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Report



## RIPPLE TRANSPORT IN INTOR

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by

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(Euratom/UKAEA Fusion Association)ABSTRACT

The possibility of controlling the power output from INTOR and preventing 'thermal-runaway' by utilising the effect of magnetic field ripple on thermal transport has been investigated with a 1-D transport code; the code uses analytic expressions derived from current theories including the effects of localised particles.

Any significant control by ripple effects appears impractical for two reasons:

- (1) The  $T^{7/2}$  temperature dependence initially predicted is inhibited by energetic particles at  $T \gtrsim 10$  keV and eventually the thermal conductivity decreases with temperature.
- (2) The effect of magnetic field ripple on transport is small. For a ripple amplitude of 2% at the plasma's edge, which would be difficult to achieve in practice, the decrease in confinement time is only 10-20%.





## 1 INTRODUCTION

Increased particle and energy losses caused by introducing ripple into the magnetic field of a tokamak reactor is of interest because of its potential for controlling both 'thermal-runaway' and power output.[1,2] Control of 'thermal-runaway' depends on the prediction that the ion loss due to ripple will vary as  $T_i^{7/2}$  and thus increase rapidly as the temperature rises. Recently however, Yushmanov [3] and subsequently Hastie and Hitchon [4] have queried this prediction. Hastie and Hitchon for example have calculated the total ion thermal flux resulting from five mechanisms, all of which contribute, namely:

- (i) ripple plateau transport [5]:
- (ii) } two regimes of banana-drift transport [6, 7, 8]:
- (iii) }
- (iv) ripple-trapped transport [9, 10]:
- (v) hybrid orbit transport [11, 3]:

and have concluded that losses vary as  $T^{7/2}$  for a limited temperature range only.

This note considers the effect of terms (iv) and (v), which according to Hastie and Hitchon [4], tend to dominate the loss process. Both terms have been incorporated into the 1-D transport code HERMES which has been used to evaluate the effect of ripple losses in INTOR.

## 2 THE DOMINANT RIPPLE TERMS

The two terms (iv) and (v) are due to ripple trapped particles which can be divided into two energy groups. The lower energy group, treated by Connor and Hastie [10], suffer many collisions in the time they take to drift poloidally around a flux surface. For the remaining high energy particles the opposite is true; they can drift far enough to 'de-trap' collisionlessly and

then perform 'hybrid' orbits [3, 11] involving several collisionless 'trapping' and 'de-trapping' events.

Loss terms due to the electrons are also dominated by ripple-trapped particles but virtually all of them fall into the low energy group so that 'hybrid' orbit transport is negligible for electrons.

The thermal heat fluxes  $Q_i$  and  $Q_e$ , for ions and electrons respectively due to the above effects, may be written

$$Q_i = \frac{0.4}{\alpha^{3+0.02}} \delta^{3/2} \tau_i \left( \frac{kT_i}{eBR} \right)^2 n F(u) \frac{dT_i}{dr} \quad (1)$$

$$Q_e = \frac{0.2}{\alpha^{3+0.02}} \delta^{3/2} \tau_e \left( \frac{kT_e}{eBR} \right)^2 n \left\{ \frac{(T_e + T_i)}{n} \frac{dn}{dr} + 4.5 \frac{dT_e}{dr} + \lambda \frac{dT_i}{dr} \right\} \quad (2)$$

where

$$F(u) = (4.5-\lambda)e^{-u} (4.5-\lambda) \left\{ 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \frac{u^4}{24} + \frac{u^5}{120} \right\} - \frac{u^5}{120} (\frac{1}{2}+\lambda)e^{-u} \quad (3)$$

$$\frac{u^5}{6} \left( \frac{\tau_i}{RNq} \right)^2 \left( \frac{kT_i}{eBR} \right)^2 = 1$$

$$\alpha = r/(RNq\delta)$$

$$\delta = \delta_0 [(R+r)/(R+a)]^N$$

$$\delta_0 = (B_{\max} - B_{\min}) / (B_{\max} + B_{\min}) \text{ at the plasma edge}$$

$$R = \text{major radius}$$

$$r = \text{minor radius}$$

$$a = \text{minor radius of the plasma boundary}$$

$$q = \text{safety factor}$$

$$N = \text{number of ripple periods in toroidal direction}$$

$$\tau_e = \frac{3}{4} \left( \frac{m_e}{2\pi} \right)^{\frac{1}{2}} \frac{(kT_e)^{3/2}}{ne^4 \ln \Lambda}$$

$$\tau_i = \frac{3}{4} \left( \frac{m_i}{2\pi} \right)^{\frac{1}{2}} \frac{(kT_i)^{3/2}}{ne^4 \ln \Lambda}$$

