, we can assume without loss of generality that the incident beam is initially parallel to the x, y plane with the centre of the beam in the plane. The angle of incidence, THETA, is then defined with respect to the x-axis (Fig.1).

One particular feature of the results should be mentioned. Focusing is observed in the x, y plane at the turning point for $E > 1$. The effect does not occur in the y,z plane (the necking observed in Fig.4 is just due to the 3-D projection) showing that it cannot be explained as a form of astigmatism rotating the rays into the z direction. Care must be taken however in ascribing physical importance to the effect as neither diffraction nor phase information is included in the program. A more detailed examination of the problem is in progress.

The results also show, for $E > 1$, symmetrical refraction about an axis parallel to the x direction (Figs.2 and 3) and the dispersion of the beam for non-planar geometry.

The program appears to be applicable to any case where refraction through a continuous medium needs to be examined. In the listing given here, the refractive index and its derivatives are analytic functions, but the method could easily be extended to deal with a mesh of refractive index points.

Due to its compactness and relatively modest amount of computation time, we believe that this program could be incorporated into numerical simulation routines for plasma heating by laser interaction. A particular feature of such a scheme would be the ability to incorporate the different absorption processes, and their variation along the ray path. At the present time the existing absorption codes would limit the model to two dimensions. Another possibility that has been suggested is the use of the program to examine the propagation of electromagnetic radiation, due to electron cyclotron emission, through the DITE plasma. In this application the refractive index would enter through the dispersion relation for electromagnetic waves in a warm magneto-plasma.

In conclusion we would like to thank D.N. Wall, A. Sanderson and T.J. Martin for computational help during this work and Drs. I.J. Spalding, R.N. Franklin and A.C. Walker for useful discussions.

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4. L.D. Landau and E.M. Lifshitz, Electrodynamics of Continuous Media (Vol.8 of course on Theoretical Physics) Ch.X, Pergamon Press 1960; M. Born and E. Wolf, Principles of Optics, 4th Edition, Ch.III, Pergamon Press 1970; see also K.G. Budden, Radio Waves in the Ionosphere, Ch.XIV, CUP 1961, and references therein, where ray-tracing in horizontally stratified ionospheric media is discussed.

