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LASER DRIVEN PELLET REFUELING FOR JET (AND REACTOR) USES

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ABSTRACT

Published estimates of pellet sizes and velocities required to refuel JET and post-JET experiments are summarized. Possible advantages and difficulties of accelerating solid, unconstrained hydrogenic (and also jacketed) pellets to these velocities using laser techniques are then discussed. An essential problem to be solved is adequate axial guidance of the pellet during its acceleration, since laser pulse durations of many sound-transit times (in the solid D₂) are necessary to avoid shock-heating the pellet. It is shown that Culham's multi-kilojoule CO₂ TROJAN laser facility is well suited to testing many of the concepts proposed. In particular it is shown that successful verification, and subsequent optimization, of such (novel) techniques would permit single shot tests of contemporary pellet ablation theories by the injection of ~ 1 mm diameter D₂ pellets at velocities $\leq 10^6$ cm s⁻¹ into the JET plasma. Means for scaling these techniques to repetition rates of order 10 Hz, and to the 1 cm pellet diameters possibly required in a working Tokamak reactor, are also discussed.

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INTRODUCTION

1. The problem of sustaining plasma density against particle losses in present day divertor Tokamaks is assuming increasing experimental importance.^(1,2) The technique of gas-puffing, to augment the plasma created during the initial electrical break-down of the gas prefill is now widely used, but with larger devices places increasingly onerous demands on pumping facilities.⁽¹⁾

2. In large toroidal reactors, where refuelling is desired to sustain fusion burn subsequent to ignition, it has been questioned whether gas-puffing can ever provide an acceptable refuelling technique.⁽²⁾ For related reasons, full penetration of thermonuclear plasmas having reactor-like dimensions by neutral (atomic or light-cluster, ie ≤ 100 amu) injection requires energies in the range 0.1 - 1 MeV/atom;^(1,3) conceptual toroidal reactors are usually designed to work at mean temperatures of 10-20 keV, so that neutral injection of this sort would imply heating rather than simply particle replacement. Whatever the mean working temperature, such neutral refuelling techniques would certainly necessitate high D-T burn-up to (economically) sustain the very high powers recirculating within the reactor complex.

3. It has been suggested that injection with heavier clusters at velocities of $\sim 10^6$ cm s⁻¹ might conceivably be used for refuelling.⁽⁴⁾ Neutral clusters having the suggested mass of $\sim 10^9$ amu can be produced by supersonic gas flow through cooled Laval nozzles.⁽⁵⁾ Electrostatic acceleration using accelerators of ≥ 1 MV are envisaged; the minimum voltage will depend on the maximum charge which can actually be deposited on the cluster before its physical disintegration (due to the resultant internally repulsive Coulomb force). It is not established with any certainty how far such clusters could penetrate into a thermonuclear plasma; the technological problems associated with (continuous) gas pumping required for cluster-injection into the high vacuum reactor vessel should also be noted.

4. If cluster penetration at $\sim 10^9$ amu should prove inadequate or technologically inconvenient, what other options remain? As noted by Spitzer, and Rose (among others) the injection of larger entities, ie liquid droplets or solid pellets, increases the possibilities for self-shielding during passage through the thermonuclear plasma. However, it can be shown using the quite general arguments given in Appendix 1, that the acceleration

of larger (unconstrained) droplets having a radius r of 100-1000 μm to velocities of $\geq 10^5 \text{ cm s}^{-1}$ would probably necessitate acceleration flight-paths exceeding 0.1-10 km, to avoid break-up during acceleration. Now stabilizing forces on liquid hydrogen in this size range, arising from a surface tension stress $s/r \sim 3000-30 \text{ dynes cm}^{-2(6)}$ are much less than the tensile strength of solid H_2 ($\sim 4 \times 10^6 \text{ dynes cm}^{-2(6)}$). It follows that pellet (rather than droplet) acceleration should offer the advantage of significantly greater mechanical strength (and hence shorter flight-paths), as well as less arduous toroidal pumping requirements (because of the lower initial fuel temperature). On the other hand, a general disadvantage of working with solid, rather than liquid, hydrogen is that means must be devised for storing (or otherwise making accessible) pellets at the repetition rates of $\sim 10-1000 \text{ Hz}$ required for typical fuelling operations.

