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Report

# ON THE INTERPRETATION OF NEUTRON SPECTRA TO DETERMINE THE CENTRE-OF-MASS MOTION OF FUSING DEUTRONS

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ON THE INTERPRETATION OF NEUTRON SPECTRA TO DETERMINE

THE CENTRE-OF-MASS MOTION OF FUSING DEUTERONS

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## ABSTRACT

The interpretation of neutron energy spectra to determine the centre-of-mass velocity of the fusing plasma deuterons in high-intensity pulsed discharges is described.

An approximate theory is presented enabling the shape of the neutron spectra from mass movement of fusing deuterons to be predicted.

This theory is applied to the interpretation of experimental neutron spectra in terms of simple models of the movement of the reacting ions.

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## INTRODUCTION

1. Neutrons emitted from high-intensity pulsed discharges in deuterium[1] are the result of the reaction $D(d,n)He^3$. The presence of anisotropy of the energy of the neutrons is an indication of a net centre-of-mass motion of the fusing deuterons and eliminates the possibility of a stationary thermonuclear plasma contributing a significant part of the neutron yield. The energy spectrum of neutrons to be expected from a thermonuclear plasma is given elsewhere[2].

2. The energy spectra of neutrons emitted at a fixed angle to the direction of the motion of the centre of mass of their fusing deuteron source are defined by the parameters of location and width. These parameters can be used to measure the centre-of-mass velocity of the deuterons as follows:-

   (a) Location; anisotropy of the mean energy of the neutron spectra is an indication of ordered motion among the fusing deuterons but does not eliminate the possibility of a random motion superimposed on the ordered motion.

   (b) Width; measured neutron spectrum width is the resultant of instrumental width, deuteron energy spectrum width, which includes both random and ordered velocity distribution, and a distribution of the direction of the ordered motion.

3. The use of these two parameters to measure deuteron energy is now discussed. The complexity of the expression for the energy of neutrons emitted by fusing deuterons can be reduced by assuming that the kinetic energy of the deuteron system is small compared with the energy released by their fusion. The resultant expressions for velocity are correct to within 10% for velocities of the centre-of-mass of the deuteron system less than $4 \times 10^8$ cms/sec. Interpretation of the results in terms of the target deuteron at rest will also give values for the centre-of-mass velocity which will be applicable, within the limits of experimental error, even if the target feuteron is not stationary. Other assumptions made in the analysis are:-

   (a) The effect of the random motion of the deuterons is negligible.
   (b) The deuterons are impinging on a target of uniform density.
   (c) The majority of neutrons are produced by deuteron systems with the same centre-of-mass velocity components.
   (d) The paths of neutrons entering the detector are parallel.
   (e) The toroidal nature of the discharge, where applicable, has a negligible effect.

## THEORY

4. Consider the fusion of two deuterons of masses $2M_p$, $M_p$ being the proton mass, $1.67 \times 10^{-24}$ gms. The incident deuteron has energy $E_d$ and velocity $V_d$, the target deuteron is at rest. The neutron is emitted with energy $E_n$ at an angle $\eta$ to $V_d$. The reaction is exoergic releasing energy Q. Conservation of energy and momentum give the equation

$$3Q = 4E_n - E_d - 2(2E_d E_n)^{\frac{1}{2}} \cos \eta \qquad (1)$$

5. The velocity of the centre-of-mass $\underset{\sim}{V}$, of the fusing deuterons is half the laboratory velocity of the incident deuteron, thus

$$E_d = 4M_p \underset{\sim}{V}^2 \qquad (2)$$

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In cylindrical geometry $\underset{\sim}{V}$ can be resolved into its three components

$$V_{\mathbf{z}} = |\underset{\sim}{V}| \cos \alpha$$

$$V_{\theta} = |\underset{\sim}{V}| \cos \beta$$

$$V_R = |\underset{\sim}{V}| \cos \gamma \tag{3}$$

$\alpha$, $\beta$ and $\gamma$ are the angles between the centre-of-mass velocity vector and the cylindrical co-ordinate axes shown in figure 1. Consider a neutron detector placed at $D_0$ at an angle $\xi$ to the z-axis. From equation 1 using the assumption that $E_d \ll Q$, then the neutron energy seen by the detector is given by

$$E_n = \frac{3}{4} Q + \frac{1}{4} \cos \eta \left[ \frac{6 Q E_d}{M_p} \right]^{\frac{1}{2}} \tag{4}$$

where $\quad \cos \eta = \cos \alpha \cos \xi + \sin \alpha \cos \phi \tag{5}$

$\eta$ being the angle between $\underset{\sim}{V}$ and the neutron entering $D_0$, $\phi$ the angle between the radius vector and the reference radial axis through $D_3$, as shown in figure 1.

