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Report

THE EFFECT OF FINITE ION AND ELECTRON TEMPERATURES ON THE ION CYCLOTRON RESONANCE INSTABILITY

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THE EFFECT OF FINITE ION AND ELECTRON TEMPERATURES ON THE ION CYCLOTRON RESONANCE INSTABILITY

by

E. G. HARRIS*

ABSTRACT

The effect of finite ion and electron temperatures on the ion cyclotron resonance instability is investigated for the Burt/Harris model of a confined plasma cylinder. This is a cylinder of plasma in which the ions move in concentric Larmor orbits with their centres on the axis of the cylinder.

By discussing changes in particle orbits produced by a self-consistent potential perturbation, a dispersion relation is derived, which has been studied analytically and numerically. Results are presented graphically for cold electron, cold ions, and equal ion/electron temperatures, showing stability criteria and growth rates.

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CONTENTS

INTRODUCTION

1. Burt and Harris[1] have investigated unstable cyclotron oscillations in a simple model in which the plasma occupies a cylindrical shell with axis parallel to the magnetic field. The ions were assumed to move initially in concentric Larmor orbits with their centres on the axis of the cylinder. The electrons were assumed to be cold. The criterion for instability was found to be

$$\ell\omega_{ci} < \omega_{pe} \left[1 + \left(\frac{m}{M}\right)^{\frac{1}{3}}\right]^{\frac{3}{2}} \approx \omega_{pe} \qquad \ldots.(1)$$

where ℓ is an integer, ω_{pi} = ion cyclotron frequency, ω_{pe} = electron plasma frequency and m and M are the electron and ion masses respectively. This criterion is essentially the same as that found for the infinite, homogeneous plasma when $T_{\parallel}$, the temperature parallel to the field is zero[2] (i.e. there is no thermal spread in velocities parallel to the field).

2. The effect of non-zero $T_{\parallel}$ on the instability criterion has been investigated by Drummond, Rosenbluth and Johnson[3], by Timofeev[4] and by Kammash and Heckrotte[5]. These were all numerical calculations carried out with the aid of a digital computer. The plasma was assumed to be infinite and homogeneous.

3. In this note we investigate temperature effects in the Burt-Harris model. This is a sufficiently simple problem that it can be treated without the aid of a computer. Furthermore, it seems likely that for some thermonuclear machines the Burt-Harris model is closer to reality than the infinite homogeneous plasma. We have in mind machines such as DCX-1 in which the radius of the ions orbit is about the same as the radius of the machine.

DERIVATION OF THE DISPERSION RELATION

4. The equation of motion for an ion is

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \frac{e}{M} [\vec{E} + \frac{1}{c} \vec{v} \times \vec{B}], \qquad \ldots.(2)$$

where

$$\vec{B} = (0, 0, -B).$$

Equation (2) is linearized by writing:-

$$\vec{v} = V_z \vec{e}_z + V_\perp \vec{e}_\theta + \vec{v}' = \vec{V} + \vec{v}', \qquad \ldots .(3)$$

where:-

$$V_\perp = \omega_{ci} r = \left(\frac{eB}{Mc}\right) r, \qquad \ldots .(4)$$

and $\vec{v}'$ and $\vec{E}$ are considered to be small quantities whose space and time dependence is given by a factor:-

$$e^{ik_z z - i\ell\theta + i\omega t}.$$

5. When linearized equation (2) may be solved for $\vec{v}'$ with the result:-

$$v'_r = \frac{i \frac{e}{M} E_r (\omega - \ell\omega_{ci} + k_z V_z) + \frac{e}{M} \omega_{ci} E_\theta}{\omega_c^2 - (\omega - \ell\omega_{ci} + k_z V_z)^2}, \qquad \ldots .(5)$$

$$v'_\theta = \frac{- \frac{e}{M} \omega_{ci} E_r + i \frac{e}{M} E_\theta (\omega - \ell\omega_{ci} + k_z V_z)}{\omega_c^2 - (\omega - \ell\omega_{ci} + k_z V_z)^2}, \qquad \ldots .(6)$$

$$v'_z = \frac{\frac{e}{M} E_z}{i(\omega - \ell\omega_{ci} + k_z V_z)}. \qquad \ldots .(7)$$