5. This paper first summarizes the pellet sizes and velocities likely to be of interest in JET and subsequent larger devices. It is noted that for such machines alternative methods of electrical or mechanical acceleration present formidable technological problems; the advantages and some possible difficulties associated with laser pellet acceleration techniques are then discussed in subsequent sections.

PELLET ACCELERATION REQUIREMENTS

6. Table 1 summarizes recent^(1,7) calculations of the minimum velocity (v) required by a D_2 pellet of initial radius r_m to penetrate a depth of $2/3$ of the minor radius in four representative Tokamaks (DITE, ASDEX, JET and a Culham 5 GW conceptual reactor⁽⁸⁾). Here, the maximum acceptable value of r_m is determined by the requirement that each cryogenic pellet should contribute no more than 5% of the particle inventory in the confined toroidal plasma, to reduce the possibility of thermal and MHD instabilities. The wide range of velocities tabulated against a specific axial density (n_a) electron temperature (T_{ea}) and ion temperature (T_{ia}) appropriate to any of these four torii arises for the following reasons:

(a) two alternative profiles of the magnetically confined density and temperature are postulated (modified-parabolic,⁽¹⁾ and Golovin⁽⁹⁾);

(b) three alternative models of the pellet-ablation are considered.

The validity of the three alternative (bare-pellet-neutral-evaporating,⁽²⁾

neutral-shielded-pellet⁽¹⁰⁾ and bare-pellet-ion-evaporation⁽¹¹⁾ models need not be discussed here; it is sufficient to note from Table 1 that both JET and post-JET devices may require the injection of quasi-spherical pellets having diameters between 1 and 10 mm and velocities of order 10^6 cm s^{-1} in order to test pellet-ablation theories, and to assess the potential of such techniques for refuelling Tokamaks or any other magnetic-confinement device utilizing a divertor. Although the energy invested in directed motion of a pellet moving at 10^6 cm s^{-1} is relatively modest ($\sim 1 \text{ eV/atom}$), we should note that

- (a) for a large D_2 pellet (eg $r \geq 2.3 \text{ mm}$, mass $\geq 10^{-2} \text{ gm}$) the kinetic energy will exceed $\frac{1}{2} \text{ kJ}$;
- (b) the binding energy of the solid is only $\sim 10^{-2} \text{ eV}$. (Thus provision for radial confinement and guidance may well be needed).

7. Appendix 1 conveniently highlights a major difficulty of accelerating pellets of radii $\geq 1 \text{ mm}$ to such velocities by any method which utilizes a localized stress; either the acceleration flight-path L must exceed 10m or the pellet must be radially confined. (It is similarly clear why smaller droplets at low velocities, eg 10^5 cm s^{-1} for 10^9 amu clusters⁽⁵⁾ or 10^4 cm s^{-1} for $10\text{-}100 \mu\text{m}$ droplets,⁽¹²⁾ have already been produced using relatively straightforward gas dynamic techniques.) Alternative techniques for accelerating (non-conducting) cryogenic hydrogen include:

- 7.1 'Electrostatic' Here the ultimate limit is set by the maximum sustainable electrostatic stress at the surface of the charged pellet; if we set $E_r^2/8\pi \sim 4 \times 10^6 \text{ cgs}$, or $E_r \sim 3 \times 10^6 \text{ v cm}^{-1}$, one concludes⁽¹¹⁾ that 200 MV accelerators are needed for 1 mm pellets at 10^6 cm s^{-1} .
- 7.2 'Mechanical' Relatively slow accelerations to limiting velocities of $(1 \sim 3) \times 10^5 \text{ cm s}^{-1}$ are possible using light-gas guns (driver sound-speed limit $\sim 10^5 \text{ cm s}^{-1}$) or centrifuges (limited by strength of materials, eg 1 m diameter epoxy matrix rotating at 1 kHz).
- 7.3 'The Rocket Effect' Here an appropriate energy source (eg laser, electron or ion beam) is used to propel the pellet by directional ablation. Thermal conduction in the corona may automatically provide some radial confinement of the pellet, but it is important to ensure that the resulting ablation is suffi-

ciently asymmetric to give a significant net forward reaction (ie some 'shadowing') if a reasonable driver efficiency is to be achieved. From a practical point of view it is particularly convenient to use a laser since it can maintain a carefully profiled intensity distribution over significant axial distances. The subsequent discussion will therefore concentrate on the use of CO₂ lasers, which are conveniently energy scalable and have usefully high efficiencies.

