CLM-R 38

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United Kingdom Atomic Energy Authority

RESEARCH GROUP

Report

PLASMA IN MIRROR MACHINES

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1964

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by

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ABSTRACT

The behaviour of plasma in mirror machines has been, so far, determined by the instabilities which develop and, in the case of high energy ion injection devices, the occurrence of space charge potential. Steady state plasma waves out of which instabilities grow, as well as the theoretically predicted instabilities themselves are discussed. Experimental results which are presented show limited agreement with theory. Some comments are made on space charge and plasma build-up in high energy injection devices. The Culham Laboratory Resonance Trapping Mirror Machine is described briefly and its plasma behaviour anticipated.

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June, 1964 (C/18 - ED)

C O N T E N T S

T A B L E S

1. INTRODUCTION

1. The term mirror-machine is used here in a somewhat restricted sense, being applied only to devices which aim to use a conventional type of field configuration (Fig.1) to confine charged particles for a relatively long period of time. Cusp devices and those employing short period oscillatory fields, such as the θ-pinch, are excluded. The machines in which we are interested fall into two classes, depending on whether a complete plasma is trapped in the bottle at the start of the operating cycle or is built up gradually during the cycle. The class of machines beginning their cycle of operation by trapping an existing plasma, which can either be formed within the bottle, or injected into it in a single shot, we shall call neutral plasma devices. The other class, relying on the ionization of neutral gas by trapped energetic positive ions to build up a plasma, are referred to as high energy injection devices.

2. Ideally the containment of plasma in a thermonuclear reactor employing mirror fields is limited by scattering losses, mainly through the mirrors, and by some loss due to charge exchange between positive ions and background gas molecules. However, we are not immediately concerned with thermonuclear reactors as such, but rather with problems which are at present standing in the way of the achievement of quite modest particle densities and containment times. The most serious problems appear to be associated with plasma instability.

3. Many plasma instabilities are oscillations which have "blown-up", and it is often easier to visualize the physical processes involved in an instability by using a picture of the corresponding wave motion. For this reason we shall consider waves in plasma before dealing with instabilities themselves. As well as facilitating an appreciation of the stability problem it is hoped that this will also help to place limits on the types of permissible waves and instabilities which might occur under particular experimental conditions. In so far as the theory of instabilities is concerned we shall do little more than present the results of theoretical treatments. Following a discussion of instabilities, comments on some experimental results will be presented.

4. In any discussion of results from high energy injection machine experiments, the topic of space charge potential will arise. The occurrence of this additional complicating factor in the study of plasma behaviour has not been satisfactorily explained, and the inability to build a neutral plasma by retention of sufficient electrons within the mirror system to neutralize the positive ion content can be a serious problem. It appears

to be connected with instability but whether as a cause or effect, or merely through observational correlation, has yet to be determined. Since the doubt as to its precise relationship to instability exists, we shall discuss space-charge separately in section 5.

5. On the whole, discussion of experiments will be limited to work which has been published. However, because of our own particular interests, some remarks will be offered, in section 6, concerning the Culham Laboratory Resonance Trapping Mirror Machine which has recently come into operation.

2. WAVES IN PLASMAS

6. Steady state wave motion in a plasma generally involves an oscillatory interchange of energy between fields and particles. When the energy flow becomes unbalanced, so that there is a net flow of energy in one direction, instability can occur. In the case of small amplitude oscillations the variables concerned, such as electric field, magnetic field, particle velocity, particle density or pressure can be expressed in the form $A = A_o$ exp. $i(\bar{k}.\bar{r} - \omega t)$, where the angular frequency is $\omega = a + ib$ and the wave number is $k = c + id$. Whether instability exists depends on the behaviour of ω, a wave of constant amplitude being maintained only while b is zero. With a and b both non-zero a positive value of b indicates a wave of growing amplitude, a case of "over-stability". Should a become zero the wave motion turns into a non-oscillatory disturbance, varying exponentailly with time, exhibiting damping or instability depending on whether b is negative or positive. However, the term "instability" is generally used to cover any situation in which b is positive, whether a is zero or not. In situations where wave propagation is the main concern, interest is centred on k. A positive value of d indicates that the wave amplitude decreases with distance, and propagation is not spatially maintained.

7. Since plasma is a dispersive medium the wave number and frequency of a wave are not simply and linearly related. The form used for the dispersion relation linking them will depend on plasma conditions and whether one wishes to determine the behaviour of k or ω as the dependent variable. For electromagnetic waves a determination of the dielectric constant, K, or index of refraction $\mu = K^{\frac{1}{2}} = \frac{c}{V_\varphi}$ where V_φ is the phase velocity of the wave, provides a dispersion relation.

WAVES WITH NO D.C. MAGNETIC FIELD

8. When a plasma experiences no D.C. magnetic field the dielectric constant is readily

obtained from consideration of the induced electric dipole moment per unit volume, in the presence of an electric field given by $\bar{E} = \bar{E}_o \exp(i\omega t)$. As shown, for example by Chandrasekhar[1], $K = 1 - 4\pi N_e e^2/m_e\omega^2$. Without a magnetic field being present three types of simple waves can be supported. Longitudinal electron oscillations, with a frequency $\omega = \omega_{oe} = \frac{(4\pi n_e e^2)^{\frac{1}{2}}}{m_e}$ do not propagate since both K and the group velocity $d\omega/dk$ are zero for $k\lambda_{De} \ll 1$ where λ_{De} is the Debye length. Transverse electromagnetic waves, with a dispersion relation $c^2k^2 = K\omega^2$, or $\omega^2 = \omega_{oe}{}^2 + c^2k^2$ can be propagated without attenuation when $\omega > \omega_{oe}$. Acoustic waves are propagated at velocities determined by the temperature of the plasma components. If ions and electrons are at temperatures T_i and T_e respectively, each plasma component might by expected, in principle, to support an acoustic wave for which the dispersion relation is $\frac{\omega}{k} = (\frac{\gamma T}{m})$, where T includes the Boltzmann constant. The phase velocities would be equal to the group velocities, being the speeds of sound in the ion and electron gases, respectively $c_i = (\frac{\gamma T}{m_i})^{\frac{1}{2}}$ and $c_e = (\frac{\gamma T_e}{m_e})^{\frac{1}{2}}$. However both ion and electron components are generally involved in an acoustic wave, and restoring forces involve electric fields as well as pressure. As shown by Spitzer[2] the velocity of a low frequency acoustic wave is given by:-

$$V^2 = \frac{\gamma_e T_e + \gamma_i T_i}{m_i}$$

THE EFFECT OF A MAGNETIC FIELD

9. The immersion of a plasma in a magnetic field greatly complicates the behaviour of waves. As well as replacing a scaler pressure by a tensor and the dielectric constant by a tensor, the field introduces two additional basic frequencies, the ion and electron cyclotron frequencues, $\omega_{ci} = B_o e/m_i c$, and $\omega_{ce} = B_o e/m_e c$ respectively. Any wave is now strongly influenced by relations between ω and ω_{ci} and ω_{ce}, as well as between the cyclotron frequencies and the electron plasma frequency, ω_{oe}. Treatments of wave behaviour are simplified if $\bar{k}$ is taken perpendicular or parallel to $\bar{B}_o$. Where the angle between $\bar{k}$ and $\bar{B}_o$ is neither zero nor $\pi/2$, dispersion relations can be carried into tractability only by making various approximations and imposing particular conditions on the relations between basic frequencies and the variables concerned.

THE USE OF HYDROMAGNETICS IN THE ABSENCE OF COLLISIONS

10. Where plasma densities are sufficiently high for the collision frequency to be of the same order as other frequencies of interest the dielectric constant becomes complex and wave behaviour is considerably modified. In mirror machines, at particle densities so far

envisaged, it is legitimate to neglect collisions. However, much of the theoretical work dealing with wave and instabilities is based on the hydromagnetic equations which, in themselves, rest on an assumption of high collision frequency. Chew et al.[3], attempted to resolve this difficulty. They showed that for low density plasmas in the presence of a magnetic field the Larmor radius of particles might be considered as their effective mean free path for motion perpendicular to the field but that, in general, there is no justification for using a hydromagnetic description unless some restriction can be placed on particle motion parallel to the field. Hydromagnetic descriptions are sufficient in some instances, particularly for obtaining an overall picture of steady state oscillations associated with plasma. It is customary to regard two fluid hydromagnetics as applicable to plasmas for which the Alfvén speed, $c_A = \frac{1}{c}(1 + 4\pi n_o m_i c^2/B_0^2)^{-\frac{1}{2}}$ can be approximated as $c_A^2 = B_0^2/4\pi n_o m_i$, where $n_o = n_i = n_e$ is the particle density and B_0 is the D.C. magnetic field.

DISPERSION RELATIONS FOR LOW DENSITY UNBOUNDED PLASMA

11. If a hydromagnetic description of plasma is accepted dispersion relations determining ω^2 can be obtained by using the equations of hydrodynamics for electron and ion gases, together with Maxwell's equations, and the assumption that variables are of the form $\exp. i(\bar{k}.\bar{r} - \omega t)$. Writing $B_o = B_o\bar{h}$, where $h^2 = h_x^2 + h_y^2 = 1$, and $\bar{k} = k_x\,\bar{1}$, where $\bar{1}$ is the unit vector in the x-direction, it is only a matter of algebra to arrive at the two equations for particle velocities, v_e and v_i, determined by the wave motion. Thus:-

$$\omega^2\bar{v}_e - \frac{\omega_{oe}^2}{\omega^2 - c^2k^2}\left[c^2k^2(v_{ix} - v_{ex})\,\bar{1} - \omega^2(\bar{v}_i - \bar{v}_e)\right] - c_e^2k^2v_{ex}\bar{1} + i\omega\omega_{ce}\bar{v}_e \times \bar{h} = 0, \tag{1}$$

and

$$\omega^2\bar{v}_i + \frac{\omega_{oi}^2}{\omega^2 - c^2k^2}\left[c^2k^2(v_{ix} - v_{ex})\,\bar{1} - \omega^2(\bar{v}_i - \bar{v}_e)\right] - c_i^2k^2v_{ix}\bar{1} - \omega\omega_{ci}\,\bar{v}_i \times \bar{h} = 0, \tag{2}$$

where $\omega_{oi}^2 = \frac{m_e}{m_i}\,\omega_{oe}^2$. In these equations the first term in the square bracket arises from the presence of an oscillating space charge while the second term depends on the presence of conduction current. The equations are essentially the same as those used by Braginskii[4]. They offer six solutions for ω^2: three at high frequencies and three at low frequencies. For high frequency oscillations ion motion is neglected (i.e. $v_i = 0$) and the Appleton-Hartree formula is obtained, with additional terms in c_e^2. Braginskii assumes $\omega_{oe}^2 \gg \omega_{ce}^2$ in obtaining his high frequency solutions, in order to simplify the problem. Using the Appleton-Hartree formula directly, Mitra[5] has given dispersion curves including regions for $\omega_{ce} > \omega > \omega_{oe}$. This dispersion relation, cubic in ω^2, has roots corresponding to right-hand and left-hand polarized E-M waves and longitudinal plasma oscillations. In the low frequency range it is assumed that electrons will have time to neutralize any space charge and that displacement current can be neglected in comparison with conduction current. This frequency region has been investigated for special conditions by Alfvén[6], Åström[7] and Baños[8,9], more generally by Braginskii, and over a wider range of conditions by Stringer[10]. Low frequency oscillations have so far appeared to be the more important from the point of view of instabilities.

12. Hydromagnetics does not cover the entire range of plasma conditions met with in mirror machines. This is shown in Fig.1, where relationships between plasma parameters are plotted as functions of n_o/B_o^2 (cm^{-3} $gauss^{-2}$). In the figure it has been assumed, in determining c_s^2/c_A^2 and β that one plasma component contributes predominantly to the pressure term. Thus if c_i^2 and $\frac{m_e}{m_i} c_e^2$ are not greatly different n_o should be taken as $n_o = n_i + n_e$ instead of $n_o = n_i = n_e$. Also, in order to have the c_s^2/c_A^2 curves asymptotic to the β-curves for the same temperature, the assumption is made that $\gamma = 2$. Names of some mirror machines are entered opposite their appropriate values of n_o/B_o^2, corresponding to the approximate operating conditions of these devices shown in Table I.

13. In the non-hydromagnetic region for, say, $n_o/B_o^2 < 10$, equations (1) and (2), used without terms in c_e^2 and c_i^2, yield two simple solutions which are of interest for low frequency oscillations. When $\lambda \gg \lambda_{De}$, for propagation parallel to B_o, ω^2 is given by:-

$$\omega^2 = c^2k^2/(1 + \omega_{oi}^2/\omega_{ci}^2). \tag{3}$$

This is the E-M equivalent of the shear Alfvén wave which occurs at higher n_o/B_o^2 values. As n_o/B_o^2 increases c^2/c_A^2 is asymptotic to $\omega_{oi}^2/\omega_{ci}^2$ (see Fig.1) and equation (3) becomes $\frac{\omega^2}{c^2k^2}$ to be neglected, conduction and displacement current terms in modified equations (1) and (2) become unimportant and only time-varying electrostatic fields interact with the plasma. The dispersion relation for these short wave oscillations is

$$\omega^2 = \tfrac{1}{2}\left\{(\omega_{ci}^2 + \omega_{oi}^2 + \omega_{oe} h_x^2) \pm \sqrt{(\omega_{ci}^2 + \omega_{oi}^2 + \omega_{oe}^2 h_x^2)^2 - 4\,\omega_{ci}^2 \omega_{oe}^2 h_x^2}\right\} \tag{4}$$

14. Some low frequency dispersion relations are listed in Table II. Relations (i) to (vi) are Stringer's transition region relations for the three low frequency branches of the dispersion curve, while (vii) and (viii) apply to straight portions of the dispersion curve. As oscillation wavelength increases relation (iii) goes over to (vii), the Shear Alfvén wave, and relation (v) becomes Stringer's low frequency electro-acoustic wave, relation (viii). Equations (3) and (4) above are included in Table II as (ix) and (x). The

relations (vii) to (x) are stable counterparts of instabilities discussed in the next section.