The equation of continuity is:-

$$\frac{\partial n}{\partial t} + \vec{v} \cdot \nabla n = - n \nabla \cdot \vec{v}. \qquad \ldots .(8)$$

Equation (8) is linearized by writing:-

$$n = N + n'. \qquad \ldots .(9)$$

then:-

$$n' = \frac{- N \nabla \cdot \vec{v}'}{i(\omega - \ell\omega_{ci} + k_z V_z)}. \qquad \ldots .(10)$$

We let:-

$$\vec{E} = - \nabla \phi, \qquad \ldots .(11)$$

and find from equations (5), (6), (7), (10) and (11):-

$$4\pi e\, n' = \frac{4\pi N e^2}{M} \left\{ \frac{\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) - \frac{\ell^2}{r^2} \phi}{\omega_c^2 - (\omega - \ell\omega_{ci} + k_z V_z)} + \frac{k_z^2 \phi}{(\omega - \ell\omega_{ci} + k_z V_z)^2} \right\}. \qquad \ldots .(12)$$

A similar expression is found for the electron contribution to the charge density, but since $V_\perp = 0$ for electrons the term $\ell\omega_{ci}$ which occurs in the denominator of equation (12) is absent.

6. The equation:-

$$\nabla^2\phi = - \Sigma\ 4\pi en, \qquad ....(13)$$

becomes:-

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \frac{\ell^2}{r^2}\phi - k_z^2\phi$$

$$= \sum_{ions}\frac{4\pi Ne^2}{M}\left\{\frac{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \frac{\ell^2}{r^2}\phi}{(\omega - \ell\omega_{ci} + k_zV_z)^2 - \omega_c^2} - \frac{k_z^2\phi}{(\omega - \ell\omega_{ci} + k_zV_z)^2}\right\}$$

$$+ \sum_{elec}\frac{4\pi Ne^2}{m}\left\{\frac{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \frac{\ell^2}{r^2}\phi}{(\omega + k_zV_z)^2 - \omega_{ce}^2} - \frac{k_z^2\phi}{(\omega + k_zV_z)^2}\right\}. \qquad ....(14)$$

In equation (14) the summations are over electron and ion groups. Each group has its own N and V_z. Equation (14) may be written:-

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \frac{\ell^2}{r^2}\phi - k_z^2\left(\frac{1-G}{1+W}\right)\phi = 0, \qquad ....(15)$$

where:-

$$G = \sum_{ions}\frac{4\pi Ne^2}{M}\frac{1}{(\omega - \ell\omega_{ci} + k_zV_z)^2} + \sum_{elec}\frac{\frac{4\pi Ne^2}{m}}{(\omega + k_zV_z)^2}, \qquad ....(16)$$

$$W = -\sum_{ions}\frac{4\pi Ne^2}{M}\frac{1}{(\omega - \ell\omega_{ci} + k_zV_z)^2 - \omega_{ci}^2}$$

$$-\sum_{elec}\frac{4\pi Ne^2}{m}\frac{1}{(\omega + k_zV_z)^2 - \omega_{ce}^2}. \qquad ....(17)$$

The solutions of equation (15) are Bessel functions:-

$$\phi(r) \sim \begin{cases} J_\ell(k_\perp r) \\ N_\ell(k_\perp r) \end{cases}, \qquad ....(18)$$

and

$$\left[\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{\ell^2}{r^2}\right]\begin{Bmatrix} J_\ell \\ N_\ell \end{Bmatrix} = - k_\perp^2 \begin{Bmatrix} J_\ell \\ N_\ell \end{Bmatrix}. \qquad ....(19)$$

