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Report

RELATIVISTIC CORRECTIONS IN ECRH CURRENT DRIVE

M. R. O'BRIEN
D. F. H. START
P. M. V. GRACE

CULHAM LABORATORY
Abingdon Oxfordshire

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M R O'Brien, D F H Start and *P M V Grace

Culham Laboratory, Abingdon, Oxon OX14 3DB
(Euratom/UKAEA Fusion Association)
*St John's College, Cambridge, UK.

ABSTRACT

This report supplements a previous paper (1), in which we used a full Fokker-Planck treatment to find the effects of a relativistic resonance condition on ECRH current drive. We tabulate current drive efficiencies for the modes and resonances of interest, in particular for the O-mode at the fundamental resonance which was not treated in (1). The results have been compared with a Lorentz gas model which gives good scaling under certain conditions. A ray tracing study has shown the size of these effects in present-day tokamaks.

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1. INTRODUCTION

In a previous paper [1] we investigated the effect of the relativistic correction to the electron mass on ECRH current drive by numerically solving the linearised, steady-state electron Fokker-Planck equation with full electron-electron collisions included. That paper was confined to the X-mode and only results for X-mode absorption at the fundamental resonance were tabulated. In this report we extend the treatment to the O-mode and tabulate current drive efficiencies for those modes which might be used for current drive, namely the O- and X-modes at the fundamental resonance and the X-mode at the 2nd harmonic resonance. This report is only intended as a supplement to [1] and much of the mathematics, the discussion of the resonance condition and the details of the numerical calculations which are in that paper are not reproduced here. For convenience, we include a glossary of symbols used in [1], and which are not redefined in the text (Appendix 1).

2. THE FOKKER-PLANCK EQUATION

The steady-state electron Fokker-Planck equation may be written schematically as:

$$0 = \left(\frac{\partial f_e}{\partial t}\right)_{collisions} + \left(\frac{\partial f_e}{\partial t}\right)_{ECRH} \qquad (1)$$

By assuming that the perturbation f_e' to the electron distribution function f_e is small and expanding f_e' in Legendre polynomials, that is $f_e' = F_{me} \sum_n a_n \left(\frac{v}{v_e}\right) P_n \left(\frac{v_\parallel}{v}\right)$ with F_{me} the background Maxwellian, equation (1) may be reduced to a series of uncoupled integro-differential equations for the coefficient functions $a_n(x)$. By solving the equation for $a_1(x)$ we calculate the current $J = -\frac{4e}{3\pi^{1/2}} n_e v_e \int_0^\infty a_1(x)\, x^3 e^{-x^2} dx$. The power absorbed per unit volume from the wave is found from $P = \frac{1}{2} m_e \int v^2 \left(\frac{\partial f_e}{\partial t}\right)_{ECRH} d^3v$.

The usual dimensionless current drive efficiency J/P is then calculated by expressing J and P in the units $-n_e e v_e$ and $n_e m_e v_e^2 \nu_o$ respectively. The equation for $a_1(x)$ is [1]:-

$$a_1'' + P(x)a_1' + Q(x)\, a_1 - \frac{16}{3\pi^{1/2}\Lambda} \left[x I_3(x) - 1.2 x I_5(x) - x^4(1-1.2x^2)\left(I_0(x) - I_0(\infty)\right)\right] = R(x) \qquad (2)$$

with the functions P, Q and Λ and the integrals I_n as defined in [1]. The left hand side of this equation comes from $\left(\frac{\partial f}{\partial t}\right)_{collisions}$ and the right hand side, R(x), from $\left(\frac{\partial f}{\partial t}\right)_{ECRH}$.

The driving term in equation (1) may be written [2]

$$\left(\frac{\partial f_e}{\partial t}\right)_{ECRH} = \left(\frac{\partial f_e}{\partial t}\right)_0 + \left(\frac{\partial f_e}{\partial t}\right)_1 + \left(\frac{\partial f_e}{\partial t}\right)_2 \qquad (3)$$

with

$$\left(\frac{\partial f_e}{\partial t}\right)_j = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left\{ v_\perp \left(\frac{v_\perp}{v_e}\right)^{2(\ell-1)} D_j \left(\frac{v_\parallel}{v_e}\right)^j \delta\left(\frac{\omega}{k_\parallel} - \frac{\ell\Omega}{k_\parallel}\left(1 - \frac{v^2}{2c^2}\right) - v_\parallel\right) \frac{\partial F_{me}}{\partial v_\perp} \right\} \qquad (4)$$

where $D_0 = \dfrac{\pi e^2}{4k_\parallel m_e^2} \cdot \dfrac{1}{((\ell-1)!)^2} \left(\dfrac{k_\perp v_e}{2\Omega}\right)^{2(\ell-1)} |E^-|^2$

$$D_1 = \frac{\pi e^2}{k_\parallel m_e^2} \frac{1}{\ell!(\ell-1)!} \left(\frac{k_\perp v_e}{2\Omega}\right)^{2\ell-1} Re(E^- E_z^*)$$

and $D_2 = \dfrac{\pi e^2}{k_\parallel m_e^2} \dfrac{1}{(\ell!)^2} \left(\dfrac{k_\perp v_e}{2\Omega}\right)^{2\ell} |E_z|^2.$

$E^- = E_x - iE_y$ with $\underline{E}$ the wave electric field vector. For the X-mode, $D_0 >> D_1 >> D_2$ and only the first term on the right hand side of equation (3) need be retained: this is the case considered in [1]. However for the O- mode all three terms must be retained.

