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Report

THE EFFECT OF A FINITE TEMPERATURE ON THE ELECTRON CYCLOTRON RESONANCE INSTABILITY

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THE EFFECT OF A FINITE TEMPERATURE ON THE ELECTRON CYCLOTRON RESONANCE INSTABILITY

by

J.G. CORDEY

A B S T R A C T

The effect of Landau damping on the electron cyclotron resonance instability is investigated for a Burt-Harris type model of a confined plasma cylinder. The axis of the cylindrical shell of plasma lies along a uniform magnetic field.

The plasma is assumed to consist of three groups of particles, a group of electrons which move in concentric Larmor orbits about the axis of the cylinder, another group of electrons with no component of velocity perpendicular to the magnetic field and a group of ions also with no component of velocity perpendicular to the magnetic field.

A dispersion relation is then deduced, for electrostatic perturbations of this system. Analytical and numerical results are presented for the stability criteria and growth rates of the instability.

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C O N T E N T S

1. INTRODUCTION

Cyclotron resonance instabilities are found to occur in a number of situations where a plasma is immersed in a magnetic field. They arise from the coupling between electrostatic waves in the plasma and the cyclotron motion of the particle and are fed by anisotropy in velocity space(1,2,3,4,5). Ion cyclotron resonance instabilities have become of increasing importance in a number of thermonuclear experimental assemblies in recent years.

In order to investigate cyclotron resonance instabilities, an experiment (CYREX) has been set up, using electrons rather than ions, so as to obviate the large magnetic fields required for ion cyclotron resonance.

The theoretical model most closely resembling the experimental conditions is one similar to the Burt-Harris(1) model. The model we consider is an infinite cylindrical shell of plasma whose axis is along a uniform magnetic field. The plasma which is neutral is assumed to consist of three groups of particles, a group of electrons which move in concentric Larmor orbits about the axis of the cylinder, another group of electrons and a group of ions both having no component of velocity perpendicular to the axis of the cylinder. Fig. 1 shows a cross-section of the cylindrical shell of plasma whose inner and outer radii are r_1 and r_2 respectively.

In the next section the dispersion relation is derived for electrostatic perturbations of the above model. In sections 4 and 5 we examine the effect of Landau damping on the growth rate and on the onset of the electron cyclotron resonance instability.

2. DERIVATION OF THE DISPERSION RELATION

In the unperturbed state, the density of the group of electrons (beam electrons) which move in concentric Larmor orbits with their centres on the axis of the cylinders is taken as N_b. The density of the second group of electrons (plasma electrons) which have no component of velocity in a direction perpendicular to the magnetic field is N_p. The ions (density N_i) are assumed to be stationary, and to be unaffected by the perturbation, and the plasma is assumed to be neutral in the unperturbed state. In the remainder of this section the dispersion relation is deduced from the equations of motion coupled with Poisson's equation, using a method similar to that of Harris(2).

The equation of motion for a beam electron is

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\, \underline{v} = -\frac{e}{m}\left(\underline{E} + \frac{\underline{v} \wedge \underline{B}}{c}\right) \qquad \ldots (1)$$

where

$$\underline{B} = (0,\ 0,\ -\ B)$$

Equation (1) may be linearised by writing

$$\underline{v} = V_{zb}\ \underline{e}_z + V_\perp\ \underline{e}_\theta + \underline{v}' = \underline{V} + \underline{v}'$$

where

$$V_\perp = \omega_{ce}\ r = -\ \frac{eB}{mc}\ r\ ,$$

and $\underline{v}'$ and $\underline{E}'$ are considered to be small quantities whose space and time dependence is given by a factor:-

$$e^{i\ K_z z\ -\ i\ \ell\theta\ +\ i\omega t}\ .$$

When linearised, equation (1) may be solved for $\underline{v}'$ and the result is

$$v_r' = \frac{-\ \frac{ie}{m}\ E_r'\ (\omega + K_z V_{zb} - \ell\ \omega_{ce})\ -\ \frac{e}{m}\ E_\theta'\ \omega_{ce}}{\omega_{ce}^2 - (\omega + K_z V_{zb} - \ell\ \omega_{ce})^2} \qquad \ldots (2)$$

$$v_\theta' = \frac{\frac{e}{m}\ \omega_{ce}\ E_r' - \frac{ie}{m}\ E_\theta'\ (\omega + K_z V_{zb} - \ell\ \omega_{ce})}{\omega_{ce}^2 - (\omega + K_z V_{zb} - \ell\ \omega_{ce})^2} \qquad \ldots (3)$$

$$v_z' = \frac{-\ \frac{e}{m}\ E_z'}{i\ (\omega + K_z V_{zb} - \ell\ \omega_{ce})} \qquad \ldots (4)$$