The boundary conditions will determine a set of allowed values of $k_\perp$. In fact, we are not particularly interested in what these values are. Equation (15) becomes:-

$$\left\{- k_\perp^2 - k_z^2\left(\frac{1-G}{1+W}\right)\right\}\phi = 0, \qquad ....(20)$$

from which:-

$$1 = \frac{k_z^2}{k^2}G - \frac{k_\perp^2}{k^2}W, \qquad ....(21)$$

where:-

$$k^2 = k_\perp^2 + k_z^2 . \qquad ....(22)$$

7. We now let $f_i(V)dV$ be the fraction of ions with V_z in dV and similarly for $f_e(V)$. The summations in equations (16) and (17) become integrations and equation (21) becomes:-

$$1 = \frac{k_z^2}{k^2}\left\{\omega_{pi}^2\int\frac{f_i(V)\,dV}{(\omega - \ell\omega_{ci} + k_zV)^2} + \omega_{pe}^2\int\frac{f_e(V)\,dV}{(\omega + k_zV)^2}\right\}$$

$$+ \frac{k_\perp^2}{k^2}\left\{\omega_{pi}^2\int\frac{f_i(V)\,dV}{(\omega - \ell\omega_{ci} + k_zV)^2 - \omega_{ci}^2} + \omega_{pe}^2\int\frac{f_e(V)\,dV}{(\omega + k_zV)^2 - \omega_{ce}^2}\right\}. \qquad ....(23)$$

Equation (23) is the dispersion relation which we shall study.

ANALYSIS OF THE DISPERSION RELATION

8. First we consider the case of "cold" ions and electrons. That is:-

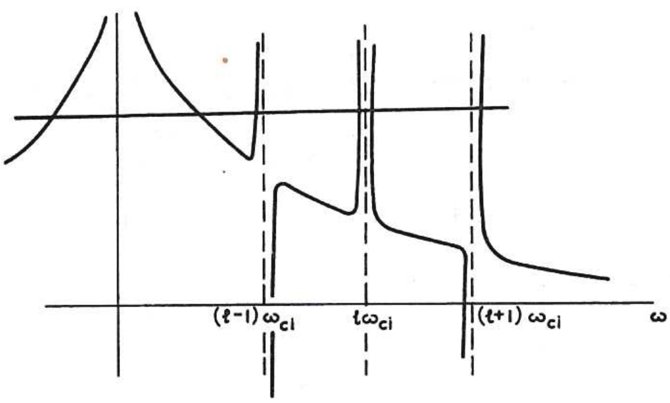
$$f_i(V) = f_e(V) = \delta(V). \qquad ....(24)$$

Then

$$\frac{1}{\omega_{pe}^2} = \frac{k_z^2}{k^2}\left\{\frac{\frac{m}{M}}{(\omega - \ell\omega_{ci})^2} + \frac{1}{\omega^2}\right\}$$

$$+ \frac{k_\perp^2}{k^2}\left\{\frac{\frac{m}{M}}{(\omega - \ell\omega_{ci})^2 - \omega_{ci}^2} + \frac{1}{\omega^2 - \omega_{ce}^2}\right\}. \qquad ....(25)$$

The real roots of equation (25) may be found by plotting the right hand side against ω and seeing where the curve cuts a horizontal line at $\frac{1}{\omega_{pe}^2}$. This is done in Fig.1.



CLM-R32 Fig. 1

9. For sufficiently small ω_{pe} (small density) all roots are real. As ω_{pe} increases the horizontal line drops below the loop near $(\ell - 1)\,\omega_{ci}$ and two intersections disappear indicating that two roots have become complex (hence, an instability). As ω_{pe} increases still further the intersections reappear (hence, stability again). As ω_{pe} increases still further the horizontal line drops below the loop near $\ell\omega_{ci}$ and two intersections disappear indicating an instability. The approximate condition for this to occur is

$$\frac{1}{\omega_{pe}^{2}} < \frac{k_z^2}{k^2} \frac{1}{(\ell\omega_{ci})^2}\,, \qquad \ldots.(26)$$

or

$$\ell\omega_{ci} < \frac{k_z}{k}\,\omega_{pe}. \qquad \ldots.(27)$$

This instability remains for all higher values of ω_{pe}.