5. V.J. Calderbank, W.A.J. Prior, Ghost User Manual, CLM-R177 (Part One), 1977.

```
C                        CUPID
C
C      A RAY TRACING PROGRAMME FOR CONTINUOUSLY
C      VARYING REFRACTING MEDIA.(3-D VERSION)
C
C
C      SPECIFY DOUBLE PRECISION FOR VARIABLES.
C      SEE APPENDIX 2 (PROGRAMME NOTES) FOR
C      EXPLANATION OF COMPUTER NAMES ETC.
C
       DOUBLE PRECISION X0,Y0,Z0,L0X,L0Y,L0Z,DLLX0,DLLY0,DLLZ0,
      *XIN,YIN,ZIN,ECC,ACRIT,BCRIT,L,M0,R0,THETA,OMEGA,A0,B0,C0,
      *MULT,DL,DLI,DRJ,DIA,BRAD,RI0,R1,NNORM,A,DCRIT,PSI,PHI,
      *ALPHA
C
C      INPUT INITIAL PARAMETERS
C      DIMENSIONS ARE IN MICRONS.
C
       READ(97,1)ECC,L,M0,X0,Y0,DL
       READ(97,1)DIA
       READ(97,1)XC,YC,ZC,XE,YE,ZE,RV
     1 FORMAT(60Y)
C
C      INITIALISE REFRACTIVE INDEX,DERIVATIVES OF RAY VECTOR,
C      COORDINATES OF BEAM CENTER,ANGLE OF INCIDENCE
C      STEP INCREMENT,BEAM DIMENSION AND CRITICAL
C      SURFACE DIMENSIONS
C
       DATA RI0,DLLX0,DLLY0,DLLZ0,Z0/1.0,0.0,0.0,0.0,0.0/
C
       XIN=X0
       YIN=Y0
       ZIN=Z0
       THETA=DATAN(M0)
       DLI=DL
       BRAD=DIA/2.0
       BCRIT=BRAD+50.0
       ACRIT=BCRIT/ECC
C
C      SWITCH ON GRAPHICAL DEVICE AND DRAW AXES.
C
       CALL PAPER(1)
       CALL BLKPEN
       CALL MAP3D(XC,YC,ZC,XE,YE,ZE,RV,DUM)
       CALL AXES3D
       CALL REDPEN
C
C      ROUTINE TO GENERATE RAYS DEFINING A CYLINDRICAL
C      BEAM OF SPECIFIED WIDTH,STARTING POINT AND
C      ANGLE OF INCIDENCE.
C
       DR=BRAD/10.0
       DO 40 J=1,10
       JJ=1
       IF(J.EQ.1) GOTO 220
       JJ=5*(J-1)
   220 DO 20 M=1,JJ
       IF(J.EQ.1) GOTO 5
       DRJ=DR*(J-1)
```

```
C
C        ANGLE OMEGA IS DEFINED ABOUT CENTRAL RAY
C        IN INCIDENT BEAM.
C
         OMEGA=(M-1)*(6.2832/JJ)
C
C        CALCULATE THE INITIAL COORDINATES OF A
C        PARTICULAR RAY IN THE BEAM
C
         X0=XIN+DRJ*DSIN(THETA)*DSIN(OMEGA)
         Y0=YIN-DRJ*DCOS(THETA)*DSIN(OMEGA)
         Z0=DRJ*DCOS(OMEGA)
C
C        SET PLOTTING POSITION AT INITIAL POINT ON RAY.
C
       5 CALL POSN3D(X0,Y0,Z0)
C
C        DEFINE THE RADIUS VECTOR OF THE INITIAL POINT
C
         R1=DSQRT(X0**2+Y0**2+Z0**2)
C
C        SET THE INITIAL DIRECTION OF THE RAY.
C
         L0X=-DCOS(THETA)
         L0Y=-DSIN(THETA)
         L0Z=0.0
C
C        CALCULATE NEW POINT ON RAY USING TAYLOR EXPANSION.
C
      10 X0=X0+DL*L0X+(DL**2/2)*DLLX0
         Y0=Y0+DL*L0Y+(DL**2/2)*DLLY0
         Z0=Z0+DL*L0Z+(DL**2/2)*DLLZ0
C
C        GRAPHICALLY JOIN THIS NEW POINT TO PREVIOUS ONE.
C
         CALL JOIN3D(X0,Y0,Z0)
C
C        CALCULATE NEW POSITION VECTOR AND COMPARE
C        WITH INITIAL POSITION VECTOR.
C
         R0=DSQRT(X0**2+Y0**2+Z0**2)
         IF(R0.GE.R1) GOTO 20
C
C
C        ROUTINE TO FIND DENSITY SURFACE ON WHICH NEW
C        POINT LIES.
C
         A=DSQRT(X0**2+(Y0/ECC)**2+(Z0/ECC)**2)
         NNORM=(A-ACRIT)/L
C
C        ROUTINE TO CHANGE LENGTH INCREMENT DL AS RAY
C        APPROACHES TURNING POINT.
C
         IF(NNORM.LE.3.0)GOTO 50
         DL=DLI
         GOTO 60
      50 DL=L/200.0
```

```
C
C     ROUTINE TO REFLECT RAY IF CRITICAL SURFACE
C     IS REACHED.
C
      DCRIT=2.0*DL/L
      IF(NNORM.GT.DCRIT) GOTO 60
      PSI=DATAN2(L0Y,L0X)
      PHI=DATAN2(B0,A0)
      ALPHA=2.0*PHI-PSI
      L0X=-DCOS(ALPHA)
      L0Y=-DSIN(ALPHA)
      GOTO 30
C
C     CALCULATE TANGENT TO THE RAY AT NEW POINT
C     USING TAYLOR EXPANSION.
C
   60 L0X=L0X+DL*DLLX0
      L0Y=L0Y+DL*DLLY0
      L0Z=L0Z+DL*DLLZ0
C
C     CALCULATE REFRACTIVE INDEX.
C
   30 RI0=DSQRT(1.0-DEXP(-NNORM))
C
C     CALCULATE GRADIENT OF REFRACTIVE INDEX IN
C     X,Y AND Z DIRECTIONS.
C
      A0=X0*DEXP(-NNORM)/(2*L*A)
      B0=Y0*DEXP(-NNORM)/(2*L*A*ECC**2)
      C0=Z0*DEXP(-NNORM)/(2*L*A*ECC**2)
      MULT=A0*L0X+B0*L0Y+C0*L0Z
C
C     CALCULATE DERIVATIVE OF TANGENTIAL UNIT VECTOR
C     TO RAY,USING ORIGINAL EQUATION FOR FORM OF RAY.
C
      DLLX0=(A0-L0X*MULT)/RI0**2
      DLLY0=(B0-L0Y*MULT)/RI0**2
      DLLZ0=(C0-L0Z*MULT)/RI0**2
C
C     GO BACK INTO ROUTINES TO CALCULATE NEXT POINT
C     FROM KNOWLEDGE OF PRESENT ONE.
C
      GOTO 10
C
C     START TO TRACE NEW RAY IN BEAM.
   20 CONTINUE
   40 CONTINUE
      CALL GREND
      STOP
      END
```

PROGRAM NOTES

The following notes explain the main names used in CUPID and indicate the procedure used for running the program.