Each of these three terms gives a contribution to the power absorbed and to the current driven i.e.

$$P = \sum_{j=0}^{2} P_j \text{ and } J = \sum_{j=0}^{2} J_j$$

(Expressions are given for P_0, P_1 and P_2 in Appendix 2.) Thus

$$\frac{J}{P} = \sum_{j=0}^{2} \left(\frac{J}{P}\right)_j \frac{P_j}{P} \qquad (5)$$

where $\left(\frac{J}{P}\right)_j = \frac{J_j}{P_j}$ is the current drive efficiency due to the jth term of equation (3). Hence to find a net J/P for the O-mode one needs to know $(J/P)_j$ for j=0, 1, 2 and the power absorbed due to each term.

The jth term gives a driving function in equation (2) given by

$$R_j(x) = \frac{12D_j}{v_e{}^3 v_0 \Lambda} (x^2 - x_0^2)^{\ell-1} x_0^{j+1} \times \{x_0^2 + \ell - x^2 + \frac{S}{x_0}((j+1)x^2 - (2\ell+j+1)x_0^2)\}$$

$$\{H(x-\alpha_-) - H(x-\alpha_+)\}$$

where H is the Heaviside function, $\alpha_\pm = \left|(1 \pm \sqrt{1-4u_0 S})/2S\right|$,

$x_0 = u_0 + Sx^2$ and we have introduced $u_0 = \frac{\omega - \ell\Omega}{k_\parallel v_e}$ and

$S = \frac{1}{2}\frac{\ell\Omega v_e}{k_\parallel c^2}$ which were used in [1] to parameterise the absorption. $R_j(x)$ reduces to the driving term R(x) in [1] for j=0. The value S=0 corresponds to the non-relativistic resonance condition and thus S gives a measure of the importance of relativistic effects.

3. RESULTS AND HOW TO USE THEM

We have solved equation (2) both analytically in the Lorentz gas limit (Z >> 1 - see Appendix 3) and numerically with full electron-electron collisions included. Tables 1 to 6 list the results of our numerical calculations. We find (cf. [1]) that the Lorentz gas model gives good scaling for high speed resonant electrons ($u_0 \gtrsim 2$) but overestimates J/P by a factor (Z + 5)/Z. Allowing for this factor, it underestimates J/P for low speed resonant electrons ($u_0 \lesssim 1$).

In the tables $(J/P)_j$ can be either negative or positive. This is because for even values of j, J_j can change sign and for odd values of j, P_j can change sign. Note that a negative P_1 is allowed but P must always be positive. Around the zeroes of P_1 the values of J/P in Table 2 get large and are unreliable. We also note that $(J/P)_j$ is independent of j for the non-relativistic limit S=0. This follows from the form of P_j and $R_j(x)$ for S=0. Consequently for reasonably small values of S, Table 1 gives a good approximation to Tables 2 and 3, but further down the tables, in particular near where $(J/P)_j$ changes sign, the approximation is less accurate.

To find the current drive efficiency for the O-mode at the fundamental resonance one needs to combine Tables 1, 2 and 3 by using equation (5). For example if $u_0 = -1$, S=1 and $P_0:P_1:P_2 = 1:-2:5$ then J/P = 0.64 .

These values of J/P may be converted into more useful quantities by using the expression

$$\frac{I(\text{Amps})}{P(\text{Watts})} = 0.122 \frac{T_e(\text{keV})}{n_e R_o \ln \lambda} \frac{J}{P}$$

with R_o in metres and n_e in units of $10^{20} m^{-3}$.

We have included these results in a ray tracing code to quantify the effects of the relativistic correction in an actual experiment. We used the tabulated efficiencies for $|u_o| < 3$ and the scaled-down Lorentz results for $|u_o| > 3$. Figures 1 and 2 show the effect of varying the angle of injection of the waves on the driven current in a tokamak the size of DITE for X-mode 2nd harmonic and O-mode fundamental heating. Both relativistic and non-relativistic (S=0) curves are shown. The curves deviate substantially at small angles (for which $k_\parallel$ is small and hence S large) and show that in an experiment to get significant current drive one would need an angle of injection greater than ~ 10° at T_e = 1 keV. This angle increases with the square root of the temperature.

The parameters used in the calculations shown in figs 1 and 2 give mostly similar power deposition profiles for corresponding angles of injection. Thus the similarity of the efficiencies shown in figs 1 and 2 suggest that the different velocity dependences of the driving terms (eq 4) for the 'O' mode and X mode do not, per se, have a substantial impact on the calculations.

Conclusion

We find that, for the O-mode, J/P has three terms compared to one for the X-mode. However if relativistic effects are small (small S) all three terms are approximately equal and only one calculation is required. The Lorentz model, when scaled by the factor Z/(Z+5) gives good agreement with the full treatment for resonant electrons with speeds greater than thermal. Ray tracing calculations show that the relativistic correction can be important at present laboratory temperatures.

References

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[2] R A Cairns, J Owen and C N Lashmore-Davies, Physics of Fluids, 26 (1983), P.3475.

[3] G Rowlands, V L Sizonenko and K N Stepanov, Sov Physics JETP 23 (1966) 661.

