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SOME SCALING CONDITIONS FOR TOKAMAK IGNITION

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ABSTRACT

The empirical ALCATOR correlation for the global tokamak containment time is used to derive the ignition conditions:

heating power required	=	$7.0 \times 10^{26} R/na$
density	>	$1.3 \times 10^{20}/a$ (so the heating power $< 5.4 \times 10^{6} R$)
β	>	$1.3/aB^2$

With the added MHD constraints of $\beta < a/Rq^2$ and $q > 2$,

a	>	$5.4R/aB^2$

and

I	>	$5.8 \times 10^{6} (a^2/R)^{\frac{1}{2}}$

are also required. For the typical reactor values of B = 5 and R/a = 4, these conditions become $a > 0.86$ and $I > 2.9 \times 10^{6} a^{\frac{1}{2}}$.

In addition, the Murakami density limit gives the condition

R/a	<	$1.6B/qZ_e$

which has a value of 4 for B = 5, q = 2, and $Z_e = 1$. Non-Ohmic heating processes may allow an increase in this value.

For heating by neutral beams the penetration condition gives the requirement that the density $< 5.8 \times 10^{19}/a$, which is about half the minimum ignition requirement.

In comparison, the new ALCATOR/FRASCATI empirical containment time correlation indicates a much lower heating power required for ignition of $P = 0.12\ I\ (\frac{R}{a})^2$, and no constraint in density.

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1. INTRODUCTION

In this paper conditions for thermonuclear ignition of a tokamak plasma are derived from a global power balance equation combined with the ALCATOR empirical correlation for the containment time. The added constraints of MHD stability and the Murakami density limit are then considered.

For a flat density and temperature profile and $T_e = T_i$, the power from a tokamak plasma in thermal equilibrium is

$$P_{out} = \frac{3}{2}\,\frac{neT}{\tau_{eE}}\,2\pi^2 a^2 R = P + \frac{n^2}{4} < \sigma v > eE_\alpha\,2\pi^2 a^2 R \qquad ..(1)$$

where P is the applied heating power (see the appendix for definitions and units). The neutron power is not included since it does not contribute to the heat balance. The energy containment time, including radiation, is given by the empirical correlation

$$\tau_{eE} = 5 \times 10^{-21} na^2 \qquad ..(2)$$

This correlation is based on data for ohmically heated tokamaks, and we assume that it holds for auxiliary (NI or RF) and thermonuclear heating.

Using Brunelli's[1] expression for the nuclear reaction cross section

$$< \sigma v > = 9.00 \times 10^{-22} \exp\left(- 0.476 \left| \ln \frac{T}{6.9\times10^4} \right|^{2.25} \right)$$

equations (1) and (2) can be rearranged to give

$$P = 9.488 \times 10^2 TR - 2.504 \times 10^{-33} R(na)^2 \exp\left(-0.476 \left| \ln \frac{T}{6.9\times10^4} \right|^{2.25}\right) \qquad ..(3)$$

Figure (1) shows P as a function of T for a given na. To achieve net power output (negative P) a heating power exceeding the maximum ($\hat{P}$) of the curves must be applied. An expression for the maximum is obtained by differentiating equation (3) to give

$$\hat{P} = 9.488 \times 10^2\, \hat{T}R\left(1 - 0.934 \left| \ln \frac{\hat{T}}{6.9\times10^4} \right|^{-1.25} \right). \qquad ...(4)$$

$\hat{P}$ is independent of minor radius because both the plasma volume and energy containment time scale in proportion to a^2.

The corresponding value of na can be derived from equations (3) and (4) to give

$$na = \left[3.538 \times 10^{35}\hat{T}\left|\ln \frac{\hat{T}}{6.9\times10^4}\right|^{-1.25} \exp\left(0.476\left|\ln \frac{\hat{T}}{6.9\times10^4}\right|^{2.25}\right)\right]^{\frac{1}{2}}$$

Figure (2) shows $\hat{P}\,\frac{na}{R}$ calculated from equation (4) is independent of $\hat{T}$ within 10% from a temperature of 3×10^3 to 1.5×10^4 (which is the range of interest for ignition) giving

$$\boxed{\hat{P} = 7.0 \times 10^{26}\,\frac{R}{na}} \qquad \ldots (5)$$

Since the ratio R/a is roughly independent of the reactor size, the power required for ignition is inversely proportional to only the density.