$\lambda$  = an electric field parameter such that

$$\frac{e}{T_i} \frac{d\phi}{dr} = - \left[ \frac{1}{n} \frac{dn}{dr} + \frac{\lambda}{T_i} \frac{dT_i}{dr} \right] \quad (4)$$

In deriving eqs. (1) and (2) the magnetic flux density was assumed to vary as

$$B = B_0 [1 - (r/R_0) \cos \theta - \delta e^{-\theta^2/2} \cos N\phi]$$

and the geometric factor  $G(\alpha)$ , referred to in ref [10], has been approximated by  $0.02/(\alpha^3 + 0.02)$ .

To evaluate the thermal fluxes using eqs 2 and 3 some assumption must be made about the radial electric field. In principle the ion and electron particle fluxes could be equated and the self-consistent electric field determined, however, this is not practical since the unknown 'anomalous' transport mechanism would have to be included. A reasonable assumption is to take  $\lambda$  as unity which corresponds to zero mass motion and is in rough accord with tokamak observations.

Figures 1 and 2 illustrate how  $Q_i$  and  $Q_e$  vary for representative parameters. The expressions for the fluxes have been broken down into two parts; the first, plotted in fig. 1, is the geometric factor

$$\text{i.e. } 0.4 \delta^{3/2} / (\alpha^3 + 0.02)$$

which varies rapidly with radial position; the second is the remaining part of the thermal conductivity normalised against the standard 'anomalous' electron thermal conductivity, namely  $\kappa_0 = 5 \times 10^{19} \text{ cm}^{-1} \text{ s}^{-1}$ , and plotted against temperature.

In evaluating  $\alpha$ , used in the geometric factor, some assumption is needed concerning the variation of  $q$  with  $r$ ; the following assumptions have been used,  $j \propto T^{3/2}$ ,  $T \propto [1 - (r/a)^2]^{\alpha} T$  and  $q(0) = 1$  which give

$$\frac{q(r)}{q(a)} = \frac{(r/a)^2}{1 - [1 - (r/a)^2]^{\alpha} q(a)} \quad (5)$$

The physical parameters used are appropriate to the initial INTOR specification.



Consider the conclusions that can be drawn from figs 1 and 2. As expected the geometric factor decreases rapidly away from the plasma edge, with high N-numbers showing the most rapid decrease. It is interesting that at  $r/a \approx 0.7$  the geometric factor is roughly the same for  $N = 4, 6$  and  $12$ . The curves in fig. 2 show that the  $T^{7/2}$  dependency ceases between 5 and 10 keV, depending on the value of  $N$ , thus indicating that 'thermal-runaway' will not be controlled. Comparison of both curves suggests that the energy confinement time will not be affected until  $\delta_o \geq 2\%$ . Note that the curves in Fig. 2 can be generalised by using  $T_i/(BR)^{4/7}$  instead of temperature and  $(Nnq/B)(BR)^{4/7}$  instead of the parameter  $N$ .

It should also be noted that the total thermal insulation or energy confinement time is measured by

$$\langle \kappa \rangle = \frac{\int_0^T \kappa dT}{\int_0^T dT}$$

rather than the value of  $\kappa$  at  $\langle T \rangle$  or the peak value of  $\kappa$ . Thus it is apparent that the effect of  $Q_e$  is negligible compared with the effect of  $Q_i$ .

## 2 COMPUTER MODELLING

The ripple-induced thermal fluxes described by eqs 1 and 2 were incorporated into the 1-D tokamak transport code HERMES [12]. Initial modelling experiments were carried out using the original INTOR reference parameters; these do not differ greatly from the present INTOR parameters and have been used previously in a series of computer simulations (eg refs 13-15). The major parameters are as follows:

$$\begin{aligned} B_o &= 5T, \\ R &= 4.5 \text{ m}, \\ \langle a \rangle &= 1.5 \text{ m}, \\ I &= 4MA, \\ \kappa_e &= 5 \times 10^{19} \text{ m}^{-1} \text{ s}^{-1}. \end{aligned}$$

Further details are given in refs 13-15.

