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Report

THE STABILITY OF TWISTED MAGNETIC FIELDS IN A FLUID WITH FINITE ELECTRICAL CONDUCTIVITY

Part 3

THE HARD-CORE PINCH OF LOW CONDUCTIVITY

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THE STABILITY OF TWISTED MAGNETIC FIELDS IN A FLUID WITH FINITE ELECTRICAL CONDUCTIVITY

Part 3

THE HARD-CORE PINCH OF LOW CONDUCTIVITY

by

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A B S T R A C T

The stability of the hard-core pinch of low conductivity is investigated. A variety of magnetic field configurations is studied in some of which the axial magnetic field reverses its direction inside the plasma column. Perturbation growth rates are obtained for $m = 0$ and $m = 1$ perturbations. It is shown that the introduction of low conductivity leads to instability in all cases studied but that systems with reversed axial fields are less unstable than those without field reversal. The possible validity of these results is discussed.

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1. INTRODUCTION

1. Of all the cylindrical hydromagnetic equilibria, one of the most stable according to present theoretical calculations should be the hard-core pinch. The plasma or conducting fluid is annular and a conducting rod passing through its centre carries an axial current. There is also an axial current in the fluid which flows in the opposite direction to the current in the rod but the total current in the fluid is less than that in the rod. If the fluid is ideally conducting, theory predicts(5) that the system should be stable against all perturbations regardless of the profile of an axial magnetic field that may be present, provided that the fields are such that the fluid is in equilibrium.

2. Because the requirements for a hard-core system to be stable are not very restrictive, many experiments have been performed in this geometry. Contrary to the predictions of the ideal hydromagnetic theory, the configurations have not been found to be completely stable [Aitken et al(1), Birdsall et al(3)]. There are several reasons why this may not be so. One, for example, is that the theory refers to an infinite cylindrical system while experiments must be performed in a tube of finite length; end effects associated with the non-uniformity of fields near the electrodes may be important. Another possibility is that the finite transport processes in the fluid must be included in the theory. Finite viscosity would almost certainly improve the stability of the system but instabilities could be caused by the finite value of the electrical conductivity.

3. It is with this latter possibility that the present report is concerned. As has been shown previously (Tayler(8)), the introduction of a finite electrical conductivity considerably complicates the mathematics of the problem. Inside the conducting fluid, instead of a second order differential equation having to be solved, the general solution must be obtained of a sixth order differential equation. Boundary conditions must then be applied to the solutions of this equation on both the boundaries of the annular fluid.

4. Although the solution of the complete problem would be extremely difficult, it is possible to find the growth rates of perturbations in a fluid of extremely low conductivity. If the fluid of very low conductivity is unstable, it proves that finite electrical conductivity can lead to instability. If the fluid of low conductivity is stable, there is hope that the system is stable for all values of the conductivity. In a previous problem(8), the worst instabilities did occur when the conductivity was low. The method of (8) has already been applied to the hard-core pinch by Bickerton and Spalding(2) and Tandon and Talwar(6,7). In the present report a rather more general case has been considered.

5. There are some complications associated with the introduction of a finite electrical conductivity into the fluid. There are many equilibrium configurations for a fluid of infinite conductivity but only a few of these remain equilibrium states when the conductivity is finite. As the configurations of most interest are not equilibrium states some approximation must be made. It is assumed that we have a configuration with slowly diffusing fields so that in the equilibrium state the finite conductivity is important. It is then assumed that, in the perturbed state, growth rates are such that instabilities (if present) grow rapidly in one field penetration time. This will certainly not be true in the case of arbitrary low conductivity but it was found in (8) that there exist values of the conductivity which are so high that instabilities grow rapidly in a field diffusion time and so low that the instability growth rates are very little different from those in a field of zero conductivity. In addition it is assumed that the conducting rod and outer walls have high enough conductivity to be regarded as infinitely conducting in the stability problem. This will be discussed further in the next section.

6. The remainder of this report is arranged as follows. The equilibrium configuration is described in section 2. The perturbed equations are solved

in section 3 and the derivation of the dispersion relation is described in section 4. The numerical results are given in section 5 and a short discussion of the results follows in section 6.

2. 'EQUILIBRIUM' CONFIGURATION

7. Since the principal feature of this problem is the presence of the conducting rod and the current it carries, all distances are normalised with respect to the radius of the rod and all magnetic fields with respect to that produced by the current at the surface of the rod.

8. A conducting rod of radius r_o is surrounded by a vacuum which is in turn surrounded by an annular conducting fluid of inner and outer radii $\Lambda_1 r_o$ and $\Lambda_2 r_o$. Finally the system is contained by a conducting wall of radius $\Lambda_3 r_o$. The rod and the wall are of high conductivity and the fluid has low conductivity σ. The fluid has density ρ_o.

9. The equilibrium magnetic fields in the conducting fluid and the two vacuum regions are taken to be:-

Inner vacuum $\quad \underset{\sim}{B} = B_o\ (o,\ r_o/r,\ b_1)\ ,$ (2.1)

Conducting fluid $\quad \underset{\sim}{B} = B_o\ (o,\ b_2 r_o/r + b_3\ r/r_o,\ b_4 + b_5 \ell n\ r/r_o),$ (2.2)

Outer vacuum $\quad \underset{\sim}{B} = B_o\ (o,\ b_6\ r_o/r,\ b_7).$ (2.3)

The expression for the magnetic field in the conducting fluid satisfies the equation

$$\text{curl}\ \underset{\sim}{j} \equiv \text{curl curl}\ \underset{\sim}{B} = o \tag{2.4}$$

and for that reason appears to be a true equilibrium field. However this is illusory. In equilibrium there would be an associated electrostatic field which would have an azimuthal component infinite on the axis of the system. Equations (2.1) to (2.3) are thus supposed to be instantaneous values of time varying fields and the particular algebraic forms have been chosen for the simplicity of the resulting work. The liklihood of such instantaneous fields occurring is perhaps remote but it is hoped that the results of the stability calculation will be qualitatively similar to those that would be obtained with other field profiles.

10. The pressure in the conducting fluid can be obtained from the equation

$$dp/dr = -\ B_z\ dB_z/dr\ -\ B_\theta\ dB_\theta/dr\ -\ B_\theta^2/r, \tag{2.5}$$

where B_θ and B_z are given in equation (2.2). Thus

$$p = p_o - B_o^2[b_3^2(r^2/r_o^2) + (2b_2b_3 + b_4b_5)\ell n(r/r_o) + b_5^2\{\ell n(r/r_o)\}^2/2]. \tag{2.6}$$

11. There are five algebraic relations between the ten constants Λ_1, Λ_2, Λ_3, b_1, b_2, b_3, b_4, b_5, b_6, b_7. Four of these arise because both B_θ and B_z must be continuous at the two surfaces of the fluid. Thus

$$1 = b_2 + b_3\ \Lambda_1^2\ , \tag{2.7}$$

$$b_1 = b_4 + b_5\ \ell n\ \Lambda_1, \tag{2.8}$$

$$b_6 = b_2 + b_3\ \Lambda_2^2\ , \tag{2.9}$$

$$b_7 = b_4 + b_5\ \ell n\ \Lambda_2. \tag{2.10}$$

In addition the pressure of the fluid must be zero at both of its surfaces. These two conditions serve to determine p_o and to give another relation between the Λ's and the b's. Thus

$$b_3^2(\Lambda_1^2 - \Lambda_2^2) + (2b_2b_3 + b_4b_5)\ell n(\Lambda_1/\Lambda_2) + b_5^2[(\ell n\ \Lambda_1)^2 - (\ell n\ \Lambda_2)^2]/2 = 0. \tag{2.11}$$

12. The equations (2.7) to (2.11) show that five of the ten parameters can be chosen arbitrarily and then the other five can be determined. In the present work the free parameters are taken to be $\Lambda_1, \Lambda_2, \Lambda_3, b_1$ and b_6. There are however further restrictions on the values of these free parameters. Not only must the pressure vanish at both surfaces of the fluid but it must also be positive everywhere within the fluid; thus only those values of the parameters which lead to such a positive pressure can be allowed. In addition, so that the system is obviously stable when the conductivity is infinite, b_6 must be positive and less than unity: for that reason it is convenient to take b_6 as one of the free parameters.

3. SOLUTION OF THE PERTURBED EQUATIONS

13. Perturbations about equilibrium are considered in which any physical variable q has the form:

$$q = q_o + q_1(r) \exp\{i(m\theta + kz) + \omega t\}, \tag{3.1}$$

where m and k are given and may be either positive or negative and ω is to be determined. The linearised hydromagnetic equations in the conducting fluid take the form

$$\rho_o \omega \underset{\sim}{v}_1 = - \text{grad } p_1 + \text{curl } \underset{\sim}{B}_o \times \underset{\sim}{B}_1 + \text{curl } \underset{\sim}{B}_1 \times \underset{\sim}{B}_o , \tag{3.2}$$

$$\text{div } \underset{\sim}{v}_1 = 0, \tag{3.3}$$

$$\omega \underset{\sim}{B}_1 + (1/\sigma) \text{ curl curl} \underset{\sim}{B}_1 = \text{curl } (\underset{\sim}{v}_1 \times \underset{\sim}{B}_o). \tag{3.4}$$

The units are rationalised Gaussian units with the velocity of light put equal to unity.