Figure 1 also shows that to achieve ignition

$$\boxed{na > 1.26 \times 10^{20}} \qquad \ldots (6)$$

is required. This follows from the scalings of the energy containment time and cross section. Combining equations (5) and (6) then gives a condition on the heating power for ignition

$$\boxed{\hat{P} < 5.4 \times 10^{6} R} \qquad \ldots (7)$$

For example, at the minimum ignition density, the power required to ignite INTOR (R = 4.8) is $< 26 \times 10^6$ and TIGER (R = 7.8) is $< 42 \times 10^6$. These values are, of course, an average of power deposited in the plasma.

Further constraints on achieving ignition may be obtained from the MHD stability conditions on β and q, the Murakami density limit and, for NI auxiliary heating, beam penetration.

2. β LIMIT

At ignition, the plasma loss is sustained bv thermonuclear power alone. so $P = 0$ in equation (1). Figure 3 shows the ignition curve of $\beta B^2 a$ plotted as a function of T from equation (1) with $P = 0$. It shows that for ignition

$$\beta B^2 a \quad > \quad 1.34 \qquad \ldots (8)$$

This condition may be combined with the theoretical MHD atability condition for ballooning modes of

$$\beta \quad < \quad \frac{a}{Rq^2} \qquad \ldots (9)$$

to give the condition on the toroidal current for ignition

$$\boxed{I \quad > \quad 5.79 \times 10^6 \left(\frac{a^2}{R}\right)^{\frac{1}{2}}} \qquad \ldots (10)$$

which, for constant R/a, is proportional to $a^{\frac{1}{2}}$. For example, with INTOR parameters ($a = 1.2$, $R/a = 4$), $I > 3.2 \times 10^6$ is required and with TIGER parameters ($a = 2$, $R/a = 4$), $I > 4.1 \times 10^6$ is required for ignition. Because of the smaller aspect ratio in JET ($a = 1.2$, $R/a = 2.44$), an $I > 4.1 \times 10^6$ is required for ignition and the extended performance JET ($I = 4.8 \times 10^6$) just fulfills this condition.

It is worth noting that the ballooning limit of equation (9) has been marginally exceeded in an experiment, and there are some theoretical grounds for a region of stability at considerably higher β, so reducing the current required for ignition.

3. q LIMIT

There is good theoretical and experimental support for the MHD requirement

$$q \quad > \quad 2 \qquad \ldots (11)$$

Combining equations (10) and (11)gives the ignition condition

$$a > \frac{5.36}{B^2} \frac{R}{a} \qquad \ldots (12)$$

For reactor values of B = 5 and R/a = 4, equation (12) gives a > 0.86 for ignition.

If the minor radius is given (constrained, for example, by cost considerations), equation (12) may be rewritten as an ignition condition for the toroidal magnetic field:

$$B > 2.32 \left(\frac{R}{a^2} \right)^{\frac{1}{2}}$$

which is proportional to $a^{-\frac{1}{2}}$ for constant R/a. For example with R/a = 4, B > 4.2 is required for ignition in INTOR (a = 1.2) and B > 3.3 is required for ignition in TIGER (a = 2). From this point of view, ignition is rather easier to achieve in a larger device.

4. DENSITY LIMIT

The Murakami[3] empirical correlation gives the maximum density of ohmically heated tokamak plasmas as

$$n = 2 \times 10^{20} \frac{B}{RqZ_e} \qquad \ldots (13)$$

where the coefficient is a value from recent experiments [4]. Combining equations (6) and (13) gives the ignition condition

$$\frac{R}{a} < 1.6 \frac{B}{qZ_e} \qquad \ldots (14)$$

For a reactor with B = 5, q = 2 and Z_e = 1, this condition gives R/a < 4. This is marginal on engineering and cost grounds. However it is possible that the Murakami limit will not hold for NI or RF heated plasma (or at least be relaxed).