ECC = Eccentricity of ellipses defining surfaces of constant density (and hence refractive index).

ACRIT, BCRIT = Dimensions of critical surface ellipse in x and y directions respectively.

L = Density Scale length.

MØ = Initial gradient of beam (with respect to x axis).

THETA = Angle of incidence of beam (= arctan (MØ))

(XØ, YØ, ZØ) = Initially the starting point of central ray in beam, subsequently a general co-ordinate point on ray.

DIA = Diameter of incident beam (BRAD = DIA/2).

DL = Taylor expansion increment.

LØX, LØY, LØZ = $\underline{1}$ in x, y and z directions respectively at a point on the ray.

DLLXØ, DLLYØ, DLLZØ = $\frac{d\underline{l}}{dl}$ in x, y and z directions respectively at a point on the ray.

A = Dimensions of ellipse in x direction used for specifying a particular refractive index surface.

NNORM = ℓn (N/Nc) where Nc = critical density.

RIØ = Refractive index.

AØ, BØ, CØ = Normalised derivatives of refractive index.

Graphical Routines

MAP3D (XC, YC, ZC, XE, YE, ZE, RV, DUM) is a subroutine of the Culham conical projection package CON3D (System Information Note 1/74).

(XC, YC, ZC) = Co-ordinates of centre of sphere of interest.

(XE, YE, ZE) = Co-ordinates of position of viewer.

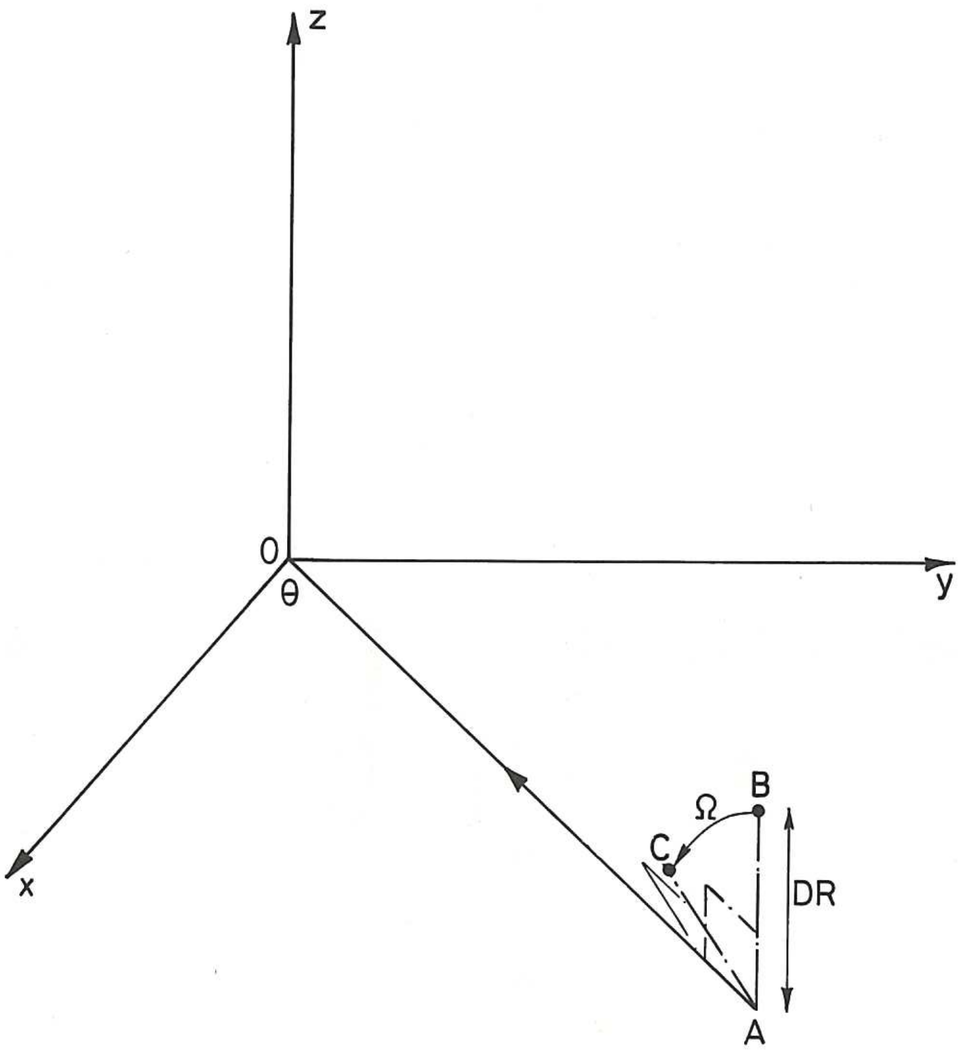
RV = Radius of interest.