Figure 3 summarizes the results of these computer simulations; it shows how the ripple-induced loss, characterised by a confinement time  $\tau_R$  and normalized against the case with no ripple, varies with the ripple depth  $\delta_o$  and the periodicity  $N$ . For those points marked 'no alpha heating' 50 MW of power were supplied by a 'magic' axial source to the electrons plus a small



contribution from ohmic-heating. In this way the mean ion temperature was kept in the range 10-15 keV which is representative of a reactor. The possibility that alpha-heating would change the temperature sufficiently to drastically alter the result was checked by including alpha-heating and then using the 'magic' heating to bring the total power input up to 50MW. Note the density used, namely  $\langle n \rangle = 8.6 \times 10^{19} \text{ m}^{-3}$ , was carefully chosen to be just below the minimum in the ignition curve for  $\delta_0 = 0$ . The resulting points are also shown in Fig. 3.

Further results of computer modelling are shown in figs 4 and 5. The current INTOR parameters were used in these computations namely:

$$\begin{aligned} B_0 &= 5.5 \text{ T}, \\ R &= 5.3 \text{ m}, \\ \langle a \rangle &= 1.52 \text{ m (ie } \sqrt{ab}, b/a = 1.6), \\ q(a) &= 2.1 \text{ (Equivalent to } I = 5.7 \text{ MA)} \\ N &= 12 \text{ (Ripple periodicity)} \end{aligned}$$

Since the INTOR plasma parameters can be expected to result in 'thermal-runaway' some stabilising influence is required if an equilibrium is to be modelled. The following technique was adopted. Of the alpha power produced only a fraction  $F$  was deposited in the plasma; the value of  $F$  was feed-back controlled to maintain the average ion temperature  $\langle T_i \rangle$  at the desired value of 10 keV. By using this artifice steady operation was obtained at  $\langle n_e \rangle = 1.57 \times 10^{20} \text{ m}^{-3}$  which, with zero ripple, resulted in  $\langle \beta \rangle = 4.13\%$  (the pressure from alpha particles was ignored) and an alpha power of 88 MW.

Figure 4 shows how the profiles of  $T_i$  and  $T_e$  varied as the ripple amplitude was changed; the changes in  $\langle \beta \rangle$ ,  $\langle T_e \rangle$ ,  $F$  and alpha power are also shown. As would be expected from the previous modelling  $\delta_0 > 2\%$  is needed before gross changes occur. At  $\delta_0 = 3\%$  the value of the fraction  $F$  exceeds unity which is of course impossible; what it means is that the density used is too small. Note that if the temperature is constant then the alpha power varies as  $n^2$  while the anomalous loss is constant and the ripple loss increases with  $n$ .

The main effect of ripple loss is to cause the profile of  $T_i$  to peak. The profile of  $T_e$  remains flat since the thermal diffusivity is set equal to the Bohm value inside  $q=1$ . Note that although

$$\hat{T}_i > \hat{T}_e$$

the reverse is true for the mean temperatures

ie  $\langle T_i \rangle < \langle T_e \rangle$

These later results can also be described by a normalised ripple time  $\tau_R/\tau_0$  as in Fig. 3. As before  $\tau_R/\tau_0 \propto \delta_0^{-3}$  but the curve is about a factor three lower; this reduction is mainly because the ellipticity of the plasma cross-section has been allowed for in these results when calculating the variation of ripple amplitude with mean radius.

Figure 5 shows the effect of ripple on 'thermal-runaway'. These results were established by the following procedure. Firstly one of the equilibria shown in Fig 4 was established; next the fraction  $F$  was increased by 10% and at the same time the feed-back control on  $F$  was switched-off. As a result the alpha power heating the plasma exceeded the plasma losses and 'thermal-runaway' occurred. The three graphs in Fig. 5 show how the value of  $\beta$ , the plasma losses and  $\langle T_i \rangle$  all increase with time as they approach new equilibrium values.

Although ripple does tend to inhibit the extent of 'thermal-runaway' even with  $\delta_0 = 3\%$  the mean ion temperature increases from 10 keV to about 15 keV with a corresponding increase in  $\beta$ .