10. The instability near $(\ell - 1)\,\omega_{ci}$ occurs for only a very small range of ω_{pe} and is probably not very important. In what follows we will assume that $k_\perp^2 \ll k_z^2$ so that the terms which give rise to it drop out of the dispersion relation which now becomes:-

$$\frac{1}{\omega_{pe}^{2}} = \frac{m}{M}\int \frac{f_i(V)\,dV}{(\omega - \ell\omega_{ci} + k_z V)^2} + \int \frac{f_e(V)\,dV}{(\omega + k_z V)^2}\,. \qquad \ldots.(28)$$

This is just the "two-stream" dispersion relation with $\ell\omega_{ci}$ replacing the kV_o term which usually occurs when ions stream through the electrons with the velocity V_o. The Landau prescription must be followed in doing the integrals.

11. We now assume Maxwellian distributions for the ions and electrons.

$$f_i(V) = \frac{1}{\sqrt{\pi}\,\alpha_i}\, e^{-v^2/\alpha_i^2}, \qquad \ldots.(29)$$

$$f_e(V) = \frac{1}{\sqrt{\pi}\,\alpha_e}\, e^{-v^2/\alpha_e}. \qquad \ldots.(30)$$

The thermal velocities, α_i and α_e, are related to the ion and electron temperatures by:-

$$\alpha_i = \sqrt{\frac{2kT_i}{M}}, \qquad \ldots.(31)$$

$$\alpha_e = \sqrt{\frac{2kT_i}{m}} \qquad \ldots.(32)$$

Equations (29) and (30) may be used in equation (28) to obtain the dispersion relation:-

$$\frac{1}{\delta z_p^2} = Z'(z - z_c) + \frac{1}{\theta} Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}z), \qquad \text{....(33)}$$

where:-

$$z = \frac{\omega}{k_z \alpha_i}, \qquad \text{....(34)}$$

$$z_p = \frac{\omega_{pe}}{k_z \alpha_i}, \qquad \text{....(35)}$$

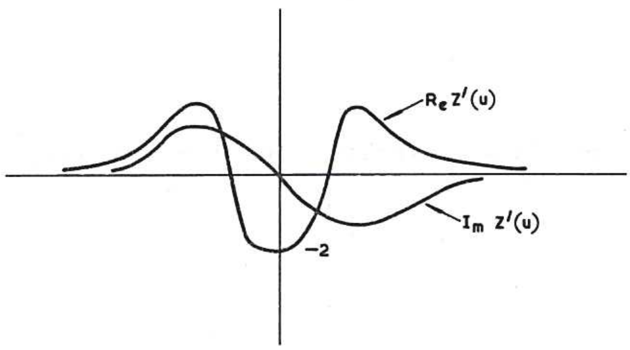
$$z_c = \frac{\ell\omega_{ci}}{k_z \alpha_i} \qquad \text{....(36)}$$

$$\theta = \frac{T_e}{T_i}, \qquad \text{....(37)}$$

$$\delta = \frac{m}{M}, \qquad \text{....(38)}$$

$$Z'(u) = -2 + 4ue^{-u^2} \int_o^u e^{+t^2} dt - i2\sqrt{\pi}ue^{-u^2}. \qquad \text{....(39)}$$

The function Z′ has been tabulated by Fried and Conte. For our present purposes we need to know the properties of Z′(u) for real values of u. In Fig.2 we sketch the real and imaginary parts of Z′.



CLM-R32 Fig. 2

For large u

$$Z'(u) \xrightarrow[u \to \infty]{} \frac{1}{u^2} - i\, 2\sqrt{\pi}\, ue^{-u^2} \qquad \text{....(40)}$$

If equation (33) is solved for z we must find:-

$$z = z(z_c, z_p, \delta, \theta), \qquad \ldots.(41)$$

or, letting $z = x + iy$,

$$x = x(z_c, z_p, \delta, \theta), \qquad \ldots.(42)$$

$$y = y(z_c, z_p, \delta, \theta). \qquad \ldots.(43)$$

For a given δ and θ we can draw curves of constant x and y in the $z_c - z_p$ plane. The curve $y = 0$ is particularly important since along it z is real and y must change sign as the curve is crossed. It divides the stable and unstable regions of the $z_c - z_p$ plane.