14. The solutions of equation (3.2) to 3.4), for low values of the conductivity, are expanded in the form:

$$\begin{aligned} \underset{\sim}{B}_1 &= \underset{\sim}{B}_{10} + \sigma \underset{\sim}{B}_{11} , \\ \underset{\sim}{v}_1 &= \underset{\sim}{v}_{10} + \sigma \underset{\sim}{v}_{11} , \\ p_1 &= p_{10} + \sigma p_{11} . \end{aligned} \tag{3.5}$$

Using the technique described in (8), only the first term in the expansion (3.5) is kept in the present report. The equations satisfied by the zero order quantities are then

$$\text{curl curl } \underset{\sim}{B}_{10} = 0 , \tag{3.6}$$

$$\rho_o \omega \underset{\sim}{v}_{10} = - \text{grad } p_{10} + \text{curl } \underset{\sim}{B}_o \times \underset{\sim}{B}_{10} + \text{curl } \underset{\sim}{B}_{10} \times \underset{\sim}{B}_o \tag{3.7}$$

and

$$\text{div } \underset{\sim}{v}_{10} = 0. \tag{3.8}$$

Equation (3.7) and (3.8) can be conveniently combined to give

$$\nabla^2 p_{10} = - 2 \text{ curl } \underset{\sim}{B}_o \cdot \text{curl } \underset{\sim}{B}_{10}. \tag{3.9}$$

As the perturbed quantities are only solved to zero order in the present report, the suffix zero is emitted in what follows. Thus we write $\underset{\sim}{B}_1, \underset{\sim}{v}_1, p_1$.

15. Equation (3.6) can be solved immediately to give:

$$
\begin{aligned}
B_{1r} &= \alpha_2\, I_{|m+1|}\{|k|r\} + \alpha_3\, I_{|m-1|}\{|k|r\} \\
&\quad + \beta_2\, K_{|m+1|}\{|k|r\} + \beta_3\, I_{|m-1|}\{|k|r\}, \\
B_{1\theta} &= - i\alpha_2 I_{|m+1|}\{|k|r\} + i\alpha_3 I_{|m-1|}\{|k|r\} \\
&\quad - i\beta_2 K_{|m+1|}\{|k|r\} + i\beta_3 K_{|m-1|}\{|k|r\}, \\
B_{1z} &= \frac{i|k|}{k}(\alpha_2 + \alpha_3) I_{|m|}\{|k|r\} - \frac{i|k|}{k}(\beta_2 + \beta_3) K_{|m|}\{|k|r\}.
\end{aligned}
\tag{3.10}
$$

The equation (3.9) must now be solved for p_1. This equation has a complementary function

$$p_{1cf} = iB_o\,[\alpha_4\, I_{|m|}\{|k|r\} + \beta_4\, K_{|m|}\{|k|r\}] \tag{3.11}$$

and a particular integral. This particular integral can only readily be obtained when $m = 0$ or ± 1. Thus we will restrict ourselves to these cases in what follows. The results obtained are

$$m = 0 \quad p_{1p.i.} = \frac{2ib_3B_or}{r_o}\,[(\alpha_2 - \alpha_3) I_1\{|k|r\} + (\beta_2 - \beta_3) K_1\{|k|r\}]. \tag{3.12}$$

$$
\begin{aligned}
|m| = 1 \quad p_{1p.i.} &= \frac{ib_5\, B_om}{kr}\,[(\alpha_2 - \alpha_3) I_o\{|k|r\} + (\beta_2 - \beta_3) K_o\{|k|r\}] \\
&\quad + \frac{2ib_3B_or}{r_o}\,[(\alpha_2 - \alpha_3) I_2\{|k|r\} + (\beta_2 - \beta_3) K_2\{|k|r\}].
\end{aligned}
\tag{3.13}
$$

Only the radial component of the velocity enters into the boundary conditions and this is now obtained from equation (3.7). After some complicated algebra there results

$$
\begin{aligned}
m = 0,\ \rho_o\omega v_{1r} &= - iB_o|k| \quad [\alpha_4 I_o'\{|k|r\} + \beta_4 K_o'\{|k|r\}] \\
&\quad - \frac{ib_3B_or|k|}{r_o}[(\alpha_2 - \alpha_3) I_o\{|k|r\} - (\beta_2 - \beta_3) K_o\{|k|r\}] \\
&\quad + \frac{2ib_3B_o}{r_o}\quad [(\alpha_2 - \alpha_3) I_o'\{|k|r\} - (\beta_2 - \beta_3) K_o'\{|k|r\}] \\
&\quad - \frac{i|k|B_ob_5}{kr}\,[(\alpha_2 + \alpha_3) I_o\{|k|r\} - (\beta_2 + \beta_3) K_o\{|k|r\}] \\
&\quad + \frac{i|k|B_ob_2r_o}{r}\,[(\alpha_2 - \alpha_3) I_o\{|k|r\} - (\beta_2 - \beta_3) K_o\{|k|r\}],
\end{aligned}
\tag{3.14}
$$

where the prime denotes differentiation with respect to the argument of the Bessel Function.

$$|m| = 1,\ \rho_o\omega v_{1r} = -\ iB_o|k|[\alpha_4\ I_1'\{|k|r\} + \beta_4\ K_1'\{|k|r\}]$$

$$+ \frac{imkB_o}{|k|r}[b_4 + b_5\ell n(r/r_o)][(\alpha_2 - \alpha_3)I_1\{|k|r\} + (\beta_2 - \beta_3)K_1\{|k|r\}]$$

$$- i|k|B_o[b_2r_o/r][(\alpha_2 - \alpha_3)I_1\{|k|r\} + (\beta_2 - \beta_3)K_1\{|k|r\}]$$

$$+ b_3[\alpha_2\{-\frac{2iB_o|k|r}{r_o}I_2' - \frac{2iB_o}{r_o}I_2 + \frac{2iB_o}{r_o}I_{|m+1|} + \frac{iB_or}{r_o}|k|I_1\}$$

$$+ \alpha_3\{\frac{2iB_o|k|r}{r_o}I_2' + \frac{2iB_o}{r_o}I_2 - \frac{2iB_o}{r_o}I_{|m-1|} - \frac{iB_or}{r_o}|k|I_1\}$$

$$+ \beta_2\{-\frac{2iB_o|k|r}{r_o}K_2' - \frac{2iB_o}{r_o}K_2 + \frac{2iB_o}{r_o}K_{|m+1|} - \frac{iB_or}{r_o}|k|K_1\}$$

$$+ \beta_3\{\frac{2iB_o|k|r}{r_o}K_2' + \frac{2iB_o}{r_o}K_2 - \frac{2iB_o}{r_o}K_{|m-1|} + \frac{iB_or}{r_o}|k|K_1\}]$$

$$+ b_5[\alpha_2\{-\frac{i|k|B_o}{kr}(m+1)I_1 + \frac{iB_om}{kr^2}I_o\}$$

$$+ \alpha_3\{\frac{i|k|B_o}{kr}(m-1)I_1 - \frac{iB_om}{kr^2}I_o\}$$

$$+ \beta_2\{\frac{i|k|B_o}{kr}(m+1)K_1 + \frac{iB_om}{kr^2}K_o\}$$

$$+ \beta_3\{-\frac{i|k|B_o}{kr}(m-1)K_1 - \frac{iB_om}{kr^2}K_o\}], \tag{3.15}$$

where the arguments of all the Bessel functions, whether stated or not, are $|k|r$.

4. DERIVATION OF THE DISPERSION RELATION

16. The solutions of the perturbed equations given in the previous section contain ten constants which are arbitrary at the moment. However, boundary conditions must be imposed on the solutions both on the perturbed surfaces of the conducting fluid and on the rod and the wall. In all there are just ten boundary conditions which give homogeneous linear equations between the constants. A consistency condition is obtained when these equations are solved. This is the dispersion relation.

17. The boundary conditions are:

On $r = r_o$, $B_r = 0$.

On $r = \Lambda_1 r_o$, p and $\underset{\sim}{B}$ continuous*

On $r = \Lambda_2 r_o$, p and $\underset{\sim}{B}$ continuous*

On $r = \Lambda_3 r_o$, $B_r = 0$

---

*Note that these 8 conditions must be applied on the perturbed surfaces of the plasma which differ slightly from the equilibrium surfaces $r = \Lambda_1 r_o$, $r = \Lambda_2 r_o$.