15. The discussion of waves has involved no terms in ∇B_o, while ∇p has been included only as a small amplitude time-variable. In experimental situations both ∇B_o and ∇p_o can occur as non-oscillatory terms, leading to drifts in particle motions. These drifts are important when considering instability. Another condition, important in a mirror machine, but ignored above, is anisotropy of temperature. Although it has been recognised that the magnetic field constrains the motion of particles, a scalar pressure was assumed, and no allowance was made for the fact that $T_\perp$ can differ from $T_\parallel$, where $T_\perp$ and $T_\parallel$ are the temperatures depending on particle energy of motion perpendicular and parallel to B_o respectively. In a mirror machine containment depends on $T_\perp > T_\parallel$. These points, omitted here, are necessarily included in any discussion of instabilities. The unrealistic assumption of an unbounded plasma, made in order to simplify treatments of waves, is also found in treatments of some instability problems.

3. INSTABILITIES

16. For the purpose of discussion, instabilities will be divided into the two causal classes used in Table III: that including instabilities which arise as a result of an anisotropic velocity distribution, the so-called "velocity-space instabilities" and that covering instabilities having their origin in the existence of gradients in configuration space.

VELOCITY SPACE INSTABILITIES

17. Instabilities which develop as a result of an anisotropic velocity distribution provide a mechanism for developing isotropy. If the plasma volume is infinite the instability would be expected to die out as velocity isotropy is reached: for plasma confined in a finite volume of not very great extent the instability leads to particle loss. Three types of velocity space instability which have received considerable attention will be discussed.

MIRROR INSTABILITY

18. Using a quasi-hydromagnetic description of a plasma, Rudakov and Sagdeev[11] examined the effect of perturbations in such quantities as pressure, magnetic moment of plasma per unit volume, and magnetic field on oscillations in the frequency region well below ω_{ci},

and for wavelengths greater than the ion Larmor radius (r_{Li}). For a sufficiently dense plasma this places the wave motion in the region referred to by equation (viii) in Table II. The dispersion relation they obtained admitted of aperiodic solutions ($\omega^2 < 0$) when the velocity distribution was sufficiently anisotropic. For $T_\perp > T_\parallel$, as would be the case in a mirror machine, the situation corresponds to an unstable hydromagnetic sound wave, with $\bar{k}$ directed nearly perpendicular to $\bar{B}_o$. The condition for instability was found to be

$$e \frac{4\pi\mu}{B_o} > 3 \frac{4\pi p}{B_o^2} + \left[9 \left(\frac{4\pi p}{B_o^2}\right)^2 + 3 \left(\frac{4\pi p}{B_o^2}\right) \right]^{\frac{1}{2}}$$

where μ is the magnetic moment per unit volume of plasma.

19. The same problem has been treated by Vedenov and Sagdeev[12], using the Boltzmann equation instead of a hydromagnetic description of plasma. The condition found for instability in this case was

$$\frac{8\pi n_o k' T_\perp}{B_o^2} \frac{T_\perp}{T_\parallel} > 1 + \frac{8\pi n_o k' T_\perp}{B_o^2}$$

or

$$\frac{p_\perp}{(B_o^2/8\pi) + p_\perp} > \frac{p_\parallel}{p_\perp}$$

This result, differing slightly from the previous one is likely to be more applicable in the case of a low density plasma. Vedenov and Sagdeev also show that the instability, in a system whose dimensions are much greater than the perturbation wavelength, which in its turn is much greater than the ion Larmor radius, will lead to a transfer of energy from motion perpendicular to $\bar{B}_o$ to motion parallel to the field.

20. A qualitative picture of the instability can be obtained in terms of the oscillation which is its stable counterpart. Consider an ion acoustic wave whose frequency is well below the ion cyclotron frequency and whose wave vector is directed at an angle θ, of nearly $\pi/2$, to the static magnetic field. This wave causes both bending and compression of field lines, producing a system of micromirrors, as well as variation in particle density, as shown in Fig. 2. With this type of oscillation field lines and plasma can be described as moving together, compression being opposed by both magnetic and plasma pressure, the "restoring forces". The plasma ions with a sufficiently large ratio of $v_\perp/v_\parallel$ will tend to follow helical paths from the regions of compression to those of rarefaction, thus withdrawing their contribution to the restoring force and at the same time increasing the pressure in the wave regions of rarefaction. A reduction in restoring force means a reduction in ω. With sufficient anisotropy so much plasma will be transferred to the "magnetic bottles" and retained there that these will become the regions of high plasma pressure,

while the mirror throats are left practically devoid of particles. Under these conditions, providing $p_\perp$ is sufficiently large compared with B_o, ω can become negative with resulting instability. Owing to the acceleration of ions along the field lines from the mirror throats, energy of transverse motion will be converted to energy of longitudinal motion. In addition, as a result of ion bunching between the mirror regions, electrons, free to move along magnetic field lines, could well develop electrostatic oscillations about the positive potential wells so formed.

Alfvén (or EM) Wave Resonance Instability

21. Whereas, in the "fire-hose" instability, an Alfvén wave type of perturbation transfers longitudinal energy to transverse energy, in other circumstances, where there is initially an excess of transverse energy, this type of wave can transfer energy in the reverse direction. This occurs when there is a Doppler-shifted resonance between the wave frequency and the cyclotron frequency of particles with a particular velocity component parallel to the wave vector, and $T_\perp$ is greater than $T_\parallel$. Rosenbluth[13] drew attention to the possibility of such an instability if the condition $\omega + k_z v_z = \omega_{ci}$ was met, where $\bar{B}_o = B_z$ and V_z is the velocity of "resonant ions". In the case of mirror machines with low density plasmas, true Alfvén waves will not occur, being replaced by electromagnetic waves (see equations vii and ix, Table II). Instability under these circumstances has been examined by Sagdeev and Shafranov[14]. They found that for $T_\perp > T_\parallel$, the case of interest in mirror machines, a circularly polarized wave builds up, for ions or electrons which are in resonance, when the electric wave vector rotates in the direction of Larmor gyration of the particles concerned. The wave grows as a result of a drift of particles in a direction opposed to their acceleration by the electric field of the wave. In the case of ion resonance it was shown that the maximum growth rate occurs for a frequency given by

$$\omega = \omega_{oi} \frac{T_\perp - T_\parallel}{T_\perp} + i\, w_{oi} \frac{T_\perp}{\sqrt{2\, T_\parallel m_i c^2}} \exp. \left\{ - \frac{B_o^2}{8\pi n k T_\parallel} \frac{T_\parallel^2}{(T_\perp - T_\parallel)^2} \right\}$$

Electron resonance has maximum effect for

$$\omega = \omega_{oe} \frac{T_\perp - T_\parallel}{T_\perp} + i\, \frac{\pi^{1/2}}{4}\, \omega_{oe} \frac{(T_\perp - T_\parallel)^{3/2}}{T_\perp} \frac{T_\perp}{T_\parallel} \left(\frac{2\, T_\parallel}{m_e c^2}\right) \exp. \left\{ - \frac{B_o^2}{8\pi n k T_\parallel} \frac{T_\perp}{T_\perp - T_\parallel} \right\}$$

These equations are based on the assumption of a Maxwellian type velocity distribution of the form

$$f = \frac{M}{2\pi T_\perp} \left(\frac{m}{2\pi T_\parallel}\right)^{1/2} \exp. \left(- \frac{\varepsilon_\perp}{T_\perp} - \frac{\varepsilon_\parallel}{T_\parallel} \right)$$

and in obtaining the dispersion relation for electrons ω_{oe} is taken as much greater than ω_{ce}. The relationship between the wave vectors and particle velocity vectors for $T_\perp > T_\parallel$, is shown, for the case where ions contribute to the growing wave, in Fig.3. With v_z and k in opposite directions, and $\omega \propto \frac{T_\perp - T_\parallel}{T_\perp}$, the resonant value of v_z will increase as the velocity distribution moves towards isotropy. Thus in Fig.3 it will be the velocity vectors below the z-axis which are concerned in the process. Vectors above the z-axis correspond to particles whose change in velocity produced by the B-field vector takes them to low v_z values, thus dropping them quickly from resonance, while particles with velocity vectors below the z-axis will provide a source of resonant ions which contribute to the E-field increase ($\bar{v}_\perp$ opposite $\bar{E}$) and at the same time have their transverse velocities converted to longitudinal velocities.

22. The minimum growth-time for the instability involving ions can be expressed as

$$t_i = \frac{m_i \, c \, \alpha^{\frac{1}{2}}}{\sqrt{2} \, \pi^{\frac{1}{2}} n^{\frac{1}{2}} e T_\perp^{\frac{1}{2}}} \exp. \left\{ \frac{\alpha}{(1-\alpha)^2} / \beta_\perp \right\},$$

where $\alpha = T_\parallel / T_\perp$. Taking the ratio of this to the ion-ion collision time, t_c Spitzer[2], we have

$$t_i/t_c = 3.3 \times 10^{-6} \ln \Lambda \, \frac{\alpha^{\frac{1}{2}} n^{\frac{1}{2}}}{(1+\alpha)^{3/2} \, T_\perp^{2}} \exp. \left\{ \frac{\alpha}{(1-\alpha)^2} / \beta_\perp \right\},$$

with $T_\perp$ in ergs. Since

$$\ln \Lambda = O(10)$$

and

$$\frac{\alpha^{\frac{1}{2}}}{(1+\alpha)^{3/2}} = O(10^{-1})$$

for the range of reasonable α values, the effect of the instability might be regarded as negligible provided

$$\frac{n^{\frac{1}{2}}}{T_\perp^{2}} \exp. \left\{ \frac{\alpha}{(1-\alpha)^2} / \beta_\perp \right\} > 10^5,$$

with $T_\perp$ in electron-volts. In Fig.4 curves are shown giving maximum values of $\beta_\perp$, as a function of $n^{\frac{1}{2}}/T_\perp^{2}$, with α as a parameter for which the above condition is satisfied. From the figure it appears unlikely that this type of instability will be a problem in mirror machines for some time to come. Another fact tending to preclude it is that possible frequencies of E-M waves, set by the dimensions of apparatus, will make resonance with ion cyclotron frequency unlikely with present B_o values and ion energies.

Coupled Ion and Electron Oscillations (Harris Instability)

23. In the presence of velocity distribution anisotropy there is the possibility of resonant coupling between ion and electron oscillations. If in equation (x), Table II, ω_{oi} is much smaller than the other terms two frequencies, ω_{ci} and $\omega_{oe} h_x$, appear as separate solutions. This topic has been treated by Harris[15-18] who examined small amplitude oscillations arising as a result of a small perturbation in a plasma particle distribution function. Instability was indicated for both Maxwell-Boltzmann and a δ-function type of distribution. When ω_{oe} is slightly greater than ω_{ci} a Doppler shifted resonance can permit coupling between the ion cyclotron motion and electrostatic electron oscillations. Similar coupling between electron cyclotron motion and plasma oscillations was predicted for ω_{oe} close to ω_{ce}. Drummond et al[19] have pointed out that a relatively high degree of anisotropy is called for in order to trigger the instability, giving the requirements $(\frac{T_\perp}{T_\parallel})$ (ions) > 8 - 9, and $(\frac{T_\perp}{T_\parallel})$ (electrons) > 2.

24. When the above conditions are met a perturbation in ion phase distribution with respect to its cyclotron motion will grow giving ion phase bunching. For electrons having a longitudinal velocity component relative to that of the ions, such that they see the electric field of the ions as having a period equal to the plasma frequency, there is coupling. Energy from the transverse motion of the ions is fed into the longitudinal oscillations of bunches of electrons. This type of instability has been a fashionable topic in association with some fairly recent experimental results and will be discussed again later.

Phase Bunching (Negative Mass Instability)

25. With reference to DCX 1 Fowler[20] showed that for monoenergetic ions, orbiting with their orbit radii antiparallel to the D.C. field gradient, bunching in azimuth will occur. The bunching in this case is the same as the well-known phase bunching of ions in some cyclic accelerators, such as the proton synchrotron. If an ion has a velocity slightly in excess of the mean of the bunch it will move to a larger radius where the field is weaker and thus its angular frequency is reduced, bringing it back, in azimuth, towards the bunch. In the case of an ion of reduced energy it tends to move to a region of higher field thus achieving an increase in angular frequency. Fowler and Harris[21] have concluded, from further theoretical investigation, that the "negative-mass" instability and the Harris instability are more or less independent : the former requiring $k_y \gg k_z$, the latter, $k_z \gg k_y$ where k_z is parallel to B_o and k_y taken along the ion orbit.

INSTABILITIES DUE TO GRADIENTS IN CONFIGURATION SPACE

26. We shall discuss the flute instability and then consider two types of plasma misbehaviour which are claimed to be universal.