12. We will find the $y = 0$ curves with θ as a parameter and $\delta = m/M = 1/1836$.

THE BOUNDARY BETWEEN STABLE AND UNSTABLE REGIONS

13. For real z we may write:-

$$\frac{1}{\delta z_p^2} = \text{Re } Z'(z - z_c) + \frac{1}{\theta} \text{ Re } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}z), \qquad \ldots.(44)$$

$$0 = \text{Im } Z'(z - z_c) + \frac{1}{\theta} \text{ Im } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}z). \qquad \ldots.(45)$$

We first consider some limiting cases.

(a) $T_i = \alpha_i = 0 \quad , \quad T_e = \text{finite}.$

Then:-

$$z, z_p, z_c \to \infty,$$

$$\theta \to \infty.$$

But $\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}z = \text{finite} = \frac{\omega}{k_z \alpha_e}$.

Equation (45) becomes:- $\text{Im}(\frac{\omega}{k_z \alpha_e}) = 0, \qquad \ldots.(46)$

hence:- $\omega = 0. \qquad \ldots.(47)$

Equation (44) becomes:- $\frac{1}{\delta z_p^2} = \frac{1}{z_c^2} - \frac{2}{\theta}. \qquad \ldots.(48)$

$$z_c = \frac{\delta^{\frac{1}{2}} z_p}{\sqrt{1 + \frac{2}{\theta}\delta z_p^2}} \qquad \ldots.(49)$$

$$\ell\omega_{ci} = \frac{\frac{m}{M}\omega_{pe}}{\sqrt{1 + 2\frac{\omega_{pe}^2}{k_z^2 \alpha_e^2}}} . \qquad \ldots.(50)$$

The right hand side of equation (50) is just the frequency of an ion sound wave. It approaches:-

$$\sqrt{\frac{kT_e}{M}}\, k_z$$

for long wave lengths. The system is unstable when $\ell\omega_{ci}$ exceeds the ion sound frequency.

(b) $T_e = \alpha_e = 0$, T_i = finite,

$$\theta \to 0.$$

Equation (45) becomes:- $\mathrm{Im}\, Z'(z - z_c) = 0,$ ....(51)

hence:- $z = z_c.$ ....(52)

Equation (44) becomes:-

$$\frac{1}{\delta z_p^2} = -2 + \frac{1}{\delta z_c^2}, \qquad \text{....(53)}$$

$$z_c = \frac{z_p}{\sqrt{1 + 2\delta z_p^2}} \xrightarrow[z_p \to \infty]{} \frac{1}{\sqrt{2\delta}} = \sqrt{\frac{M}{2m}}\,. \qquad \text{....(54)}$$

The system is stable if z_c is less than the value given by equation (54).

14. Next we consider the case of equal temperatures, $\theta = 1$. Equation (45) becomes:-

$$\mathrm{Im}\, Z'(z - z_c) = -\,\mathrm{Im}\, Z'(\delta^{\frac{1}{2}} z), \qquad \text{....(55)}$$

which has the solution:-

$$z - z_c = -\,\delta^{\frac{1}{2}} z, \qquad \text{....(56)}$$

or

$$z = \frac{z_c}{1 + \delta^{\frac{1}{2}}}\,. \qquad \text{....(57)}$$

Then equation (44) becomes:-

$$\frac{1}{\delta z_p^2} = 2\,\mathrm{Re}\, Z\left(\frac{\delta^{\frac{1}{2}} z_c}{1 + \delta^{\frac{1}{2}}}\right). \qquad \text{....(58)}$$

This curve is drawn in Figs. 3 and 4. Since the right hand side becomes negative for arguments less than 0.92, this sets a lower limit on z_c. For large arguments:-

$$\frac{1}{\delta z_p^2} \to \frac{2}{\left(\frac{\delta^{\frac{1}{2}} z_c}{1 + \delta^{\frac{1}{2}}}\right)^2}, \qquad \text{....(59)}$$

$$z_c = \sqrt{2}\, z_p\, (1 + \delta^{\frac{1}{2}}) \approx \sqrt{2}\, z_p. \qquad \text{....(60)}$$

It is seen that asymptotically the boundary curve approaches a straight line with slope approximately equal to $\sqrt{2}$.