18. The boundary conditions can be written in an alternative form as:-

$$B_{1r}^{I} = 0, \qquad r = r_o \qquad (4.1)$$

$$B_{1r}^{I} = B_{1r}, \qquad r = \Lambda_1 r_o \qquad (4.2)$$

$$B_{1\theta}^{I} = B_{1\theta} + B_o v_{1r}[1 + b_3\Lambda_1^2 - b_2]/\Lambda_1^2\omega r_o, \qquad r = \Lambda_1 r_o \qquad (4.3)$$

$$B_{1z}^{I} = B_{1z} + b_5 B_o v_{1r}/\Lambda_1\omega r_o, \qquad r = \Lambda_1 r_o \qquad (4.4)$$

$$p_1 = B_o^2 v_{1r}[2b_2b_3 + b_4b_5 + b_5^2\ell n\Lambda_1 + 2\Lambda_1^2 b_3^2]\ \Lambda_1\omega r_o, \qquad r = \Lambda_1 r_o \qquad (4.5)$$

$$B_{1r}^{O} = B_{1r}, \qquad r = \Lambda_2 r_o \qquad (4.6)$$

$$B_{1\theta}^{O} = B_{1\theta} + B_o v_{1r}[b_6 + b_3\Lambda_2^2 - b_2]/\Lambda_2^2\omega r_o, \qquad r = \Lambda_2 r_o \qquad (4.7)$$

$$B_{1z}^{O} = B_{1z} + b_5 B_o v_{1r}/\Lambda_2\omega r_o, \qquad r = \Lambda_2 r_o \qquad (4.8)$$

$$p_1 = B_o^2 v_{1r}[2b_2b_3 + b_4b_5 + b_5^2\ell n\Lambda_2 + 2\Lambda_2^2 b_3^2]/\Lambda_2\omega r_o, \qquad r = \Lambda_2 r_o \qquad (4.9)$$

$$B_{1r}^{O} = 0. \qquad r = \Lambda_3 r_o \qquad (4.10)$$

19. The equations can be slightly rearranged. Using equations (2.6) and (2.8), equations (4.3) and (4.7) can be re-written:-

$$B_{1\theta}^{I} = B_{1\theta} + 2b_3 B_o v_{1r}/\omega r_o \qquad (4.11)$$

and

$$B_{1\theta}^{O} = B_{1\theta} + 2b_3 B_o v_{1r}/\omega r_o. \qquad (4.12)$$

Equations (4.3), (4.4) and (4.5) and (4.7), (4.8) and (4.9) can be combined to give:-

$$p_1 + B_o B_{1\theta}/\Lambda_1 + B_o b_1 B_{1z} = B_o B_{1\theta}^{I}/\Lambda_1 + B_o b_1 B_{1z}^{I} \qquad (4.13)$$

and

$$p_1 + b_6 B_o B_{1\theta}/\Lambda_2 + B_o b_7 B_{1z} = b_6 B_o B_{1\theta}^{O}/\Lambda_2 + B_o b_7 B_{1z}^{O}, \qquad (4.14)$$

where the equilibrium conditions have again been used. Finally equations (4.11) and (4.4) and equations (4.12) and (4.8) are combined to give:-

$$b_5 B_{1\theta} - 2b_3\Lambda_1 B_{1z} = b_5 B_{1\theta}^{I} - 2b_3\Lambda_1 B_{1z}^{I} \qquad (4.15)$$

and

$$b_5 B_{1\theta} - 2b_3\Lambda_2 B_{1z} = b_5 B_{1\theta}^{O} - 2b_3\Lambda_2 B_{1z}^{O}. \qquad (4.16)$$

If equations (4.1), (4.2), (4.4), (4.6), (4.8), (4.10), (4.13), (4.14), (4.15) and (4.16) are taken as the ten independent boundary conditions and the expressions for $\underset{\sim}{v}_1$ and $\underset{\sim}{B}_1$ are used, it can be seen that ω^2 only enters into two of them. Thus the dispersion relation can be seen to be a quadratic equation for ω^2. These two roots for ω^2 arise because the plasma has two free surfaces.

20. Because of the complexity of the expressions derived in Section 3 for the quantites p_1 and v_{1r} and because of the number of algebraic equations involved, no further algebraic reduction of the dispersion relation will be made here. It is more convenient for the whole of the remainder of the problem to be handled by a computer. In fact a programme to solve the dispersion relation was written by one of the authors (F.R.A.Hopgood) and solutions were obtained on the Mercury computer at A.E.R.E. Harwell. These are reported in the next section.

21. It is however worth considering some symmetry properties of the equations. The present description of the problem involves specifying values of the constants Λ_1, Λ_2, Λ_3, b_1 and b_6. The equations are then solved for $m = 0, \pm 1$ and all $|k|$ for $k = \pm |k|$. In fact all these cases are not distinct. For $m = 0$,

the dispersion relation does not depend on the sign of k so that a complete solution can be obtained by taking k positive. For $|m| = 1$, both signs of k must be considered but it is easy to show that changing the sign of b_1 (and hence of b_4, b_5 and b_7) is equivalent to changing the sign of k. A complete solution can be obtained by taking $m = 1$, $k = 1$ and by considering both signs b_1 simultaneously. This is a result that has often been used in simpler problems; physically it divides perturbations into two classes which are helices either in the same sense or the opposite sense to the magnetic field helix at any point in the system. Of course in the present problem there is no need for the magnetic field helix to have the same sense everywhere as b_1 and b_7 need not have the same sign.

5. SOLUTION OF DISPERSION RELATION

22. Because of the number of free parameters involved in this problem, it is clearly not practicable to obtain a complete survey of all possible solutions of the dispersion relation. It is hoped that sufficient cases have been considered to give a reasonable idea of possible results. The overall geometry of the problem was fixed from the outset. Thus we have considered

$$\Lambda_1 = 2, \ \Lambda_2 = 3, \ \Lambda_3 = 4. \qquad (5.1)$$

This means that the separation of the plasma from the rod and from the outer wall and the thickness of the plasma are all equal. This should give a reasonable idea of results when these three distances are of the same order of magnitude but there may be other interesting cases when one or other of the distances is very small.

23. The values of the parameters b_1 and b_6 considered are:-

$$b_1 \text{ or } b_6 = 0.2, \ 0.4, \ 0.6, \ 0.8. \qquad (5.2)$$

In addition, for $m = 1$, corresponding negative values of b_1 have also been studied, though these results have subsequently been displayed as belonging to negative values of k. Finally for given values of b_1 and b_6 there are two possible sets of values for b_4, b_5 and b_7. This occurs because equation (2.11) is quadratic in the b's. These solutions have equal and opposite values of b_7 but there is no such simple relation between the values of b_4 and b_5. Thus, for each pair of values of b_1 and b_6, we have obtained one configuration with a reversed axial field and one without.

24. The full results of all these cases are summarised briefly in Tables I to IV. In Table I the values of b_1 to b_7 for the 32 cases are listed. The $m = 0$ results are shown in Table II and the $m = 1$ results for positive and negative values of k in Tables III and IV. It should be noted, as mentioned in Section 4, that there are two solutions for each case.

25. The general character of the results can be summarised as follows:-

(i) For both $m = 0$ and $m = 1$, one solution of the equations is always stable.

(ii) For both $m = 0$ and $m = 1$, the other solution is always unstable for small values of kr_0. If the axial field in the outer vacuum is opposite in direction to the axial field in the inner vacuum (reversed axial field), the system is stable for large values of kr_0, otherwise large k modes are unstable.

(iii) For $m = 0$ modes the growth rate - wave number curves are smooth but there are considerable irregularities in the $m = 1$ curves. These seem in some way to be associated with relationship between the pitch of the perturbation and the magnetic field, but it is difficult to be certain because of the variable nature of the field pitch especially when the axial field reverses.

(iv) For $m = 1$ modes when the axial field is not reversed (so that the field pitch does not change sign), there is some tendency for the perturbations antiparallel to the field to be more unstable than those parallel to it. A result of this nature has previously been reported by Bickerton and Spalding[2] and Jukes[4]. This result is most likely to be true for small values of $|k|$ and there is perhaps some tendency for the opposite to be true for large $|k|$.

The main overall characteristic of the results is that the degree of instability seems to be considerably reduced by the presence of a reversed a ial field.

26. One other aspect of the results deserves comment. When it was observed that the $m = 1$ results predicted a finite growth rate at zero wave-number, we were predisposed to disbelieve the results. However, after detailed consideration of the behaviour of the equations for small values of k, it seems that the result is correct. As this probably represents bodily movement of the low conducting fluid with respect to the rod and the walls it is perhaps not too surprising.*

27. Although the results shown in Tables III and IV give a reasonable idea of the behaviour of the solutions for $Y_0^2 (= 4\pi\rho\ \omega^2 r_0^2/B_0^2)$ as a function of $X_0(kr_0)$ a finer grid of values of X_0 is required before accurate curves can be drawn. For this reason more detailed results have been obtained for two cases only. The magnetic field and pressure profiles for these cases are shown in Figs.1 and 4 and growth rates for $m = 0$ and $m = 1$ perturbations in Figs.2,3,5 and 6. One of these cases has a reversed magnetic field and the other does not.

6. DISCUSSION

28. The conclusions that can be drawn from the calculations of a present report are:-

(i) The introduction of a finite and low electrical conductivity does lead to instabilities.

(ii) For wave-numbers which should not be seriously affected by viscosity, the growth rates can be an appreciable fraction of a typical hydromagnetic growth rate ($Y_0 \sim 1$).

These results are essentially similar to those obtained for the pinch in (8) and for the hard-core pinch in (2),(6) and (7). Provided the results for arbitrary finite conductivity bear the same relation to these low conductivity results as they did in the similar pinch problem studied in (8), there is a possibility that the basic requirement that instability growth times should be shorter than field diffusion times can be satisfied. Thus finite conductivity must be regarded as a possible cause of instability.

The final conclusion is new because the possibility of a reversed axial field was not allowed for in (2),(6) and (7).

(iii) The system is much less unstable if the axial field is reversed.

29. The present results suggest two further lines of study:

(a) An attempt to solve the theoretical problem with a truly finited value of the conductivity.

(b) A hard-core experiment with reversed axial field.

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*Bickerton and Spalding[2] obtain a similar result.

Jukes[4] has already made some progress with (a) and Dr. P. Reynolds informs us that there is some prospect of experiment (b) being performed. Meanwhile it is of course still possible that a completely different mechanism accounts for the hard-core instabilities.