Flute Instability

27. The flute instability owes its origin to the fact that when a plasma is held by a confining magnetic field whose gradient is directed towards the plasma, an effective interchange between the field containing plasma and the vacuum field is possible. The problem was initially treated by Rosenbluth and Longmire[22], who showed that from considerations of first orbit theory, and from energy considerations, plasma in a mirror type confining field would develop unstable flute-like deformations of its surface. By first order orbit theory it is readily shown that for an initial surface perturbation $\Delta x = a \sin ky$, where $\bar{B}_o = B_z$ and the plasma surface is taken as parallel to the y-z plane, an equation of motion $d^2a/dt^2 = gka$ can be written where g expresses a gravitational-type force which is roughly equivalent to $\frac{\bar{R}}{R^2}\ (\upsilon_{\parallel}^2 + \frac{1}{2}\upsilon_{\perp}^2)$, where $\bar{R}$ is the radius of curvature of the magnetic field lines. If g is negative an oscillatory motion of the plasma surface occurs with $\omega = |gk|^{\frac{1}{2}}$, but if g is positive, as it is when ∇B is directed towards the plasma, the perturbation grows exponentially. The growth of the flute instability is shown more rigorously in terms of the energy of the system of the field and plasma, the instability providing a mechanism for lowering the potential energy of the system.

28. Kadomtsev[23,24] has also analysed the behaviour of plasma under these conditions, arriving at results in agreement with Rosenbluth and Longmire. In another treatment[25] of the problem he shows the analogy between the flute instability and thermal convection in an incompressible fluid, the fluid being represented by the magnetic field while the plasma pressure plays the part of the fluid temperature. In this way he arrives at a "containment time" for a plasma in a mirror machine with conducting walls. We shall refer to this situation when discussing experimental results.

29. Although the flute instability stands as a serious obstacle to mirror confinement of plasmas there appear to be conditions under which its effects can be considerably reduced if not eliminated. One obvious method, of course, is to reverse the direction of ∇B. This is just what is attempted in cusp-geometries. Kadomtsev[25] suggested the employment of a B_z field whose value increases with radius together with a B_θ field produced by an axial current. The azimuthal field reverses the gradient of B for smaller radii, producing a toroidal region of confinement whose z-dimension is greater than its radial one,

having the confining field increasing away from the plasma in all directions. As far as is known this field configuration has not as yet received experimental attention. The use of external current carrying conductors, parallel to the machine axis, in conjunction with a mirror type meridian field has been employed experimentally to convert a normal mirror field to one in which ∇B is directed away from the machine axis. This experimental arrangement due to Ioffe will be discussed later.

30. Even when the magnetic field configuration is conducive to flute instability it will not necessarily occur if other conditions prevail against it. Some of these will be discussed with experimental results. One circumstance which is believed to preclude all but $m = 1$ flute-instability has been discovered by Rosenbluth et al[26]; this is the condition of "finite Larmor radius". Where the ion Larmor radius is sufficiently large compared with that of the electrons, the two particle species will experience slightly different perturbed electric fields, where the perturbation is due to the drift of particles in the presence of a magnetic field gradient. The different rate of drifts of the particles in the perturbed fields they experience can build up a charge separation out of phase with that which drives the flute, thus cancelling it. For this type of flute stabilization to be effective the Debye radius associated with the plasma is required to be considerably less than the ion Larmor radius.

31. It was mentioned above that finite Larmor radius effects will not necessarily suppress the $m = 1$ instability. This situation is shown in the theory by the fact that the perturbed electric field which drives the instability, taken as derived from a potential ψ, is uniform, independent of position, for the case where $m = 1$. Thus the effect of difference in mean position of particles due to their differing Larmor radii, produces no difference in average electric field which is experienced. Rosenbluth suggests that an external conducting wall, whose radius is not much greater than the plasma surface radius, may suppress the $m = 1$ instability.

Short Wave Electrostatic Instability Due to ∇B

32. Krall and Rosenbluth[27] have examined the stability of a plasma experiencing a small magnetic field gradient perpendicular to the D.C. confining field. The co-ordinate system and parameters are shown in Fig.5. The behaviour of electrostatic modes, propagating in the y-direction is considered, where the wave amplitude is proportional to $\exp(i\omega t)$, with real $\omega \simeq \omega_{ci}$. A stable electrostatic mode would conform to a dispersion relation of the type given by equation (x) in Table II. Due to the field gradient particles have a drift

velocity v_{yD}. When $v_{yD} \sim \frac{\omega}{k}$ there is the possibility of unstable oscillations. If the magnetic field $\bar{B} = B_o\,(1 + ax)$ and r_i is the ion Larmor radius, it is found that for relatively large non-uniformities, for which

$$ar_i > \left(\frac{m_e}{m_i}\,\frac{T_i}{T_e}\right)^{\frac{1}{2}},$$

or $kr_e < 1$, growth rates of the order

$$(ar_i)^2 \left(\frac{T_e}{T_i}\right)^3 \omega_{ci}.$$

For smaller non-uniformities, such as when $kr_e > 1$ or

$$ar_i < \left(\frac{m_e}{m_i}\,\frac{T_i}{T_e}\right)^{\frac{1}{2}}$$

the growth rate is

$$(m_e/m_i)\;(T_e/T_i)^{\frac{1}{2}}\;\omega_{ci}.$$

It was suggested that if ∇B has a component in the direction of B_o the resonance might be destroyed by a variation in ω_{ci} as particles moved along the field lines. Further, it was pointed out, even if this instability were to grow, because such short wavelengths are involved, no gross motion perpendicular to B_o will be expected. The instability would cause enhanced diffusion in velocity space.

Low Frequency E-M Instability due to ∇P

33. In this case an electromagnetic wave, propagated in a direction nearly perpendicular to B_o, is unstable in the presence of particle density gradients or temperature gradients. Fig.6 shows the co-ordinates and conditions pertaining to this type of instability, as discussed by Mikhailovskii and Rudakov[28]. Acted upon by the fields E_1 and B_1 of the wave, electrons with a z-component of velocity equal to $-\omega/k_z$ will drift in the direction of $-\nabla p$. Thus, for example, with ∇p due to ∇n, the number of resonant particles at any x-plane is continuously increasing. The energy, transmitted to the wave by electrons moving in the direction of $-Ez$, and in phase with E, builds up, giving conditions of instability.

34. On the basis of intuitive considerations, the condition for instability is shown to be $1 - \frac{\bar{\omega}}{\omega} < 0$ where

$$\bar{\omega} = \frac{k_y\, cT_e(\partial n/\partial x)}{e\,B_o\,n}\left\{1 - \tfrac{1}{2}\,\frac{\partial \ell n.T_e}{\partial \ell n.n}\right\}$$

In deriving the above condition for instability it was assumed that $T_e = T_i$, and that the electron velocity distribution was Maxwellian in V_z. With a pressure gradient due to the ions, as well as that of the electrons, the ions will drift across the magnetic field with a velocity

$$v_{id} = \frac{c}{en} \frac{\bar{B}_o}{B_o^2} x\nabla p_i .$$

Equating the ion drift velocity with the y-component of the phase velocity of the wave, ω/k_y, the instability condition becomes

$$T_e\ (\partial n/\partial x)\ (1 - \tfrac{1}{2} \frac{\partial \ell n.T_e}{\partial \ell n\ n}) > \nabla p_i ,$$

Or, if we take $\nabla p_i = T_i(\partial n_i/\partial x)$, and $(\partial n_i/\partial x) = (\partial n_e/\partial x)$, with $T_e = T_i$, the condition is

$$\frac{\partial \ell n.T_e}{\partial \ell n.n} > 2, \text{ or } < 0,$$

as found by Rudakov and Sagdeev(29).

35. The same kind of intuitive arguments can be applied when the ion drift is due to a magnetic field gradient, in which case the condition for instability (taking $\omega/k_y = v_{id}$) becomes

$$\frac{T_e(\partial n/\partial x)}{n} \left(1 - \tfrac{1}{2} \frac{\partial \ell n.T_e}{\partial \ell n.n} \right) < \varepsilon_\perp \frac{\nabla B}{B} ,$$

where $\varepsilon_\perp$ is the ion energy due to motion perpendicular to B_o.

36. The last two instabilities we have discussed are two of the many "universal instabilities" which have been foreshadowed by theory. Some of these are listed without comment, by Rostoker(30).

CLASSIFICATION OF INSTABILITIES

37. There is some justification for dividing instabilities into two main groups (see for example Kadomtsev(31)), "hydromagnetic", and "kinetic" or "micro-". However, apart from the possibility of the overlapping of the hydromagnetic and kinetic treatment of problems, theoretically, there is also doubt as to whether instabilities naturally belonging to the kinetic class will not have large scale effects. The difficulty with this terminology is that theoretical methods of treatment, initial conditions and results of the instabilities are all involved.

38. The instabilities we have discussed are listed in Table III. The Table also indicates the conditions required by theory for the instabilities to develop, their frequency and growth rates, the sources from which driving energy comes, as well as the physical processes involved. In the remarks column, where experimental results are noted, it is not assumed that the particular instability is operative, but merely that the results are consistent

with it being so. The references given are those appearing in the text.

4. DISCUSSION OF SOME EXPERIMENTAL RESULTS

39. Although there is a wealth of experimental information from mirror machines, much of it exists in a rather depreciated currency. For this reason we shall only discuss some of those results which appear meaningful and reliable, either as evidence bearing on the theory of containment or as posing definite problems whose solution must yet be found. Much of the discussion will deal with instabilities, some will anticipate the following section on space charge neutralization problems. For the sake of simplicity experimental material will be presented according to the devices responsible for it, following which some summarizing comments will be made at the end of the section.

ION MAGNETRON - (U.S.S.R.)

40. In this device[32-35] breakdown of neutral gas and the production of plasma are accomplished by the application of a radial electric field between a central arc discharge and the walls of the vacuum chamber. The plasma behaviour has been studied after removal of the breakdown field. Detection of ions escaping to the walls and through the mirrors has indicated that more than 80% of the ions were lost across the field lines. By placing detectors at different azimuthal positions, as well as z-positions, on the vacuum chamber wall it was established that the escaping ions arrived in phase to detectors having the same azimuthal position, independently of their z-position, whereas there was no obvious correlation between the arrival of ions at various θ-positions. It was thus suggested that the particle loss was due to a flute-type instability. On this basis it was claimed that life-times for plasma containment were about two orders of magnitude longer than would be expected according to the theory of Rosenbluth and Longmire[22].

41. The time of containment was examined in the light of Kadomtsev's theory, taking into account the effect of conducting walls, assumed to lie roughly parallel to the magnetic field lines. Qualitatively the effect of the walls is to reduce the E_θ field driving the instability in the vicinity of the walls, it being, of course, zero at the wall surface. Unfortunately the rather nice agreement between Kadomtsev's theory and Ioffe's results hangs on the chosen value of two parameters which cannot be readily or satisfactorily evaluated. Were the conducting walls to reduce the flute instability they could not, of course, suppress it completely as the effect is the removal of ions which are driven to the wall by the instability.

42. A preliminary report of a further experiment aimed at suppressing the flute instability was communicated by Ioffe[36] at Salzburg, a fuller report appearing in 1963[37]. This included the reversal of ∇B. So called "Ioffe windings", were added to the basic field of his ion magnetron, roughly following the lines of force of the basic field, around the axis of the trap, as sketched in Fig.7. With this field configuration measurements of escaping neutral particle flux indicated a considerable increase in plasma lifetimes, as well as a great reduction in plasma density fluctuations. One effect of this type of field, due to the fact that field lines, apart from those passing close to the axis of the machine, are likely to cut the vacuum chamber wall, is to reduce the allowed radius of the plasma containment region compared with what it would be in the absence of the Ioffe windings. Altogether the field configuration is complex and the multiple windings need to be taken into account in initial design of any device, rather than to be added to an existing device, before proper comparisons can be made.

43. It has been claimed from Los Alamos[38] that the addition of Ioffe windings to "Scylla III" reduced by a factor of two the frequency of plasma distortion, restrained the distortion to be symmetric about the axis, delayed its onset and reduced its growth rate. These results are consistent with Ioffe's observations on the modified ion-magnetron.

"TABLE-TOP": SINGLE STAGE COMPRESSION DEVICE

44. This single stage compression machine receives its plasma charge in a single shot from a hydrogen (or deuterium) loaded titanium washer plasma source. In 1960[39,40] it was believed that in this device the mirror instability had been induced and observed under controlled conditions, by varying the β-value associated with the contained electron content of the plasma. The electron temperature exceeded that of the ions for the experiment. More recently[41] it has been established that the observed escape of electrons occurred at a much lower β-value than was credited in the earlier experiments thus ruling out the likihood of mirror instability. The importance of the correction of this error lies not so much in its relation to mirror instabilities as in the fact that there is no longer the necessity to explain the inordinately long containment times for particle densities associated with the experiment. The long containment time was originally explained on the grounds that the plasma was electrically shorted across the field lines because these intersected the conducting vacuum walls outside the mirror regions.

45. The more recent experiments have shown results consistent with the production of a flute type instability in a rotating plasma. The instability appears to set in earlier

for lower background gas pressure, and is stated not to occur at all if the pressure is as high as 4×10^{-6} mm Hg. In terms of plasma density, assumed proportional to the source discharge potential, the plasma appeared stable at low and high densities (0.5 kV and 5.0 kV respectively, applied to the plasma source), being unstable at intermediate densities. At higher source potentials more neutral gas is injected into the system but, it is assumed, the effect of this is cancelled by the fast pumping system employed. An interesting feature of these experiments was the onset of instability, in many instances as though it had been triggered by a mechanism originating outside the plasma, for example by the discharge of a spark gap near the apparatus. In subsequent experiments[(42)] for the purpose of studying the instability in plasmas having positive and negative space charges, triggering was accomplished by pulsing a nearby axial magnet coil. These experiments appeared to confirm that rotation of the plasma was due to an electric field, arising from an excess of ions or electrons in the plasma, crossing with B_0 (rotation being in opposite senses for plasma with opposite space charge potentials), as well as to indicate that the unstable plasma, consistent with an $m = 1$ mode flute-instability, rotated as an off-axis column, rather than as a "spinning arm" or "rotating wave on a disc" (see Figs. 1-4 ref (41)).