DUM = Dummy parameter.

AXES3D draws and annotates positive half of each of the axis.

POSN3D, JOIN3D, have same function as their 2-D counterparts described in Ref.5.

To run the program, the user specifies the parameters indicated in lines 1900 - 2100 of the listing. ZØ is not input in this case, but has been set to zero in program along with DLLXØ, DLLYØ, DLLZØ. RIØ is set equal to unity. The program generates co-ordinate points for each ray, joins them graphically and creates a GRAPHS file which can then be assessed via a Tektronics T4010 console or copied on a chart recorder.



A – Starting point of first ray in beam
B – Starting point of second ray
Rotation through angle Ω gives next point **C** etc.

Fig. 1 Geometry of model.

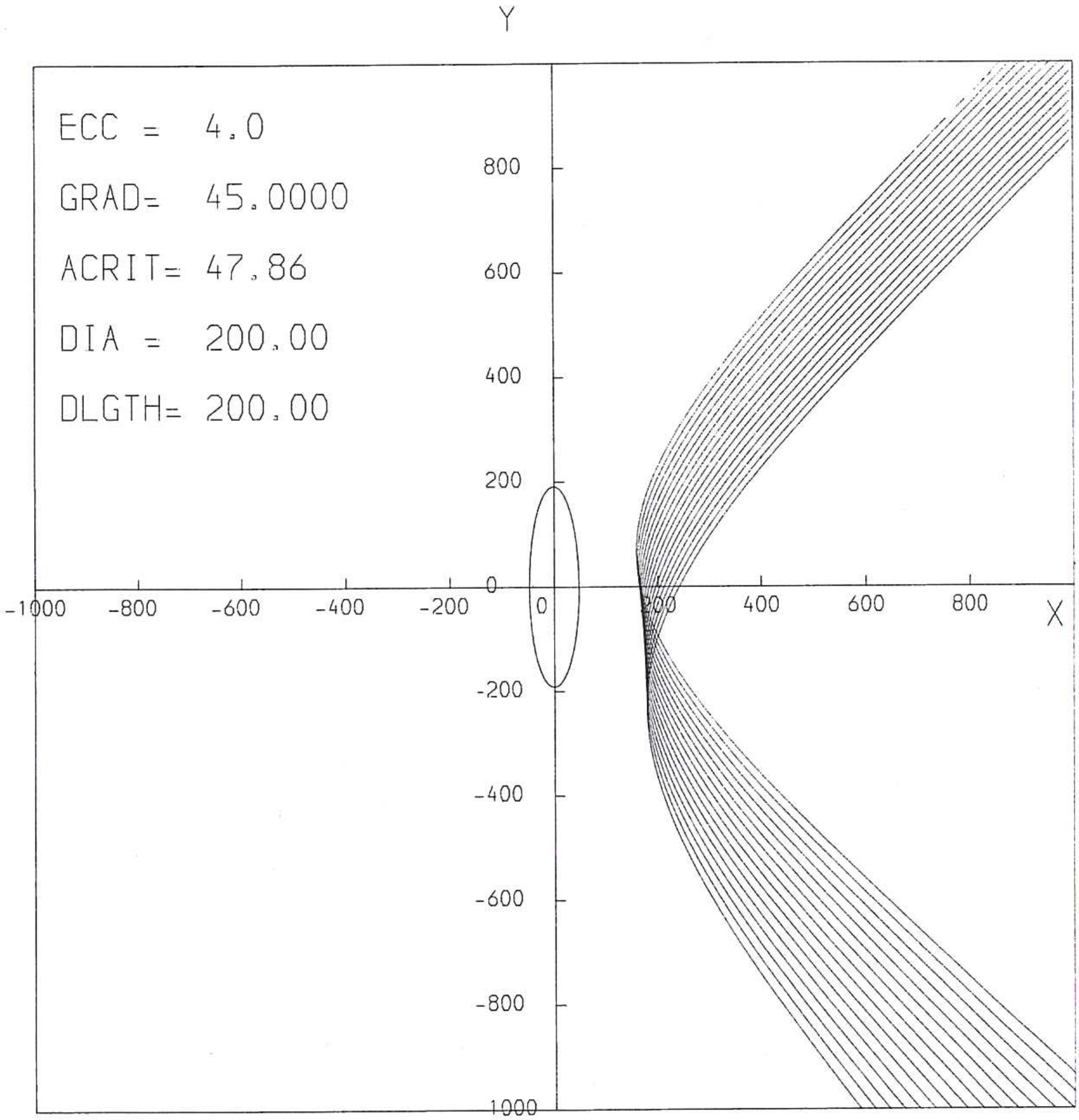


Fig. 2 Trace, using 2-D plotting facility, showing beam incident on a plasma whose critical surface is represented by an ellipse of eccentricity = 4·0