### 3 CONCLUSIONS

The computational results support the conclusions drawn from figs. 1 and 2, namely that a value of  $\delta_0 > 2\%$  is needed to affect the energy confinement time; however, the computations of Erb [16] suggest that this level of ripple is probably impractical in INTOR. Figs. 1 and 2 also show that there is no reason to suppose that ripple losses will significantly suppress 'thermal-runaway'; this conclusion is supported by the modelling results shown in Fig. 5.



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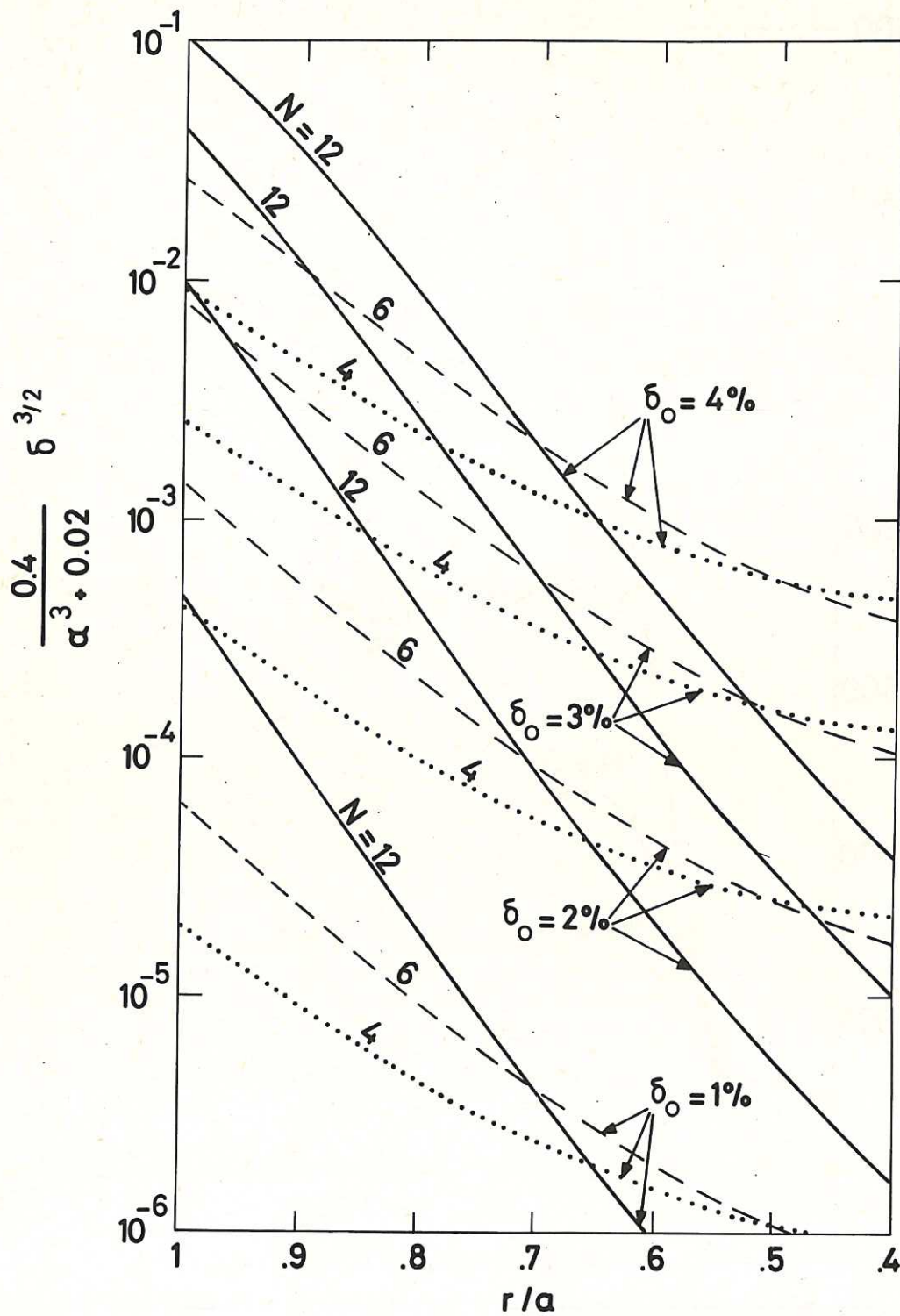


Fig.1 The geometric factor  $0.4 \delta^{3/2} / (\alpha^3 + 0.02)$  plotted against  $r/a$  with the ripple periodicity  $N$  and the edge ripple amplitude  $\delta_0 = (B_{\max} - B_{\min}) / (B_{\max} + B_{\min})$  as parameters.  $R = 4.5 \text{ m}$ ,  $a = 1.5 \text{ m}$ ,  $q(a) = 3$ .