Appendix 1 - Glossary of Symbols

$\underline{B}$	magnetic field
v	electron speed
$v_\parallel, v_\perp$	components of electron velocity parallel to and perpendicular to $\underline{B}$
m_e	electron rest mass
T_e	electron temperature
v_e	$= \left(\frac{2T_e}{m_e}\right)^{1/2}$
F_{me}	electron Maxwellian distribution
e	electron charge
n_e	electron density
$\ln \lambda$	Coulomb logarithm
ν_0	$= \frac{e^4 n_e \ln\lambda}{4\pi\varepsilon_0^2 v_e^3 m_e^2}$ = collision frequency
ω	wave angular frequency
$k_\parallel$	component of wave vector parallel to $\underline{B}$
$k_\perp$	component of wave vector perpendicular to $\underline{B}$
ℓ	harmonic number
Ω	electron rest gyrofrequency $= \frac{e\lvert\underline{B}\rvert}{m_e}$
c	speed of light
R_0	major radius

Appendix 2 - Expressions for P_j

P_j, the power absorbed due to the jth term in equation (3), is given by

$$P_j = \tfrac{1}{2} m_e \int v^2 \left(\frac{\partial f_e}{\partial t}\right)_j d^3v \qquad \text{(A2.1)}$$

with $\left(\frac{\partial f_e}{\partial t}\right)_j$ given by equation (4).

By writing A2.1 in the variables $x = v/v_e$ and $\xi = v_\parallel/v$ and integrating over ξ one gets

$$P_j = \frac{4D_j n_e m_e}{(\pi^{\frac{1}{2}} v_e)} I_p \quad \text{where}$$

$$I_p = \int_{\alpha_-}^{\alpha_+} x^3 (x^2-x_0^2)^{\ell-1} x_0^j \{x^2-x_0^2-\ell + 2\ell x_0 S - \frac{jS}{x_0}(x^2-x_0^2)\} e^{-x^2} dx$$

with $x_0 = u_0 + Sx^2$.

It is convenient to express I_p in terms of the integrals K_n where $K_n = \int_a^b y^n e^{-y} dy$, $a = \alpha_-^2$ and $b = \alpha_+^2$ and then use the recurrence relation $K_n = nK_{n-1} + a^n e^{-a} - b^n e^{-b}$.

We give I_p for the four cases of interest:-

<u>ℓ=1, j=0</u>

$$I_p = \tfrac{1}{2}\,[-S^2K_3 + (1-2u_oS + 2S^2)\,K_2$$
$$- (1 + u_o^2 - 2u_oS)\,K_1]$$

<u>ℓ=1, j=1</u>

$$I_p = \tfrac{1}{2}\,[-S^3K_4 + S(1-3u_oS + 3S^2)K_3$$
$$+ u_o - 2S - 3u_o^2S + 6u_oS^2)K_2$$
$$- u_o\,(u_o^2 + 1 - 3Su_o)K_1]$$

<u>ℓ=1, j=2</u>

$$I_p = \tfrac{1}{2}[-S^4K_5 + S^2(1-4u_oS + 4S^2)K_4$$
$$+ S(2u_o - 3S - 6u_o^2S + 12u_oS^2)K_3$$
$$+ u_o\,(u_o - 4S - 4u_o^2S + 12u_oS^2)K_2$$
$$- u_o^2\,(u_o^2 + 1 - 4u_oS)K_1]$$

<u>ℓ=2, j=0</u>

$$I_p = \tfrac{1}{2}\,[S^4K_5 - 2S^2\,(1-2u_oS + 2S^2)K_4$$
$$+ (1-4u_oS + 6S^2 + 6u_o^2S^2 - 12u_oS^3)K_3$$
$$- 2\,(1 + u_o^2 - 4u_oS - 2u_o^3S + 6u_o^2S^2)K_2$$
$$+ u_o^2\,(2 + u_o^2 - 4u_oS)K_1]$$

Appendix 3 - Lorentz Gas Results

In the Lorentz gas limit ($Z >> 1$) equation (2) reduces to

$$a_1(x) = \frac{-R_j(x)\Lambda(x)}{2Z}$$

$$= \frac{-6D_j}{v_e^3 v_o Z} (x^2-x_o^2)^{\ell-1} x_o^{j+1} \; x\{x_o^{2+\ell} - x^2 + \frac{S}{x_o}((j+1)x^2 - (2\ell+j+1)x_o^2)\}$$

for the jth term. The current can be expressed in terms of $a_1(x)$ as

$$J = -\frac{4e}{3\pi^{\frac{1}{2}}} n_e v_e \int_0^\infty a_1(x) x^3 e^{-x^2} dx, \text{ which becomes}$$

$$J = \left(\frac{8D_j n_e e}{\pi^{\frac{1}{2}} v_e^2 v_o Z} \right) I_J$$

where $I_J = \int_{\alpha_-}^{\alpha_+} (x^2-x_o^2)^{\ell-1} x_o^{j+1} x^4 \{x_o^{2+\ell} - x^2 + \frac{S}{x_o}((j+1)x^2 - (2\ell+j+1)x_o^2)\} e^{-x^2} dx$

We give I_J for the 4 cases of interest in terms of the integrals K_n defined in Appendix 2.

