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A COMMENT ON THE TOKAMAK DENSITY LIMIT

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ABSTRACT

It is shown that in the limit of radiation dominated power loss, the maximum average density of an ohmically heated plasma in thermal equilibrium is given by

$$n = Kf\,\frac{B}{Rq} = \frac{\mu_o Kf}{2\pi}\,\frac{I}{a^2}$$

where K is a weak function of electron temperature and depends on the impurity species, and f is a function of Z_e. For $Z_e \gtrsim 1.5$, the maximum density depends only on the current density, within a factor of x2.

This equation is consistent with experimentally observed scaling and quantatitive observations summarized in the Murakami correlation.

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Murakami's correlation(1) for the maximum average electron density in ohmically heated Tokamaks has been recently(2) summarized as (see the Appendix for definitions and units)

$$n \sim 2 \times 10^{20} \frac{B}{RqZ_e} \qquad .. (1)$$

where q and Z_e have been added to Murakami's original formula. This is rather less than the minimum ignition density(3) and it is hoped that the necessary density increase can be made during the additional heating phase.

Gibson(16) has shown that the Murakami correlation is consistent with a simple global balance between ohmic power and impurity radiation power for discharges where the global power loss is primarily radiative. This note is to point out that the quantitative agreement is quite good over the experimental range of electron temperatures. This does not exclude the possibility that conduction is important in restricted regions of the plasma, but overall it is assumed that radiation dominates.

The global power balance between ohmic dissipation and radiation from an impurity of density n_z is

$$\eta_{\parallel} \left(\frac{I}{\pi a^2} \right)^2 = n\, n_z L \qquad .. (2)$$

where L is the sum of the line, recombination and bremsstrahlung radiation factors(5) given in figure (1) for the common impurities Mo, Fe, O and C, assuming coronal equilibrium.

Using the classical resistivity(4) (neglecting the trapped electron correction since high density discharges are being considered)

$$\eta_{\parallel} = \left(\frac{em}{2\pi}\right)^{\frac{1}{2}} \frac{Z_e \gamma \ln\Lambda}{64\varepsilon_o^2 T_e^{3/2}}$$

equation (2) becomes

$$\boxed{n = K\, f\, \frac{B}{Rq}} \qquad .. (3)$$

where

$$K = \frac{2}{\mu_o} \left[\left(\frac{em}{2\pi}\right)^{\frac{1}{2}} \frac{z^2 \ln\Lambda}{64\epsilon_o^2\, L\, T_e^{3/2}} \right]^{\frac{1}{2}}$$

which is a function only of T_e, since $[\ln \Lambda]^{\frac{1}{2}}$ is very insensitive to n, and

$$f = \left[\gamma \left(1 + \frac{1}{\sigma Z^2}\right)\right]^{\frac{1}{2}} = \left[\gamma \left(1 + \frac{1}{Z_e - 1}\right)\right]^{\frac{1}{2}}$$

for $\sigma Z << 1$, which is a function only of Z_e.

The important points about equation (3) are:

a) The scaling with B, R and q are the same as in the Murakami experimental correlation. Since $B/Rq \propto I/a^2$, the scaling implies that the maximum density is proportional to the global current density.

b) Equation (3) does not depend on MHD effects, although lack of thermal equilibrium in regions of the plasma (associated, for example, with impurities from the limiter or with the ionization energy of a rapidly injected neutral gas) may produce a loss of MHD stability because of the development of an adverse current density profile.(15) Equation (2) shows that loss of thermal stability may also occur if the slope of the Log L/Log T plot of figure 1 is less than -1.5 for ohmically heated plasma.

c) K is shown in figure (2) calculated using the impurity radiation factors of figure (1). In the experimental range of electron temperatures, K for each type of impurity is not strongly sensitive to temperature within the accuracy of Murakami's correlation (∿ x 2).

d) Although both the correlation equation (1) and the theoretical equation (3) are global and disregard many details, the quantitative agreement is quite good. This is illustrated by the experimental points on the $K(T_e)$ plot of figure 2, calculated using the value of $K = Rqn/Bf$ using experimental results from Table I. It can be seen that the experimental points lie near $K \sim 4 \times 10^{19}$, which is between the single species curves calculated for heavy impurities and light impurities; this is to be expected since a mixture of heavy and light impurities is usually present. Note that equation (3) contains no fitted parameters.