6.  Thus, with the detector placed at $D_1$, $\eta = \alpha$ and $\xi = 0°$ and the neutron has energy $E_{n1}$. Neutrons entering a detector at $D_2$ have energy $E_{n2}$, given by $\xi = \pi$ and $\eta = \pi - \alpha$ in equation 4. If $\delta = E_{n1} - E_{n2}$, the separation of the peaks of the neutron spectra from the two views of the fusion reactions, then the axial component of the centre-of-mass velocity of the deuterons is obtained from equations 3 and 4.

$$\text{i.e.} \quad V_{\mathbf{z}} = \left[ \frac{G}{6M_pQ} \right]^{\frac{1}{2}} \cdot \delta \quad \text{cm/sec} \tag{6}$$

where $\delta$ and $Q$ are in MeV and $G = 1.59 \times 10^{-6}$ ergs/meV.

7.  Following the argument presented by Jones et al[3], a detector placed at $D_3$, receiving neutrons only from a radial plane of the discharge, subtends an angle $\phi$ with the radius vector from the discharge centre line through the fusion deuterons, $\phi$ taking all values from 0 to $2\pi$. A range of reaction angles will be observed, giving a neutron spectrum of finite width. The neutron spectrum is given by

$$N(E_n) = K \left| \frac{d\phi}{dE_n} \right| dE_n$$

From figure 1, $\cos \eta = \cos \gamma \cos \phi + \cos \beta \sin \phi$

Using equations 1, 2 and 3,

$$E_n \simeq \frac{3}{4}Q \pm \left[ \frac{3M_pQ}{2} \right]^{\frac{1}{2}} \cdot (V_R \cos \phi + V_\theta \sin \phi) \tag{7}$$

and $\quad \dfrac{d\phi}{dE_n} = \dfrac{1}{V_\theta \cos \phi - V_R \sin \phi} \cdot (2M_pE_n)^{\frac{1}{2}} \cdot \left[ \dfrac{1}{2} + \dfrac{3Q}{8E_n} \right] \tag{8}$

The energy limits of the spectrum are thus given by,

$$V_\theta \cos \phi = V_R \sin \phi$$

i.e. where $\quad E_n \simeq \dfrac{3Q}{4} \pm \left\{ \dfrac{3M_pQ}{2}[V_\theta{}^2 + V_R{}^2] \right\}^{\frac{1}{2}} \tag{9}$

8.  The functions $\phi(E_n)$ and $\dfrac{d\phi}{dE_n}$ have the form shown in figure 2, the width $\Omega$ being given by equation 9 as

$$\Omega = [6Q M_p (V_\theta{}^2 + V_R{}^2)]^{\frac{1}{2}}$$

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A more general analysis, with cloud chamber placed at Do gives

$$\Omega \,(\xi) \;=\; [\, 6 \; M_p \; Q \; (V_\theta{}^2 + V_R{}^2) \,]^{\frac{1}{2}} \; \sin \xi \qquad\qquad (10)$$

Thus a change in neutron spectrum width with observation angle can be an indication of deuteron motion.

9. If only half the radial plane of the discharge is viewed by the detector so that ∅ varies from 0 to π, the intrinsic energy spectrum of the neutrons detected takes the form shown in figure 3a as the top-half view. Those neutrons emitted from regions with π < ∅ < 2 π give the energy spectrum for the bottom half view shown in figure 3b. $E_b$ and $E_c$ are the energies equivalent to ∅ = 0 and ∅ = π respectively, the separation λ, being given by

$$\lambda \;=\; (6 Q M_p)^{\frac{1}{2}} V_R \qquad\qquad (11)$$

10. In practical units the expressions for the centre of mass velocity components are thus:

$$V_z{}^2 \;=\; \frac{G}{6 M_p Q} \, \delta^2$$

$$V_\theta{}^2 \;=\; \frac{G}{6 M_p Q} (\Omega^2 - \lambda^2) \qquad\qquad (12)$$

$$V_R{}^2 \;=\; \frac{G}{6 M_p Q} \, \lambda^2$$

## DERIVATION OF THE INTRINSIC SPECTRA FOR SIMPLE MODELS

11. The above analysis can be used to predict the shape of the intrinsic spectra from simple deuteron fusion systems. The simplest model consists of a stream of mono-energetic deuterons following similar paths along the discharge tube and impinging on a 'cold' deuterium plasma. Thus all fusion reactions would exhibit the same components of centre-of-mass velocity. Construction of the radial spectra ($\xi = 90^o$) expected for given values of $V_\theta$ and $V_R$ is straightforward using equations 7, 8 and 9. Conversely, interpretation of spectra obtained experimentally, in terms of the above simple model, is effected by measuring δ, λ and Ω and using equation 12. The presence of azimuthal deuteron motion ($V_\theta$) is best exhibited and measured by constructing the 'difference spectrum'; this consists of subtracting the 'bottom-half view' from the 'top-half view' as shown in figure 3c.