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TABLE I

MAGNETIC FIELD PARAMETERS FOR THE CASES CONSIDERED

Case	b_1	b_2	b_3	b_4	b_5	b_6	b_7
1	0.200	1.640	-0.160	-0.241	0.636	0.200	0.458
2	0.200	1.640	-0.160	1.325	-1.622	0.200	-0.458
3	0.200	1.480	-0.120	-0.191	0.565	0.400	0.429
4	0.200	1.480	-0.120	1.275	-1.551	0.400	-0.429
5	0.200	1.320	-0.080	-0.114	0.453	0.600	0.384
6	0.200	1.320	-0.080	1.198	-1.440	0.600	-0.384
7	0.200	1.160	-0.040	-0.003	0.284	0.800	0.315
8	0.200	1.160	-0.040	1.080	-1.270	0.800	-0.315
9	0.400	1.640	-0.160	0.102	0.429	0.200	0.574
10	0.400	1.640	-0.160	2.065	-2.402	0.200	-0.574
11	0.400	1.480	-0.120	0.141	0.373	0.400	0.551
12	0.400	1.480	-0.120	2.026	-2.346	0.400	-0.551
13	0.400	1.320	-0.080	0.200	0.289	0.600	0.517
14	0.400	1.320	-0.080	1.968	-2.262	0.600	-0.517
15	0.400	1.160	-0.040	0.283	0.168	0.800	0.468
16	0.400	1.160	-0.040	1.884	-2.141	0.800	-0.468
17	0.600	1.640	-0.160	0.382	0.315	0.200	0.728
18	0.600	1.640	-0.160	2.870	-3.275	0.200	-0.728
19	0.600	1.480	-0.120	0.412	0.271	0.400	0.710
20	0.600	1.480	-0.120	2.839	-3.231	0.400	-0.710
21	0.600	1.320	-0.080	0.457	0.206	0.600	0.684
22	0.600	1.320	-0.080	2.794	-3.166	0.600	-0.684
23	0.600	1.160	-0.040	0.519	0.117	0.800	0.647
24	0.600	1.160	-0.040	2.733	-3.077	0.800	-0.647
25	0.800	1.640	-0.160	0.629	0.246	0.200	0.900
26	0.800	1.640	-0.160	3.706	-4.192	0.200	-0.900
27	0.800	1.480	-0.120	0.654	0.211	0.400	0.885
28	0.800	1.480	-0.120	3.681	-4.157	0.400	-0.885
29	0.800	1.320	-0.080	0.690	0.159	0.600	0.864
30	0.800	1.320	-0.080	3.645	-4.105	0.600	-0.864
31	0.800	1.160	-0.040	0.738	0.089	0.800	0.836
32	0.800	1.160	-0.040	3.597	-4.035	0.800	-0.836

TABLE II

GROWTH RATE ($Y_o^2 = 4\pi\rho_o\omega^2 r_o^2/B_o^2$) AS FUNCTION OF WAVE NUMBER ($X_o = kr_o$) FOR m = 0 PERTURBATIONS

Case \ X_o	0.1	0.2	0.5	1.0	2.0	5.0	10.0
1	0.0015 -0.0389	0.0057 -0.0425	0.0249 -0.0589	0.0603 -0.0924	0.1242 -0.1737	0.3097 -0.4565	0.6251 -0.9401
2	0.0008 -0.2130	0.0031 -0.2136	0.0145 -0.2192	0.0147 -0.2492	-0.1082 -0.3787	-0.6096 -1.0563	-1.4173 -2.2944
3	0.0015 -0.0297	0.0055 -0.0332	0.0231 -0.0485	0.0549 -0.0782	0.1127 -0.1480	0.2794 -0.3849	0.5604 -0.7879
4	0.0007 -0.1928	0.0028 -0.1932	0.0128 -0.1982	0.0122 -0.2252	-0.1039 -0.3413	-0.5821 -0.9471	-1.3535 -2.0570
5	0.0014 -0.0189	0.0048 -0.0220	0.0189 -0.0343	0.0439 -0.0573	0.0903 -0.1091	0.2250 -0.2802	0.4510 -0.5697
6	0.0006 -0.1647	0.0022 -0.1650	0.0103 -0.1688	0.0079 -0.1910	-0.0994 -0.2861	-0.5455 -0.7832	-1.2611 -1.7049
7	0.0011 -0.0077	0.0034 -0.0099	0.0118 -0.0174	0.0265 -0.0305	0.0550 -0.0584	0.1396 -0.1469	0.2812 -0.2958
8	0.0004 -0.1274	0.0016 -0.1275	0.0069 -0.1300	0.0028 -0.1460	-0.0897 -0.2148	-0.4923 -0.5605	-1.1244 -1.2278
9	0.0026 -0.0227	0.0084 -0.0281	0.0307 -0.0478	0.0704 -0.0826	0.1506 -0.1569	0.4005 -0.3973	0.8267 -0.8064
10	0.0014 -0.4600	0.0053 -0.4604	0.0232 -0.4685	0.0050 -0.5246	-0.3361 -0.7673	-1.8724 -1.9346	-4.2030 -4.2957
11	0.0027 -0.0168	0.0082 -0.0220	0.0282 -0.0398	0.0634 -0.0704	0.1343 -0.1344	0.3536 -0.3366	0.7250 -0.6787
12	0.0013 -0.4366	0.0050 -0.4370	0.0219 -0.4446	0.0041 -0.4985	-0.3222 -0.7316	-1.7710 -1.8717	-3.9411 -4.1950
13	0.0025 -0.0104	0.0071 -0.0148	0.0227 -0.0288	0.0499 -0.0522	0.1051 -0.0999	0.2755 -0.2473	0.5630 -0.4954
14	0.0012 -0.4040	0.0046 -0.4044	0.0199 -0.4115	0.0028 -0.4622	-0.3002 -0.6826	-1.5975 -1.8056	-3.5542 -4.0499
15	0.0019 -0.0044	0.0047 -0.0071	0.0137 -0.0152	0.0292 -0.0284	0.0613 -0.0545	0.1611 -0.1329	0.3290 -0.2640
16	0.0011 -0.3613	0.0040 -0.3616	0.0175 -0.3683	0.0022 -0.4153	-0.2650 -0.6229	-1.3574 -1.7222	-3.0344 -3.8490
17	0.0035 -0.0169	0.0102 -0.0232	0.0335 -0.0441	0.0747 -0.0791	0.1609 -0.1506	0.4357 -0.3744	0.9048 -0.7546
18	0.0025 -0.8498	0.0094 -0.8505	0.0411 -0.8658	0.0049 -0.9743	-0.6243 -1.4513	-3.2283 -3.9553	-7.2130 -8.8352
19	0.0036 -0.0125	0.0098 -0.0184	0.0305 -0.0371	0.0668 -0.0678	0.1423 -0.1295	0.3808 -0.3189	0.7854 -0.6387
20	0.0024 -0.8248	0.0091 -0.8256	0.0400 -0.8407	0.0049 -0.9473	-0.6045 -1.4177	-3.1037 -3.9025	-6.9395 -8.7168
21	0.0033 -0.0080	0.0083 -0.0127	0.0242 -0.0272	0.0520 -0.0507	0.1101 -0.0969	0.2923 -0.2364	0.6003 -0.4706
22	0.0023 -0.7903	0.0088 -0.7910	0.0383 -0.8059	0.0052 -0.9102	-0.5738 -1.3724	-2.9183 -3.8311	-6.5372 -8.5487
23	0.0023 -0.0037	0.0052 -0.0065	0.0142 -0.0147	0.0300 -0.0279	0.0631 -0.0534	0.1671 -0.1289	0.3424 -0.2552
24	0.0022 -0.7452	0.0083 -0.7463	0.0363 -0.7611	0.0063 -0.8628	-0.5297 -1.3166	-2.6699 -3.7381	-6.0016 -8.3238
25	0.0041 -0.0145	0.0112 -0.0212	0.0348 -0.0425	0.0767 -0.0776	0.1656 -0.1477	0.4517 -0.3640	0.9402 -0.7311
26	0.0040 -1.3891	0.0154 -1.3905	0.0673 -1.4169	0.0097 -1.6014	-0.9999 -2.4210	-5.0712 -6.7848	-11.3781 -15.1166
27	0.0042 -0.0108	0.0107 -0.0170	0.0316 -0.0359	0.0683 -0.0667	0.1459 -0.1273	0.3928 -0.3111	0.8121 -0.6210
28	0.0039 -1.3635	0.0151 -1.3645	0.0662 -1.3911	0.0102 -1.5740	-0.9774 -2.3882	-4.9437 -6.7283	-11.0988 -14.9895
29	0.0037 -0.0071	0.0088 -0.0120	0.0249 -0.0265	0.0529 -0.0500	0.1122 -0.0956	0.2994 -0.2317	
30	0.0039 -1.3280	0.0147 -1.3293	0.0647 -1.3556	0.0112 -1.5366	-0.9432 -2.3441	-4.7548 -6.6523	-10.6892 - 4.8105
31	0.0025 -0.0034	0.0054 -0.0062	0.0145 -0.0145	0.0303 -0.0277	0.0638 -0.0530	0.1695 -0.1274	0.3477 - 0.2517
32	0.0037 -1.2824	0.0143 -1.2838	0.0629 -1.3100	0.0130 -1.4888	-0.8956 -2.2895	-4.5032 -6.5547	-10.1467 -14.5751

TABLE III

GROWTH RATE ($Y_0^2 = 4\pi\rho_0\omega^2 r_0^2/B_0^2$) AS FUNCTION OF WAVE NUMBER ($X_0 = kr_0$) FOR m = 1 PERTURBATIONS AND $X_0 > 0$