e) The impurity fraction, σ, is only involved via f, which is plotted as a function of Z_e is figure (3). For high impurity fraction, when $Z_e > \sim 1.5$, f is constant within the accuracy of Murakami's correlation, and so the maximum density is independent of impurity fraction and Z_e. This is because both the ohmic power and the radiated power are proportional to the impurity fraction at high Z_e. This behaviour at high Z_e differs from the correlation equation (1) in which the density falls as $1/Z_e$.

f) For low impurity fractions, when $Z_e \rightarrow 1$, $f \rightarrow \infty$ and the theoretical equation (3) gives $n \rightarrow \infty$ whereas the correlation equation (1) tends to a constant density. This is reasonable because for very pure plasmas, impurity radiation will not dominate the overall power loss and other processes such as conduction will then limit the density.

Conclusions

A global energy balance between ohmic power and impurity radiation is in reasonable qualitative and quantitative agreement with the Murakami global empirical correlation for the maximum density. For $Z_e > \sim 1.5$, the maximum density depends only on the current density within a factor of x 2; when Z_e approaches unity, the maximum density increases and is sensitive to Z_e.

TABLE I

EXPERIMENTAL VALUE OF nRq/Bf

Reference	DITE	ATC	PLT	TFR	ORMAK	PULSATOR	T10	ALCATOR	
	6	7	8	9	10	11 12	13	14	
R	1.17	0.9	1.3	0.98	0.8	0.7	1.5	0.54	
a	0.27	0.17	0.4	0.20	0.23	0.11	0.37	0.09	
B	1.8	1.5	3.2	4	2.5	2.7	3	6	
I	1.5	0.78	3.84	2	1.75	0.88 0.55	4.2	1.45	x10^5
q	3.74	3.2	6.67	4.08	4.7	2.7 4.2	3.26	3.10	
Limiter	Mo	Fe(ss)	Fe(ss)	Ni/Fe	W	Mo Fe	W/Mo	Mo	
Gettering	× ✓	× ✓	× ✓	dirty clean	×	× ×	×	×	
Z_e	6 2	5.4 1.2	3.4 1.1	5.5 1.2 to 1.5	7.9	2.3 2.0	1.2 to 1.5	<1.2	
T_{eo}	9 6	11 7	21 8	17 7	15.3	6.5 7	10	7.5	x10^2
T	(5.4)(3.6)	(6.6)(4.2)	(12.6)(4.8)	(10.2)(4.2)	6.8	(3.9)(4.2)	(6)	(4.5)	x10^2
n	1.6 2.8	1.9 2.1	3.3 9.3	2 7	2.8	7 10	5	.55	x10^{19}
f	1.2 1.7	1.2 3	1.4 3.8	1.2 3 to 2.2	1.16	1.6 1.7	3 to 2.2	>3	
nRq/Bf	3.2 4.0	3.0 1.3	6.4 6.9	1.7 2.4 to 3.2	3.7	2.7 6.5	3.7 to 5.1	<5.1	x10^{19}

$(T_e) = 0.6\ T_{eo}$

REFERENCES

1. MURAKAMI M et al. : Nuclear Fusion 16, 2, 347 (1976).
2. BICKERTON R. J. TIGER (79)112.
3. REYNOLDS J.A., MORGAN J.G.: TIGER (79)109.
4. BRAGINSKI S I : Rev. Plasma Phys. 25, 205 (1965).
5. JENSEN R V et al. : PPL Report 1334 (1977).
6. AXON et al. IAEA InnsbruckConf.CN-37/A3(1978).
7. STOTT P et al. Nuclear Fusion 15, 431 (1975).
8. BOL et al. : IAEA Innsbruck Conf. CN-37/A1(1978).
9. EQUIPE TFR : IAEA InnsbruckConf. CN-37/A6(1978).
10. BERRY L A et al.: IAEA Berchtesgarten Conf. CN-35/A4-1(1976).
11. ENGLEHARDT W et al.: IAEA InnsbruckConf. CN-37/A5(1978).
12. MEISEL D et al.: IAEA Berchtesgarten Conf. CN-35/A6(1976).
13. BAGDASAROV A A et al.:IAEA InnsbruckConf. CN-37/A2(1978).
14. GONDHALEKAR A et al:IAEA InnsbruckConf. CN-37/C4 (1978).
15. FURTH et al.: Phys Fluids 13, 3020 (1970)
16. GIBSON A: Nuclear Fusion 16, 546(1976).