5. NEUTRAL BEAM PENETRATION

The condition [5] that the neutral beam is dissociated in the plasma is

$$na = 3.6 \times 10^{14} E$$

For deuterium, NI beams can be produced efficiently for a beam energy $E < \sim 1.6 \times 10^5$,

$$\therefore \boxed{na < \sim 5.8 \times 10^{19}} \qquad \ldots (15)$$

is required.

This is about half the ignition limit of equation (6) so ignition with a neutral beam is difficult.

6. EFFECT OF THE NEW ALCATOR CONTAINMENT TIME

The ALCATOR and the Frascati [6] groups now favour a new empirical correlation

$$\tau_{eE} = \frac{8 \times 10^{-24} na^2 T}{B_p} \; \frac{a}{R}$$

$$= \frac{4 \times 10^{-17} na^4 T}{RI}$$

Substituting this into equation (1), the power required to sustain the plasma in thermal equilibrium is

$$P = 0.12 \; I \left(\frac{R}{a}\right)^2 - \frac{n^2}{4} < \sigma \; v > eE_\alpha \; 2\pi^2 a^2 R$$

Since the first term is independent of temperature and density, the ignition condition is simply

$$P > 0.12 \; I \left(\frac{R}{a}\right)^2$$

For example, the power required for JET ($R/a = 2.44$, $I = 3 \times 10^6$) is 2.1×10^6.

Apart from the limitations of the applicability of the new containment time scaling law, the minimum current is determined by the requirement to confine α particles.

7. CONCLUSIONS

In this note we have considered the scaling of the power required to ignite a tokamak assuming a flat profile with circular cross-section and the empirical energy containment time. Equilibrium conditions are taken, so the time taken to reach ignition is not calculated.

The main conclusions are that for ignition:

a) power required to heat the plasma is $7.0 \times 10^{26} \frac{R}{na}$

b) density $> \frac{1.3 \times 10^{20}}{a}$, so the power required is $< 5.4 \times 10^{6} R$

c) $\beta > \frac{1.3}{aB^2}$

d) $I > 5.8 \times 10^{6} \left(\frac{a^2}{R}\right)^{\frac{1}{2}}$

} Derived by adding the MHD condition $\beta < a/Rq^2$.

e) $a > \frac{5.4R}{aB^2}$

} Derived by adding the MHD condition $q > 2$.

f) $\frac{R}{a} < 1.6 \frac{B}{qZ}$

} Derived by adding the empirical correlation $n < 2 \times 10^{20} B/_{RqZ}$

For neutral beam auxiliary heating, the penetration condition is $na < \sim 5.8 \times 10^{19}$ which fails to meet the ignition condition (b).

In constrast, the new ALCATOR scaling indicates a lower power required of $0.12I(R/a)^2$ and no constraint on the density.

The main uncertainties of these estimates are the accuracy of the empirical correlation for energy containment time and maximum density and whether they are applicable to plasmas dominated by non-ohmic heat input profile effects and any lack of thermal equilibrium.

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APPENDIX

SYMBOLS AND UNITS

a	m	minor radius
B	T	toroidal magnetic field
B_p	T	poloidal magnetic field
E	eV	kinetic energy of neutral beam atoms
E_α	eV	α energy from D-T fusion, 3.52×10^6
e	C	electron charge, 1.602×10^{-19}
I	A	toroidal current
n	m^{-3}	number density
P	W	power required for ignition
P_{out}	W	total power from the plasma (ex.neutron power)
$\hat{P}$	W	maximum power required for ignition
q	-	$2\pi\ a^2B/\mu_oRI$
R	m	major radius
T	eV	plasma temperature ($= T_e = T_i$)
$\hat{T}$	eV	T corresponding to maximum power required for ignition
Z_e	-	effective resistance anomaly
β	-	$2neT/(B^2/2\mu_o)$
$<\sigma v>$	m^3/s	average D-T thermonuclear reaction cross section x ion velocity for a Maxwellian plasma
τ_{eE}	s	ALCATOR correlation for electron energy containment time
μ_o	H/m	$4\pi \times 10^{-7}$