46. The triggering of a flute-like instability by external means has been observed by Sweetman[(43)] in plasma contained in "Phoenix" at AWRE. The fact that an instability can be triggered in this way, by subjecting it to pulsed electrostatic or electromagnetic fields suggests that a quasi-stable condition has persisted up to the instant of triggering. Although this may be regarded as a hopeful sign it is rather early for optimism on this account.

OGRA

47. The important experiments, experimental parameters and techniques, and results associated with "OGRA" have been reported by Kurchatov[(44)] and Bogdanov et al[(45,46)]. It has been found that for ion densities between 10^6 and $10^7 \mathrm{cm}^{-3}$, collective motion of the plasma leads to a loss of ions to the vacuum chamber walls in a manner consistent with a flute-type instability. At the same time electron loss, either across or along the field lines - in spite of and contributing to the large positive space charge - is considerable. By measuring the energy of secondary plasma ions passing through the mirrors, the plasma potential relative to the chamber walls can be inferred. The results obtained in this way are in reasonable agreement with estimates of an inferred radial electric field, based on measurements of the plasma rotation frequency.

48. A degree of stabilization against the flute-type instability, was obtained by applying positive potentials of 10 kV - 12 kV to electrodes placed near the mirrors, outside the trapping region for fast ions. The following possible explanations of enhanced stability were offered:-

(a) By draining practically all the electrons from the outer regions of the plasma, conditions for the formation of electrically polarized flutes are absent.

(b) The critical density for flute formation, as pointed out by Kadomtsev(31), increases with decreasing ratio of n_e/n_i.

(c) If electrons are removed from outer layers of plasma the $\bar{E}_r \times \bar{B}_o$ drift of electrons, at a slightly smaller radius, could exceed the total drift of the ions ($\bar{E}_r \times \bar{B}_o + \bar{B} \times \nabla B$) which, because of their larger radii, would experience a weaker average electric field. Thus a stabilizing effect could be produced.

49. In the absence of end electrodes, in the presence of the considerable positive space charge associated with the plasma, electrons would need energies at least comparable with the space charge potential in order to escape from the plasma potential well along the B_o field. One method of producing such energies might be a coupling between electrostatic electron oscillations and ion cyclotron motion, ω_{oe} and ω_{ci}. The ion gyro frequency is accurately known and estimates of ω_{oe} put it reasonably close to ω_{ci} (see Table II). In addition, the ratio of $v_\perp$ to $v_{\parallel}$ for ions, with injection at 70^o to B_o, is close to that required for the Harris instability. Although radiation from the plasma has been detected at the ion cyclotron frequency and harmonics, no positive identification of oscillating electric fields parallel to B_o as have been found with "Phoenix"(43), has been reported. However, by measuring the energy spread in a beam of electrons which has passed through the plasma, parallel to the axis, the presence of standing waves of the form $E_o \exp(i\theta + i(\omega t - kz) + i(\omega t + kz))$ has been inferred(47). The energy spread in a beam, of initial energy 1 kV, amounted to several hundred volts after a single passage through the plasma. This still leaves unsolved the question as to how the phasing of electrons from one transit to the next can be such that very large longitudinal energies can be built up.

OGRENOK

50. OGRENOK(48) is a scaled down OGRA-type device into which 10 keV atomic hydrogen ions are injected. The following measurements which are of interest have been made.

(a) Distribution of fast ions over the volume of the trap.

(b) Drifts of fast ions perpendicular and parallel to B_o.

(c) Plasma space charge potential.

(d) Density fluctuations of plasma at radii less than injection radius.

(e) Time resolved measurements of escaping secondary plasma components arriving at the mirror regions.

51. Although the high energy ion density is quite low (7×10^5 cm^{-3} - 6×10^6 cm^{-3}) collective behaviour of ions is apparent and there is evidence of flute-type instability. The high energy ions were found to precess about the axis of the machine at a higher rate than would be accounted for by magnetic field gradients alone, the precession frequency being that expected of a plasma cylinder charged to a potential of +100 volts. Fast ion current, detected in azimuth, showed oscillations at 10 - 20 kc/s which was regarded as consistent with rotation of a plasma column having a fluted perturbation, as well as oscillations at 100-200 kc/s and several Mc/s which correlated with modulations observed in the injected beam. However the secondary plasma, exhibiting a radial oscillation of 10-20 kc/s, showed no variation in θ.

52. The detection of low energy ions and electrons moving longitudinally in the mirror regions, suggest that the low energy ion content pulsated at the frequency of rotation of the high energy ion content, with flow to the mirrors being 180^0 out of phase with the flow to radial probes. Electrons were detected travelling into the mirrors as the low energy ions were moving to the walls. The current detectors measuring the arrival of particles showed depths of amplitude modulation as follows: ions moving radially, 100%; ions to the mirrors, 10%-20%; electrons to the mirrors, 30%-50%. At $\theta = 0$, the azimuthal position of the injection channel, electrons escaping to the mirrors showed only those modulation frequencies which were present in the injection beam, indicating that they were predominantly secondaries from the shell of the injector.

<u>DCX 1</u>

53. By injecting molecular ions and trapping only the atomic ions formed by dissociation the disadvantage of an injector snout is avoided in this machine. A beam of up to 5 mA 600 kV H_2^+ ions is injected into the vacuum chamber and passes through a 300 amp carbon arc which follows the magnetic field lines for some 10 ft and brings about dissociation of some of the molecular ions. 300 keV H_1^+ ions are trapped into a ring, with particle Larmor orbit radii of 8-9 cm, encircling the magnetic axis of the apparatus. Descriptions and discussion of apparatus and recent experiments are given in ORNL semi-annual progress reports[49-51] of the last two years. It is important to note that because the ion Larmor

radius is nearly the same as the radius of the plasma, the model used by Longmire and Rosenbluth for flute-type instability is not applicable to this experiment.

54. The lifetime of H_1^+ ions in DCX 1, at pressures and particle densities so far obtained, is limited by charge exchange alone and has a value extending up to some 37 seconds. Although instabilities are observed they are not responsible for loss of high energy ions, and the escape of secondary ions and electrons, on average at equal rates, takes place along the magnetic field lines through the mirrors. The small volume of plasma of one to two litres with the primary H_1^+ ions limited to short z-dimension, low base pressures of order 10^{-9} mm Hg, together with the fact that the plasma does not come into serious contact with the vacuum chamber walls, confer a simplicity on this experiment which gives it a considerable advantage for the study of a limited number of experimental phenomena. Whether the presence of the D.C. arc has any major unfavourable effect on the plasma behaviour is not known, although it would be expected to define a potential value at one azimuthal position of the ion ring. As a generator of large amplitude R.F. noise signals it is a disadvantage and sometimes when making R.F. measurements it has been found necessary to use gas dissociation of H_2^+ rather than suffer the arc.

55. Experimental observations have shown a considerable spread in energies of the circulating protons, inferred from measurements of energy of escaping charge-exchange neutrals. A tentative explanation of this was offered in terms of charge bunching in the plasma ring, with subsequent acceleration of protons by the potential well associated with each bunch. Evidence for the presence of bunches was also provided by R.F. measurements and high frequency fluctuations in measurements of the potential of the plasma which was probed by means of a Lithium beam travelling along the magnetic field lines. It is found that the charge bunching is more intense during injection, it being suggested that bunching requires a supply of monoenergetic protons.

56. More recent experiments[52] suggest the presence of an instability involving both energetic protons and secondary electrons. Accompanying increases in R.F. signals consistent with ion bunching there is a rise in plasma potential, as well as an increase in slow electron flow to collectors at the mirrors, followed by a reduced flow. Taking into account a rise in charge exchange neutral counting rate during the period of perturbed plasma potential followed by a slow return to normal it is suggested that degradation of proton energy occurs as a result of an increase in electron heating rate, during periods of instability. As all measurements do not fully support this picture it has only been

put forward by the laboratory as a tentative explanation of the processes involved.

57. Another effect attributed to the presence of ion bunches is the expansion of the plasma volume by increase in its z-extent.

SUMMARY AND COMMENTS

58. From the rather limited amount of more or less reliable experimental evidence available what conclusions can be drawn concerning the behaviour of plasma in mirror machines? In most devices there appears to be evidence of an azimuthal variation in particle density which can, in some instances, be put down to a fluted deformation of the plasma surface; a plasma potential, differing from zero, appears in all cases, most markedly in Ogra: primary plasma (or particle) lifetime, when measured for reasonable densities, cannot be accounted for in terms of charge exchange alone, except in the case of DCX 1: there appears to be some improvement in stability in the presence of conducting walls contiguous with magnetic field lines (Ion Magnetron), when electrodes in the mirror regions are positively charged (Ogra) and considerable improvement where ∇B is reversed (Ion Magnetiron). The loss of plasma particles, both primary and secondary, appears in all cases to be associated with instability, either flute or electrostatic or both.

59. In the two "pyrotrons", Toytop and Table Top it has been an $m = 1$ instability which was observed. As these devices deal with fairly dense plasmas purely hydromagnetic effects, in terms of diamagnetic currents reacting with non-uniform magnetic fields, can be used to explain the bodily movement from the machine axis, without recourse to the flute-instability theory. On the other hand the polarization which would meet the requirements of the flute-instability has been detected. Whichever driving mechanism is invoked, however, the instability has its roots in the inwardly directed magnetic field gradient. With the MTSE* apparatus(52) at Culham an $m = 1$ mode is most common, $m = 2$ sometimes occurs and there are suggestions that higher modes occasionally develop.

60. The Ion Magnetron plasma is born in quite a turbulent state and the plasma boundary is in all probability well deformed from the start. Here perhaps there is reasonable certainty that a growing flute-instability has been observed and, of course, similar devices using much denser plasmas(54) have produced deformations which have been photographed, convincingly indicating fluting. The success of suppression methods, particularly if the agreement with Kadomtsev's theory can be taken seriously, strongly supports the

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* Magnetic Trap Stability Experiment.

flute instability theory as explaining the mechanism here operating.

61. The H.E. injection devices have presented a different picture from the Ion-Magnetron, and the pyrotrons. DCX 1 from which primary ions are not lost by instabilities, has enabled some excellent observations to be made. However, even in this case a single satisfactory model for describing the behaviour of the plasma is lacking. Electrostatic instabilities appear responsible for passing energy to the electrons from the ions; ion bunching in orbit, which might be expected to lead to flute-type instabilities, has not done so in this case but, of course, in this device the ion orbits are concentric with the machine axis and the plasma density quite low. In the case of the neutral injection devices electrostatic oscillations have been observed in conjunction with electron loss and possible deformation of outer plasma boundary. As in the case of DCX 1 the instability, observed during injection, might rely on the supply of monoenergetic ions to the system. However, in both cases the onset of electrostatic instability may simply depend on the build up of electron density to a value that brings the plasma frequency sufficiently near the ion-cyclotron frequency for resonance coupling. The possibility of externally triggering the instability does not necessarily relate it to what is observed in Table-Top.

62. So many factors are involved on the Ogra and Ogrenok devices that reasonable plasma models are not possible. Here a flute-type instability is apparent as a θ-variation in high energy ion density, and electromagnetic oscillations and electron loss are also observed. If the electron loss is due to electrostatic instability, though evidence does not necessarily support this possibility, it could be expected that regions limited in θ would be electron depleted at one time and this would give rise to E_θ fields which would deform the plasma boundary into flutes. On the other hand there are quite serious field asymmetries, as well as plasma density asymmetry due to method of injection, which could be responsible for initiating a flute-like instability.

63. At this stage, in spite of some agreement between theory and experimental results, it is not possible to use experimental observations to give a reliable overall instability picture. However, it is reasonable to assume that any lack of symmetry about the z-axis in magnetic field, initial plasma production, or residual gas pressure is likely to produce radial motion in the plasma. In more recent experiments with neutral injection, electrostatic oscillations and electron loss appear to be displacing the "flute-instability" as the number-one problem. Indeed, except where turbulent conditions exist initially it is possible that the true flute-instability is not yet a limiting factor in plasma containment.

One advantage offered in the case of H.E. injection devices, but not often enough exploited, is the possibility of observing the onset of instability during ion injection.

64. Table IV shows the plasma configuration and appraent behaviour, and the methods employed for observing plasma behaviour in the mirror machines which have been mentioned. In the last column of the Table some conjectures are made as to possible causes of instabilities and particle losses which have been observed.

5. SPACE CHARGE AND PLASMA BUILD-UP IN H.E. INJECTION DEVICES

65. One of the assumptions underlying the operation of high energy injection mirror machines is that the positive space charge due to the accumulation of positive ions will be almost completely neutralized by electrons collected from ionizing events between the energetic ions and the neutral gas molecules in the system. In the absence of accelerating mechanisms electrons would escape from the plasma only by virtue of thermal energies which would be expected to lie between a few and several tens of electron volts, depending upon the ion injection energy. On the other hand, low energy secondary ions should be driven from the system by the small positive space charge which, after electron accumulation, according to the ideal picture, would be of the order of the electron thermal energy.