CO₂ LASER PELLET-ACCELERATION

8. As with inertial confinement fusion, detailed optimization of the laser pellet interaction will be very open ended, because there are so many variables at the designer's disposal. An illustrative example of the type of experiment which might be explored with only minor modification to the TROJAN multi-kilojoule CO₂ laser facility at Culham will serve, however, to highlight several important practical considerations. Suppose we wish to accelerate a D₂ pellet of radius $r = 0.5$ mm and mass $m \sim 10^{-4}$ gm to a terminal velocity of $v = 10^6$ cm s⁻¹ over a flight-path $L = 10$ cm, by means of a linearly rising and falling laser-induced acceleration. Assuming a symmetrical laser pulse, the peak acceleration $v^2/2L \sim 5 \times 10^{10}$ cm s⁻² occurs half way through a pulse of total duration $\tau = 4L/v \sim 40$ μ s; the corresponding maximum stress P is of order 6×10^8 dynes/cm². (We note from Appendix 1 that this stress significantly exceeds the tensile strength of the pellet, so that we wish to ensure a gentle, ie quasi-isentropic, compression and decompression of the pellet during its acceleration.)

8.1 Equation of State

Ignoring the electron contribution appropriate to the higher temperatures (or pressures) obtaining in ionized plasmas, the internal pressure (p) is given as a function of the specific volume (V) and temperature (T) by:

$$p = pc(V) + \Gamma_0 \frac{c_v T}{V} \quad \dots(1)$$

where $pc(V)$, the elastic component at $T = 0$, is of order 10^9 cgs; c_v the specific heat at constant volume varies from 10^7 ergs/mole K at low temperatures to $\sim 1.5 \times 10^8$ ergs/mole K at the triple point; and Γ_0 , the Grüneisen coefficient, is of order 1. Thus the temperature rise associated with an

isentropic compression to pressures as high as $\sim 2.5 \times 10^9$ dynes/cm² should only be of the order 10K, the exact value depending on the ratio of ortho/para D₂ and the initial pellet temperature.

8.2 Avoidance of Shocks

The longitudinal sound speed⁽⁶⁾ in solid D₂ is $< 2 \times 10^5$ cm s⁻¹, so that the pressure pulse takes > 40 sound transit times to reach a maximum, and sharply rising shocks may be avoided using well-tailored incident intensities.

Approximately $\frac{1}{2}$ of the energy transmitted to any pellet by a strong shock could appear as internal heat rather than as directed kinetic energy; at velocities of order 10^6 cm s⁻¹ this would clearly produce an unacceptably high transverse divergence of the accelerated material.

8.3 Radial Containment

Lateral flow of an unconstrained pellet would occur at the envisaged acceleration, with the stresses becoming proportionately greater for larger pellet masses. However, radial confinement and axial guidance might be simultaneously achievable using an appropriately contoured (far field) spatial intensity distribution;⁽¹³⁾ here, the coronal temperature (which determines the thermal 'conductivity') must clearly be chosen to avoid too-symmetrical ablation. Alternatively and more simply, a thin cylindrical jacket (of radius $\sim r$ and thickness t) can be used to contain the hoop stress

$$\sigma = P(r/t) \quad \dots(2)$$

The tensile strength (σ_T) of Al or Be at cryogenic temperatures is $\sim (1-2) \times 10^9$ dynes cm⁻², with Mo and some steels having a strength ≥ 5 higher and materials such as mylar, nylon, borosilicate glass or polyethylene a factor of 2-10 lower. A jacket of thickness

$$t \geq (P/\sigma_T)r \quad \dots(3)$$

thus provides useful mechanical containment for modest accelerations ($\leq 10^{10}$ cm² s⁻¹). However the mass of the jacket (of density ρ_1) exceeds that of the pellet (density $\rho_0 \sim 0.2$ gm cm⁻²) when

$$\frac{2\pi r t \rho_1}{\pi r^2 \rho_0} \sim 2 \left\{ \frac{P}{\sigma_T} \right\} \left\{ \frac{\rho_1}{\rho_0} \right\} \geq 1 \quad \dots(4)$$

As a specific example a 3×10^{-3} cm thick Ni jacket could provide adequate strength for the 100 μgm D_2 pellet of the present discussion, but the target mass would be increased by ~ 500 μgm . Eqn.A1 shows that P scales as rV^2/L , so that the mass of the jacket becomes more of a problem at higher accelerations, or with larger pellets. However, metallic jackets might be stripped from such a pellet by eddy currents induced during transit through an appropriately orientated magnetic field (B) of order 10^5 gauss and physical extent $\geq LP(B^2/8\pi)^{-1}$. (The joule heating induced in such a jacket is discussed in Appendix 2.)

