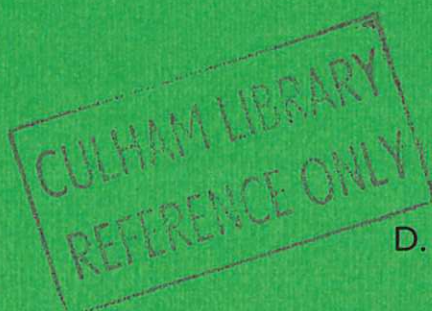




U K A E A

Report

A CURRENCY EXCHANGE RATE  
FOR USE IN TECHNICAL COMPARISONS



D. E. T. F. ASHBY



CULHAM LABORATORY  
Abingdon Oxfordshire

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## A CURRENCY EXCHANGE RATE FOR USE IN TECHNICAL COMPARISONS

by

D E T F Ashby

Culham Laboratory, Abingdon, Oxon, OX14 3DB, UK  
(Euratom/UKAEA Fusion Association)

### ABSTRACT

When two or more technical projects which have been costed in different currencies at different times are to be compared the straightforward application of international exchange rates and inflation indices does not necessarily give a unique answer. A procedure for generating an average exchange rate that overcomes this problem is proposed.

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## 1. INTRODUCTION

A problem arises when two or more technical projects costed in different currencies are to be compared. Suppose that there are two different designs of some large item such as a fission reactor which have been generated and costed in separate countries at different times. Neither design has been built; they are paper studies only in local currencies. The objective is to compare the two designs. A straight forward application of exchange rates with an allowance for inflation does not give a unique answer. One could go through a fresh costing exercise and arrive at new prices in a common currency. But this procedure would be costly and time-consuming for a large item such as a fission reactor. (Although it would have to be done in this final analysis). Another method of making a comparison is to take a major component common to each design (eg., the turbo-alternators in a fission reactor) and compare the relative costs of this component to establish a purchasing power parity. But if the relevant exchange rates and inflation indices are known as functions of time then this represents a lot of information about the behaviour of the currencies and relative costs. How can this information best be used in a comparison?

This problem was pointed out by P.I.H. Cooke<sup>1</sup> in connection with the costs of conceptual fusion reactors. He wished to compare the direct capital costs from a study by Hollis<sup>2</sup> in 1977 pounds sterling and the STARFIRE<sup>3</sup> study which was costed in 1980 US dollars. Cooke pointed out that there are two ways of comparing costs; in the first the 1977 pounds are inflated to 1980 pounds and then converted to 1980 dollars using the mean exchange rate for 1980 (2.3277\$/£); alternatively 1977 pounds are converted to 1977 dollars (1.7456\$/£) and then inflated to 1980 dollars. The latter method leads to a result which is 34% less than the former.

Figure 1 neatly illustrates the problem by plotting logarithmically the UK and US inflation indices and the exchange rate as functions of time. The exchange rate curve immediately shows the danger in using costs derived in 1977 or 1980 when the exchange rate was at an extreme. Intuitively one feels that some average or standard exchange rate should be used which circumvents the cost uncertainty

arising from inflation and fluctuations in the exchange rate. This report suggests such an exchange rate.

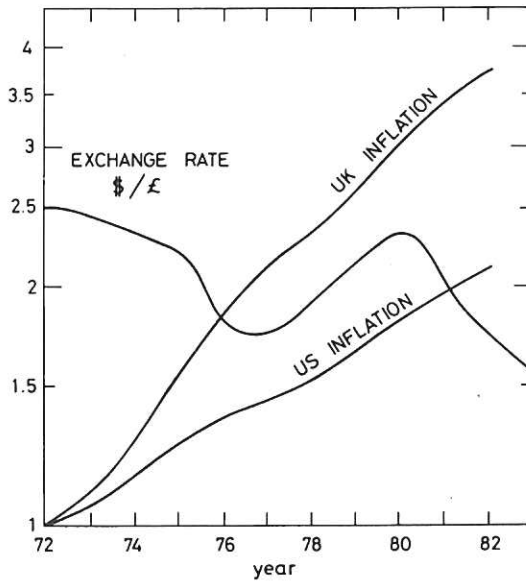


Fig.1 The exchange rate and normalised inflation indices plotted logarithmically against time. The Retail Price Index has been used to show UK inflation while the GNP Price Level Deflator is used for US inflation.