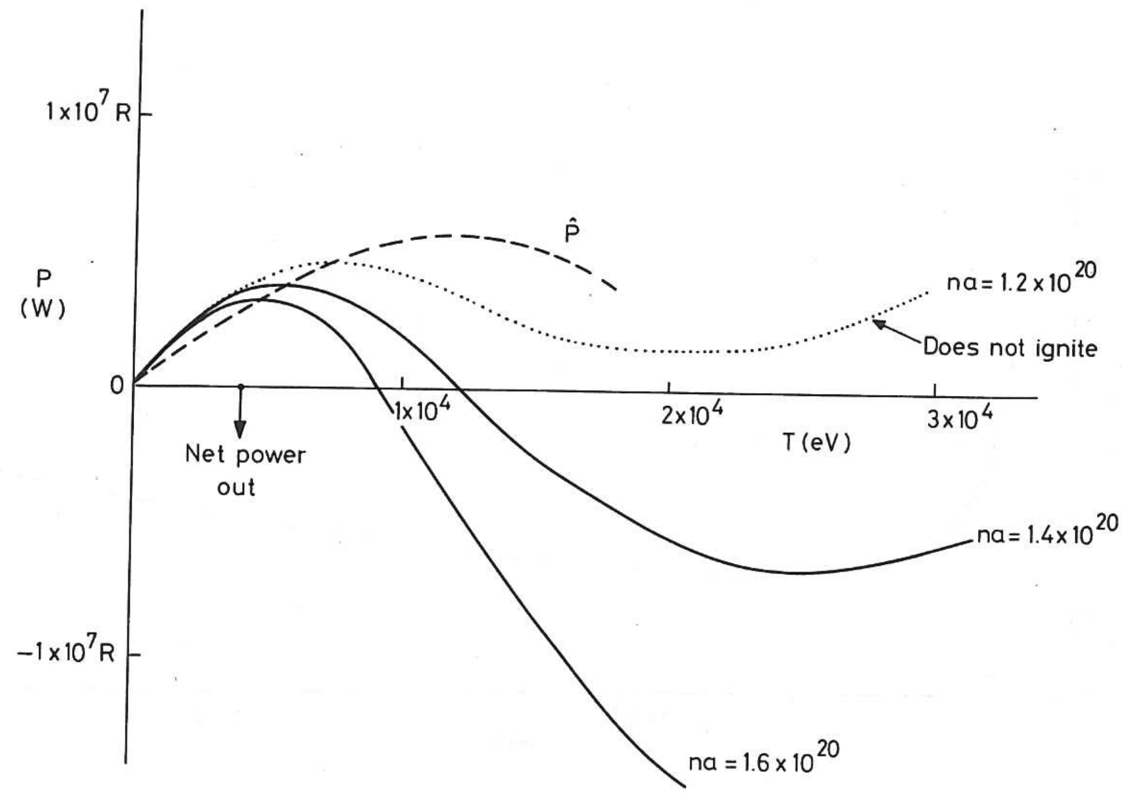

Fig.1 Power required to sustain the plasma (Eq.3).