15. We can find the asymptotic behaviour for other values of θ. Assume that as $z_c, z_p \to \infty$, then z, $z - z_c \to \infty$. Then the solution of equation (44) is approximately:-

$$\delta^{\frac{1}{2}} \theta^{-\frac{1}{2}} z = -(z - z_c), \qquad \ldots.(61)$$

or

$$z = \frac{z_c}{1 + \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}}. \qquad \ldots.(62)$$

Equation (45) becomes:-

$$\frac{1}{\delta z_p^2} = (1 + \frac{1}{\theta}) \text{ Re } Z' \left(\frac{\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}} z_c}{1 + \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}}\right), \qquad \ldots.(63)$$

$$= \frac{1}{\delta z_c^2} (1 + \theta) (1 + \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}})^2 . \qquad \ldots.(64)$$

$$z_c = z_p \sqrt{1 + \theta} (1 + \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}). \qquad \ldots.(65)$$

For $\theta = 1$, equation (65) agrees with equation (60). For $\theta = 0$, or $\theta = \infty$ the slope becomes infinite.

16. For θ other than 0, 1 or ∞, equations (44) and (45) must be solved numerically. This is easily done in the following way. Let:-

$$u_p = \delta^{\frac{1}{2}} z_p, \qquad \ldots.(66)$$

$$u_c = \delta^{\frac{1}{2}} z_c, \qquad \ldots.(67)$$

$$u = z - z_c. \qquad \ldots.(68)$$

Equations (44) and (45) become:-

$$\text{Im } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}} z) = -\theta \text{ Im } Z'(u), \qquad \ldots.(69)$$

$$\frac{1}{u_p^2} = \text{Re } Z'(u) + \frac{1}{\theta} \text{ Re } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}} z). \qquad \ldots.(70)$$

The procedure is as follows:-

(1) Choose a value of u.

(2) Using the tables of Fried and Conte solve equation (69) for z.

(3) Find z_c from equation (68).

(4) Calculate u_p (hence z_p) from equation (70).

This gives one point on the curve. The above procedure works best for $\theta < 1$. For $\theta > 1$ it is better to choose z first and calculate u from equation (69). The remaining steps are the same as before. The results are given in Figs. (3) and (4).

GROWTH RATES

17. The same procedure can be followed in drawing curves for constant y with $y \neq 0$. We write:-

$$0 = \text{Im } Z'(x - z_c, y) + \frac{1}{\theta} \text{Im } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}x, \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}y). \qquad \ldots.(71)$$

For a given value of y we choose $x - z_c$ and calculate x. We then use this value of x in:-

$$\frac{1}{\delta z_p^2} = \text{Re } Z'(x - z_c, y) + \frac{1}{\theta} \text{Re } Z'(\delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}x, \delta^{\frac{1}{2}}\theta^{-\frac{1}{2}}y),$$

to calculate z_p. This gives us one point on the y = const. curve in the $z_p - z_c$ plane.

18. This has been done for the case $\delta = \theta = 0.01$ (admittedly unrealistic) for the purpose of seeing how the real and imaginary parts of z varied as one passed through the unstable region. In Fig.5 we have plotted x and y vs. z_p for $z_c = 7$ = const. It is seen that x follows the line $x = z_p$ into the unstable region ($y > 0$) and then levels off at $x = z_c$.