Case \ X_0	0.1	0.2	0.5	1.0	2.0	5.0	10.0
1	0.0168 -0.0987	0.0188 -0.0702	0.0032 -0.0071	0.0508 -0.0437	0.1324 -0.1329	0.3380 -0.4143	0.6623
2	0.0133 -0.1877	0.0156 -0.2388	0.0224 -0.3202	-0.0111 -0.3612	-0.2121 -0.4547	-0.8157 -1.0789	
3	0.0130 -0.0724	0.0129 -0.0473	0.0074 -0.0043	0.0574 -0.0460	0.1291 -0.1230	0.3114 -0.3598	0.5993 -0.7623
4	0.0124 -0.1558	0.0142 -0.2046	0.0194 -0.2826	-0.0097 -0.3235	-0.1912 -0.4095	-0.7764 -0.9474	-1.5597 -2.6263
5	0.0087 -0.0463	0.0071 -0.0264	0.0160 -0.0083	0.0544 -0.0405	0.1106 -0.0981	0.2560 -0.2699	0.4865 -0.5593
6	0.0113 -0.1218	0.0134 -0.1675	0.0179 -0.2401	-0.0049 -0.2795	-0.1572 -0.3575	-0.7209 -0.7660	
7	0.0041 -0.0213	0.0024 -0.0090	0.0162 -0.0077	0.0385 -0.0260	0.0722 -0.0573	0.1622 -0.1466	0.3062 -0.2956
8	0.0093 -0.0845	0.0124 -0.1260	0.0181 -0.1916	0.0048 -0.2290	-0.1029 -0.3021	-0.5244 -0.6483	-1.1959 -1.2844
9	0.0207 -0.1102	0.0271 -0.0912	0.0219 -0.0394	0.0189 -0.0175	0.1124 -0.0978	0.3762 -0.3392	0.0882 -0.7483
10	0.0156 -0.2265	0.0218 -0.3223	0.0431 -0.5237	-0.0071 -0.6767	-0.4063 -0.9516	-1.9184 -2.2242	-4.2356 -4.6326
11	0.0163 -0.0826	0.0198 -0.0657	0.0113 -0.0206	0.0299 -0.0241	0.1116 -0.0935	0.3417 -0.2974	0.7180 -0.6398
12	0.0147 -0.1942	0.0197 -0.2862	0.0386 -0.4810	-0.0044 -0.6338	-0.3700 -0.9077	-1.7695 -2.1630	-3.9415 -4.5133
13	0.0113 -0.0547	0.0126 -0.0415	0.0036 -0.0067	0.0318 -0.0240	0.0947 -0.0761	0.2726 -0.2253	0.5633 -0.4738
14	0.0138 -0.1602	0.0185 -0.2480	0.0354 -0.4336	0.0014 -0.5840	-0.3192 -0.8559	-1.5634 -2.0842	-3.5246 -4.3461
15	0.0057 -0.0270	0.0057 -0.0191	0.0010 -0.0006	0.0230 -0.0163	0.0592 -0.0450	0.1628 -0.1246	0.3323 -0.2562
16	0.0124 -0.1241	0.0177 -0.2072	0.0341 -0.3815	0.0115 -0.5276	-0.2511 -0.7967	-1.2983 -1.9789	-2.9788 -4.1184
17	0.0249 -0.1187	0.0359 -0.1074	0.0403 -0.0669	0.0061 -0.0119	0.0845 -0.0709	0.3688 -0.2976	
18	0.0180 -0.2725	0.0294 -0.4271	0.0716 -0.8054	-0.0016 -1.1380	-0.6824 -1.7246	-3.2520 -4.3826	
19	0.0198 -0.0900	0.0269 -0.0792	0.0259 -0.0432	0.0051 -0.0056	0.0888 -0.0721	0.3348 -0.2644	0.7431 -0.5851
20	0.0169 -0.2393	0.0263 -0.3884	0.0649 -0.7569	0.0011 -1.0895	-0.6381 -1.6777	-3.0883 -4.3116	-6.9354 -9.1565
21	0.0138 -0.0602	0.0177 -0.0516	0.0140 -0.0235	0.0133 -0.0107	0.0774 -0.0609	0.2649 -0.2029	0.5753 -0.4380
22	0.0161 -0.2050	0.0241 -0.3483	0.0595 -0.7041	0.0066 -1.0339	-0.5791 -1.6223	-2.8656 -4.2195	-6.4930 -8.9624
23	0.0072 -0.0302	0.0086 -0.0250	0.0051 -0.0088	0.0122 -0.0091	0.0486 -0.0370	0.1553 -0.1141	0.3317 -0.2409
24	0.0151 -0.1692	0.0229 -0.3066	0.0556 -0.6470	0.0159 -0.9717	-0.5039 -1.5587	-2.5834 -4.1026	-5.9203 -8.7085
25	0.0294 -0.1259	0.0453 -0.1213	0.0587 -0.0918	0.0287 -0.0408	0.0549 -0.0465	0.3470 -0.2672	0.8388 -0.6356
26	0.0207 -0.3241	0.0391 -0.5514	0.1090 -1.1646	0.0041 -1.7463	-1.0592 -2.7670	-5.0959 -7.3286	-11.4221 -15.7092
27	0.0233 -0.0956	0.0343 -0.0904	0.0402 -0.0629	0.0121 -0.0179	0.0655 -0.0531	0.3175 -0.2408	0.7393 -0.5520
28	0.0192 -0.2898	0.0345 -0.5095	0.0998 -1.1099	0.0064 -1.6919	-1.0087 -2.7162	-4.9197 -7.2502	-11.0903 -15.5536
29	0.0164 -0.0645	0.0228 -0.0597	0.0239 -0.0376	0.0015 -0.0030	0.0607 -0.0478	0.2515 -0.1871	0.5699 -0.4166
30	0.0183 -0.2547	0.0311 -0.4666	0.0916 -1.0509	0.0113 -1.6304	-0.9437 -2.6567	-4.6372 -7.1501	-11.0630 -15.3440
31	0.0086 -0.0325	0.0113 -0.0295	0.0104 -0.0164	0.0033 -0.0027	0.0393 -0.0302	0.1468 -0.1065	0.3258 -0.2313
32	0.0175 -0.2186	0.0289 -0.4227	0.0850 -0.9879	0.0194 -1.5621	-0.8631 -2.5888	-4.3874 -7.0262	-10.0386 -15.0758

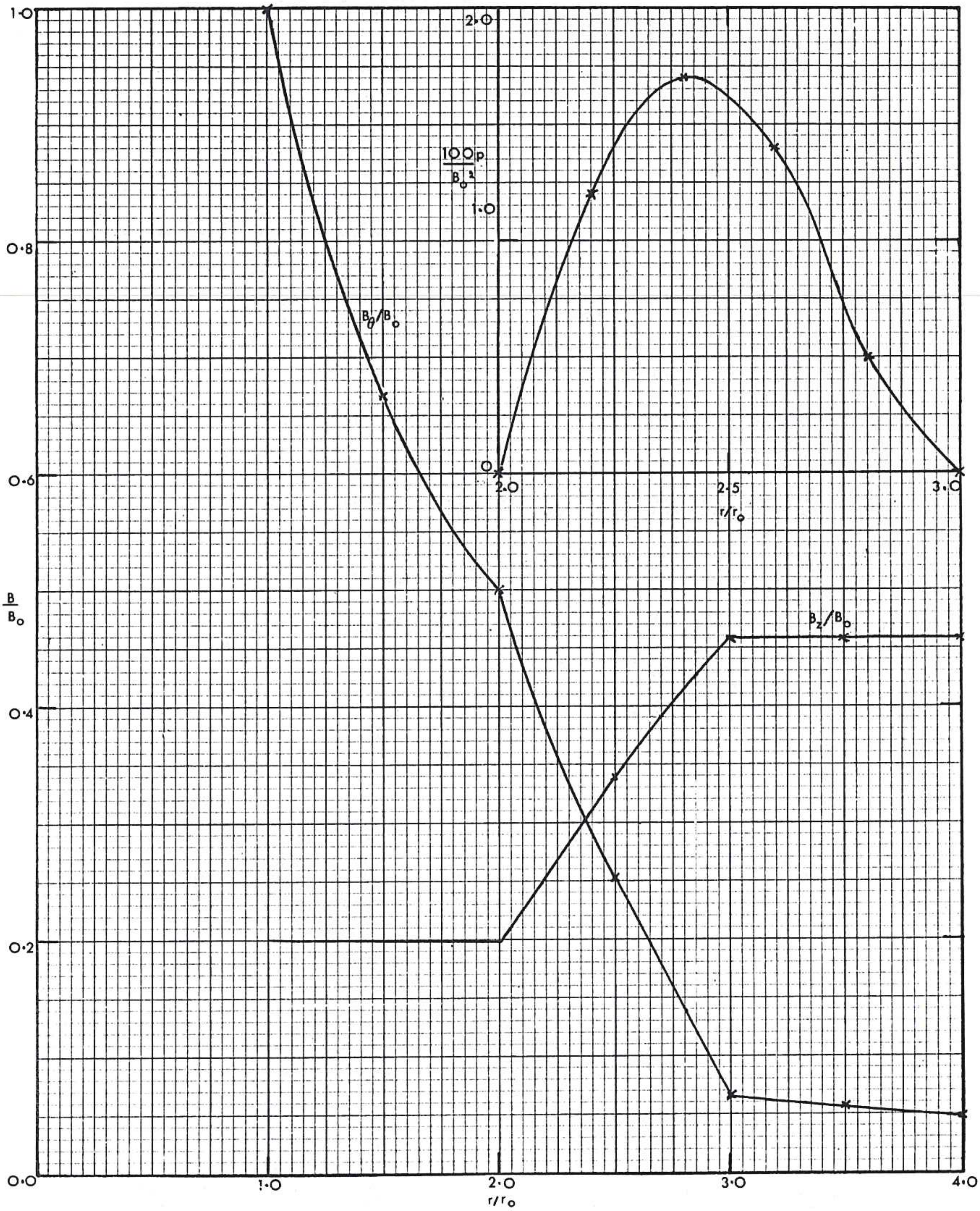
TABLE IV

GROWTH RATE ($Y_0^2 = 4\pi\rho_0\omega^2 r_0^2/B_0^2$) AS FUNCTION OF WAVE NUMBER ($X_0 = kr_0$) FOR m = 1 PERTURBATIONS AND $X_0 < 0$