12. A more complex model of high-energy deuteron motion in a plasma which can be interpreted by the above methods consists of two groups of deuterons, each group being mono-energetic and having its own centre-of-mass velocity components. The expected spectra are simply the sum of the spectra computed for each group separately. Interpretation of experimental spectra, however, in terms of two groups of deuterons, is more difficult. For example, should the two 'half views' be identical so as to give a difference spectrum which is virtually zero, then interpretation could suggest a one group model with zero azimuthal motion or alternatively, a two group situation, the groups having identical energies but having opposing $V_\theta$ components (e.g. deuteron paths being left-hand and right-hand helices). This ambiguity can be eliminated by examining the 'radial' neutron spectra from unequal areas of the plane parallel to the tube bore. In this case ∅ would vary from -Δ to π + Δ for the 'top-half view', the bottom-half view' being complementary. Δ is the azimuthal angle governing the inequality of the areas of the plane viewed and hence the difference in the yield of neutrons from each view. In this case the 'difference'
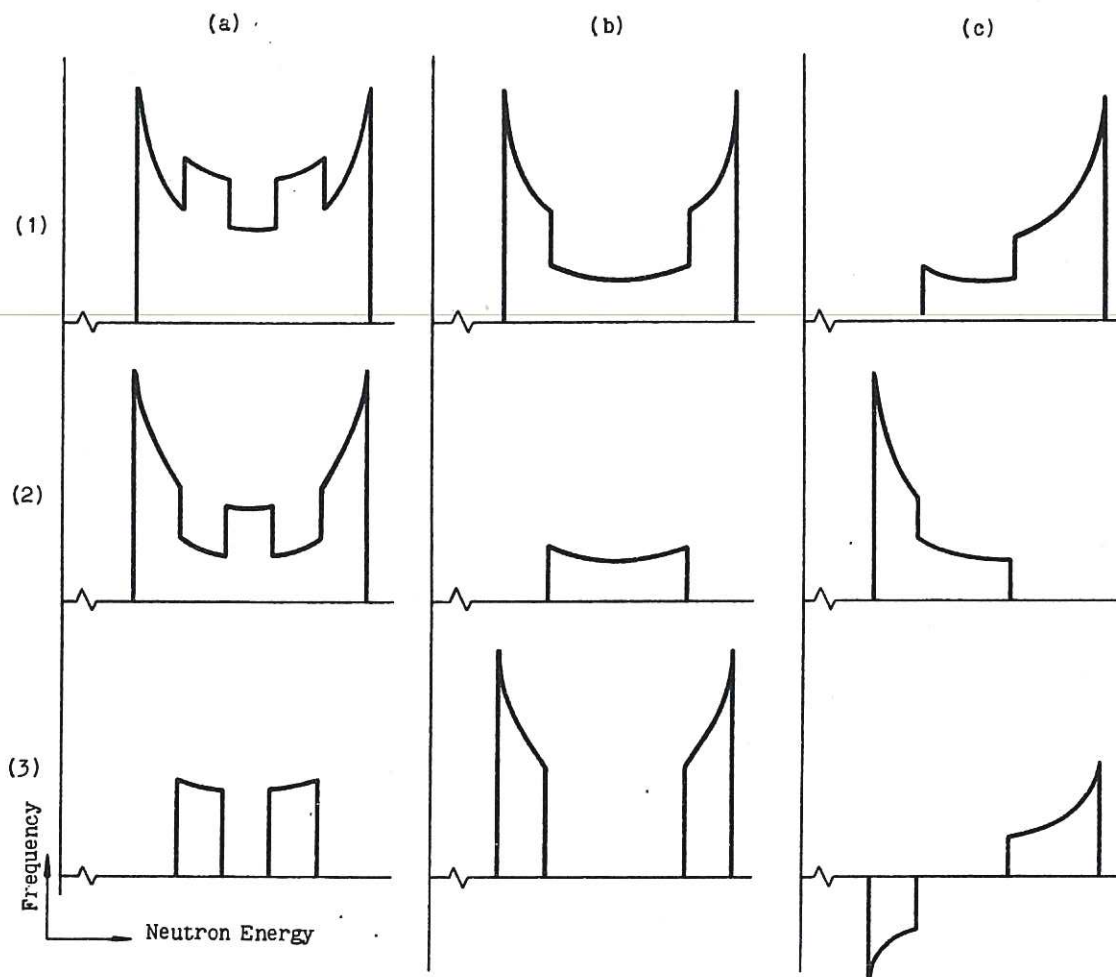
CLM—R 20  Fig. 4.

The 'top half view' (1), the 'bottom half view' (2) and the difference spectra (3) expected for (a) two groups of deuterons having identical radial components of their centre—of—mass velocities but having opposing azimu—thal components $V_\theta$.  (b) one group of deuterons with $V_\theta = 0$ and (c) two groups of deuterons having opposing radial components.