The equation of continuity is

$$\frac{\partial n}{\partial t} + (\underline{v} \cdot \underline{\nabla})\ n = -\ n\ \underline{\nabla} \cdot \underline{v}\ . \qquad \ldots (5)$$

Equation (5) is linearised by writing

$$n = \varepsilon(r)\ N_b + n_b'\ ,$$

where

$$\varepsilon(r) = \begin{cases} 1 & \text{if}\quad r_2 > r > r_1 \\ 0 & \text{if}\quad r > r_2,\ r < r_1\ . \end{cases}$$

$$(\text{Note}\ \frac{d\varepsilon}{dr} = \delta(r - r_1) - \delta(r - r_2)\ .).$$

From the linearised form of (5) we deduce that

$$n_b' = \frac{-\ \varepsilon(r)\ N_b\ \underline{\nabla} \cdot \mathbf{v}' - v_r\ N_b\ \frac{d\varepsilon}{dr}}{i\ (\omega - \ell\ \omega_{ce} + K_z V_{zb})}\ .$$

Since curl E = 0 (we assume the magnetic field is undisturbed by the perturbation) we may take

$$\underline{E} = -\ \underline{\nabla}\varphi \qquad \ldots (7)$$

Equations (2), (3), (4), (5) and (7) then give

$$4\pi\, e\, n_b' = \frac{-4\pi\, N_b\, e^2}{m}\ \varepsilon(r) \left\{ \frac{\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial\varphi}{\partial r}\right) - \frac{\ell^2}{r^2}\varphi}{\omega_{ce}^2 - (\omega + K_z V_{zb} - \ell\, \omega_{ce})^2} + \frac{K_z^2\, \varphi}{(\omega - \ell\, \omega_{ce} + K_z V_{zb})^2} \right\}$$

$$+ \frac{4\pi\, N_b}{m}\, e^2 \frac{d\varepsilon}{dr} \left\{ \frac{\frac{\ell\varphi}{r}\omega_{ce} - (\omega - \ell\, \omega_{ce} + K_z V_{zb})\frac{\partial\varphi}{\partial r}}{[\omega_{ce}^2 - (\omega - \ell\, \omega_{ce} + K_z V_{zb})^2]\,[\omega - \ell\, \omega_{ce} + K_z V_{zb}]} \right\} \qquad \ldots (8)$$

A similar expression is found for the plasma electrons, but since $V_\perp = 0$ for this group of electrons, the term $\ell\, \omega_{ce}$ which occurs in equation (8) may be omitted.

After substituting for n_b' and the similar term for the plasma electrons, Poisson's equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) - \frac{\ell^2}{r}\varphi - K^2\left(\frac{1-G}{1+W}\right)\varphi = \frac{F(r)}{1+W(r)}\cdot\frac{d\varepsilon}{dr} \qquad \ldots (9)$$

where

$$W(r) = \sum_{be} \frac{4\pi\, e^2\, N_b\, \varepsilon(r)}{m\,\{\omega_{ce}^2 - (\omega - \ell\, \omega_{ce} + K_z V_{zb})^2\}} + \sum_{pe} \frac{4\pi\, e^2\, N_p\, \varepsilon(r)}{\omega_{ce}^2 - (\omega + K_z V_{zp})^2}$$

$$G(r) = \sum_{be} \frac{4\pi\, e^2\, N_b\, \varepsilon(r)}{m\,(\omega - \ell\, \omega_{ce} + K_z V_{zb})^2} + \sum_{pe} \frac{4\pi\, e^2\, N_p\, \varepsilon(r)}{m\,(\omega + K_z V_{zp})^2} \qquad \ldots (10)$$

$$F(r) = \sum_{be} \frac{\{\frac{\ell\varphi}{r}\omega_{ce} - (\omega - \ell\, \omega_{ce} + K_z V_{zb})\frac{d\varphi}{dr}\}\, 4\pi\, e^2\, N_b}{\{\omega_{ce}^2 - (\omega + K_z V_{zb})^2\}\,(\omega + K_z V_{zb})} + \sum_{pe} \frac{4\pi\, e^2 [\frac{\ell\varphi}{r}\omega_{ce} - (\omega + K_z V_{zb})\frac{\partial\varphi}{\partial r}] N_p}{\{\omega_{ce}^2 - (\omega + K_z V_{zp})^2\}\,(\omega + K_z V_{zp})}$$