8.4 Required Laser Intensity

Prediction of the laser intensity required to heat coronal plasma at a critical density n_c to a 'temperature' of order $(P/n_c) \sim 30$ eV is affected by the effective charge \bar{Z} of the corona if a jacket is used, by the absorption and heat transport mechanisms obtaining near the critical surface, and by the subsequent multi-dimensional flow of the ablating plasma. Many of these issues have been addressed elsewhere (see for example Refs.14-19). For the present we shall postulate that an incident $\lambda = 10.6$ μm intensity of order 10^9 W cm^{-2} is required, and conclude that an absorbed laser power of order 8 MW and energy 0.3 kJ would be needed to give the 100 μgm pellet a kinetic energy of $\sim 5\text{J}$. A pulse duration of ~ 40 μs , with sufficient power and energy reserves for spatially contouring the beam, can be provided with relatively minor changes to the TROJAN laser facility, which could thus provide a very important test of whether a pellet can be adequately guided over the distance L without unacceptable deviations of the pellet away from the axial focus.

8.5 Illustrative Focusing Requirements

If a (single mode) gaussian beam is focused by aberration free optics to a spot size (full width to $1/e^2$ intensity) of $\sim 2r$, the intensity is constant to a factor of ~ 2 over an axial distance

$$2Z \sim 2\pi r^2/\lambda \sim 15 \text{ cm} \quad \dots(5)$$

where Z is the 'depth of focus'. Neglecting truncation and aberrations, the f-number (F) required of a spherical mirror focusing this mode is given by:

$$4 \lambda F / \pi \leq 2r \quad \dots(6)$$

Thus the optical system has an $F \leq 83$, corresponding to a focal length of $\sim 16.5\text{m}$ for a TROJAN beam diameter of $\sim 20\text{ cm}$. (It is in principle possible to utilize acceleration path-lengths greater than 15 cm with some loss of beam utilization, provided that the CO_2 laser pulse duration can be lengthened sufficiently.) In practice a toroidal mirror, zone plate or other phase-adjusting device might be utilized to provide the desired, slightly hollow far field intensity distribution.

REACTOR REQUIREMENTS

9. If the pellet ablation scaling laws implicit in Table 1 were validated experimentally in JET, a post-JET (ie reactor) requirement of pellet radii $r \leq 5\text{ mm}$ is indicated. What are the practical implications of such a ten-fold enhancement of radius?

9.1 Depth of focus is not, in principle, a problem.

From eqns.5 and 6, the maximum attainable depth of focus using diffraction limited $\lambda = 5$ or $10\ \mu\text{m}$ lasers and aberration free $F \leq 1000$ optics is 15 - 30m. However, care would be needed to ensure that the corresponding laser pulse duration ($\leq 3\text{ ms}$) did not exceed the 'burn-through' time ($\sim n_o r / n_c c_s$) of the pellet.

9.2 Axial stability of the pellet during acceleration is essential.

Assuming laser pellet guidance provided a sufficiently stable pellet trajectory,⁽¹³⁾ incident intensities similar to those discussed in para.8 should permit acceleration to velocities of $\sim 10^6\text{ cm s}^{-1}$ along much more modest path lengths (of $\geq 1\text{m}$). The stress generated can thus be sufficiently low (ie $\leq 6 \times 10^8\text{ dynes cm}^{-2}$) for radial confinement by a cylindrical jacket to be a viable technical option.

9.3 There is considerable scope for optimizing efficiency.

Detailed optimization of overall acceleration efficiency is essential and would necessitate a particularly careful balance of laser wavelength

(ie CO or CO₂) and incident intensity, and of the ablating material (ie jacket, with or without end cap, or bare D-T?). In either event the rocket equations indicate that a significant fraction of the pellet should be ablated for an optimal acceleration.