APPENDIX

SYMBOLS AND UNITS

a	m	minor radius
B	T	toroidal magnetic field
e	C	electron charge, 1.6×10^{-19}
f	-	the part of equation (3) that depends on σ
I	A	toroidal current
K	$m^{-2}T^{-1}$	the part of equation (3) that depends on T_e
L	Wm^3	radiation power function
m	kg	electron mass, 9.1×10^{-31}
n	m^{-3}	electron density
n_H	m^{-3}	ion density
n_z	m^{-3}	impurity density
q	-	$2\pi a^2 B/\mu_o RI$
R	m	major radius
T_e	eV	electron temperature
Z	-	impurity charge
Z_e	-	effective resistive anomaly = $1+\sigma Z^2/1+\sigma Z$
γ	-	e - e collision factor in resistivity= $1 + \frac{1.424}{Z_e + 1.069}$
ε_o	F/m	$1/\mu_o c^2$
$\eta_{\parallel}$	m	classical resistivity
Λ	-	screening parameter
μ_o	H/m	$4\pi \times 10^{-7}$
σ	-	impurity ion fraction = n_Z/n_H
τ_{eE}	s	ALCATOR correlation for electron energy containment time $= 5 \times 10^{-21} na^2$

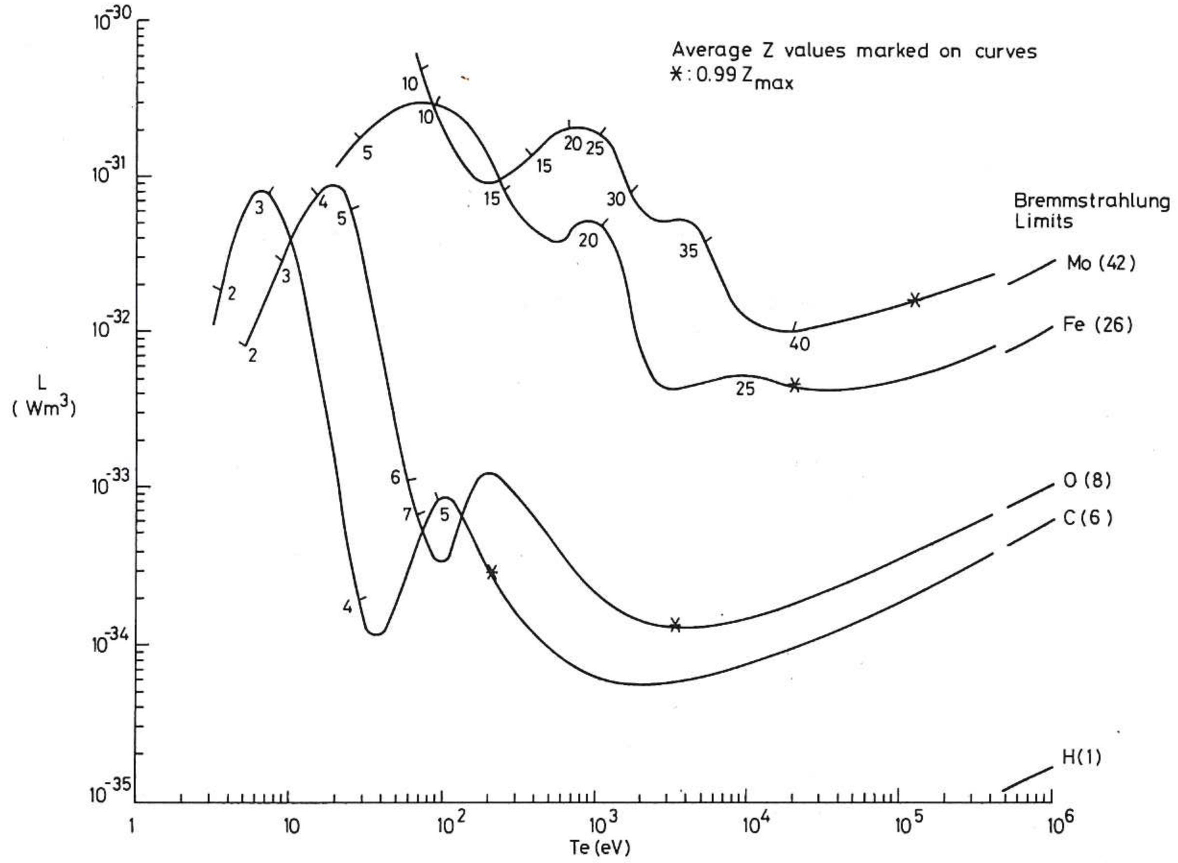

Fig.1 Total radiation power function.

