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Report

STABILITY OF THE PLASMA SHEATH

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STABILITY OF THE PLASMA SHEATH

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Abstract

The stability of a collisionless sheath joined to a plasma in the presence of secondary emission of electrons from the sheath boundary is examined in the fluid approximation.
Instability is unlikely to occur under floating conditions but if significant currents flow corresponding to increased wall-plasma potentials the system can go unstable.

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Stability of the Plasma-Sheath

1. Introduction

The existence, properties and structure of the sheath which forms adjacent to any surface in contact with a plasma have been studied in considerable detail in recent years. For instance, in the case of low density plasmas when the space charge dominated region of the sheath is of the order of several Debye lengths in thickness and the charged particle motion in the region is collisionless, the requirement that the potential behave monotonically in space leads in the simple model to the Bohm criterion[1]. This has been generalized to include an ion distribution[2]. The requirement imposed relates to spatial and not temporal variations and so the stability of the solution is not investigated.

Stability analysis of the sheath is inherently complicated in comparison with situations in uniform plasma because, even in the simplest fluid model of ion motion, the charged particle densities and the ion speed are all functions of position. However, such analyses have in principle been carried out for conventional situations in plane and cylindrical geometry. At high frequencies comparable with the electron plasma frequency an examination of the behaviour of the so-called Tonks-Dattner resonances both experimentally[3] and at various levels of sophistication theoretically[4,5] has shown that electron plasma waves undergo absorption as they propagate into the sheath and are reflected. Corresponding work at frequencies of the order of and below the ion plasma frequency has also shown that ion modes propagating on the ion "beam" which leaves the plasma and traverses the sheath are absorbed[6]. One is therefore led to the conclusion that the conventional plasma sheath is stable.

There is a situation which arises in high temperature plasmas, e.g. fusion reactors or when the bounding surface is electron emissive which is a priori prone to instability. Under such circumstances there will be an electron 'beam' leaving the surface and interpenetrating with the ion 'beam' travelling in the opposite direction. The steady state analysis of this two stream situation has been given Hobbs & Wesson[7] and in the hot cathode configuration by Prewett and Allen[8]. Recent extensions have been described by Harbour[9]. It is clearly of interest to those engaged in the design of divertors and fusion reactors to know if these steady state solutions are physically significant, or as a result of instability, merely mathematical curiosities.

2. The model

A complete analytical solution to the stability problem is clearly impossible and so resort has to be had to computer simulation or some approximate model. This paper describes the results of such a model and is based on the following assumptions.

1. The situation is stable so that the steady state solutions for number density and particle velocities are meaningful.
2. The variation of the 'plasma' parameters within the sheath is not so rapid as to render the use of uniform plasma theory corresponding to the local parameters invalid.
3. Instability will result at any frequency if a disturbance at that frequency grows sufficiently rapidly spatially as the sheath is traversed. Following usual conventions a growth by a factor of e^3 or e^π is taken as a criterion.

The justification for assumption (2) is the success of WKB methods in relation to the propagation of electron and ion waves mentioned earlier and exemplified by Crawford[10] and can be sustained a posteriori.

If one then sets up a plane geometry model, the following equations describe the steady state

$$n_{ep} = n_o \exp(-\eta) \qquad \text{for the 'plasma' electrons}$$

$$n_{ip} = n_o (1 + \alpha) \left(1 + \frac{2\eta}{u_{io}{}^2}\right)^{-\frac{1}{2}} \qquad \text{for the 'plasma' ions}$$

$$n_{eb} = \alpha n_o \left(\frac{\eta_c}{\eta_c - \eta} \right)^{\frac{1}{2}} \qquad \text{for the 'emitted' electrons}$$

where η is the potential normalized to the electron temperature, n_o is the density of plasma electrons and αn_o the density of surface-derived electrons in the plasma, u_{io} is the ion speed on leaving the sheath normalized to the ion acoustic speed $c_s = \left(\frac{kT_e}{M} \right)^{\frac{1}{2}}$, η_c is the normalized potential difference between plasma and surface. The equations are closed by solving Poisson's equation

$$\frac{d^2\eta}{d\zeta^2} = \frac{1 + \alpha}{\left(1 + \frac{2\eta}{u_{io}^2} \right)^{\frac{1}{2}}} - \exp{-\eta} - \alpha \left(\frac{\eta_c}{\eta_c - \eta} \right)^{\frac{1}{2}}$$

with boundary conditions $\eta = 0$ and $\frac{d\eta}{d\zeta} = 0$ as $\zeta \to \infty$.