REFERENCES

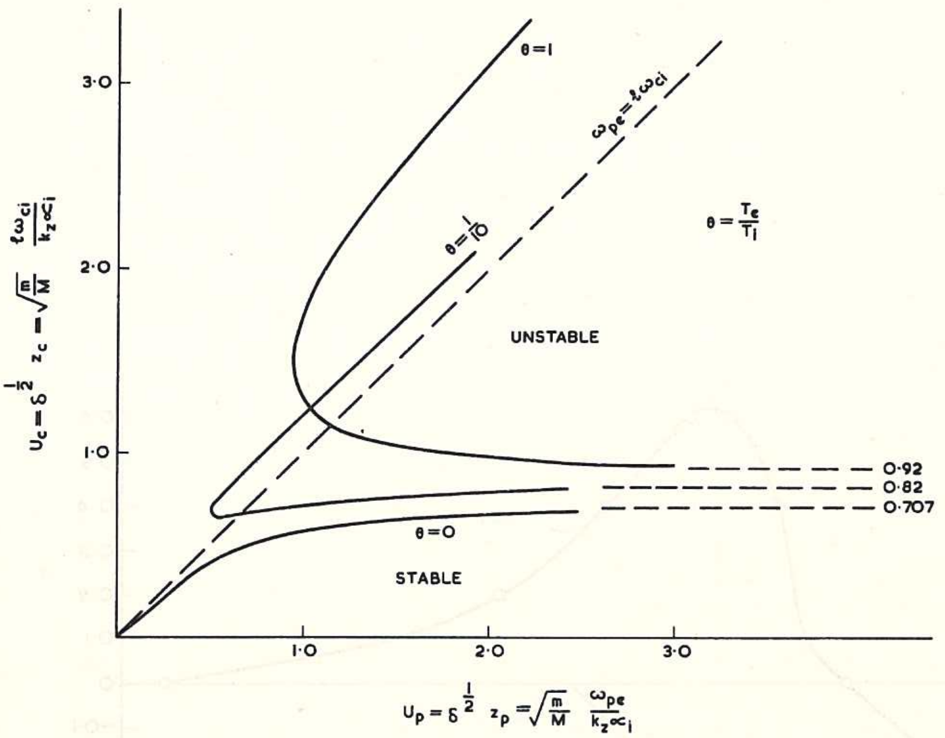
1. BURT, P. and HARRIS, E.G. Unstable cyclotron oscillations in a cylindrical plasma shell. Phys. Fluids, vol.4, no.11, pp.1412-1416, November, 1961.

2. HARRIS, E.G. Plasma instabilities associated with anisotropic velocity distributions. J. Nucl. Energy, Part C, vol.2, pp.138-145, 1961.

3. DRUMMOND, W.E., ROSENBLUTH, M.N. and JOHNSON, M.L. Instabilities due to anisotropic velocity distribution. Bull. Amer. Phys. Soc., series 2, vol.6, no.2, pp.185, 20 March, 1961. (Abstract only).

4. TIMOFEEV, A.V. Build up of ion acoustic vibrations in an anisotropic plasma. Soviet Physics - J.E.T.P., vol.12, no.2, pp.281-282, February, 1961.

5. KAMMASH, T. and HECKROTTE, W. Longitudinal oscillations in multicomponent plasma with anisotropic velocity distributions. IN:- University of California. Lawrence Radiation Laboratory. Controlled thermonuclear research semiannual report, July to December, 1962. UCRL 10607.