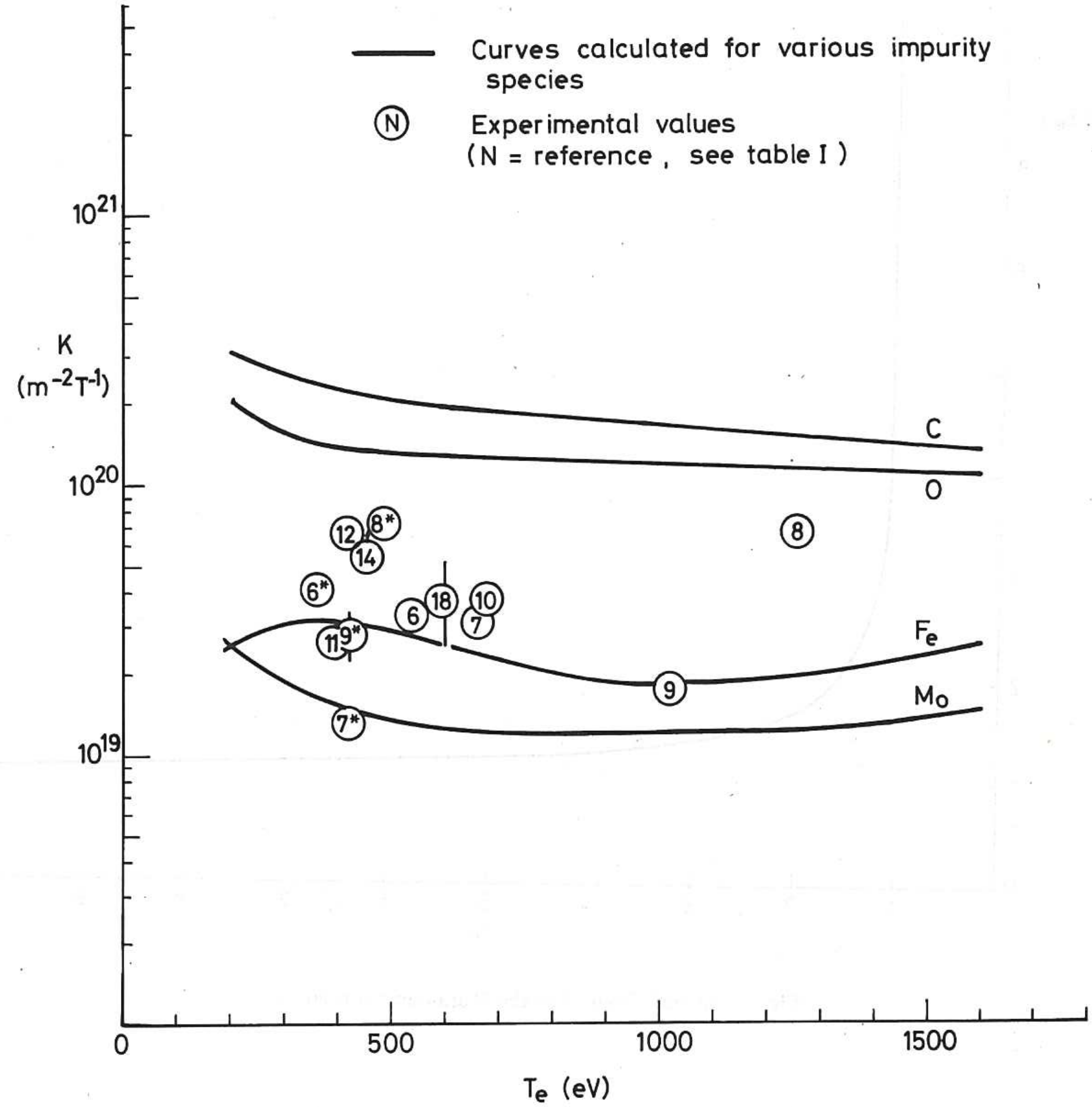

Fig.2 The coefficient K in the Murakami correlation.