9.4 Very large lasers are required; these can probably be built.

For these very arduous (reactor-like) requirements a laser pulse duration of up to 400 μ s, with energies and peak powers of ≥ 300 kJ and ≥ 0.8 GW, might be necessary. At a total injection rate of 20 Hz this would imply a mean laser power of order 6 MW during the fuelling cycle. Discharge efficiencies of up to 60% have already been reported for sub-kilojoule, ~ 100 μ s duration, electron beam, pre-ionized CO lasers;⁽²⁰⁾ thus electrical power input requirements for the refuelling cycle might be ~ 20 MW if the assumptions discussed above are experimentally validated.

9.5 Batching (of the technique proposed for JET) can be envisaged.

As noted in para.4, on-line production and acceleration of the targets could well prove technically inconvenient. If this were so, one could envisage the use of 4 or more refuelling devices spaced around the reactor, each serviced by its own laser. This would clearly reduce the mean power requirements on individual windows/focusing optics, etc., and would facilitate the positioning of batches of, say, 100 stationary solid pellets using a (mechanical) carousel. Each pellet in the (stationary) batch could then be irradiated by suitably deflecting the laser beam. Such a procedure would have the advantage of more uniformly distributing the pellets within the torus; however, care would be required in the design of the carousel to ensure that irradiation of one pellet did not perturb its neighbours. As noted elsewhere thermally induced stresses, due to the β decay of DT pellets,⁽²¹⁾ should set an upper limit of $r \leq 1$ cm on the size of any (unconstrained) solid pellet which is to be stored indefinitely. If larger pellets were needed, storage as a liquid in metallic jackets near the triple point can be envisaged, with relatively rapid freezing if necessary before use.

CONCLUSIONS

10.1 Useful tests of speculative laser acceleration ideas, which if successful

would permit the measurement of pellet ablation rates in JET, are possible with minor modifications to equipment and techniques now being employed for laser-plasma filling⁽¹⁴⁾ of CLEO stellarator. If the outcome of the JET work were encouraging, the general technique would appear to be scalable to Tokamak reactor requirements.

10.2 It should be noted that for such post-JET applications, the energy pulse from the laser would in all probability be comparable to that from the ANTARES CO₂ laser now under construction at Los Alamos. However, the peak power would be very much less, thus simplifying window and capacitor bank requirements. The longer pulse duration might, however, pose spatial mode problems associated with acoustic waves generated in the laser - a problem of less importance for short pulse, inertial confinement fusion drivers. Finally, the mean electrical input required for driving this 'acceleration laser' would certainly be less than that envisaged in inertial confinement reactors (where, of course, additional laser energy is used for compression and ignition of the pellet); in the examples discussed the gross energy input for refuelling would be ~ 1 keV/atom (using a CO₂ laser).

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NOTE ADDED IN PROOF My attention has been drawn to reference 22, which provides an extensive summary of alternative refuelling techniques. The main conclusions of the present work remain unaltered.

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TABLE 1

	5 GW(t) Reactor ⁽⁸⁾	JET	ASDEX	DITE
n_a (10^{13} cm^{-3})	18	3	2	4.3
T_{ea} keV	20	10	4	0.8
T_{ia} keV	20	10	2	0.25
Pellet r_m (mm)	4.8	0.84	0.35	0.31
$V \times 10^6$ (cm s^{-1})	1 - 2	0.2 - 3	0.02 - 0.6	0.003 - 0.03

APPENDIX 1

Stress Induced in Targets Accelerated for Refuelling

1. The stress associated with accelerating a sphere of radius r and density ρ to terminal velocity v in a distance L (by localised, rather than uniform volumetric, forces) is of order:

$$P = \frac{4}{3} \pi r^3 \rho \left(\frac{v^2}{2L} \right) / \pi r^2 \sim r \rho v^2 / 2L \quad \dots(A1)$$

2. Unconstrained targets might be expected to deform under the following conditions:

(i) Inviscid liquids: if P exceeds the pressure due to $S^{(6)}$ the surface tension

$$P > S/r \quad \dots(A2)$$

(ii) Solid pellets: when P exceeds the (temperature sensitive) yield strength, which lies in the range $(0.5 \sim 2) \times 10^6$ dynes/cm² for para and normal H₂⁽⁶⁾ to $(3 \sim 5) \times 10^6$ dynes/cm² for normal D₂.