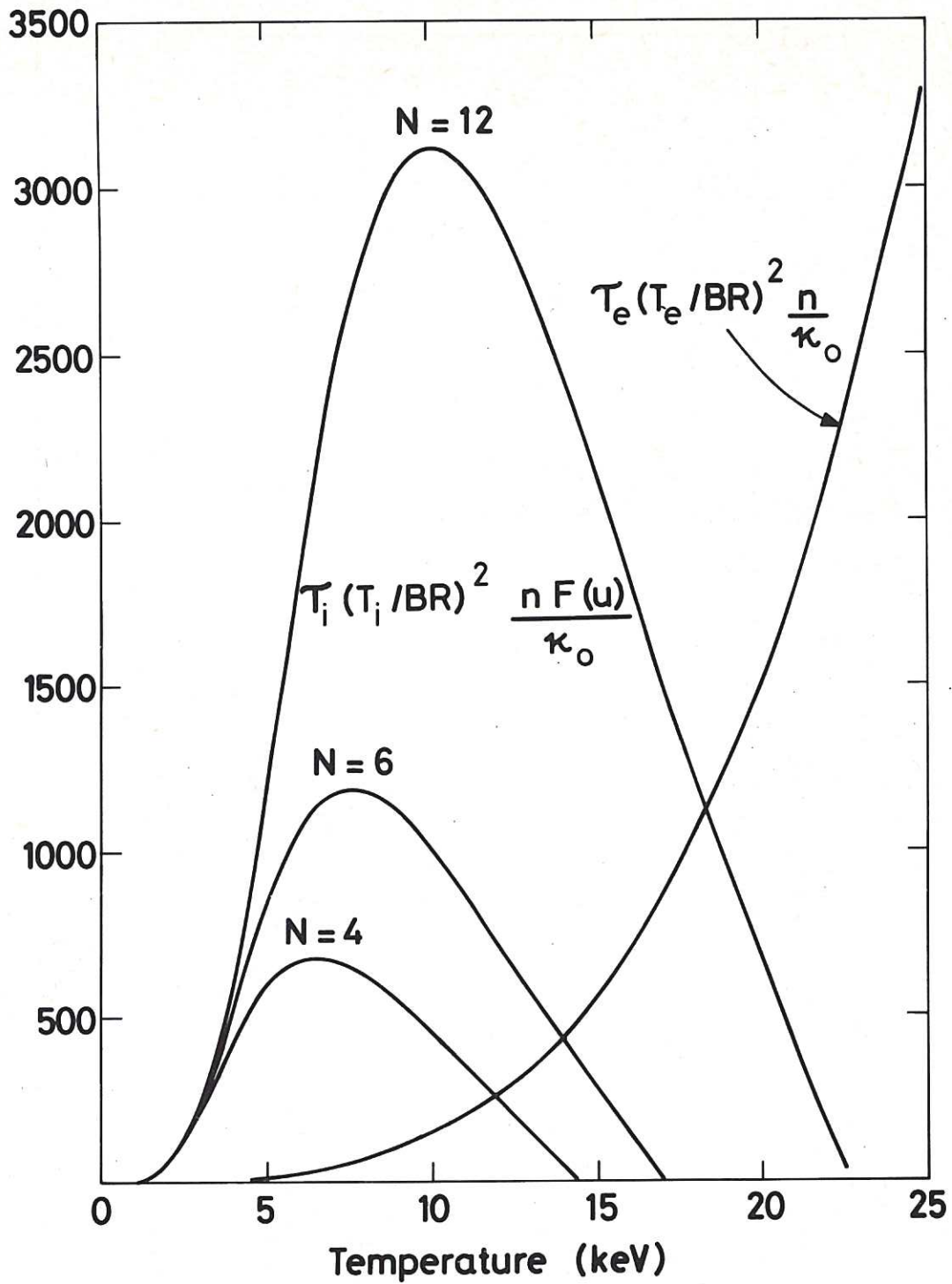


Fig.2 The normalized thermal conductivities  $\tau_i (T_i / BR)^2 \frac{n F(u)}{\kappa_0}$  and  $\tau_e (T_e / BR)^2 \frac{n}{\kappa_0}$  plotted against temperature, with the periodicity  $N$  as a parameter. The other parameters are:  $R = 4.5 \text{ m}$ ,  $\lambda = 1$  (zero mass motion)  $n = 5 \times 10^{19} \text{ m}^{-3}$ ,  $q = 3$ ,  $B = 5 \text{ T}$ ,  $\kappa_0 = 5 \times 10^{19} \text{ m}^{-1} \text{ s}^{-1}$ .