66. Let us consider the case of H_2^+ ions injected into a trap in which the background neutral gas density remains constant. During injection we have the number densities of H_2^+, e, and H_1^+ given by

$$n_{H_2^+} = \frac{N_i}{\Omega} \tau_1 \left(1 - e^{-t/\tau_1}\right),$$

$$n_e = \frac{N_i}{\Omega} \frac{\tau_1}{\tau_2} \left[t - \tau_1\left(1 - e^{-t/\tau_1}\right) \right],$$

and

$$n_{H_1^+} = \frac{N_i}{\Omega} \frac{\tau_1}{\tau_3} \left\{ \tau_4\left(1 - e^{-t/\tau_4}\right) - \frac{\tau_1 \tau_4}{\tau_4 - \tau_1} \left(e^{-t/\tau_4} - e^{-t/\tau_1}\right) \right\},$$

where

τ_1 is mean life of H_2^+ ions in trap,

τ_2 is life-time of H_2^+ ions against ionizing collisions,

τ_3 is life-time of H_2^+ ions against production of H_1^+

τ_4 is mean life of H_1^+ ions in trap, determined by charge exchange loss.

N_i is the rate of injection of H_2^+ ions

and

Ω is the volume of the trap occupied by plasma.

If τ_1 is much less than other time constants we can express the number densities during injection as

$$n_{H_2^+} \sim \frac{N_i}{\Omega} \tau_1$$

$$n_e \sim \frac{N_i}{\Omega} \tau_1 \frac{t}{\tau_2}$$

$$n_{H_1^+} \sim \frac{N_i}{\Omega} \tau_1 \frac{\tau_4}{\tau_3} (1 - e^{-t/\tau_4}).$$

These values are plotted in Fig.6, where the following τ-values have been taken:

$$\tau_1 = 5 \times 10^{-4}$$

$$\tau_2 = 3.9 \times 10^{-2}$$

$$\tau_3 = 6.0 \times 10^{-2}$$

$$\tau_4 = 2.5 \times 10^{-2} ,$$

consistent with injection of 50 keV H_2^+ ions in the presence of a background hydrogen gas pressure of 10^{-8} mm Hg. Following "cut-off" of the injected beam we have

$$n_{H_2^+} = n_{H_{20}^+} \exp(^{-t/\tau_1})$$

and

$$n_{H_1^+} = n_{H_{10}^+} \exp(^{-t/\tau_4}).$$

These quantities are also shown in Fig.8. The rate at which electrons will leave the trap, following "cut-off" will depend on their energy and density, determining their Debye length, and the geometry of the system. For a long machine, once the plasma potential is zero or negative it might be assumed that the electrons will leave the plasma at a rate $n_e \bar{v}_{ez} A$ where v_{ez} is their mean velocity parallel to B_o and A the cross section of the plasma. Thus n_e will fall as

$$n_{eo} (1 - \frac{n_{eo} \bar{v}e}{L} t),$$

and providing

$$\frac{n_{eo}}{n_{io}} \frac{\bar{v}_e}{L} \tau_1 \geqslant 1$$

the space change potential following "cut-off" could be expected to remain at a small value. However, when the electrons suffer no collisions in their flow from the plasma their number density immediately falls by a factor of two at the ends of the plasma region, once the potential trapping them is removed, and this could well invalidate the simple picture we have assumed. Indeed, if τ_1 is sufficiently small it is possible that the remaining electron content of the plasma, after the loss of most of the H_2^+ ions, is left bunched and oscillating. Observation of the plasma potential variation with z before and during the cut-off period is needed or comparison between the behaviour of the plasma when the cut-off time constant takes various values.

67. Ionization by electrons is not included in the above and τ_1 is taken as covering all losses of injected ions. In a real experimental case n_o is unlikely to be constant, and its variation in time, depending on such things as pumping rates, and possible release from or adsorption by vacuum surfaces, as well as rate of inflow of gas to the plasma region from vacuum region is in general beyond relaible estimation. When molecular hydrogen ions are injected into a mirror machine some fifteen atomic reactions between neutral particles and ions are involved (excluding those associated with negative ion formation). These are shown, together with cross sections for 50 keV H_2^+ and 25 keV H_1^+, where they have been measured, in Table V. Fig.9 shows the variation of some important cross sections with particle energy. Cross sections for a considerable energy range are published by Oak Ridge(55); other sources used for Table V are references (56-60). As shown by the Table the experimental values for various cross sections are not entirely consistent with each other, nor is there always agreement between different determinations of the same cross section. However, the information available, and the expected exponential nature of decay rates of plasma contents, enable observation-based estimates to be made of the extent to which the plasma behaviour departs from ideal. The cross section values also enable estimates to be made of plasma build-up, and conditions necessary for "burn-out", in idealized models.

68. Although it has not been established definitely that inability to retain electrons in the potential well of high energy ions in H.E. injection mirror machines will place an upper limit on available plasma densities, it is to be noted that the densities achieved so far appear to stop at values for which the plasma frequency is of the order of the ion cyclotron frequency. This fact, together with observed electrostatic radiation at the ion cyclotron frequency and modulation in electron currents to the mirrors, is consistent with electrostatic instabilities being responsible for the loss. Results with DCX 1 indicate that the instability will grow only while the high energy ions are monoenergetic. This suggests that an induced energy spread in the injection beam would be an advantage. However, introducing a beam of particles which are not monoenergetic is a problem in itself. Apart from this, conditions in DCX 1 are so different from those pertaining to other machines that observations here cannot be generalised. Longer devices, even in the presence of an ion energy distribution, offer conditions for a Doppler shifted resonance between ions and electrons. Also, in machines where ion orbit centres are not concentric

with the axis, electrostatic instabilities limited in θ are likely to be more serious in that electron loss here could possibly enhance flute-type growth.

69. Apart from electron oscillations within the plasma which can develop at the plasma frequency and are likely to resonate with the ion cyclotron motion when ω_{oe} and ω_{ci} are close enough in value, any electrons produced in the presence of a potential well will in general oscillate about the centre of it. Referring to Fig. 9 which shows diagrammatically an idealized positive space charge potential well, electrons formed in the regions AB and CD will oscillate through the well, undergoing acceleration in the uniform field regions, and travelling the distance ℓ at a constant, maximum velocity. Those formed between B and C will only undergo oscillations by virtue of thermal energies, or energies gained by mechanisms in the BC region. Considering an electron with zero velocity at a distance z from B, in a uniform field region, the time it will take to reach a point distant z beyond C is

$$t_{\frac{1}{2}} = 1.06 \times 10^{-8} \left(\frac{\ell}{V_o} \right)^{\frac{1}{2}} \left\{ z^{\frac{1}{2}} + 1.59 \frac{\ell}{z^{\frac{1}{2}}} \right\}$$

where the $z^{\frac{1}{2}}$ term arises from times spent being accelerated and decelerated and the $\ell/z^{\frac{1}{2}}$ term is the time spent in transit through the field full region BC. Thus providing $\ell \gg z$ the "period" of oscillations is determined by the value of ℓ and we have

$$t_{\frac{1}{2}} = 1.7 \times 10^{-8} \left(\frac{\ell}{V_o} \right)^{\frac{1}{2}} \frac{\ell}{z^{\frac{1}{2}}}$$

or if ℓ and z have nearly the same value

$$t_{\frac{1}{2}} = 1.7 \times 10^{-8} \frac{\ell}{V_o^{\frac{1}{2}}}$$

Thus the frequency of oscillation, as would be measured by observing times of arrival of the electron back at its starting point would be

$$\omega_{\ell} = 1.8 \times 10^{8} V_o^{\frac{1}{2}} \ell$$

70. In the case of Ogra, with ℓ about 1200 cm, ω_{ℓ} will equal ω_{ci} for V_o of about 10 kV while for Ogrenok, taking ℓ = 200 cm, ω_{ℓ} and ω_{ci} become equal when V_o is about 200 volts. These values of V_o are in quite good agreement with the observed, or inferred, space charge potentials of these machines. Indeed the values of ω_{ℓ}, based on observed V_o, are closer to ω_{ci}, for each machine, than are the values of ω_{oe}, estimated from plasma densities.

71. The above does not indicate how electrons escape from the body of the plasma to produce the space charge potential though it is possible that mechanisms arise during initial injection or are associated with "cut-off". However, two points are worth making. Firstly, if electrons are accelerated by an electrostatic, Harris-type of instability, which is essentially a short wave phenomenon, then in a long mirror machine the unstable oscillations must be well beyond the region of linearity at quite low electron energies. Secondly, until estimates made of the electron density escaping through the mirrors show that it is otherwise, it is possible that their escape is across the magnetic field lines to the walls, and the observed oscillating electron flux in the mirrors is due, to a great extent, to electrons formed initially in the mirror regions: if the background pressure were considerably higher in the mirror regions than nearer the median plane it is conceivable that ionization by electrons is responsible for electron multiplication in these regions. Some bunching mechanism would be required, of course, to produce what would be essentially a single group of the higher energy electrons moving backwards and forwards through the main plasma region.

72. If the space charge potential which occurs during injection of positive ions into long machines should prove intolerable and cannot be neutralized by electron build-up, consideration might be given to the injection of negative hydrogen or deuterium ions as well, during the initial stages of the main injection. The negative ions, responding much more slowly than electrons to local electric fields would be less likely to be carried from the plasma by large amplitude oscillations, and might enable charge neutrality to be conserved until a reasonably high plasma density had been achieved. The main difficulties would probably be associated with the production of a sufficiently intense source of negative ions. Cross sections are not generally well known for atomic reactions involving these ions, but the fact that the charge loss rate in the presence of neutrals is about an order of magnitude higher than for positive ions at the same energy is not serious as the ions departing as neutrals will leave their electrons behind. Were a negative ion content used to partially neutralize the positive ion content, a much reduced ejection of electrons would be required at "cut-off". The extent to which space charge occurs during injection could also be reduced by programming the turn-on of the ion beam so that the injected current rose exponentially with a time constant not very different from that associated with the electron density growth rate.

73. The existence of a considerable positive space charge associated with plasma in a mirror machine may not be without some advantage. This aspect will be discussed in the next section.

6. THE CULHAM LABORATORY RESONANCE TRAPPING MIRROR MACHINE

74. The principle of the method of trapping particles in this machine was first put forward by Sinelnikov(61,62), and is discussed in some detail by Laing and Robson(63). The main departure from conventional mirror machines in this device is that high energy ions are injected along the B_o field which is space modulated, rather than across an approximately uniform field. The significant design parameters are as follows:-

Distance between mirrors	308 cm.
Radius of vacuum chamber	11 cm.
Radius at which ions injected	5 cm.
Main magnetic field ($\bar{B}_o$)	14 K.G.
Mirror field (B_M)	38.5 K.G.
Field at Ion Source (B_{oo})	9.1 K.G.
Modulation field ($\bar{B}_r$)	500 gauss at 5 cm.
Injection energy for H_2^+ ions	55 KeV.
Base vacuum system pressure	10^{-9} mm.Hg.

Modulation coil system: 2 groups of 7 coils, separation of coil centres 10 cm, with 40 cm between the group end coils nearest median plane. The current through the field modulation coils flows oppositely in each consecutive coil.

Under these conditions ions entering the machine with $v = v_z$ gain $v_\perp$ as they pass through regions presenting a component B_r of magnetic field. When the input energy of the ions is optimized, resonance is achieved between the particle ion cyclotron motion and the modulation field of wavelength $\lambda = 20$ cm. As v_θ is gained at the expense of v_z, ideally λ would decrease progressively down the machine in order to maintain exact resonance. However, an optimum value of v_{zo} can be chosen, slightly above that for resonance through the initial modulations, which gives a maximum overall gain of magnetic moment during the first transit. Whether any sort of resonance can be preserved during the second and subsequent transits depends on the phases of the particles coming from their mirror reflection. Since small spreads in ion launching energy and direction lead to the occurrence of large phase spreads at the mirrors it is likely that in a practical case there will be near randomization of phase before particles begin their second transit. H_1^+ ions formed from dissociation of H_2^+ by neutrals will be trapped by the mirror field and their v_z velocities will be too low for possible resonance with the fundamental of the modulated field, at their cyclotron frequency.

75. As shown by Chandrasekhar[1], if a group of particles passes through a region of magnetic field variation in a time comparable with their Larmor period, the mean magnetic moment of the group will increase. This means that although some particles will lose magnetic moment those that achieve an increase will more than compensate for the loss. Calculations by Laing[64] show that this effect leads to some particles, randomly colliding with field modulations, gaining high $v_\perp/v_\parallel$ ratios and being trapped in the machine for long times. Since the field modulations are equivalent to weak mirrors, particles with high magnetic moments will become trapped in these micro-mirrors. Thus after a period of injection the machine will contain high energy ions, H_2^+ and H_1^+, electrons, low energy ions and neutral gas atoms, with groups of ions contained in the micro-mirrors and overall containment of charged particles in the main mirror system. It is expected that, although H_2^+ ions will finish their first transit with a ratio $E_\perp/E_0$ of 0.4, after a short time the velocity distribution which will develop will lead to a $p_\perp/p_\parallel$ of about 3, with only 10-20% of ions being trapped within the micro-mirrors.

PLASMA SYMMETRY

76. The precession of primary ions due to ∇B, mainly as they enter and leave the mirror region is about $2\frac{1}{2}^\circ$ per transit. Thus if ions are injected at one value of θ a hollow cylinder of ions and developing secondary plasma will grow in θ at this precession rate. As some ions are lost at the mirrors on each transit the ion density and secondary plasma density will decrease with θ in the direction of precession, both during injection and after, until some sort of randomization has occurred. When injection occurs over an annulus there would be no initial density variations in θ.