## 2 DERIVATION OF A STANDARD EXCHANGE RATE

Let  $C_i(t_0)$  be the cost of a set of items in currency  $i$  at time  $t_0$ . If inflation increases the cost to  $C_i(t)$  at time  $t$  then the inflation index is given by

$$I_i(t) = C_i(t)/C_i(t_0) \quad (1)$$

where the index is normalised to time  $t_0$ . Now consider a second currency  $j$  in which the cost of the same items is  $C_j(t)$ ; the inflation index in this currency is given by

$$I_j(t) = C_j(t)/C_j(t_0) \quad (2)$$

The problem to be solved can be formulated in the following way. Given two inflation indices  $I_i(t)$  and  $I_j(t)$  together with the actual exchange rate  $r_{ij}(t)$ , all as functions of time, how can the cost  $C_j(t_2)$  be derived from  $C_i(t_1)$ ? A simple application of the exchange rate to this problem fails to give a unique answer as is seen by comparing the result of first exchanging the currency, and then inflating, with the result of inflating before exchanging; in general

$$C_i(t_1) r_{ij}(t_1) I_j(t_2)/I_j(t_1) \neq C_i(t_1) I_i(t_2)/I_i(t_1) r_{ij}(t_2) \quad (3)$$

since actual exchange rates depend on other influences besides inflation.

Consider a standard exchange rate defined by

$$R_{ij}(t) = C_j(t)/C_i(t) \quad (4)$$

From equations (1), (2) and (4)

$$R_{ij}(t) = K_{ij} I_j(t)/I_i(t) \quad (5)$$

where

$$K_{ij} = C_i(t_0)/C_j(t_0) \quad (6)$$

is a constant. This new exchange rate changes only as a result of inflation and cannot show the hysteresis mentioned in the Introduction and formulated in equation (3) since

$$C_i(t_1) R_{ij}(t_1) \frac{I_j(t_2)}{I_j(t_1)} = C_i(t_1) \frac{C_j(t_1)}{C_i(t_1)} \frac{C_j(t_2)}{C_j(t_0)} \frac{C_j(t_0)}{C_j(t_1)} = C_j(t_2)$$

and

$$C_i(t_1) \frac{I_i(t_2)}{I_i(t_1)} R_{ij}(t_2) = C_i(t_1) \frac{C_i(t_2)}{C_i(t_0)} \frac{C_i(t_0)}{C_i(t_1)} \frac{C_j(t_2)}{C_j(t_1)} = C_j(t_2)$$

In addition  $C_j(t_2)$  can now be derived from  $C_i(t_1)$ , thus solving the problem posed, as long as the constant  $K_{ij}$  can be determined.

The evaluation of  $K_{ij}$  is based on the premiss that the actual exchange rate  $r_{ij}$  cannot continually depart from the standard exchange rate  $R_{ij}$ ; in some average fashion  $R_{ij}/r_{ij}$  must tend to be constant. This requirement can be expressed as

$$\langle R_{ij}/r_{ij} \rangle = \lim_{T \rightarrow \infty} f^{-1} \frac{\int_0^T f(R_{ij}/r_{ij}) dt}{\int_0^T dt} = 1 \quad (7)$$

where  $f$  is an unknown function which must be determined from the

properties of  $R_{ij}$  and  $r_{ij}$ . Equation (7) immediately leads to

$$\int_0^T f(R_{ij}/r_{ij}) dt = T f(1) \quad (8)$$

if the interval  $T$  is sufficiently long.