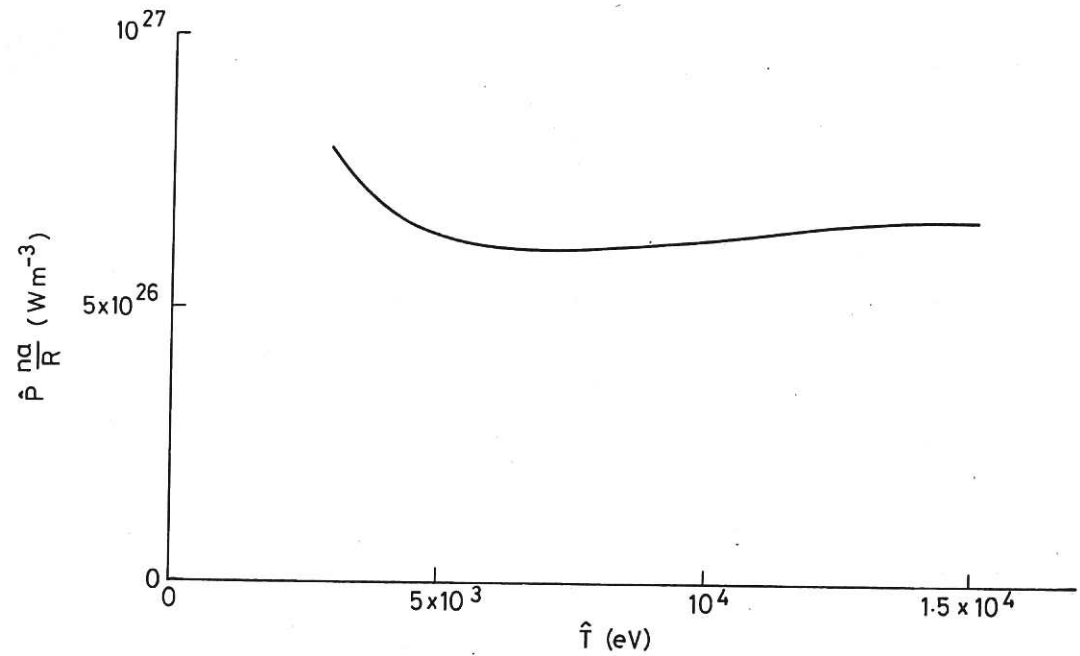

Fig.2 Maximum power required to ignite plasma.

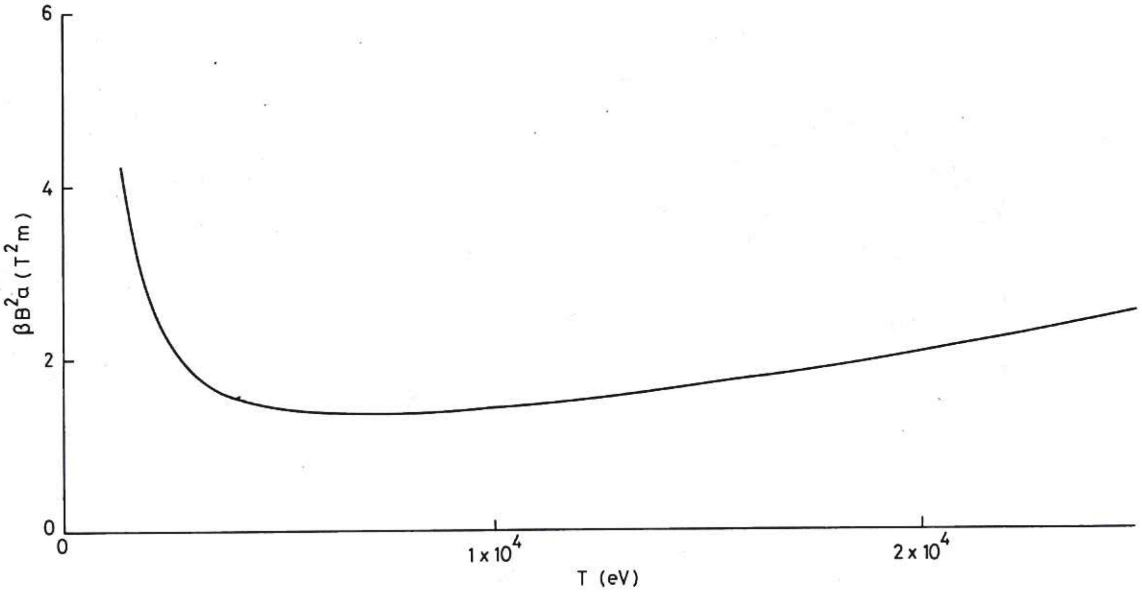

Fig.3 Ignition curve.

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