In the above expressions the first summation sign extends over all beam electrons. The summation signs may be replaced by integrals over distribution functions giving,

$$W = \omega_{be}^2\, \varepsilon \int \frac{f_b\,(V)\, dV}{\{\omega_{ce}^2 - (\omega - \ell\, \omega_{ce} + K_z V)^2\}} + \omega_{pe}^2\, \varepsilon \int \frac{f_p\,(V)\, dV}{\omega_{ce}^2 - (\omega + K_z V)^2}\,, \qquad \ldots (11)$$

$$G = \omega_{be}^2\, \varepsilon \int \frac{f_b\,(V)\, dV}{(\omega - \ell\, \omega_{ce} + K_z V)^2} + \omega_{pe}^2\, \varepsilon \int \frac{f_p\,(V)\, dV}{(\omega + K_z V)^2}\,, \qquad \ldots (12)$$

where ω_{pe} and ω_{be} are the electron plasma frequency for the plasma electrons and the beam electrons respectively. The expression for F may also be written in the above form.

The free-space boundary conditions on φ and $\frac{d\varphi}{dr}$ are

$$\varphi(r_1^+) = \varphi(r_1^-)\,, \qquad \ldots (13)$$

$$\varphi(r_2^+) = \varphi(r_2^-)\,, \qquad \ldots (14)$$

$$\left.\frac{d\varphi}{dr}\right|_{r_1^-} = \left.\frac{d\varphi}{dr}\right|_{r_1^+} + \left.\frac{P}{r}\varphi\right|_{r_1^+} , \qquad \ldots (15)$$

$$\left.\frac{d\varphi}{dr}\right|_{r_2^+} = \left.\frac{d\varphi}{dr}\right|_{r_2^-} + \left.\frac{P}{r}\varphi\right|_{r_2^-} \qquad \ldots (16)$$

where

$$P = \frac{\omega_{be}^2\, \ell\, \omega_{ce}}{1+W'} \int \frac{f_b\,(V)\; dV}{\{\omega_{ce}^2 - (\omega + K_z V)^2\}(\omega + K_z V)} + \frac{\omega_{pe}^2\, \ell\, \omega_{ce}}{1+W'} \int \frac{f_p\,(V)\; dV}{\{\omega_{ce}^2 - (\omega + K_z V)^2\}(\omega + K_z V)}$$

and W' is equal to W (see equation (11)) with $\varepsilon(r)$ replaced by unity. Equations (15) and (16) are obtained by integrating equation (9) over the discontinuities at r_1 and r_2. The component of $\underline{E}'$ normal to the surface of the plasma, $\frac{d\varphi}{dr}$, is discontinuous due to the presence of space charge.

The solutions of equation (9) which are regular for $r < r_1$ and $r > r_2$ are Bessel functions of imaginary argument $I_\ell(K_z r)$ and $K_\ell(K_z r)$, respectively. In the interior $r_1 < r < r_2$, the solution is a linear combination of Hankel functions of the first and second kind $H_\ell^{(1)}(\lambda r)$ and $H_\ell^{(2)}(\lambda r)$, where

$$\lambda^2 = -\,(1 - G')\; K_z^2/(1 + W'). \qquad \ldots (17)$$

Insertion of these functions into equations (13) - (16) leads to the dispersion relation

$$\begin{vmatrix} I_\ell(K_z r_1) & -H_\ell^{(1)}(\lambda r_1) & -H_\ell^{(2)}(\lambda r_1) & 0 \\ 0 & -H_\ell^{(1)}(\lambda r_2) & -H_\ell^{(2)}(\lambda r_2) & K_\ell(K_z r_2) \\ -\frac{dI_\ell}{dr}(K_z r_1) & \left[\frac{dH_\ell^{(1)}}{dr}(\lambda r_1) + \frac{P}{r_1} H_\ell^{(1)}(\lambda r_1)\right] & \left[\frac{dH_\ell^{(2)}}{dr}(\lambda r_1) + \frac{P}{r_1} H_\ell^{(2)}(\lambda r_1)\right] & 0 \\ 0 & \left[\frac{dH_\ell^{(1)}}{dr}(\lambda r_2) + \frac{P}{r_2} H_\ell^{(1)}(\lambda r_2)\right] & \left[\frac{dH_\ell^{(2)}}{dr}(\lambda r_2) + \frac{P}{r_2} H_\ell^{(2)}(\lambda r_2)\right] & -\frac{dK_\ell}{dr}(K_z r_2) \end{vmatrix} = 0$$