Now actual exchange rates possess a transitive property such that

$$r_{in} = r_{ij} r_{jk} \dots r_{lm} r_{mn}. \quad (9)$$

The standard exchange rate  $R_{ij}$  also has this property as a result of its definition in equation (4); consequently  $R_{ij}/r_{ij}$  is transitive as well which places a constraint on the function  $f$  namely

$$\begin{aligned} \int_0^T f\left(\frac{R_{ij}}{r_{ij}} \frac{R_{jk}}{r_{jk}} \dots \frac{R_{lm}}{r_{lm}} \frac{R_{mn}}{r_{mn}}\right) dt = \\ \int_0^T f\left(\frac{R_{ij}}{r_{ij}}\right) dt = \int_0^T f\left(\frac{R_{jk}}{r_{jk}}\right) dt = \dots = T f(1) \end{aligned} \quad (10)$$

A common function that meets the constraint imposed by equation (10) is the logarithmic function to arbitrary base since

$$\log(1) = 0$$

and  $\log(x_1 x_2 x_3 \dots) = \log x_1 + \log x_2 + \log x_3 \dots$

Using this function equation (8) becomes

$$\int_0^T \log(R_{ij}/r_{ij}) dt = 0 \quad (11)$$

which with equation (5) defines  $R_{ij}(t)$ . Equation (11) can be re-written as

$$\int_0^T (\log R_{ij} - \log r_{ij}) dt = 0 \quad (11a)$$



which shows the merit of plotting exchange rates on a logarithmic scale.

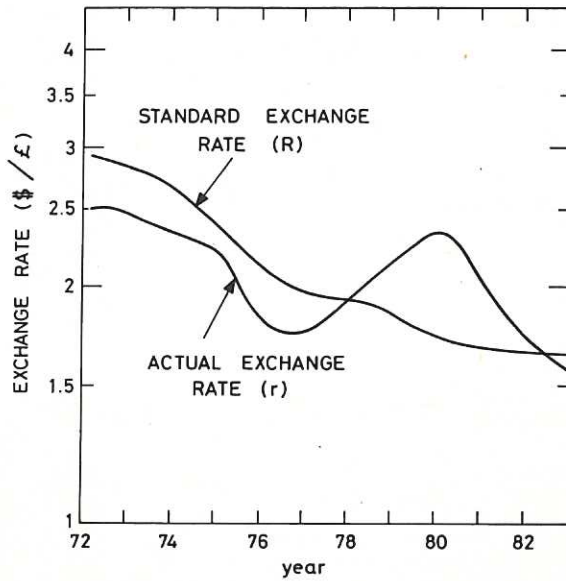


Fig.2 The standard exchange rate defined by Eqs. (4) and (8) compared with the actual exchange rate. Note that the vertical scale is logarithmic.

Figure 2 compares the actual exchange rate  $r_{ij}$  with the standard exchange rate  $R_{ij}$ , derived from equations (5) and (11), for the data shown in Figure 1.

Although the logarithmic function satisfies the requirements on the function  $f$  is it the only function that does? The following uniqueness proof was suggested by J.B. Taylor<sup>4</sup>.

Consider  $x_1(t)$  a function of  $t$  such that

$$\int_0^T f(x_1) dt = T f(1) \quad (12)$$

where  $x_1$ ,  $t$  and  $T$  are all positive real. Now take  $x_2(t)$ , a second function of  $t$  defined by

$$x_2 = 1 + \epsilon \cos (\pi m t/T) \quad (13)$$

and consider what happens as  $\epsilon \rightarrow 0$ . By Taylor's expansion

$$\begin{aligned}\int_0^T f(x_2) dt &= \int_0^T \{f(1) + \epsilon \cos(\pi m t/T) f'(1)\} dt \\ &= T f(1)\end{aligned}\quad (14)$$

where  $m$  is a non-zero integer and  $f'(x) \equiv df/dx$ . Equations (12), (14) and (10) give

$$\int_0^T f(x_1 x_2) dt = T f(1) \quad (15)$$

which with equation (13) gives

$$\int_0^T f\{x_1[1 + \epsilon \cos(\pi m t/T)]\} dt = T f(1) \quad (16)$$

By Taylor's expansion equation (16) gives

$$\int_0^T [f(x_1) + x_1 \epsilon f'(x_1) \cos(\pi m t/T)] dt = T f(1) \quad (17)$$

which with equation (12) leads to

$$\epsilon \int_0^T [x_1 f'(x_1) \cos(\pi m t/T)] dt = 0 \quad (18)$$

Since  $x_1 f'(x_1)$  is a function of  $t$  Fourier's Theorem gives

$$x_1 f'(x_1) = \text{constant.}$$

A solution to which is

$$f(x) \propto \log(x) \quad (19)$$

Now  $\log(x(t))$  satisfies equation (10) for any  $x(t)$  but it is also the only function to satisfy equation (10) when  $x(t)$  is defined by equation (13); therefore it follows that  $\log(x)$  can be the only function that satisfies equation (10) for any  $x(t)$ .