$\ell=1,\ j=0$

$$I_J = \tfrac{1}{2}\{u_o(u_o^2 + 1 - 3Su_o)K_{3/2} + (3u_o^2 S - u_o + 2S - 6u_oS^2)K_{5/2}$$
$$+ S(3u_oS - 1 - 3S^2)K_{7/2} + S^3 K_{9/2}\}$$

$\ell=1,\ j=1$

$$I_J = \tfrac{1}{2}\{u_o^2(u_o^2 + 1 - 4u_oS)K_{3/2} + u_o(-u_o + 4u_o^2S + 4S - 12u_oS^2)K_{5/2}$$
$$+ S(6u_o^2S + 3S - 2u_o - 12u_oS^2)K_{7/2}$$
$$+ S^2(4u_oS - 1 - 4S^2)K_{9/2} + S^4 K_{11/2}\}$$

$\ell=1,\ j=2$

$$I_J = \tfrac{1}{2}\{u_o^3(u_o^2 + 1 - 5u_oS)K_{3/2}$$
$$+ u_o^2(-u_o + 5u_o^2S + 6S - 20u_oS^2)K_{5/2}$$
$$+ u_oS(10u_o^2S + 9S - 3u_o - 30u_oS^2)K_{7/2}$$
$$+ S^2(10u_o^2S + 4S - 3u_o - 20u_oS^2)K_{9/2}$$
$$+ S^3(5u_oS - 1 - 5S^2)K_{11/2} + S^5 K_{13/2}\}$$

$\ell=2,\ j=0$

$$I_J = \tfrac{1}{2}\{u_o^3(5u_oS - u_o^2 - 2)K_{5/2}$$
$$+ u_o(20u_o^2S + 2u_o^2 + 2 - 5u_o^3S - 12u_oS)K_{5/2}$$
$$+ (30u_o^2S^3 + 6u_o^2S + 3S - 10u_o^3S^2 - 18u_oS^2 - u_o)K_{7/2}$$
$$+ S(20u_oS^3 + 6u_oS - 10u_o^2S^2 - 8S^2 - 1)K_{9/2}$$
$$+ S^3(5S^2 + 2 - 5u_oS)K_{11/2}$$
$$- S^5 K_{13/2}\}$$

TABLE 1 J/P for j=0 , l=1 and Z=1

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.00	-5.24	-3.92	-2.87	-2.08	-1.45	-0.82	-0.52	-0.18	0.18	0.52	0.82	1.45	2.08	2.87	3.92	5.24
0.02	-4.71	-3.59	-2.68	-1.96	-1.37	-0.76	-0.46	-0.12	0.24	0.58	0.89	1.54	2.20	3.09	4.32	5.94
0.04	-4.29	-3.32	-2.51	-1.86	-1.29	-0.69	-0.40	-0.06	0.30	0.65	0.96	1.62	2.33	3.36	4.86	6.94
0.06	-3.95	-3.09	-2.36	-1.76	-1.22	-0.63	-0.33	0.004	0.36	0.71	1.02	1.70	2.49	3.70	5.64	8.64
0.08	-3.66	-2.88	-2.22	-1.66	-1.14	-0.56	-0.27	0.06	0.43	0.77	1.09	1.79	2.66	4.17	6.98	13.51
0.10	-3.40	-2.70	-2.10	-1.57	-1.07	-0.50	-0.21	0.12	0.49	0.84	1.15	1.88	2.87	4.91	-	-
0.12	-3.18	-2.54	-1.98	-1.48	-1.00	-0.43	-0.15	0.18	0.55	0.90	1.22	1.98	3.14	6.66	-	-
0.14	-2.97	-2.39	-1.87	-1.39	-0.93	-0.37	-0.09	0.24	0.61	0.96	1.29	2.07	3.53	-	-	-
0.16	-2.80	-2.25	-1.76	-1.31	-0.86	-0.31	-0.03	0.30	0.66	1.02	1.35	2.18	4.28	-	-	-
0.18	-2.63	-2.12	-1.66	-1.23	-0.79	-0.25	0.03	0.36	0.72	1.08	1.41	2.29	-	-	-	-
0.20	-2.48	-2.00	-1.57	-1.16	-0.73	-0.19	0.08	0.41	0.78	1.14	1.48	2.41	-	-	-	-
0.24	-2.21	-1.79	-1.39	-1.01	-0.60	-0.08	0.19	0.52	0.88	1.25	1.59	2.66	-	-	-	-
0.3	-1.86	-1.50	-1.15	-0.81	-0.43	0.08	0.34	0.66	1.02	1.39	1.73	-	-	-	-	-
0.4	-1.39	-1.10	-0.82	-0.52	-0.17	0.29	0.53	0.83	1.18	1.53	1.84	-	-	-	-	-
0.6	-0.74	-0.54	-0.34	-0.12	0.15	0.51	0.72	0.98	1.31	1.68	-	-	-	-	-	-
1.0	-0.16	-0.06	0.05	0.17	0.34	0.60	0.78	1.03	1.42	-	-	-	-	-	-	-
1.5	0.04	0.10	0.16	0.25	0.37	0.60	0.75	1.01	1.43	-	-	-	-	-	-	-
2.0	0.10	0.14	0.20	0.27	0.37	0.57	0.71	0.95	1.32	-	-	-	-	-	-	-
2.5	0.13	0.16	0.21	0.27	0.36	0.54	0.67	0.88	-	-	-	-	-	-	-	-