RADIAL ELECTRIC FIELDS AND FLUTE INSTABILITY

77. The Resonance Trapping Mirror Machine, like other mirror devices which are simple magnetic bottles is hydromagnetically unstable in that its plasma exists in a region having an inwardly directed ∇B. At low densities finite Larmor radius stabilization cannot, on theoretical grounds[26], be expected to protect the plasma against flute instabilities, although at higher particle densities it should become stable for $m > 1$ perturbations*. However, this device can be expected to respond to positive electrodes

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* Since the outer unstable surface of the plasma is linked by each H_2^+ orbit with the inner stable surface some overall stability against fluting might be provided while the ion content is dominantly H_2^+.

in the mirror regions, as in the case of Ogra, and since it can be operated with a plasma symmetry unknown in Ogra a proper investigation of the stabilizing mechanism produced by the end electrodes should be possible.

78. By taking advantage of the plasma free region on the axis of the Mirror Machine for mounting a central rod electrode to which potentials can be applied, the behaviour of the plasma when subjected to controllable radial electric fields can be studied. The central potential combined with the plasma potential, possibly controlled by end electrodes, would make possible considerable variations in E_r and dE/dr. For stabilization against flute-instability on the outer surface of the plasma the drift velocity of electrons relative to the ions' drift must be in the direction of the ∇B ion drift, i.e.

$$\frac{c\bar{E}_e}{B} - \frac{c\bar{E}_i}{B} > \tfrac{1}{2}m\bar{v}_\perp^2 \frac{c}{e} \frac{\nabla B}{B^2}$$

or

$$r_{\ell i} \frac{dE}{dr} > \varepsilon_\perp \frac{\nabla B}{B^e}$$

where $\bar{E}$ is the average radial field seen by particles, $\varepsilon_\perp$ is the mean energy of ion motion perpendicular to B_o and $r_{\ell i}$ is the ion Larmor radius, that for the electrons being taken as nearly zero. Rosenbluth(65) has usggested that a plasma experiencing differential rotation might not be stable, and even in the presence of an outwardly directed ∇B, differential rotation could lead to instabilities.

ELECTROSTATIC OSCILLATIONS OF ELECTRONS

79. With $p_\perp/p_\parallel \sim 3$ conditions are well away from those believed necessary for the development of the Harris instability. On the other hand ion groups trapped in the micro-mirrors of the system would have a much larger degree of velocity anisotropy and individually might be expected to resonate with electron motion along the magnetic field. However, even if these groups of ions maintained themselves with very little velocity spread in $v_\perp$ the electron oscillation amplitude, once it exceeded $\lambda/2$, or 10cm, would cause the electron excursions to bring them under the influence of ions in adjacent micro-mirrors. Thus for a large transfer of energy to electrons to occur the phases of motion of ions in the micro-mirrors would have to be maintained in correct relation from one mirror to the next which is unlikely.

GENERAL STABILITY PICTURE

80. Table VI lists the instabilities we have discussed in relation to mirror machines with comments on the expected behaviour of the Culham Laboratory Resonance Trapping Machine.

PARTICULAR FEATURES OF THE RESONANCE TRAPPING MIRROR MACHINE

81. The main investigations which might be attempted with advantage in the device are as follows:-

(a) Sinelnikov trapping

This, of course, is the experiment of prime importance. Not only must primary ions, either H_2^+ or H_3^+ be trapped and retained in the system for times sufficient to give reasonable ionization and dissociation, but the energetic protons formed by dissociation must also be retained for times determined by charge exchange reactions. If the machine succeeds in these respects if offers considerable advantages over other methods of injection.

(b) Plasma Symmetry

Simply by operating the ion source as a complete annulus or as a small arc of a circle it will be possible to compare the behaviour of plasmas which are built up uniformly in θ with those which are not. Other devices at present in existence do not produce a plasma initially uniform in θ.

(c) Primary ion $v_\perp$ distribition

One major difference between the primary ions in this machine and other long H.E. ion injection machines is that here $v_\perp$ takes on a considerable spread in values. This places it in a better position with regard to velocity space instabilities and removes the "beam-like" properties which the primary ion content tends to have in other devices.

(d) Application of radial electric fields

As mentioned earlier a considerable amount of control can be had over radial fields at relatively low particle densities. This will make it possible to estimate their value as a means for suppressing flute instability.

82. Apart from the above experiments, for which the machine might be said to be especially suited, there is the much needed investigation of plasma behaviour over the transition region between its single particle and collective character as the Debye length decreases to below the machine dimensions. In particular it is hoped to approach this region from the low density side rather than, as is done in other long H.E. injection devices, to begin with a relatively dense, violently unstable plasma and observe its decay to calmer densities. There is a lack of information during the injection stage of plasma build-up

in H.E. injection devices, and experiments covering this phase might well lead to a better understanding of the initial stages of the development of instabilities.

83. Evaluation of the integral $\int \frac{d\ell}{rRB_0^2}$, where R is the radius of curvature of the B_0 field line and r is its distance from the machine axis, indicates that the Resonance Trapping Mirror Machine is theroetically unstable against flutings for an isotropic plasma pressure. Indeed it would be only marginally stable if all contained particles had the minimum magnetic moment necessary for trapping, with a mirror ratio of four to five. However the design of the machine lends itself to modification to one of the stable geometries discussed by Furth[(66)]. It appears that in order to convert the device to Furth's "bell-shaped" mirror machine stable against interchange modes, the major modification would be the introduction at one end of the machine, of an additional "inner-field coil" which would probably need to be superconducting. The practicability of doing this is being considered at the present time.

7. CONCLUSIONS

84. The major limitation to achieving higher particle densities in mirror machine plasmas is imposed by plasma waves which are unstable. The steady state wave picture for plasmas, having $r_0/B_0^2 \gg 10$ (for D_2^+ and H_2^+), is obtainable by applying two fluid hydromagnetics. This method has been outlined in section 2. Where $r_0/B_0^2 < 10$ and hydromagnetics does not apply only ion cyclotron/plasma oscillations are indicated (para.13).

85. The results of theoretical predictions of instabilities, discussed in section 3 suggest that for conditions met in existing machines a limited number of waves are likely to become unstable, two in particular. Where ∇B_0 is directed towards the plasma at its boundary the flute instability (para.27) is anticipated and, with high energy injection devices having a very high degree of velocity space anisotropy the electrostatic (Harris) instability (para.23) might occur. For higher β-values than at present achieved in mirror machines other velocity space instabilities such as the mirror-instability (para.18) and the electromagnetic wave resonance (para.21) are predicted. Many universal instabilities have been predicted: two of these have been discussed (paras. 32 and 33).

86. The seriousness of the flute instability is born out by experimental results (section 4), its presence having been indicated in a number of machines (paras.40, 45, 47 and 51). Some reduction in the degree of fluting appears to have been achieved without changing the

direction of ∇B_0 (paras.47 and 48) but only by reversing ∇B_0 (paras.42 and 43) has any dramatic improvement been produced.

87. Electrostatic oscillations have been observed as probable Harris instabilities in the case of short high energy injection devices, DCX 1 and Phoenix. In the long machines, as pointed out in paras.69 and 70, observed electron oscillations are likely to be caused by the space charge potential well, and may develop as a result of the space charge present during early stages of injection (paras.66 and 67).

88. The Culham Laboratory Resonance Trapping Device (section 6) makes a new approach to the method of injection and trapping of ions. Under its initial operating conditions it is likely to be flute-unstable and may have plasma potential problems, nevertheless it will have considerable possibilities as a device for investigating these problems as well as a number of others (para.81) which cannot be tackled in other existing devices.

8. ACKNOWLEDGEMENTS

I am grateful to Drs. D.R. Sweetman, G. Francis and D.W. Mason for discussions on their experimental results, to Drs. W.B. Thompson, J.B. Taylor and K.V. Roberts for discussions on instability problems, to Dr. A.E. Robson for many discussions on all aspects of mirror machines, and to Dr. R.S. Pease who suggested I write these notes.

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TABLE I

OPERATING PARAMETERS OF SOME MIRROR MACHINES

Device	Plasma Length (cm)	Plasma Radius (cm)	Magnetic field (K.Gauss)	Particle Density ($n_e=n_i$)	T_e	T_i	ω_{ce}	ω_{ci} for H_2^+	ω_{oe}
Ion Magnetron	200	25	5	10^9	20 eV	1-5 keV	8.9×10^{10}	2.5×10^7	1.9×10^9
Toy Top	120	11	17	10^{13}	100 eV	36 keV	3×10^{11}	9.5×10^7	1.9×10^{11}
Ogra	1000-1200	30	2.8	10^7	50 eV	160 keV	5×10^{10}	1.4×10^7	1.9×10^8
Ogrenok	200	17	3.0	6×10^6	20 eV	10 keV	6×10^{10}	3×10^7 *	1.4×10^8
Phoenix	5	8	50	3×10^8	10-100 eV	20 keV	8.9×10^{11}	2.5×10^8*	8.5×10^8
M.T.S.E. **	100	10	4.0	10^{11}	50 eV	1 keV	7.1×10^{10}	2×10^7	1.9×10^{10}

* H_1^+ only.

** Magnetic Trap Stability Experiment.

TABLE II

LOW FREQUENCY DISPERSION RELATIONS

Dispersion Curve Branch for Hydromagnetic Plasma	Magnitude of k^2	Type of Oscillations	Dispersion Relation	
High frequency branch	$k^2 = 0\left(\frac{\omega_{ci}^2}{c_A^2 k_x^2}\right)$	Compressional Alfven/H.F. Alfven wave transition	$\omega^2 = \frac{c_A^2 k^2}{2Q}\left[1 + h_x^2\left(1 + \frac{c_A^2 k^2}{Q\omega_{ci}^2}\right) + \sqrt{\left\{1 + h_x^2\left(1 + \frac{c_A^2 k^2}{Q\omega_{ci}^2}\right)\right\}^2 - 4h_x^2}\right]$ where $Q = 1 + c^2k^2/\omega_{oe}^2$	(i)
	$k^2 \approx \frac{\omega_{ce}^2 h_x^2}{c_s^2}$	Electron cyclotron/acoustic wave transition	$(\omega^2 - \omega_{ce}^2 h_x^2)\left[\omega^2 - c_i^2 k^2 - \left(\frac{T_e}{T_e + T_i}\right)\frac{c_s^2 k^2}{1 + k^2\lambda_{De}^2}\right] = \left[\frac{\omega^2}{\omega_{oe}^2} - \frac{m_e}{m_i}\left(1 + k^2\lambda_{Di}^2\right)\right]\frac{\omega^2(\omega^2 - \omega_{ce}^2)}{(1 + k^2\lambda_{De}^2)}$	(ii)
Intermediate frequency branch	$k^2 = 0\left(\frac{\omega_{ci}^2}{c_A^2 h_x^2}\right)$	Shear Alfven/ion cyclotron wave transition	as (i) except has negative square root	(iii)
	$k^2 = 0\left(\frac{\omega_{ci}^2}{c_s^2}\right)$	Ion cyclotron/acoustic wave transition	$\omega^2 = \frac{1}{2}\left[(\omega_{ci}^2 + c_s^2k^2) + \sqrt{(\omega_{ci}^2 + c_s^2k^2) - 4c_s^2k^2h_x^2}\right]$	(iv)
	$k^2 \approx \frac{\omega_{ce}^2 h_x^2}{c_x^2}$	Acoustic/electron cyclotron wave transition	as (ii)	(v)
Lowest frequency branch	$k^2 = 0\left(\frac{\omega_{ci}^2}{c_s^2}\right)$	Electro-acoustic/ion cyclotron wave transition	as (iv) except has negative square root	(vi)
Intermediate frequency branch	$k^2 << \frac{\omega_{ci}^2}{c_A^2 k_x^2}$	Shear Alfven wave	$\omega^2 = c_A^2 k^2 h_x^2$	(vii)
Lowest frequency branch	$k^2 << \frac{\omega_{ci}^2}{c_s^2}$	L.F. Electro-acoustic wave	$\omega^2 = c_s^2 k^2 h_x^2\left[\frac{\omega_{ci}^2 + c_s^2k^2h_x^2}{\omega_{ci}^2 + c_s^2k^2}\right]$	(viii)
Non Hydromagnetic Plasma	$k^2 << 1/\lambda_{De}^2$	L.F. Electromagnetic wave parallel to B_o^2	$\omega^2 = c^2k^2/(1 + \omega_{oi}^2/\omega_{ci}^2)$	(ix)
	$k^2 >> \frac{\omega_{ci}^2}{c^2}$	Ion cyclotron/Plasma oscillations	$\omega^2 = \frac{1}{2}\left[(\omega_{ci}^2 + \omega_{oi}^2 + \omega_{oe}^2h_x^2) \pm \sqrt{(\omega_{ci}^2 + \omega_{oi}^2 + \omega_{oe}^2h_x^2)^2 - 4\omega_{oe}^2h_x^2}\right]$	(x)

(Equations (i)-(viii) - Stringer[10]).