This latter requires $u_{io}^2 = \frac{1 + \alpha}{1 - \frac{\alpha}{2\eta_c}}$.

Fig.1 shows the variation of particle number densities and particle speeds for values of $\alpha = 0.10$ and $\eta_c = 10.0$ such that there is strong emission and therefore a large positive current to the wall. These parameters are typical for the hot cathode situation. Under these circumstances there is a region $0 < \zeta < 4$ where there are relatively high speed electron and ion beams interpenetrating and thus prone to excite instability.

Most of the results given by Harbour[9] relate to conditions for which the net current to the wall is zero, i.e. the wall is floating, or $j^+ = j^-$. Writing his currents in the present notation, we have

$$j^+ = n_{po} e u_{io} c_s, \qquad j_s^- = n_{eb} c \left(\frac{eV_s}{m} \right)^{\frac{1}{2}},$$

and $$j^- + j_s^- = n_{eo} e \frac{\bar{c}}{4} \exp - \left(\frac{eV_s}{kT_e} \right).$$

This implies that the random electron current to the wall is greater than the emitted current, otherwise j^- would be negative and when the requirement

$j^+ = j^-$ is imposed we would have a contradiction.

Thus the range of the parameter α is limited and the range of wall potentials examined is confined to values less than those for the $\alpha^f = 0$ case. where α^f is the value of α for a given η_c for which no net current flows. Fig.2 gives the value of α^f for which $j^+ = j^-$ as a function of the normalized wall potential and also gives the corresponding value α^* for zero field at the cathode. This value was calculated by Prewett and Allen[8] and higher emissions lead to the formation of a virtual cathode.

The stability analysis is carried out by calculating for a given frequency ω the imaginary part k_i of the wave number $k \equiv k_r + ik_i$ corresponding to the fluid dispersion relation

$$1 - \frac{\omega^2_{pep}}{\omega^2 - k^2 c_e^2} - \frac{\omega^2_{pip}}{(\omega + kV_i)^2} - \frac{\omega^2_{peb}}{(\omega - kV_e)^2} = 0,$$

where ω_{pep}, ω_{pip}, ω_{peb} are the local plasma frequencies corresponding to the local particle densities and V_i and V_e the speeds of the ion and electron 'beams'.

3. Results

Calculations have been performed for a range of values of α and η_c. The variation of spatial growth rate and wavelength were found to be smooth functions of the parameters involved and of the spatial coordinate justifying assumption (2) made earlier. In general the higher the values of η_c for a given α the higher the growth and for a given η_c the denser the electron beam the higher the growth. In order for the growth to achieve instability in the sense of assumption (3) it is necessary for there to be a significant positive current to the wall. This requires a parameter range somewhat different from that investigated by Harbour. Figures 3 and 4 give results for such conditions showing the different wave modes on the electron and ion beams and the fact that the most strongly growing mode with its frequency peaked near the plasma ion plasma frequency has a spatial spectrum which is a function of position.

4. Conclusions

The results given above indicate the requirements for stability in the sheath region. However, they do also point to the fact that the more critical region so far as stability is concerned may well be in the plasma beyond the sheath when the wall-derived electron beam may persist in collisionless conditions on a scale long compared with the sheath and where larger total growths may be achieved. Here it is interesting to note that the early work of Cannara and Crawford[11] showed this to be the case for the hot cathode discharge.