Case \ $\lvert X_0 \rvert$	0.1	0.2	0.5	1.0	2.0	5.0	10.0
1	0.0082 -0.1397	0.0056 -0.1441	0.0049 -0.1297	0.0221 -0.1291	0.0804 -0.2043	0.2645 -0.4951	0.5789 -0.9817
2	0.0100 -0.0575	0.0023 -0.0025	0.0716 -0.0535	0.0922 -0.1536	0.0219 -0.3359	-0.4131 -1.0285	-1.2025 -2.2616
3	0.0074 -0.1095	0.0047 -0.1138	0.0013 -0.1011	0.0147 -0.0995	0.0680 -0.1636	0.2340 -0.4066	0.5145 -0.8118
4	0.0061 -0.0320	0.0195 -0.0041	0.0846 -0.0595	0.0981 -0.1513	0.0259 -0.3173	-0.3939 -0.9419	-1.1509 -2.0484
5	0.0063 -0.0771	0.0041 -0.0811	-0.0007 -0.0704	0.0068 -0.0661	0.0500 -0.1127	0.1847 -0.2877	0.4105 -0.5787
6	0.0016 -0.0066	0.0396 -0.0098	-0.0929 -0.0606	0.0973 -0.1381	0.0240 -0.2776	-0.3708 -0.7984	-1.0759 -1.7181
7	0.0042 -0.0415	0.0031 -0.0446	0.0008 -0.0377	0.0005 -0.0311	0.0271 -0.0552	0.1119 -0.1455	0.2536 -0.2951
8	0.0179 -0.0031	0.0567 -0.0141	0.0954 -0.0548	0.0879 -0.1115	0.0155 -0.2132	-0.3390 -0.5909	-0.9639 -1.2578
9	0.0057 -0.1287	0.0024 -0.1250	0.0130 -0.1094	0.0602 -0.1285	0.1509 -0.2054	0.4071 -0.4521	0.8352 -0.8630
10	0.0063 -0.0293	0.0401 -0.0102	0.1074 -0.1191	0.0884 -0.3089	-0.1958 -0.6342	-1.5890 -1.8739	-3.9572 -4.1655
11	0.0051 -0.0995	0.0014 -0.0964	0.0071 -0.0823	0.0472 -0.0995	0.1270 -0.1654	0.3515 -0.3725	0.7247 -0.7162
12	0.0017 -0.0034	0.0632 -0.0183	0.1226 -0.1283	0.0956 -0.3083	-0.1889 -0.6169	-1.5562 -1.7844	-3.8734 -3.9359
13	0.0042 -0.0688	0.0010 -0.0665	0.0023 -0.0542	0.0322 -0.0671	0.0941 -0.1160	0.2686 -0.2666	0.5575 -0.5157
14	0.0233 -0.0040	0.0857 -0.0261	0.1343 -0.1320	0.0979 -0.2964	-0.1833 -0.5792	-1.5094 -1.6354	-3.5793 -3.7489
15	0.0028 -0.0359	0.0009 -0.0348	-0.0004 -0.0264	0.0160 -0.0330	0.0521 -0.0596	0.1543 -0.1394	0.3231 -0.2711
16	0.0499 -0.0100	0.1068 -0.0330	0.1414 -0.1290	0.0943 -0.2716	-0.1781 -0.5194	-1.4028 -1.4637	-3.0865 -3.5732
17	0.0036 -0.1219	0.0015 -0.1155	0.0299 -0.1114	0.0947 -0.1431	0.1987 -0.2194	0.4845 -0.4480	0.9574 -0.8293
18	0.0026 -0.0010	0.0812 -0.0284	0.1429 -0.2370	0.0825 -0.5892	-0.5081 -1.1764	-3.1890 -3.5146	-7.1672 -8.3605
19	0.0030 -0.0935	-0.0001 -0.0875	0.0205 -0.0833	0.0753 -0.1115	0.1652 -0.1770	0.4125 -0.3701	0.8203 -0.6908
20	0.0266 -0.0043	0.1063 -0.0397	0.1608 -0.2511	0.0908 -0.5924	-0.5011 -1.1631	-3.1043 -3.4763	-6.9357 -8.2639
21	0.0026 -0.0640	-0.0007 -0.0592	0.0119 -0.0547	0.0527 -0.0763	0.1211 -0.1251	0.3098 -0.2671	0.6201 -0.5020
22	0.0542 -0.0112	0.1314 -0.0509	0.1759 -0.2597	0.0957 -0.5850	-0.4905 -1.1324	-2.9579 -3.4218	-6.5740 -8.1203
23	0.0017 -0.0330	-0.0005 -0.0302	0.0048 -0.0265	0.0274 -0.0386	0.0662 -0.0654	0.1738 -0.1421	0.3504 -0.2687
24	0.0827 -0.0189	0.1560 -0.0614	0.1875 -0.2619	0.0966 -0.5655	-0.4751 -1.0850	-2.7460 -3.3490	-6.0764 -7.9231
25	0.0019 -0.1175	0.0032 -0.1120	0.0502 -0.1226	0.1258 -0.1620	0.2374 -0.2380	0.5381 -0.4576	1.0319 -0.8252
26	0.0286 -0.0041	0.1203 -0.0558	0.1847 -0.4211	0.0810 -1.0119	-0.9022 -1.9973	-5.0273 -6.2028	-11.3220 -14.4988
27	0.0014 -0.0895	0.0005 -0.0838	0.0365 -0.0920	0.0998 -0.1264	0.1953 -0.1916	0.4538 -0.3781	0.8774 -0.6885
28	0.0560 -0.0117	0.1478 -0.0711	0.2055 -0.4412	0.0904 -1.0202	-0.8971 -1.9883	-4.9487 -6.1654	-11.0952 -14.3986
29	0.0012 -0.0608	-0.0008 -0.0558	0.0233 -0.0608	0.0699 -0.0869	0.1418 -0.1357	0.3374 -0.2737	0.6571 -0.5025
30	0.0848 -0.0204	0.1757 -0.0864	0.2240 -0.4559	0.0974 -1.0184	-0.8853 -1.9644	-4.8079 -6.1103	-10.7371 -14.2486
31	0.0008 -0.0311	-0.0009 -0.0279	0.0109 -0.0299	0.0366 -0.0444	0.0768 -0.0714	0.1870 -0.1467	0.3668 -0.2712
32	0.1148 -0.0301	0.2036 -0.1013	0.2396 -0.4644	0.1015 -1.0055	-0.8660 -1.9253	-4.6026 -6.0360	-10.2440 -14.0446