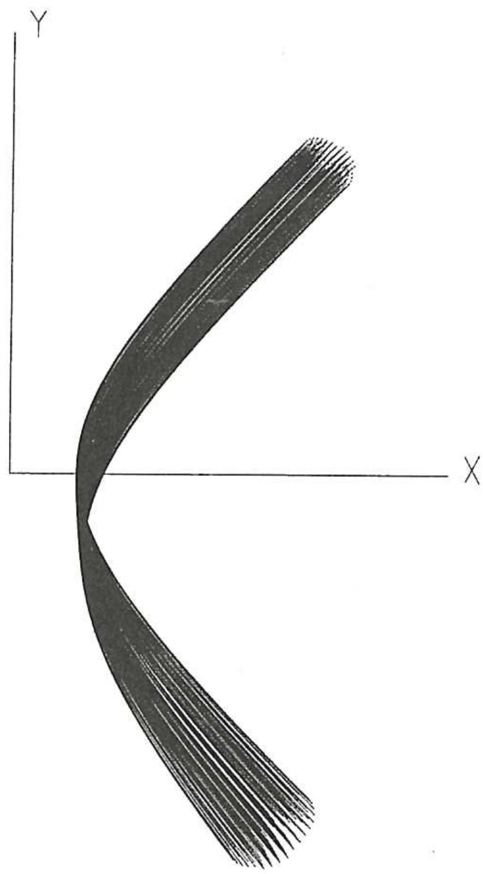


Fig. 3 As Fig. 2, but using 3-dimensional projection facility (looking along z axis).

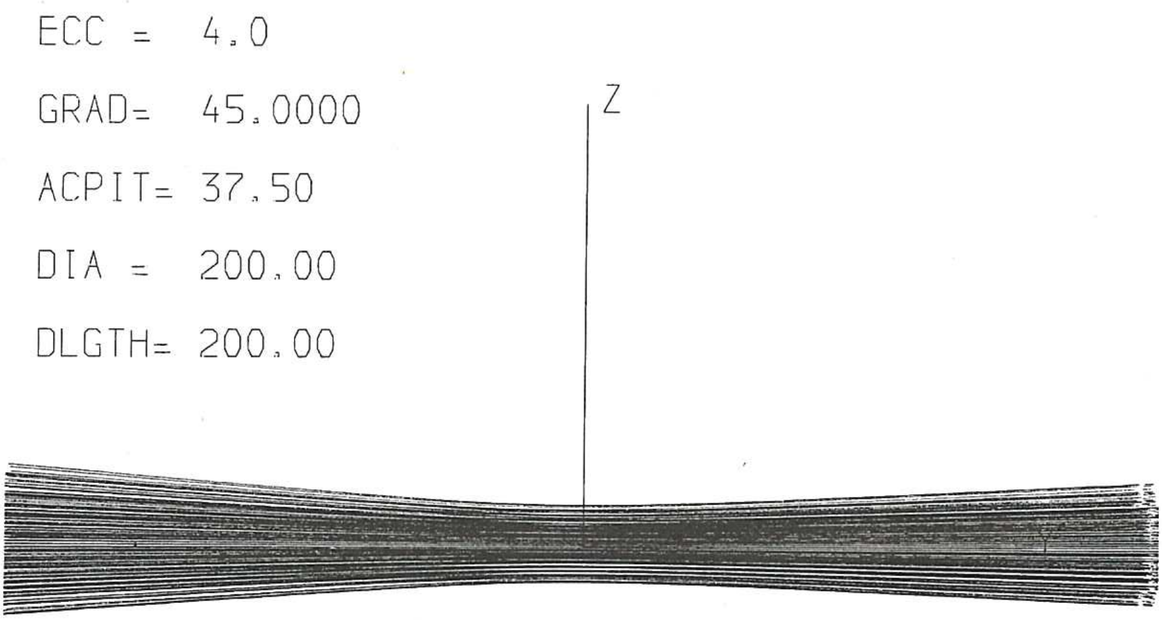


Fig. 4 As Fig. 3, but looking along x axis.

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