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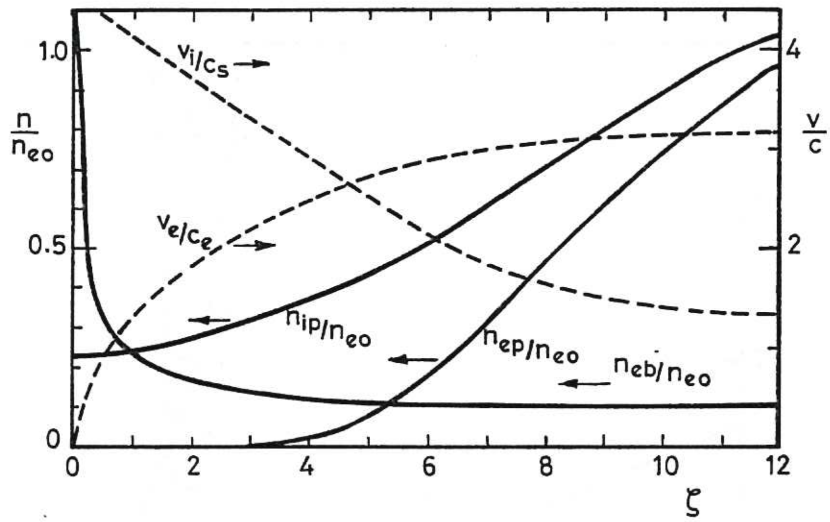

Fig.1 The variations of ion density n_{ip}, electron density n_{ep} and secondary electron beam density n_{eb} normalized to the plasma electron density n_{eo} through the sheath region as a function of ζ the distance from the wall in Debye lengths. The speed of the plasma derived ions v_i normalized to the ion sound speed c_s and the beam electrons v_e normalized to the plasma electron thermal speed c_e is also shown. The wall-plasma potential drop is $10\,kT_e/e$ and α the beam derived electron density/plasma electron density is $0{\cdot}1$.

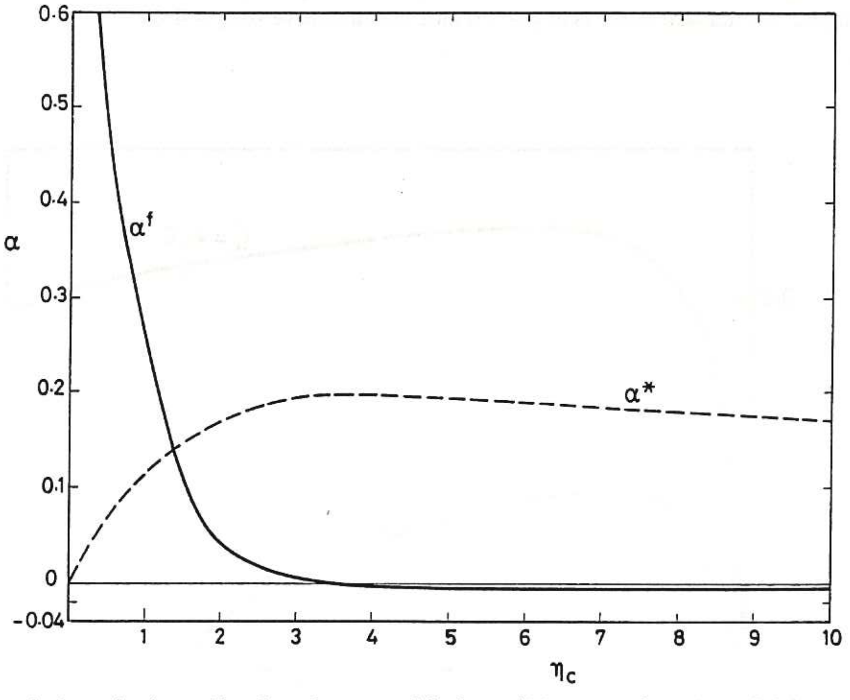

Fig.2 The variation of values of α given by two critical conditions as a function of the normalized potential drop η. α_f is the value of α which corresponds to zero net current through the sheath and α^* the value which corresponds to zero electric field at the wall. For $\alpha > \alpha^*$ space charge limitation will set in and for $\alpha > \alpha_f$ there is a net electron current leaving the wall.