TABLE 2 J/P for j=1 , l=1 and Z=1

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.00	-5.24	-3.92	-2.87	-2.08	-1.45	-0.82	-0.52	-0.18	0.18	0.52	0.82	1.45	2.08	2.87	3.92	5.24
0.02	-4.71	-3.59	-2.68	-1.96	-1.38	-0.77	-0.47	-0.15	0.24	0.58	0.88	1.53	2.21	3.10	4.33	5.94
0.04	-4.29	-3.32	-2.51	-1.86	-1.30	-0.72	-0.44	-0.37	0.31	0.65	0.95	1.62	2.34	3.37	4.87	6.94
0.06	-3.95	-3.08	-2.36	-1.76	-1.24	-0.68	-0.43	0.62	0.39	0.71	1.02	1.71	2.50	3.72	5.65	8.56
0.08	-3.65	-2.88	-2.23	-1.67	-1.17	-0.65	-0.47	0.41	0.47	0.79	1.09	1.80	2.68	4.19	7.00	11.5
0.10	-3.40	-2.71	-2.11	-1.59	-1.12	-0.65	-0.63	0.43	0.55	0.86	1.17	1.90	2.90	4.93	-	-
0.12	-3.18	-2.55	-2.00	-1.52	-1.08	-0.68	-1.13	0.49	0.63	0.94	1.24	2.00	3.17	6.69	-	-
0.14	-2.99	-2.41	-1.90	-1.45	-1.04	-0.75	-5.04	0.56	0.71	1.01	1.32	2.11	3.57	-	-	-
0.16	-2.82	-2.28	-1.81	-1.39	-1.02	-0.91	3.29	0.63	0.79	1.08	1.39	2.22	4.32	-	-	-
0.18	-2.67	-2.17	-1.73	-1.34	-1.00	-1.23	1.64	0.70	0.87	1.16	1.47	2.34	-	-	-	-
0.20	-2.53	-2.07	-1.66	-1.30	-1.00	-1.99	1.29	0.77	0.94	1.23	1.54	2.46	-	-	-	-
0.24	-2.29	-1.89	-1.54	-1.24	-1.07	46.2	1.13	0.91	1.08	1.36	1.67	2.68	-	-	-	-
0.3	-2.02	-1.70	-1.43	-1.24	-1.44	2.27	1.16	1.08	1.25	1.51	1.80	-	-	-	-	-
0.4	-1.75	-1.56	-1.45	-1.62	-5.86	1.58	1.26	1.25	1.40	1.62	1.87	-	-	-	-	-
0.6	-1.91	-2.11	-2.95	-14.6	2.97	1.42	1.30	1.31	1.45	1.70	-	-	-	-	-	-
1.0	-7.48	-55.7	7.39	3.04	1.81	1.34	1.28	1.31	1.50	-	-	-	-	-	-	-
1.5	9.85	5.19	3.30	2.29	1.70	1.35	1.28	1.30	1.50	-	-	-	-	-	-	-
2.0	5.20	3.78	2.85	2.20	1.72	1.38	1.29	1.26	1.37	-	-	-	-	-	-	-
2.5	4.33	3.44	2.76	2.22	1.78	1.41	1.29	1.21	-	-	-	-	-	-	-	-

TABLE 3 J/P for j=2 , l=1 and Z=1

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.00	-5.24	-3.92	-2.87	-2.08	-1.45	-0.82	-0.52	-0.18	0.18	0.52	0.82	1.45	2.08	2.87	3.92	5.24
0.02	-4.71	-3.59	-2.68	-1.96	-1.38	-0.78	-0.49	-0.17	0.24	0.58	0.88	1.53	2.21	3.10	4.33	5.94
0.04	-4.29	-3.31	-2.51	-1.86	-1.31	-0.74	-0.47	-0.02	0.33	0.64	0.95	1.62	2.35	3.38	4.88	6.95
0.06	-3.94	-3.08	-2.36	-1.77	-1.25	-0.72	-0.46	0.24	0.43	0.72	1.02	1.71	2.51	3.73	5.67	8.56
0.08	-3.65	-2.88	-2.23	-1.69	-1.21	-0.70	-0.41	0.40	0.54	0.81	1.11	1.81	2.70	4.21	7.01	11.2
0.10	-3.40	-2.71	-2.12	-1.61	-1.16	-0.68	-0.27	0.52	0.65	0.90	1.19	1.92	2.92	4.96	-	-
0.12	-3.19	-2.56	-2.01	-1.55	-1.13	-0.64	-0.02	0.64	0.75	0.99	1.28	2.03	3.21	6.71	-	-
0.14	-3.00	-2.47	-1.92	-1.49	-1.10	-0.55	0.26	0.75	0.86	1.09	1.37	2.15	3.61	-	-	-
0.16	-2.84	-2.31	-1.84	-1.44	-1.07	-0.39	0.51	0.85	0.96	1.18	1.46	2.27	4.35	-	-	-
0.18	-2.69	-2.20	-1.77	-1.39	-1.04	-0.16	0.70	0.95	1.06	1.28	1.55	2.40	-	-	-	-
0.20	-2.56	-2.11	-1.71	-1.35	-0.99	0.09	0.86	1.05	1.16	1.37	1.64	2.53	-	-	-	-
0.24	-2.34	-1.95	-1.60	-1.27	-0.85	0.58	1.11	1.23	1.33	1.53	1.78	2.70	-	-	-	-
0.3	-2.08	-1.75	-1.45	-1.11	-0.43	1.09	1.36	1.43	1.52	1.69	1.90	-	-	-	-	-
0.4	-1.75	-1.46	-1.12	-0.58	0.48	1.44	1.53	1.56	1.61	1.73	1.89	-	-	-	-	-
0.6	-1.00	-0.61	-0.09	0.55	1.16	1.45	1.48	1.50	1.58	1.72	-	-	-	-	-	-
1.0	0.18	0.43	0.69	0.94	1.17	1.33	1.38	1.44	1.56	-	-	-	-	-	-	-
1.5	0.50	0.63	0.77	0.93	1.10	1.28	1.35	1.43	1.55	-	-	-	-	-	-	-
2.0	0.56	0.66	0.77	0.90	1.06	1.25	1.33	1.39	1.41	-	-	-	-	-	-	-
2.5	0.58	0.66	0.75	0.87	1.03	1.22	1.30	1.34	-	-	-	-	-	-	-	-