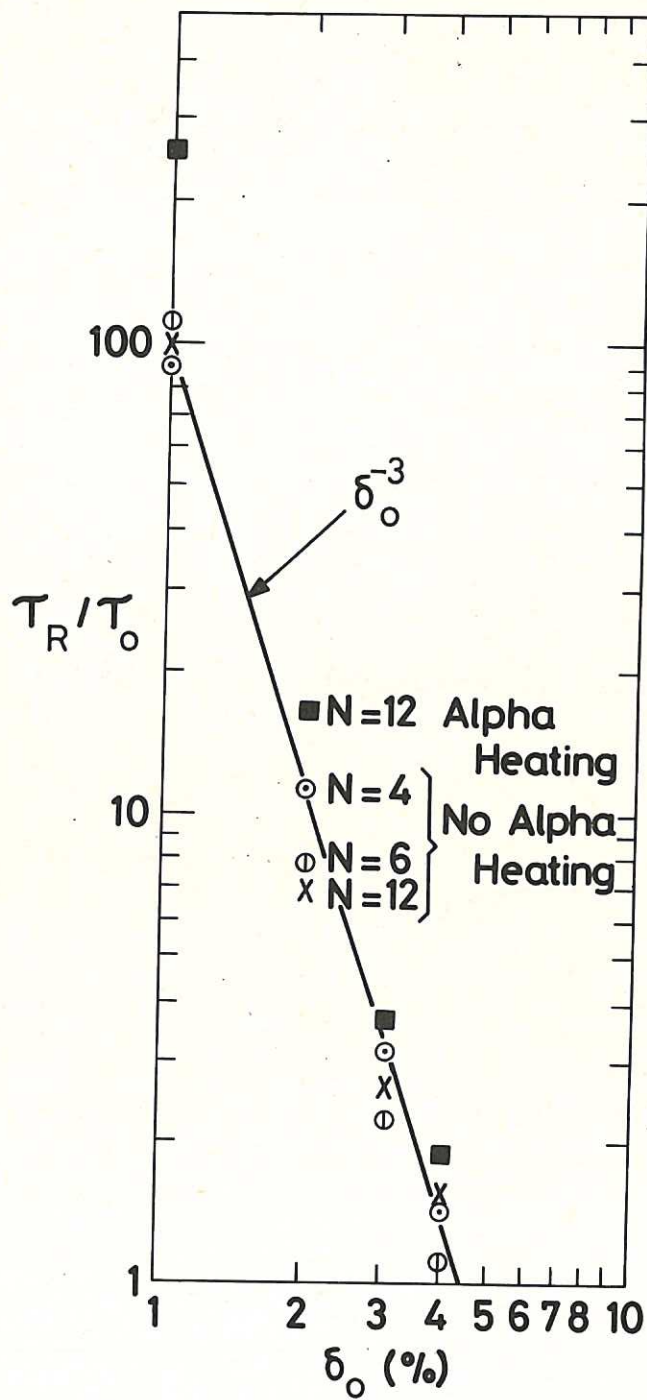


Fig.3 Ripple losses, characterized by a confinement time normalized against the confinement time with no ripple, as a function of  $\delta_0$  with  $N$  as a parameter. In each case 50MW of power was supplied; for the cases with no alpha-power this was entirely from an axial source.  $T_0 = 2.33\text{sec}$ ,  $\langle n \rangle = 8.6 \times 10^{19} \text{m}^{-3}$ .