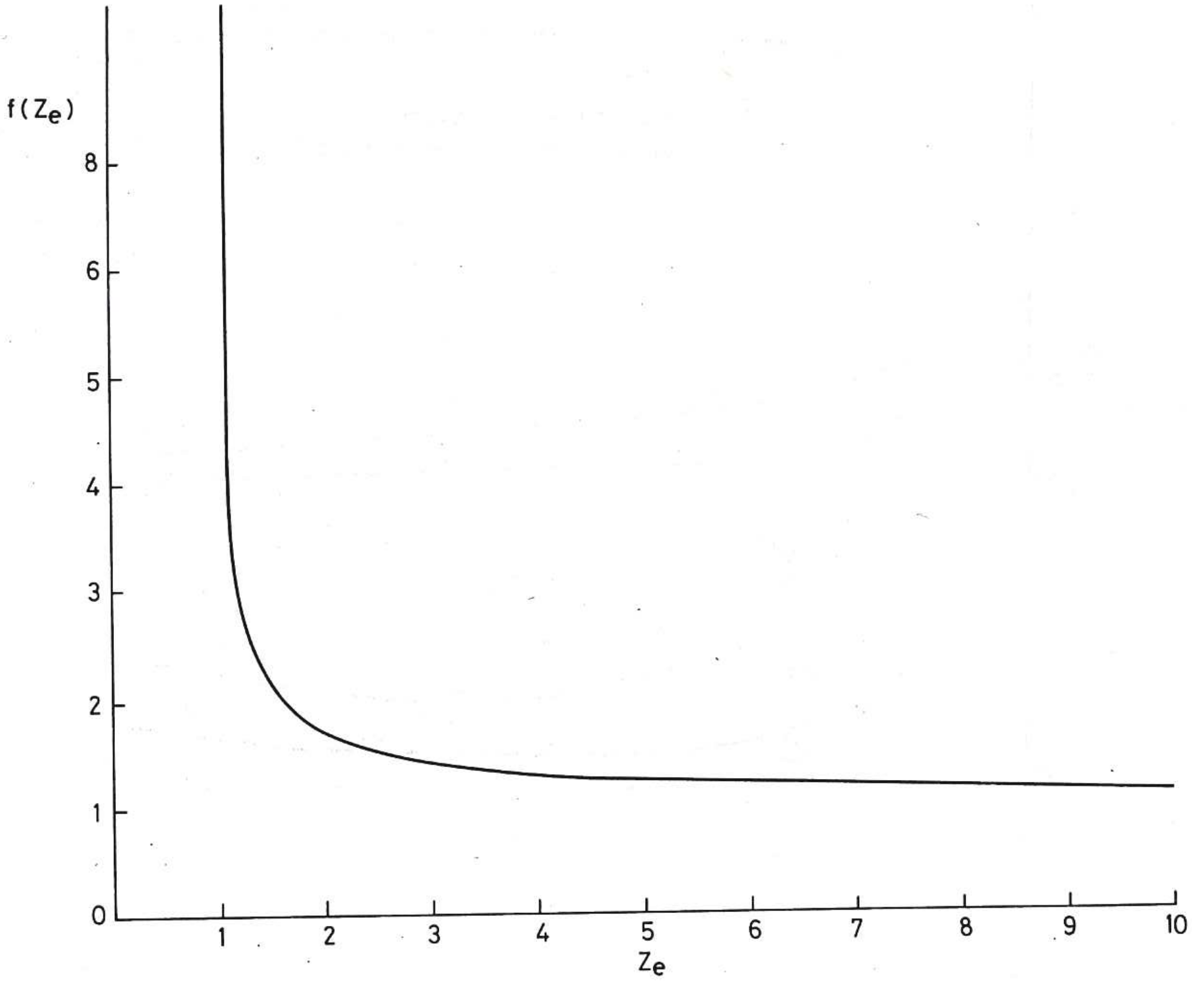

Fig.3 The coefficient f in the Murakami correlation.

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