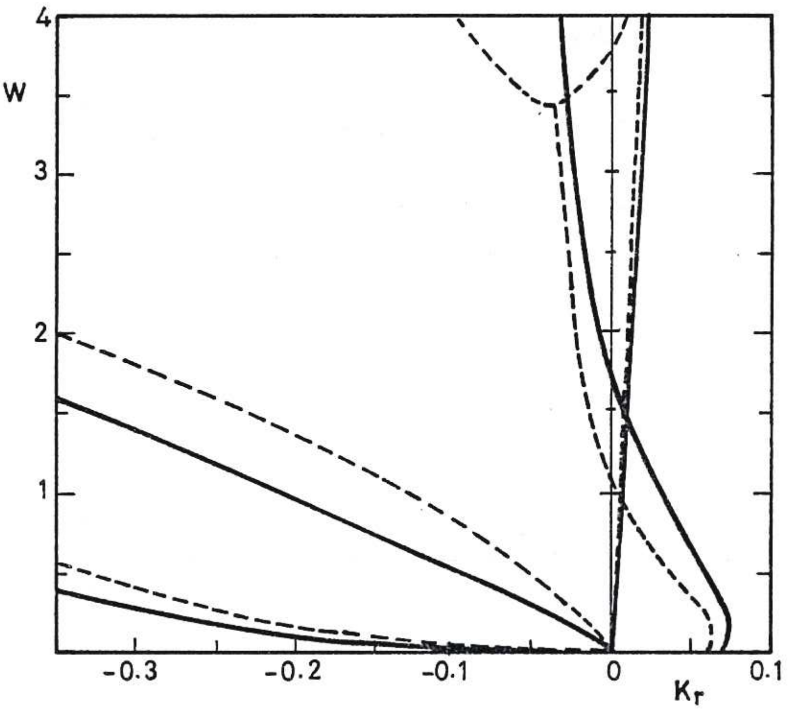

Fig.3 The dispersion curves for the real part of the modes existing in the sheath at two points for $\eta = 10{\cdot}0$. Frequency ω is normalized to the ion plasma frequency and wavenumber k to the plasma Debye length. The dashed curve is at $\zeta = 3{\cdot}0$ and the full curve for $\zeta = 4{\cdot}6$.

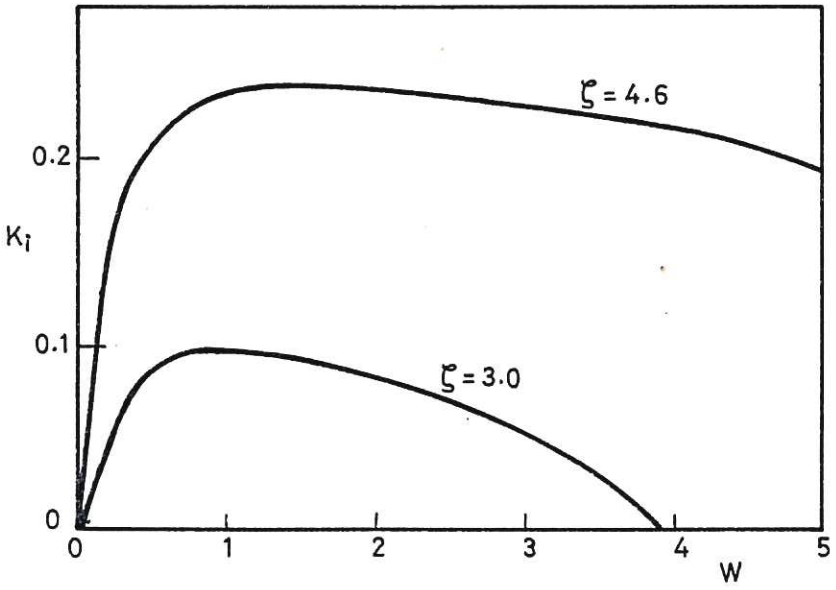

Fig.4 The spatial growth rates for the most strongly growing mode in Fig.3 which can be identified by the bifurcation for $\zeta = 3{\cdot}0$ showing how the spectrum broadens and growth rates increase as the plasma is approached. The normalization is to the plasma Debye length and ion plasma frequency.

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