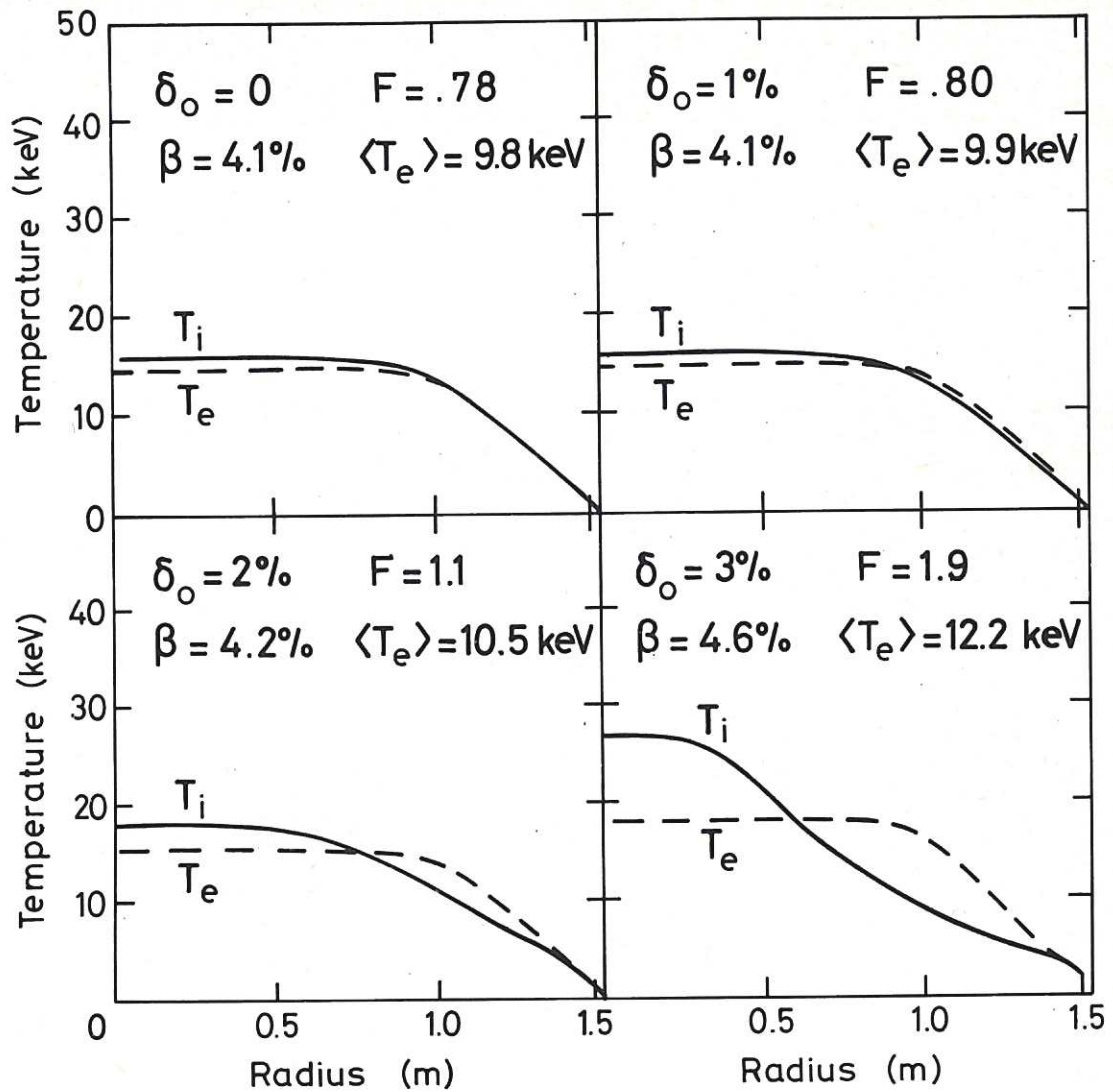


Fig.4 The effect of ripple losses on temperature profiles in INTOR: when  $F$ , the fraction of alpha power absorbed by the plasma, is feed-back controlled to maintain  $\langle T_i \rangle = 10 \text{ keV}$ . Coil periodicity  $N=12$  and  $\langle n_e \rangle = 1.57 \times 10^{20} \text{ m}^{-3}$ .