Typical values for P and S/v are tabulated for H₂[†] below:

Radius	Stress (cgs) S/r (dynes cm ⁻²)	P (dynes cm ⁻²)							
		v(cm s ⁻¹)	10 ⁴	10 ⁴	10 ⁵	10 ⁶	10 ⁴	10 ⁵	10 ⁶
		L (cm)	1	0.1	10	1000	10 ⁻²	1	100
1 μm*	3 x 10 ⁴	5 x 10 ²		5 x 10 ³		5 x 10 ⁴			
10 μm	3 x 10 ³	5 x 10 ³		5 x 10 ⁴		5 x 10 ⁵			
100 μm	3 x 10 ²	5 x 10 ⁴		5 x 10 ⁵		5 x 10 ⁶			
1 mm	3 x 10	5 x 10 ⁵		5 x 10 ⁶		5 x 10 ⁷			
10 mm	3	5 x 10 ⁶		5 x 10 ⁷		5 x 10 ⁸			

* Mass ~ 5 x 10¹⁰ amu

† Tabulated values of (S/r) should be increased by a factor of (x 1.25) for D₂; similarly the values for P should be increased by factors of approximately (x 2) for D₂ and (x 3) for T₂.

APPENDIX 2

Joule Heating in Magnetically-Decelerated Jackets

1. The circumferential emf E induced during deceleration (from velocity v , over path length x) by a thin cylindrical metal jacket of thickness w , finite resistivity ζ radius r and length $\sim 2r$ by a magnetic field $B = 10^5$ emu is of order

$$E = - \frac{d\varphi}{dt} \sim B \cdot \pi r^2 \cdot \left(\frac{v}{x}\right) \text{ emu} \quad \dots(\text{A2.1})$$

Due to ohmic heating in a skin depth Δ ($\leq w$), having an effective resistance $R \sim \pi\zeta/\Delta$, energy

$$(E^2/R)(x/v) \sim (\pi B^2 v x 10^{-16}/\zeta x)(r^4 \Delta) \text{ cgs} \quad \dots(\text{A2.2})$$

is dissipated in the jacket, producing a mean temperature rise (T) of

$$(2\pi r w \cdot 2r \cdot \rho) c T = (E^2/R)(x/v) \text{ cgs} \quad \dots(\text{A2.3})$$

where c is the specific heat. It follows that

$$T \sim (B^2 x 10^{-16}/4\zeta \rho c) (r^2 v/x)(\Delta/w) \text{ cgs.} \quad \dots(\text{A2.4})$$

2. If significant magnetic deceleration is to be achieved, the characteristic decay time associated with the self-inductance (\mathcal{L}) of the jacket must be comparable to the time taken for deceleration. Thus we require $(\mathcal{L}/R) \geq (x/v)$, or equivalently $\Delta \leq w$. Hence

$$(2\pi^2 r x 10^{-9}/\pi\zeta\Delta^{-1}) \geq (x/v)$$

$$\text{or } w \geq \Delta \geq (x/v)(1.6 \times 10^8 \zeta/r) \quad \dots(\text{A2.5})$$

This equation places a constraint on the minimum jacket thickness additional to that discussed in the main text. Nominal values of T , appropriate to the deceleration of JET-type ($r = 0.5$ mm) and reactor-type ($r = 5$ mm) jackets at cryogenic temperatures from velocities of $\sim 10^6$ cm s⁻¹ are tabulated overleaf.

3. The strong temperature dependence of ζ and c has been ignored in the above discussion for simplicity, so that these 'nominal' values of Δ and T represent respectively lower and upper bounds to real values which differ by at least an order of magnitude from those tabulated. However, it is

r (mm)	x (cm)	Jacket	t	Δ $\leftarrow 10^{-3} \text{ cm} \rightarrow$	w	Mass Ratio (Jacket w/D ₂)	T (10 ³ K)
0.5	15	Ni	3	24	24	x 43	2
0.5	15	Cu	6	0.45	6	x 11	8
5.0	150	Ni	30	24	30	x 5	15
5.0	150	Cu	60	0.45	60	x 11	8