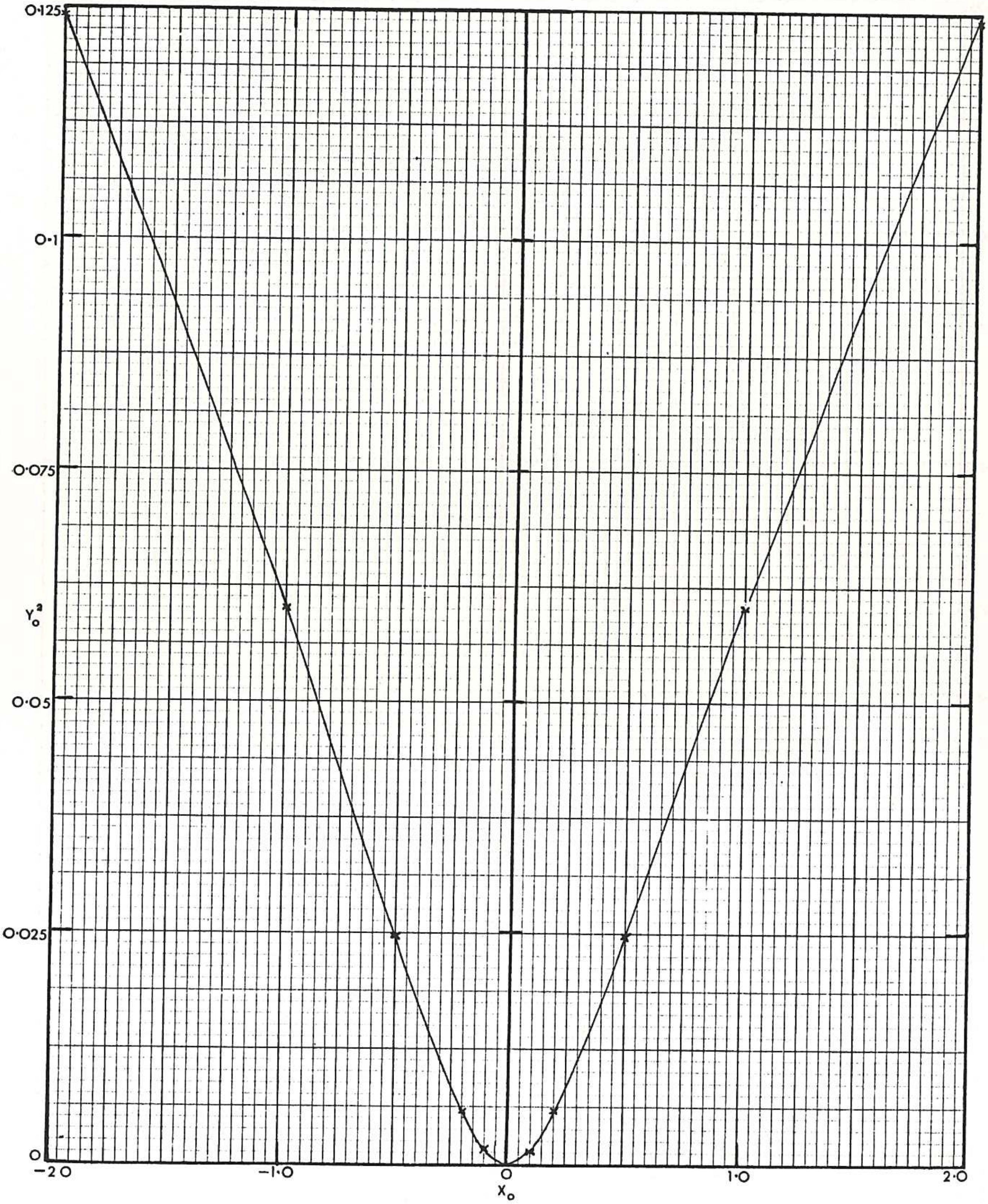
Sketch of magnetic field and pressure for a configuration without axial field reversal (Case 1).

B_θ/B_0, B_z/B_0 and $100p/B_0^2$ are shown as functions of r/r_0.



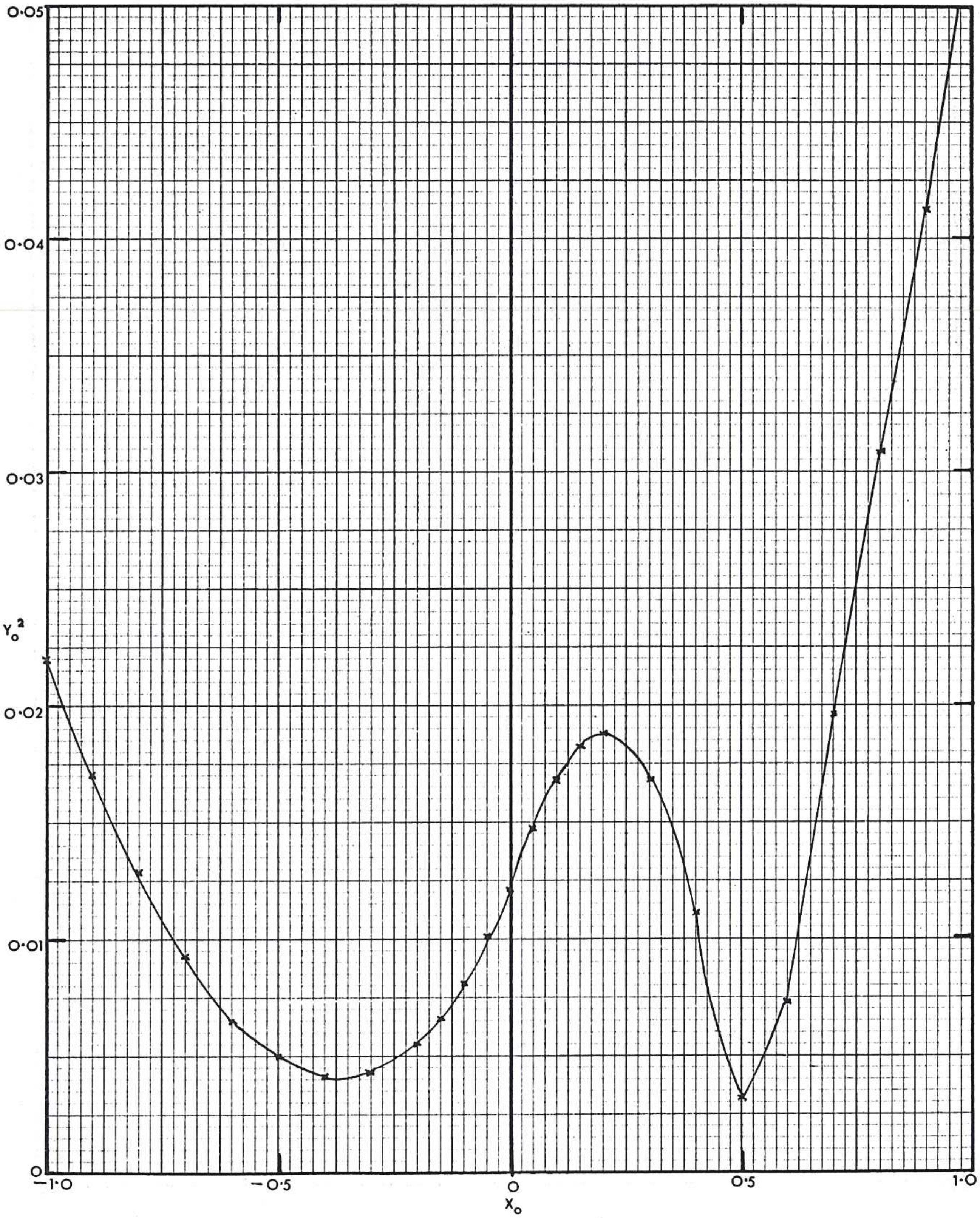
CLM-R25 FIGURE 1

Growth rate as a function of wave-number for m = 0 perturbations (Case 1).
For the fields shown in Fig. 1, the dimensionless growth rate (Y_0^2) for m = 0 perturbations is shown as a function of dimensionless wave-number (X_0).



CLM-R25 FIGURE 2

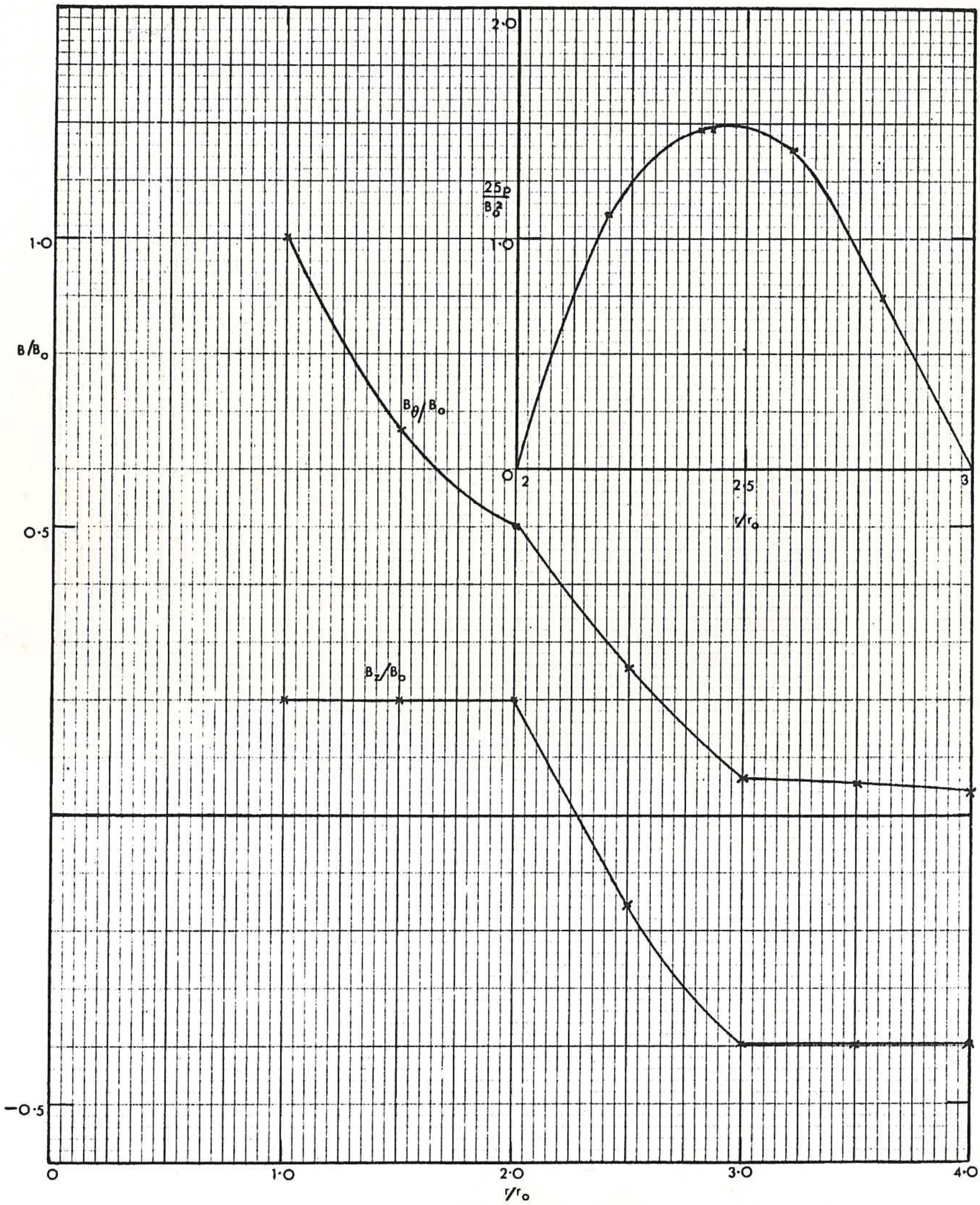
Growth rate as a function of wave-number for m = 1 perturbations (Case 1).
For the fields shown in Fig. 1, the dimensionless growth rate (Y_o^2) for m = 1 perturbations is shown as a function of dimensionless wave-number (X_o).