TABLE 4 J/P for j=0 , l=1 AND Z=2

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.00	-4.42	-3.27	-2.35	-1.65	-1.10	-0.58	-0.36	-0.12	0.12	0.36	0.58	1.10	1.65	2.35	3.27	4.42
0.02	-3.96	-2.99	-2.19	-1.55	-1.03	-0.53	-0.31	-0.07	0.17	0.40	0.63	1.16	1.76	2.55	3.62	5.02
0.04	-3.60	-2.75	-2.04	-1.46	-0.97	-0.48	-0.26	-0.03	0.21	0.45	0.68	1.23	1.87	2.78	4.09	5.89
0.06	-3.30	-2.55	-1.91	-1.38	-0.91	-0.43	-0.22	0.02	0.26	0.50	0.73	1.30	2.00	3.08	4.76	7.34
0.08	-3.05	-2.37	-1.79	-1.30	-0.85	-0.38	-0.17	0.06	0.30	0.54	0.78	1.37	2.15	3.48	5.92	11.53
0.10	-2.83	-2.22	-1.69	-1.22	-0.78	-0.33	-0.12	0.11	0.34	0.59	0.83	1.45	2.33	4.12	-	-
0.12	-2.64	-2.08	-1.58	-1.15	-0.74	-0.29	-0.08	0.15	0.39	0.63	0.88	1.53	2.57	5.64	-	-
0.14	-2.47	-1.95	-1.49	-1.08	-0.68	-0.24	-0.03	0.19	0.43	0.68	0.93	1.61	2.91	-	-	-
0.16	-2.31	-1.83	-1.40	-1.01	-0.63	-0.19	0.01	0.23	0.47	0.72	0.98	1.70	3.57	-	-	-
0.18	-2.17	-1.72	-1.32	-0.94	-0.57	-0.15	0.05	0.28	0.51	0.77	1.02	1.79	-	-	-	-
0.20	-2.04	-1.62	-1.24	-0.88	-0.52	-0.10	0.10	0.32	0.55	0.81	1.07	1.90	-	-	-	-
0.24	-1.80	-1.44	-1.09	-0.76	-0.42	-0.02	0.18	0.39	0.63	0.88	1.15	2.11	-	-	-	-
0.3	-1.51	-1.19	-0.90	-0.60	-0.28	0.10	0.28	0.49	0.72	0.98	1.24	-	-	-	-	-
0.4	-1.11	-0.86	-0.62	-0.37	-0.09	0.25	0.42	0.61	0.82	1.04	1.28	-	-	-	-	-
0.6	-0.57	-0.40	-0.23	-0.06	0.15	0.40	0.52	0.67	0.83	1.02	-	-	-	-	-	-
1.0	-0.10	-0.02	0.06	0.15	0.27	0.42	0.50	0.60	0.72	-	-	-	-	-	-	-
1.5	0.05	0.09	0.14	0.19	0.27	0.37	0.43	0.49	0.54	-	-	-	-	-	-	-
2.0	0.09	0.12	0.15	0.19	0.25	0.33	0.37	0.40	0.37	-	-	-	-	-	-	-
2.5	0.10	0.12	0.15	0.18	0.23	0.29	0.31	0.33	-	-	-	-	-	-	-	-

TABLE 5 J/P for j=0 , l=2 and Z=1

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.0	-5.44	-4.08	-2.99	-2.12	-1.41	-0.74	-0.45	-0.15	0.15	0.45	0.74	1.41	2.12	2.99	4.08	5.44
0.02	-4.86	-3.71	-2.75	-1.97	-1.30	-0.65	-0.36	-0.07	0.24	0.54	0.83	1.53	2.28	3.25	4.54	6.19
0.04	-4.40	-3.39	-2.54	-1.82	-1.19	-0.56	-0.28	0.02	0.33	0.63	0.93	1.64	2.46	3.57	5.13	7.25
0.06	-4.01	-3.12	-2.35	-1.69	-1.09	-0.46	-0.19	0.11	0.41	0.72	1.02	1.76	2.65	3.96	5.98	9.07
0.08	-3.68	-2.88	-2.17	-1.56	-0.98	-0.37	-0.10	0.19	0.50	0.81	1.11	1.88	2.86	4.48	7.32	14.25
0.10	-3.38	-2.66	-2.01	-1.43	-0.88	-0.29	-0.02	0.27	0.58	0.89	1.20	2.00	3.11	5.26	-	-
0.12	-3.12	-2.46	-1.86	-1.31	-0.78	-0.20	0.06	0.35	0.66	0.97	1.29	2.12	3.41	6.96	-	-
0.14	-2.88	-2.27	-1.72	-1.20	-0.69	-0.12	0.14	0.43	0.74	1.05	1.37	2.24	3.81	-	-	-
0.16	-2.67	-2.10	-1.58	-1.09	-0.60	-0.04	0.22	0.51	0.81	1.13	1.45	2.36	4.49	-	-	-
0.18	-2.47	-1.94	-1.45	-0.99	-0.51	0.04	0.30	0.58	0.89	1.20	1.53	2.48	-	-	-	-
0.20	-2.29	-1.79	-1.33	-0.89	-0.42	0.11	0.37	0.65	0.95	1.27	1.60	2.58	-	-	-	-
0.24	-1.96	-1.52	-1.11	-0.70	-0.27	0.25	0.49	0.77	1.07	1.38	1.71	2.72	-	-	-	-
0.3	-1.55	-1.18	-0.82	-0.46	-0.06	0.42	0.65	0.91	1.20	1.50	1.80	-	-	-	-	-
0.4	-1.01	-0.72	-0.44	-0.14	0.19	0.60	0.80	1.03	1.30	1.58	1.85	-	-	-	-	-
0.6	-0.36	-0.19	-0.01	0.19	0.41	0.72	0.88	1.09	1.35	1.68	-	-	-	-	-	-
1.0	0.06	0.14	0.23	0.34	0.48	0.72	0.87	1.09	1.43	-	-	-	-	-	-	-
1.5	0.16	0.21	0.27	0.35	0.46	0.67	0.82	1.04	1.44	-	-	-	-	-	-	-
2.0	0.18	0.22	0.27	0.33	0.43	0.62	0.76	0.97	1.35	-	-	-	-	-	-	-
2.5	0.18	0.22	0.26	0.32	0.41	0.58	0.70	0.89	-	-	-	-	-	-	-	-