### 3. DISCUSSION AND CONCLUSIONS

Obviously some form of standard exchange rate is needed when comparisons are made between costings based on different currencies; the alternative is to accept results that depend on when the conceptual exchange between currencies is made; the resulting uncertainty could, in recent years, amount to a factor of 1.5. The exchange rate  $R_{ij}$  removes this problem. For example Figure 2 shows that a comparison between 1977 dollar prices and 1980 pound prices should be made by first converting 1980 pounds to 1980 dollars, at a rate of 1.7 pounds per dollar ( $R_{ij}$ ), instead of 2.3 pounds per dollar ( $r_{ij}$ ), and then deflating to 1977 dollars; alternatively 1977 dollars could be converted to 1977 pounds and the result inflated to 1980 pounds if prices in 1980 pounds are required.

One objection to the exchange rate  $R_{ij}$  is that its rigorous derivation depends on knowing the two inflation indices  $I_i(t)$  and  $I_j(t)$  together with the actual exchange rate  $r_{ij}(t)$  over a sufficient period of time so that the averaging integral can be performed meaningfully; however even an integral over a short period, (which uses all the information available), must be preferable to using a single value of  $r_{ij}$  and then inflating the result which will lead to some unknown level of hysteresis.

A further useful feature of this standard exchange rate is that it conveniently illustrates the hysteresis involved when actual exchange rates and inflation indices are used to compare costs. Consider a comparison of costs in currency  $i$  at time  $t_1$  with costs in currency  $j$  at time  $t_2$ . The costs at time  $t_1$  could be multiplied by either

$$r_{ij}(t_1)I_j(t_2)/I_j(t_1) \text{ or } r_{ij}(t_2)I_i(t_2)/I_i(t_1)$$

depending on whether the exchange is made at time  $t_1$  or at time  $t_2$ . The ratio of these two quantities is

$$[r_{ij}(t_1)/R_{ij}(t_1)] \div [r_{ij}(t_2)/R_{ij}(t_2)]$$

and is the magnitude of the hysteresis between  $t_1$  and  $t_2$ . This important factor can be estimated by inspection when  $r_{ij}$  and  $R_{ij}$  are plotted on a logarithmic scale as in Fig.2.

To conclude, the standard exchange rate suggested is based on these requirements:

- (a) No hysteresis when costs are converted and inflated.
- (b) Exchange rates are transitive so that  $R_{ik} = R_{ij} R_{jk}$  etc.
- (c) The standard exchange rate cannot continually depart from the actual exchange rate.

It is difficult to escape the conclusion that the proposed form is the simplest that satisfies all these requirements.

#### REFERENCES

- 1 P I H COOKE, private communication.
- 2 A A HOLLIS, "An Analysis of the Estimated Capital Cost of a Fusion Reactor" AERE-R 9933 (1981).
- 3 C C BAKER et al, "STARFIRE - A Commercial Tokamak Fusion Power Plant", ANL/FPP-80-1 (1980).
- 4 J B TAYLOR, private communication.









The first part of the paper discusses the importance of the research and the objectives of the study. It then presents a literature review of the existing research on the topic. The second part of the paper describes the methodology used in the study, including the data collection and analysis techniques. The third part of the paper presents the results of the study, and the fourth part discusses the conclusions and implications of the findings.

The study was conducted using a quantitative research design. Data was collected from a sample of 100 participants using a survey questionnaire. The data was then analyzed using statistical software to determine the relationships between the variables of interest.

The results of the study indicate that there is a significant positive relationship between the variables of interest. This finding is consistent with the hypotheses of the study and contributes to the understanding of the phenomenon being investigated.

The implications of the findings suggest that there are practical applications for the research. These implications can be used to inform policy and practice in the field of study.

In conclusion, the study has provided valuable insights into the relationship between the variables of interest. Further research is needed to explore the topic in more depth and to validate the findings of this study.



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