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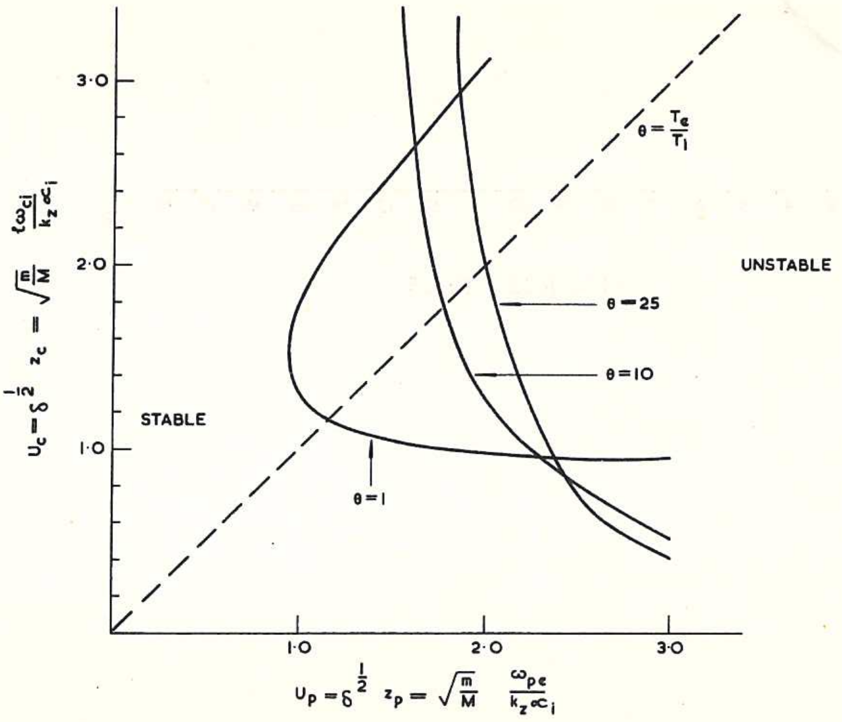

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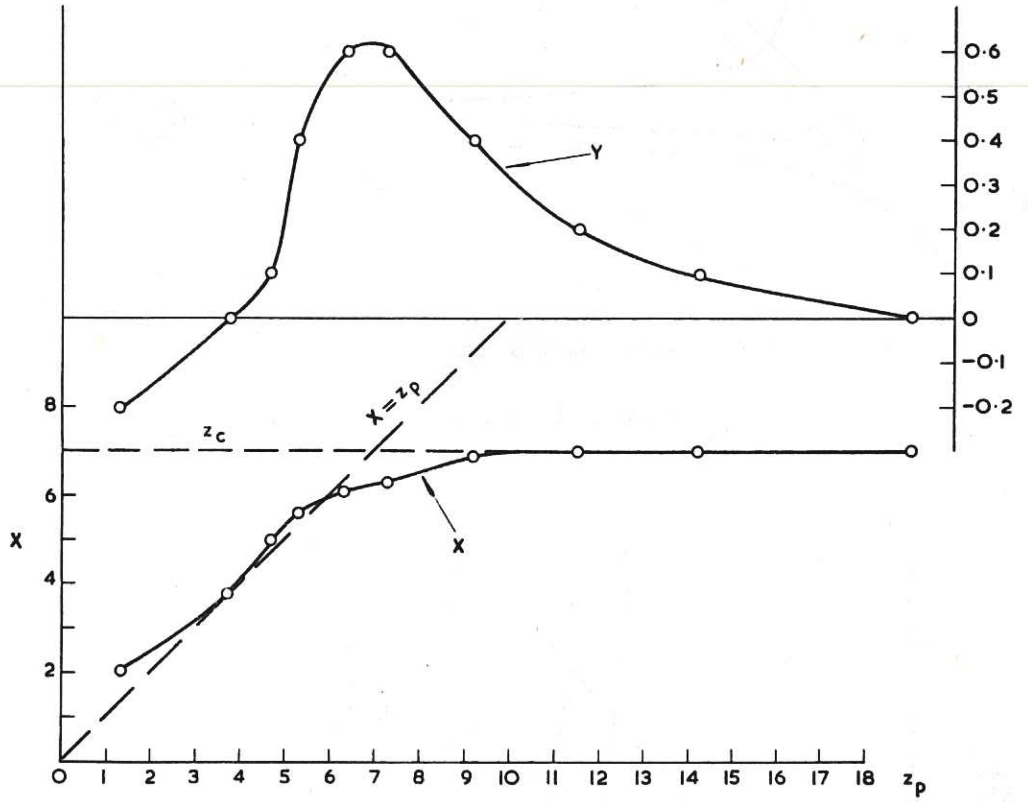

Y
X = z_p
z_c
X
0·6
0·5
0·4
0·3
0·2
0·1
0
−0·1
−0·2
Y
8
6
4
2
X
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
z_p

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