TABLE 6 J/P for j=0 , l=2 and Z=2

S \ U_o	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	-0.3	-0.1	0.1	0.3	0.5	1.0	1.5	2.0	2.5	3.0
0.00	-4.60	-3.42	-2.47	-1.72	-1.11	-0.56	-0.34	-0.11	0.11	0.34	0.56	1.11	1.72	2.47	3.42	4.60
0.02	-4.10	-3.11	-2.27	-1.59	-1.02	-0.49	-0.27	-0.05	0.18	0.41	0.63	1.20	1.85	2.70	3.82	5.24
0.04	-3.70	-2.83	-2.09	-1.47	-0.93	-0.41	-0.20	0.02	0.25	0.48	0.70	1.29	2.00	2.96	4.32	6.14
0.06	-3.37	-2.60	-1.93	-1.35	-0.84	-0.34	-0.13	0.09	0.31	0.54	0.78	1.39	2.16	3.31	5.05	7.37
0.08	-3.08	-2.39	-1.78	-1.24	-0.76	-0.27	-0.06	0.15	0.38	0.61	0.85	1.48	2.34	3.75	6.21	12.07
0.10	-2.83	-2.20	-1.64	-1.14	-0.68	-0.20	0.002	0.22	0.44	0.68	0.92	1.58	2.55	4.43	-	-
0.12	-2.60	-2.03	-1.51	-1.04	-0.60	-0.14	0.07	0.28	0.51	0.74	0.98	1.68	2.81	5.90	-	-
0.14	-2.40	-1.87	-1.39	-0.95	-0.52	-0.07	0.13	0.34	0.56	0.80	1.05	1.78	3.16	-	-	-
0.16	-2.22	-1.73	-1.28	-0.86	-0.45	-0.01	0.19	0.40	0.62	0.86	1.11	1.88	3.75	-	-	-
0.18	-2.05	-1.59	-1.17	-0.77	-0.38	0.05	0.24	0.45	0.67	0.91	1.17	1.98	-	-	-	-
0.20	-1.89	-1.47	-1.07	-0.69	-0.31	0.11	0.30	0.50	0.72	0.96	1.22	2.06	-	-	-	-
0.24	-1.61	-1.24	-0.88	-0.54	-0.18	0.21	0.39	0.59	0.81	1.04	1.29	2.16	-	-	-	-
0.3	-1.26	-0.95	-0.64	-0.34	-0.02	0.34	0.51	0.69	0.89	1.11	1.34	-	-	-	-	-
0.4	-0.81	-0.57	-0.33	-0.09	0.17	0.47	0.61	0.76	0.93	1.11	1.30	-	-	-	-	-
0.6	-0.27	-0.13	0.02	0.17	0.33	0.53	0.63	0.74	0.88	1.04	-	-	-	-	-	-
1.0	0.06	0.12	0.19	0.26	0.35	0.48	0.55	0.63	0.74	-	-	-	-	-	-	-
1.5	0.13	0.17	0.21	0.25	0.31	0.40	0.45	0.51	0.54	-	-	-	-	-	-	-
2.0	0.14	0.17	0.19	0.23	0.28	0.34	0.38	0.40	0.33	-	-	-	-	-	-	-
2.5	0.14	0.16	0.18	0.21	0.25	0.30	0.32	0.33	-	-	-	-	-	-	-	-