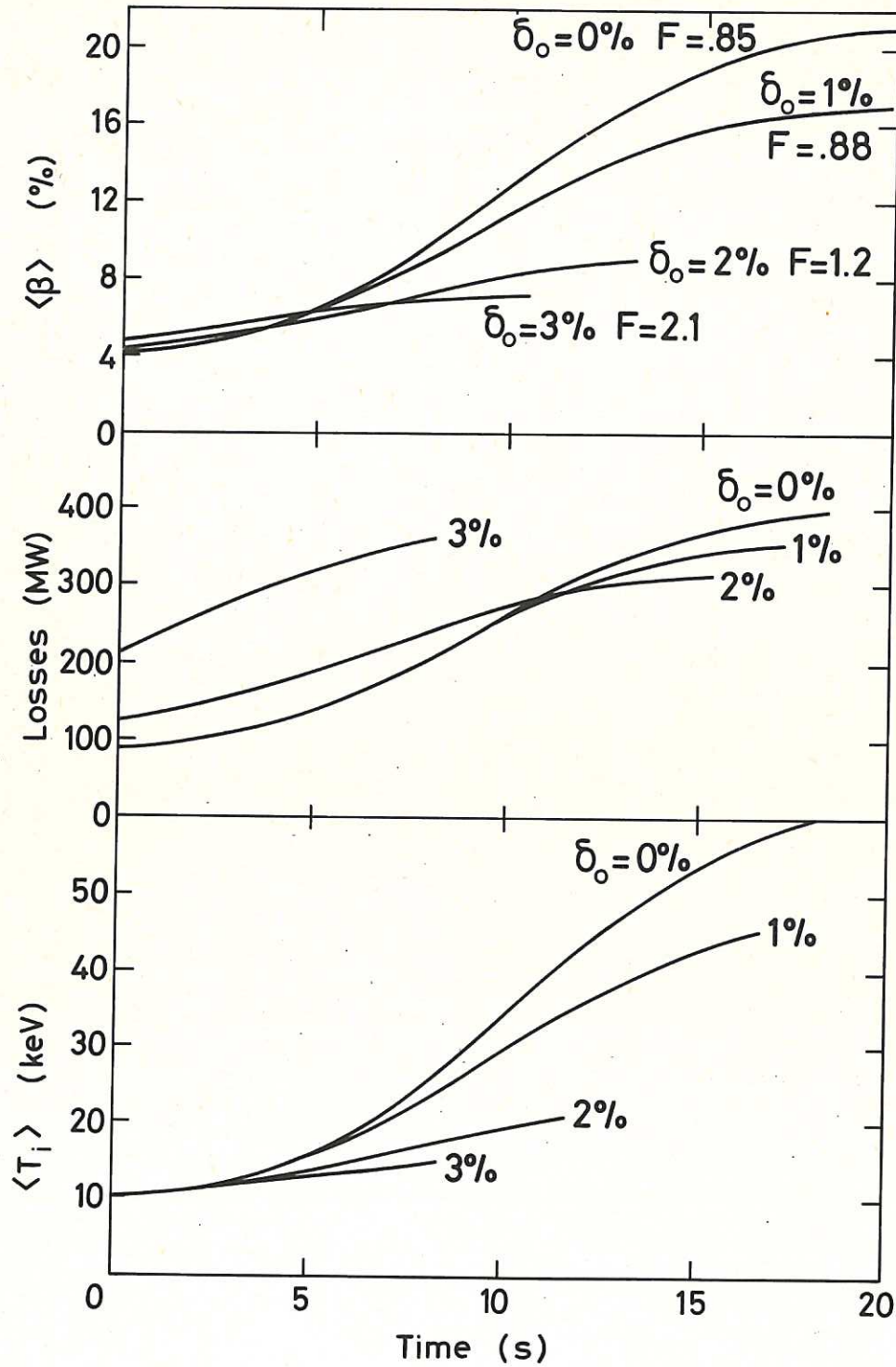


Fig.5 The effect of ripple on 'thermal-runaway'. The equilibria shown in Fig.4 were disturbed by increasing F by 10% and then removing the feedback control.







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