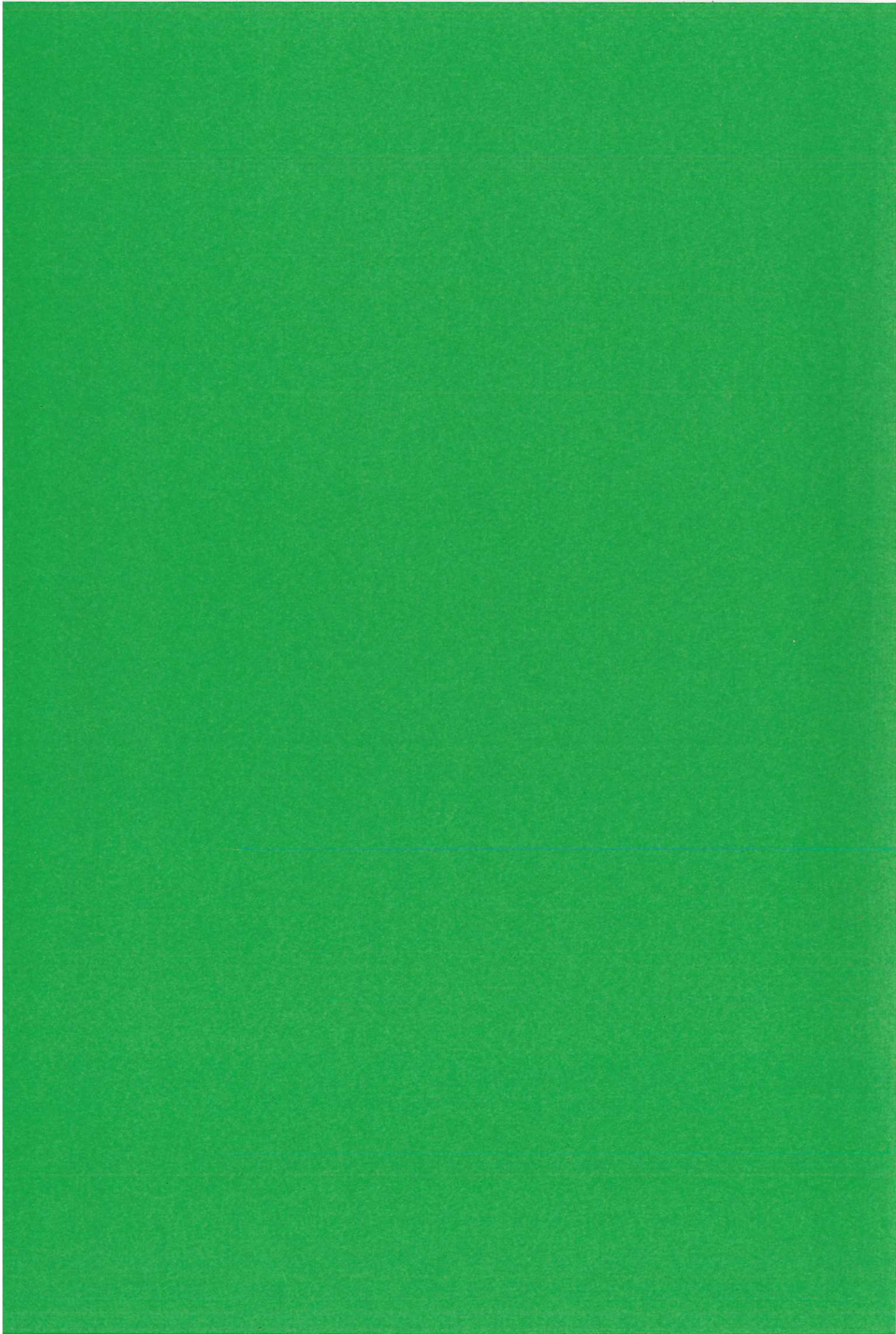
clear from the above that, excluding any consideration of superconducting jackets

3.1 the cryogenic D₂ core would rapidly become detached during deceleration of the rapidly warming jacket;

3.2 an impulsive travelling-wave magnetic accelerator is not a viable alternative to the other techniques outlined in para.7;

3.3 deceleration path-lengths of many metres, with fields of < 100 kG, are needed to avoid excessive heating of the jacket (which should be a good conductor such as copper). Deformation by buckling, or by Rayleigh-Taylor instabilities in the jacket, seems unlikely to impede the escape of the D₂ core;

3.4 optimization of the overall efficiency would necessitate careful optimization of jacket mass.



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