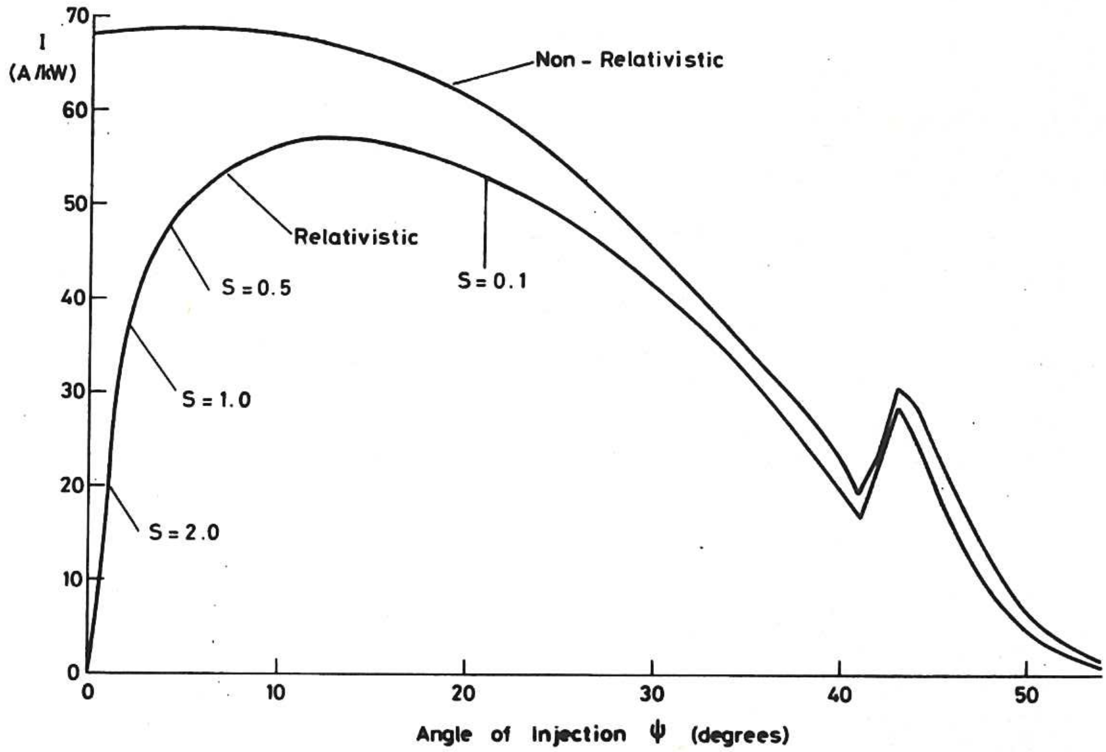

Fig. 1 Variation of driven current with angle of injection to the major radius ψ, for O-mode functional ECRH launched from the outside of a tokamak the size of DITE (a = 26 cm, R_0 = 117 cm). The ECRH grequency is 60GHZ and the central magnetic field is 21.4kG. The density and temperature profiles are parabolic with central values of $2 \times 10^{19}\ m^{-3}$ and 2keV respectively. Both non-relativistic and relativistic calculations are shown. Note that the latter give a sharp drop in current for small ψ (large S).

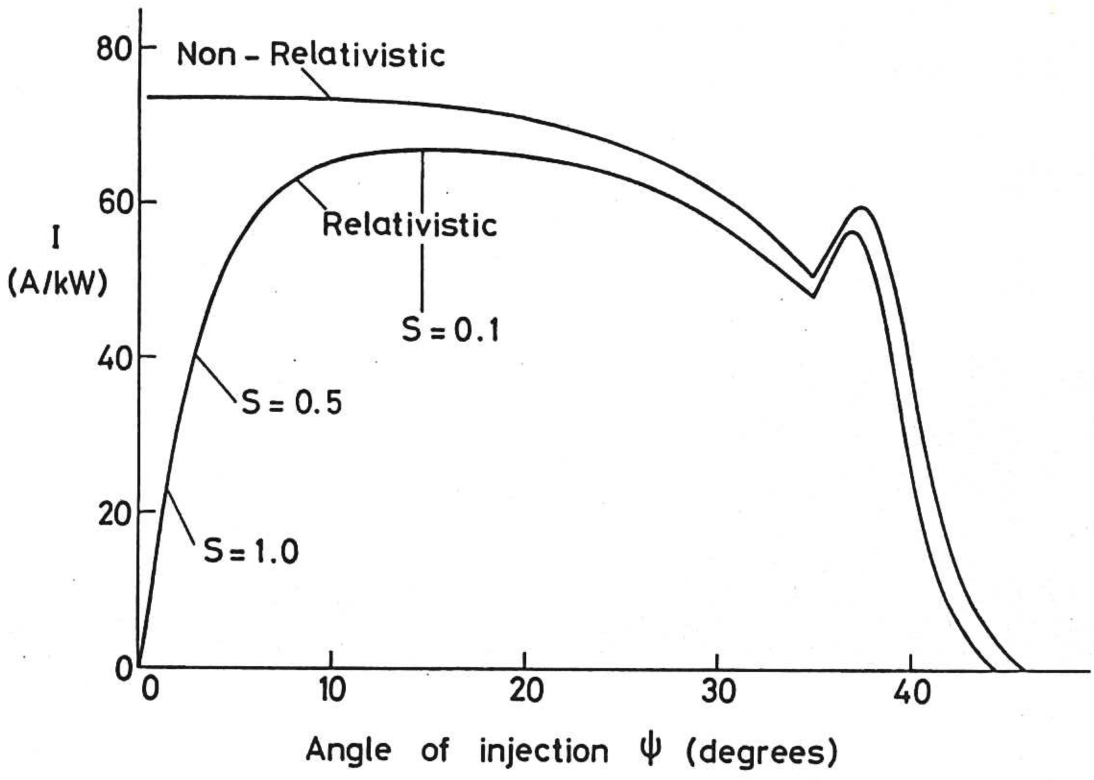

Fig. 2 As Fig. 1 but for X-mode second harmonic heating with central field 10.7kG, central density $1 \times 10^{19}\ m^{-3}$ and central temperature 1keV. Note the similarity to Fig. 1.

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