TABLE III

INSTABILITIES OF INTEREST - MIRROR MACHINES

Class	Name	Condition for Instability	Frequency $R(\omega)$ in $e^{-i\omega t}$	Growth Rate $I(\omega)$ in $e^{-i\omega t}$	Energy Source	Physical Process	Remarks
VELOCITY SPACE	Mirror	$p_\perp > p_\parallel$ $\beta_\perp > \frac{p_\parallel}{p_\perp}$ where $\beta_\perp = \frac{p_\perp}{\frac{B^2}{8\pi} + p_\perp}$ or $\beta_\perp > \beta_c = [\frac{1}{2}(\frac{\nabla p}{p})/(\frac{\nabla B}{B}) + 1]^{-1}$ ∇ taken parallel to B_o	small, $\ll \omega_{ci}$ $\omega = 0$, for instability	$< v_{i\perp}$	$\varepsilon_\perp$ (ions) $\varepsilon_\perp \to \varepsilon_\parallel$ via instability	Longitudinal oscillations of ions, almost $\perp B_o$. Diamagnetic plasma moves to reduced B-field regions of wave, further reducing field in these regions.	Erroneously believed to have been observed in "Table-Top" Refs: 11, 12, 38, 39
	"Slow Alfven" at high n; electro-magnetic at low n	$T_\perp \neq T_\parallel$ in mirror machine $T_\perp > T_\parallel$	Low density (i) $\omega_{ci} \frac{T_\perp - T_\parallel}{T_\parallel}$ (ii) $\omega_{ce} \frac{T_\perp - T_\parallel}{T_\perp}$ ($T_e = T_i$ in (ii))	$\omega_{oi} \frac{T_\perp}{(2\pi m_i c^2)}$ ½ exp( ) $\omega_{oe} (\frac{T_\perp - T_\parallel}{T_\parallel})^{3/2} \frac{T_\perp}{T_\parallel}$ $(\frac{2T_\parallel}{m_e c^2})^{1/2}$ exp ( )	$\varepsilon_\perp$ (i) ions (ii) electrons $\varepsilon_\perp \to E_\perp$ of wave.	Doppler resonance between wave and particles cyclotron motion. For $T_\perp > T_\parallel$, B-vector of wave causes bunching (in resonance) of particles which lose $\varepsilon_\perp$ to $E_\perp$ of wave. $\omega - n\,\omega_{ci} - k_\parallel v_\parallel = 0$	Although Alfven waves widely observed in high density plasmas $(\frac{4\pi nm_i c^2}{B_o^2} \gg 1)$, no observation made in low β plasma, where wave is E-M, with positive identification. Refs: 13, 14
	Harris Electro-static	(i) $\omega_{oe} > \omega_{ci}$, $(\frac{T_\perp}{T_\parallel})_i > 8\text{-}9$ (ii) $\omega_{oe} > \omega_{ce}$, $(\frac{T_\perp}{T_\parallel})_e > 2$ $\bar{k}$ has both $k_\perp$ and $k_\parallel$ components	$\sim \omega_{ci}$ $\sim \omega_{ce}$	$< \omega_{ci} (0.219 \frac{m_e}{m_i})^{1/2}$ i.e. $< \frac{\omega_{ci}}{120}$ for D_2^+	$\varepsilon_\perp$ (i) ions (ii) electrons $\varepsilon_\perp \to E_\parallel$ of wave, and $\varepsilon_\parallel$ of electrons	Coupling between ion cyclotron motion of particles, bunched in phase, and electrons oscillating parallel to B_o	Electrostatic oscillations at frequency near ω_{ci} observed in DCX 1, Ogra and Phoenix. Refs: 15-18, 19
CONFIGURATION SPACE	Flute, inter-change	$\nabla\|B\|$ directed towards plasma at boundary	0	$< v_{i\perp}$	Plasma pressure	∇B drift of ions produces charge separation at perturbed surface of plasma. Resulting E-field crossed with B_o gives rise to plasma motion $\perp$ surface.	Suppression reported as result of "Ioffe windings", also positive end electrodes. Finite Larmor radius stabilization predicted for $m > 1$ provided $r_{Li} \gg \lambda_D$. Refs: 22, 23, 24, 25, 26, 40
	∇B Universal	Presence of ∇B	ω_{ci}	( ) exp. ( )	$\varepsilon_\perp$ (ions)	Resonance between ∇B drift of ions and wave, with k nearly $\perp B_o$. Frequency near ω_{ci}.	Expected to occur for many types of wave. Seriousness of instability not fully evaluated. Ref: 27
	∇P Universal	Presence of ∇p (∇n or ∇T)	$k_\perp v_{iD}$	$\frac{\omega^2 e^2 n}{k_\parallel^3 u_e T_e} \left\{ 1 - \frac{k_\perp c Te (\partial n/\partial x)}{\omega e n B_o} \left(1 - \frac{\partial \ln .Te}{\partial \ln .n}\right) \right\}$ where u_e is mean electron thermal energy	$\varepsilon_\parallel$ (electrons) Potential energy of system with ∇p	Wave field produces drift of electrons to lower pressure regions causing an increase in number of resonant electrons giving energy to wave.	Theory does not apply where $T_i \gg T_e$. However where electrons gain high v_z due to another instability or because formed in a potential well, this type of instability might develop. Refs: 28

ε = energy of particle

E = electric field

TABLE IV

PLASMA BEHAVIOUR IN SOME MIRROR MACHINES

Device	Plasma Configuration	Behaviour	Method of Observation	Possible source of Instability or particle loss
Ion Magnetron	Cylindrical, $L/r_o \sim 8$ $n \sim 10^9$	Fluted turbulent loss of plasma to walls. Suppressed by reversing ∇B.	Ion energy: neutrals through foils. Radial escape: biassed wall-electrodes. End escape: "comb" and "sector"	Flute instability.
Table-Top	Tapered cylinder $n \sim 10^{11}$, $L/r_o \sim$ few	Plasma drift to wall, consistent with m = 1 instability.	Escaping flux: scintillators. Radial position of plasma: electrostatic pick-up electrode.	Flute instability, m = 1.
Ogra	Cylindrical, $L/r_o \sim 40$ H_2^+ beam passing through other components.	Fluted rotation. High space charge potential.	Flute and rotation: wall electrodes. Potential: Stopping potential for 2ndy ions.	Flute instability, or fluting a secondary effect, following electron loss by unknown mechanism.*
Ogrenok	Cylindrical, $L/r_o \sim 20$ H_1^+ beam passing through low energy components.	Fluted rotation. Some space charge potential.	Fast ion current: collimated current collectors. Mirror escape: biassed electrodes.	As with Ogra mechanisms not entirely clear. In both cases if electrons accelerated in pot. well, ∇p universal may operate.
DCX 1	Ring, $L/r_o < 1$. H_1^+ beam, encirling axis.	Only charge exchange loss of H_1^+. Energy spread accompanies increased electron loss to mirrors.	Monitored neutral flux, and 2ndy content flow to mirrors. Some radiation measurements.	Ion clumping during injection. Interaction between ion clumps and electrons.

* Recently reported (Semashko, S. Mirror machine conference, Paris) that stable operation achieved when Lithium arc run along axis of machine.

TABLE V

PARTICLE CROSS-SECTIONS FOR 50KeV. H_2^+ AND 25KeV. H_1^+ IONS ON HYDROGEN GAS MOLECULES

ROMAN NUMERALS ARE IN BRACKETS FOR REACTIONS HAVING RELATIVELY SMALL CROSS-SECTION

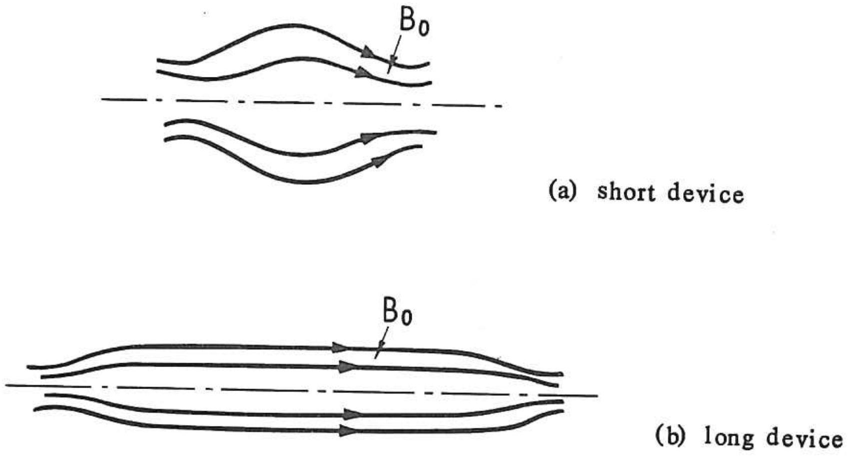
Reaction / Effective Process	$H_2^0(H_2^+;H_2^0)H_2^+$	$H_2^0(H_2^+;H_2^0)H_1^0,H_1^+$	$H_2^0(H_2^+;H_2^0)H_1^+,H_1^+,e$	$H_2^0(H_2^+;H_1^0,H_1^0)H_2^+$	$H_2^0(H_2^+;H_1^0,H_1^+)H_2^0$	$H_2^0(H_2^+;H_1^+,H_1^+)H_2^0,e$	$H_2^0(H_2^+;H_2^+)H_2^+,e$	$H_2^0(H_2^+;H_2^+)H_1^0,H_1^+,e$	$H_2^0(H_2^+;H_2^+)H_1^+,H_1^+,2e$	Σ σ units of $10^{-16}cm^2$	$H_2^0(H_1^+;H_1^0)H_2^+$	$H_2^0(H_1^+;H_1^0)H_1^0,H_1^+$	$H_2^0(H_1^+;H_1^0)H_1^+,H_1^+,e$	$H_2^0(H_1^+;H_1^+)H_2^+,e$	$H_2^0(H_1^+;H_1^+)H_1^0,H_1^+,e$	$H_2^0(H_1^+;H_1^+)H_1^+,H_1^+,2e$	Σ σ units of $10^{-16}cm^2$
	i	(ii)	(iii)	iv	v	vi	vii	(viii)	(ix)	measured	x	xi	xii	xiii	xiv	xv	measured
1 Electron production			1			1	1	1	2	3.23[56]			1	1	1	2	1.7[56] 1.7[59]
2 2^{ndy} H_2^+ production	1			1			1			6.56[56]	1			1			5.9[56] 5.0[59]
3 2^{ndy} H_1^+ production		1	2					1	2	0.48[56]		1	2		1	2	0.4[56] 0.2[59]
4 H.E. H_2^0 production	1	1	1														
5 H.E. H_1^0 production				2	1						1	1	1				
6 H.E. H_1^+ production					1	2				2.2[58] 2.2[60]							
7 Loss of Primary Ion	1	1	1	1	1	1				4.56[60]	1	1	1				
8 Electron capture	1	1	1	1						3.0[56] 4.1[57]* 3.0[60]	1	1	1				5.0[56]
Σ σ					1.4[58] 0.95[60]	0.4[58] 0.57[60]											