CLM-R25 FIGURE 3

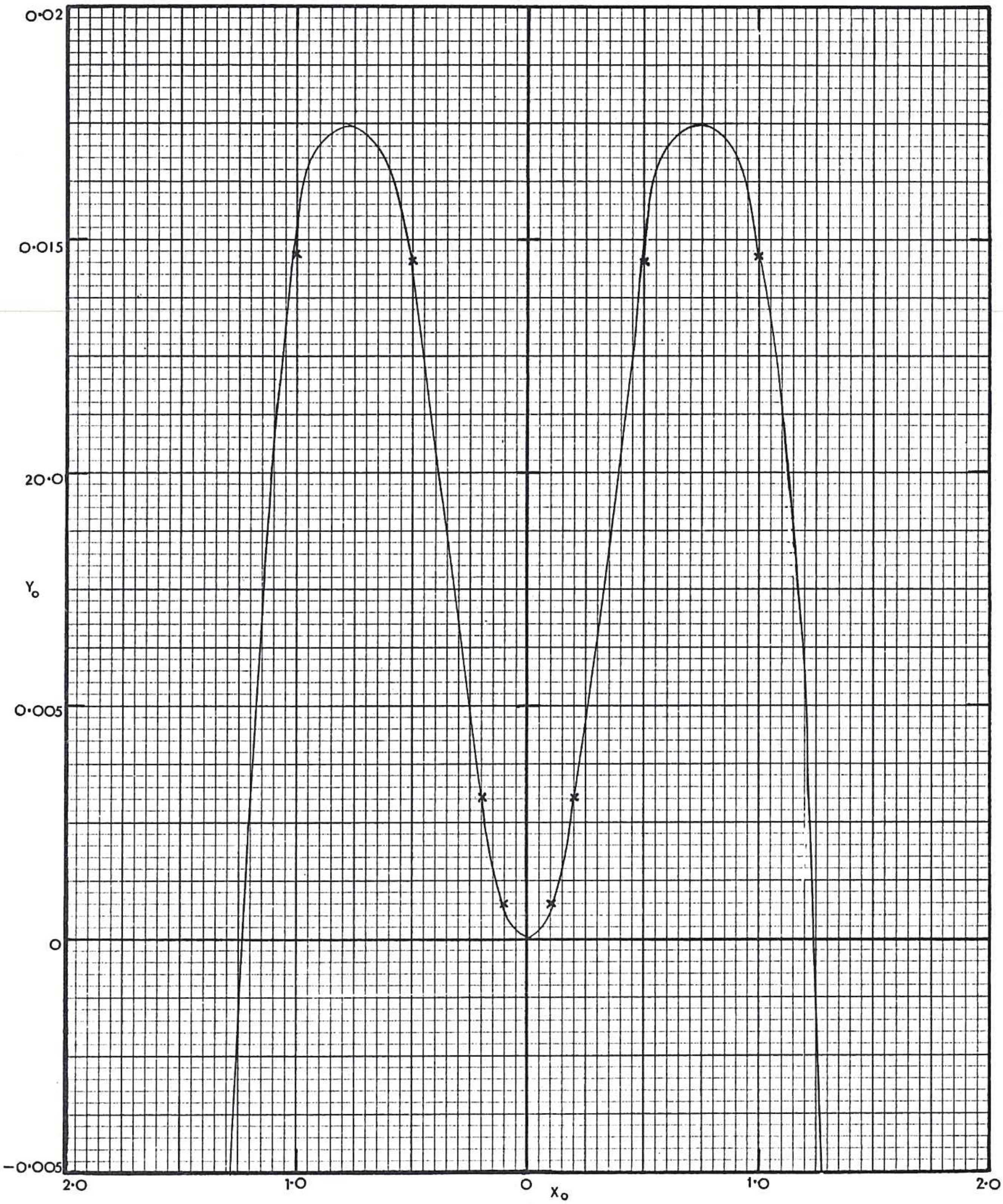
Sketch of magnetic field and pressure for a configuration with axial field reversal (Case 2).

B_θ/B_o, B_z/B_o and $25p/B_o^2$ are shown as functions of r/r_o.



CLM-R25 FIGURE 4

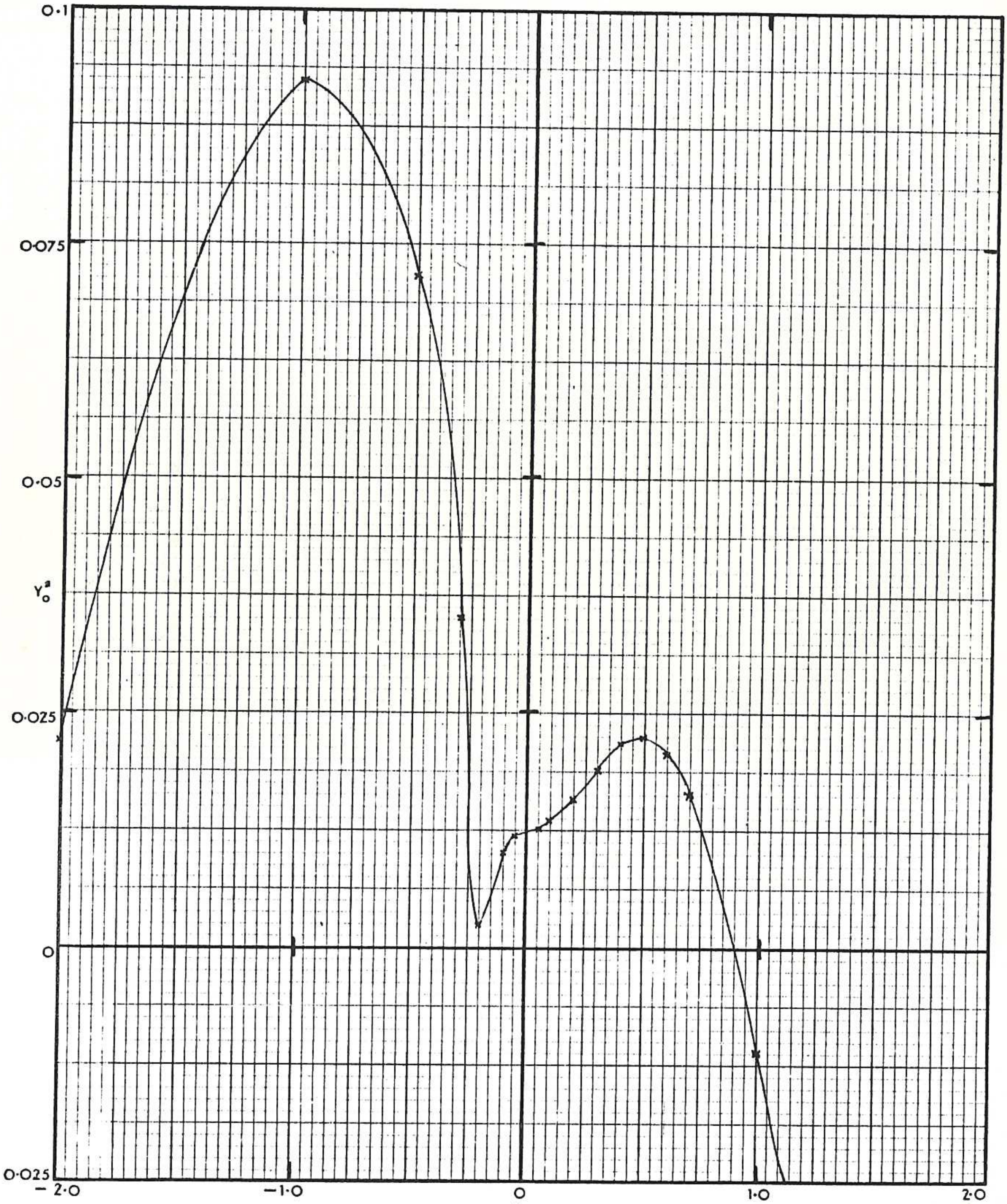
Growth rate as a function of wave-number for m = 0 perturbations (Case 2).
For the fields shown in Fig. 4, the dimensionless growth rate (Y_0^2) for m= 0 perturbations is shown as a function of dimensionless wave-number (X_0). The curve is rather schematic in the neighbourhood of the maxima.



CLM-R25 FIGURE 5

Growth rate as a function of wave-number for m = 1 perturbations (Case 2).

For the fields shown in Fig. 4, the dimensionless growth rate (Y_0^2) for m = 1 perturbations is shown as a function of dimensionless wave-number (X_0).



CLM-R25 FIGURE 6