* Extrapolated value

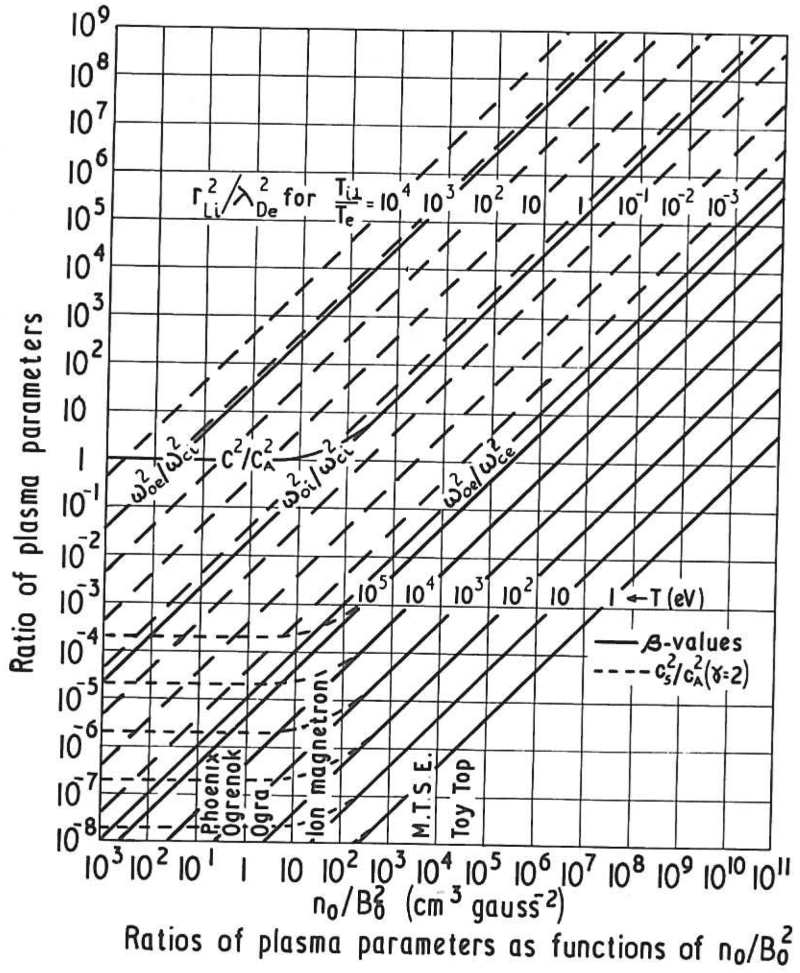
TABLE VI

CULHAM LABORATORY RESONANCE TRAPPING MIRROR MACHINE: STABILITY EXPECTATIONS

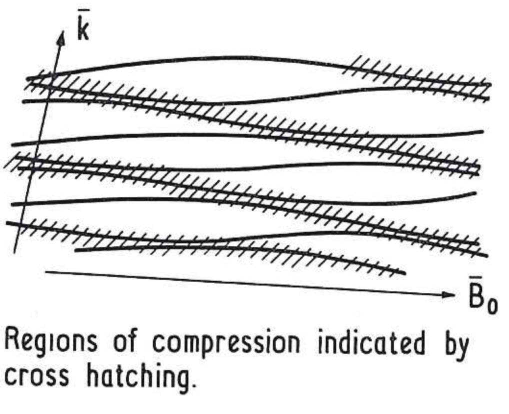
Instability	Condition for Instability	Expected Condition in Mirror Machine	What would be observed	Comments
Mirror	$p_\perp > p_\parallel$ $p_\perp/(\frac{B^2}{8\pi} + p_\perp) > \frac{p_\parallel}{p_\perp}$	$p_\perp \approx 3p_\parallel$ $p_\perp/(\frac{B^2}{8\pi} + p_\perp) < \frac{p_\parallel}{p_\perp}$ for $n < 10^{14}$	Increased scattering loss for $n > 10^{14}$	
Alfvén or E-M	$T_\perp \neq T_\parallel$	$T_\perp \neq T_\parallel$	Maybe weak E-M signals at $\omega = 2\omega_{ci}/3$	Not expected to be serious. Very slow growing.
Harris or Electrostatic	$T_\perp > 8\text{-}9\ T_\parallel$	$T_\perp < 8\ T_\parallel$	Electrostatic signals at $\omega \sim \omega_{ci}$	Unlikely in view of anticipated $v_\perp/v_\parallel$. May develop if electrons accelerated in a space-charge potential well.
Flute	∇B towards plasma at boundary	Condition as for instability	θ-variation in plasma density	Marginally stable a/c Finite Larmor radius. Application of E_r field may give stability at low densities.
∇B Universal	∇B	∇B	Weak E-M signals at $\omega \sim \omega_{ci}$	Not expected to be serious. Signal likely to be too weak for observation if other radiation at ω_{ci} present.
∇p Universal	∇p $(T_e \sim T_i)$	Without acceleration of electrons, $T_e \ll T_i$.	Low frequency E-M radiation.	Theory based on $T_e = T_i$, $u_e > \omega/k \gg u_i$. If electrons accelerated in potential well conditions might be conducive to instability.



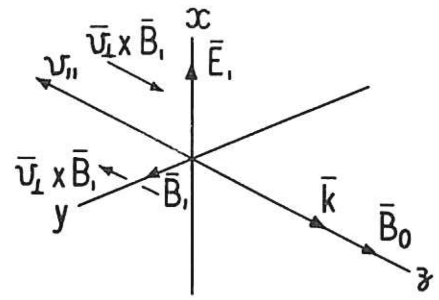
CLM-R38 Fig. 1
Diagrammatic representation of B_0 field configuration of simple mirror machines



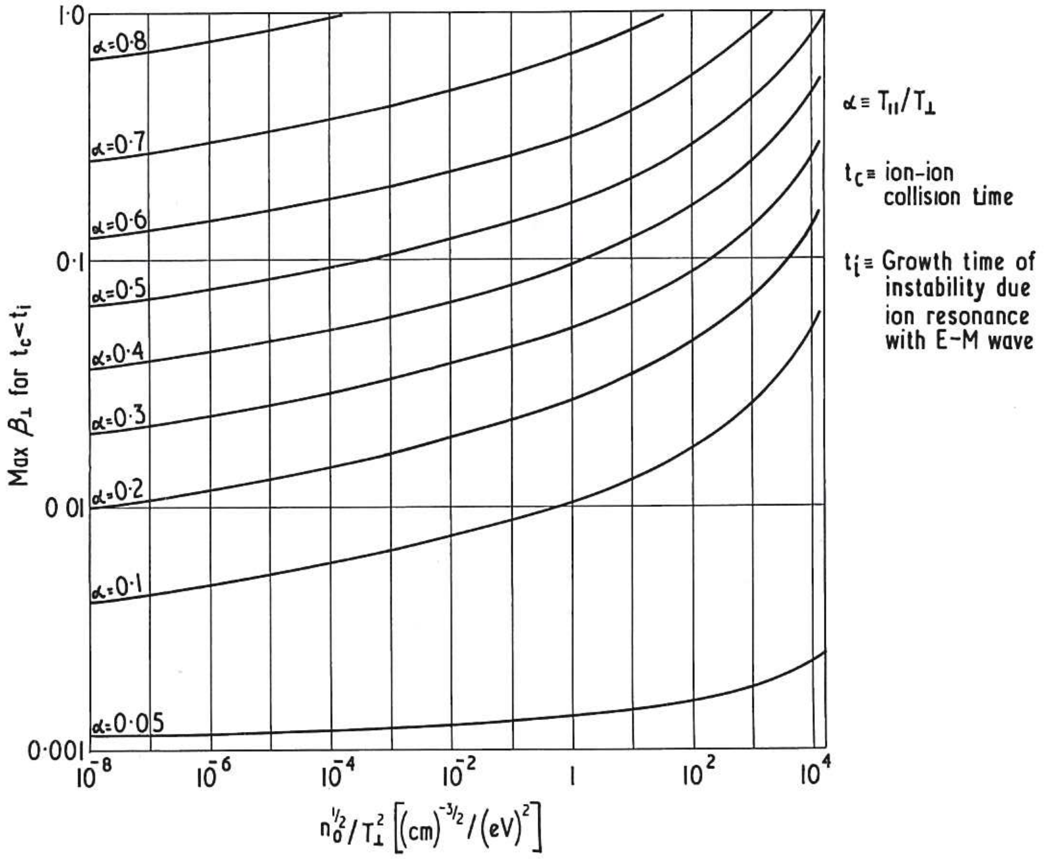
CLM-R38 Fig. 2
Ratios of plasma parameters as functions of n_0/B_0^2 (cm^{-3} gauss^{-2}) - for D_2^+ or H_2^+



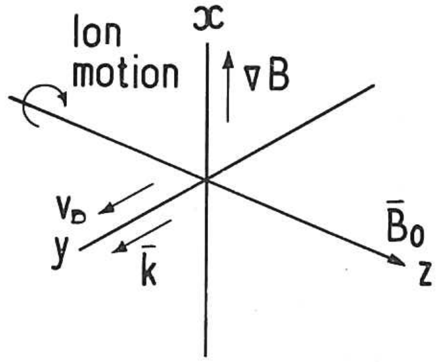
CLM-R38 Fig. 3
Diagrammatic representation of field lines and compression regions in case of mirror instability



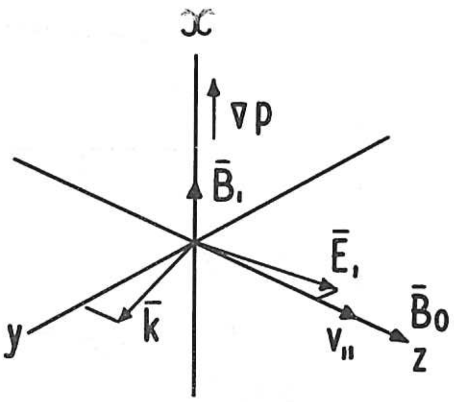
CLM-R38 Fig. 4
Vectors associated with 'slow Alfven' wave instability for ions with $T_{\perp} > T_{\parallel}$



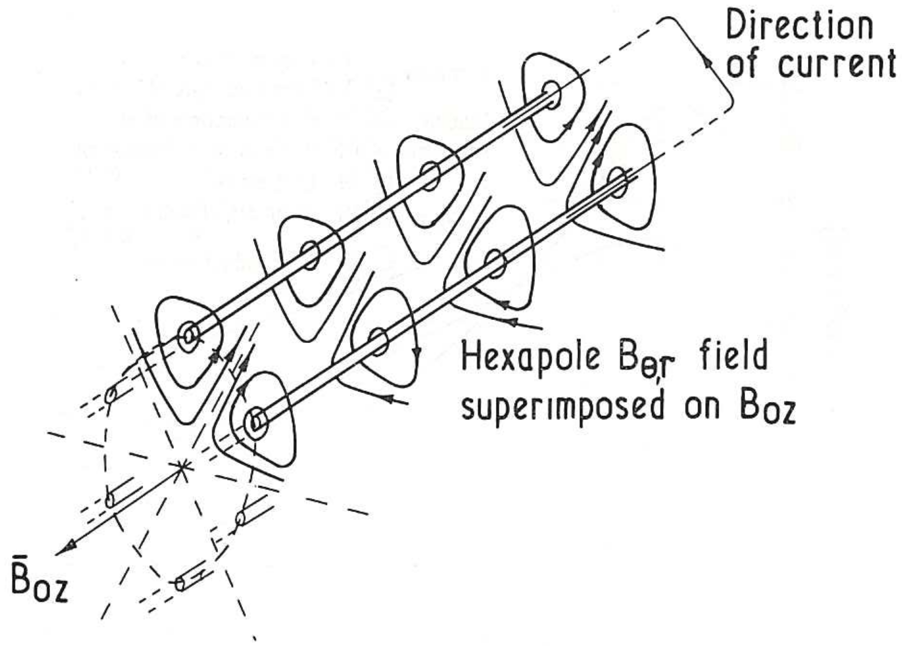
CLM-R38 Fig. 5
Maximum $\beta_{\perp}$ for which ion-ion collision time is less than growth time of 'slow Alfven' wave instability



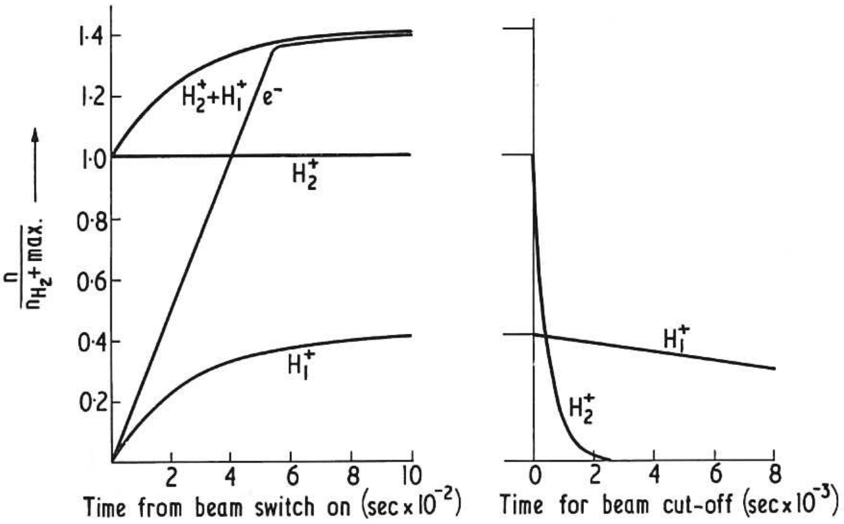
CLM-R 38 Fig. 6
Vectors associated with short wave electrostatic ∇B instability



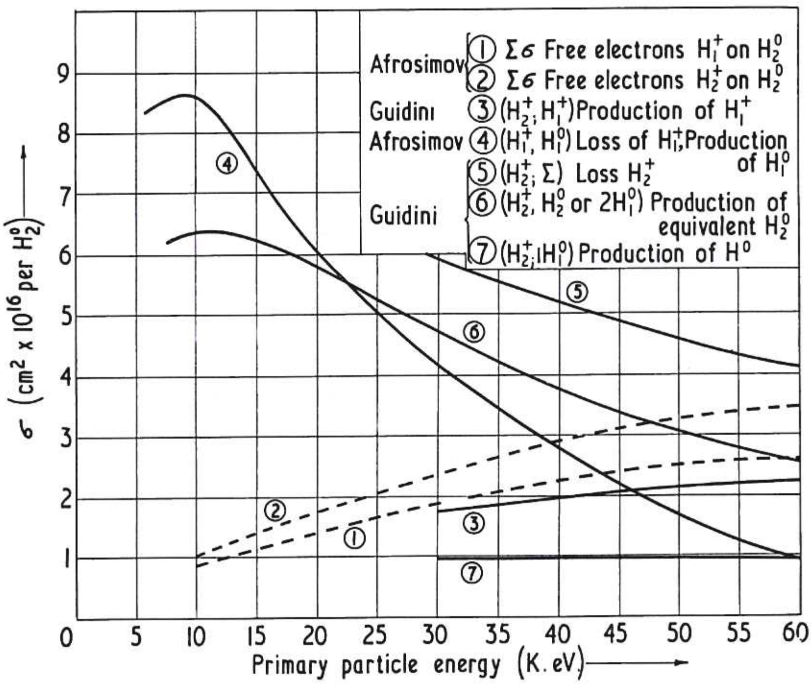
CLM-R 38 Fig. 7
Vectors associated with low frequency E-M ∇p-instability



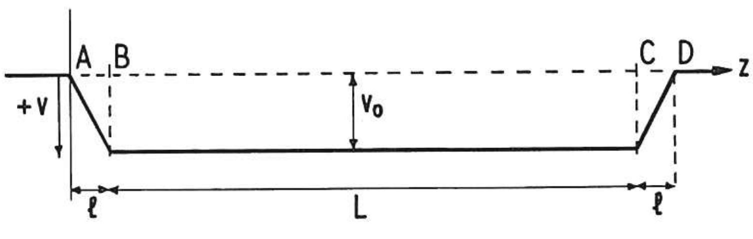
CLM-R 38 Fig. 8
Diagrammatic sketch of hexapole 'Ioffe-winding'



CLM-R38 Fig. 9
Time variation of particle density in H.E. injection mirror machine



CLM-R38 Fig. 10
Some cross sections for atomic reactions involving H_2^+ and H_1^+ with H_2^0



CLM-R38 Fig. 11
Idealized potential well for long mirror machine

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