$$\ldots (18)$$

The above dispersion is in the same form as the one given by Burt and Harris[1] for the ion cyclotron resonance instability. The λ and P of course have different meanings here.

If instead of the free-space boundary condition at r_1 and r_2 we assume the plasma is bounded by conducting walls, then the boundary conditions are

$$\varphi(r_1) = \varphi(r_2) = 0.$$

The dispersion relation in this case is

$$\begin{vmatrix} H_\ell^{(1)}(\lambda r_1) & H_\ell^{(2)}(\lambda r_1) \\ H_\ell^{(1)}(\lambda r_2) & H_\ell^{(2)}(\lambda r_2) \end{vmatrix} = 0 \qquad \ldots (19)$$

The above dispersion relation is somewhat simpler than dispersion relation (18).

3. ANALYSIS OF THE DISPERSION RELATIONS

The two dispersion relations (18) and (19) may be considerably simplified if we confine our attention to short wavelength disturbances only and replace the cylinder functions with their asymptotic forms. This requires $|\lambda r_1|$, $|\lambda r_2|$, $|K_z r_1|$ and $|K_z r_2| \gg 1$. This approximation appears to be justifiable in a number of fusion experiments.

The asymptotic forms of (18) and (19) are

$$\tan \lambda a = \frac{-2\lambda}{K_z^2 - \lambda^2} \qquad \ldots (20)$$

$$\tan \lambda a = 0 , \qquad \ldots (21)$$

respectively, where $a = r_2 - r_1$. Equation (21) has the simple solution

$$\lambda = n\pi/a \qquad (n = 1, 2, 3, \ldots)$$

Equations (20) and (21) give a set of allowable values of λ for free-space and conducting wall boundary conditions respectively. These values are then to be substituted in equation (17) which may then be regarded as the dispersion relation giving ω in terms of K_z. Rewriting (17) after substituting for W' and G' from equations (11) and (12) we have

$$\lambda^2 \left[1 + \omega_{be}^2 \int \frac{f_b (V)\, dV}{\{\omega_{ce}^2 - (\omega - \ell\, \omega_{ce} + K_z V)^2\}} + \omega_{pe}^2 \int \frac{f_p (V)\, dV}{\omega_{ce}^2 - (\omega + K_z V)^2} \right]$$
$$+ K_z^2 \left[1 - \omega_{be}^2 \int \frac{f_b (V)\, dV}{(\omega - \ell\, \omega_{ce} + K_z V)^2} - \omega_{pe}^2 \int \frac{f_p (V)\, dV}{(\omega + K_z V)^2} \right] = 0. \qquad \ldots (22)$$

The above dispersion relation is similar to the one given by Harris[(2)] for the ion cyclotron resonance instability. We may obtain the ion cyclotron resonance dispersion relation from (22) by replacing ω_{be} by ω_{pi} and $\ell\, \omega_{ce}$ by $\ell\, \omega_{ci}$.

In general we shall take $\omega_{be} \ll \omega_{pe}$; this is usually the case in most experiments.

4. NUMERICAL PROCEDURE AND SUMMARY OF RESULTS

A numerical iteration technique[6] was adopted to solve equation (22) in conjunction with either equation (20) or (21) for determining ω in terms of K_z or K_z in terms of ω. Two types of distribution were used for $f_b(V)$ and $f_p(V)$. These were the Maxwellian distribution and the resonance distribution which are as follows:-

$$f(V) = \frac{1}{\sqrt{\pi}\, V_\theta} e^{-V^2/V_\theta^2} , \qquad \text{... (23)}$$

$$f(V) = \frac{1}{\pi} \frac{V_\theta}{V^2 + V_\theta^2} \qquad \text{... (24)}$$

The results for the two distribution functions were found to be very similar and any differences were not thought to have any physical significance. No significant difference was found between using equation (20) (free-space boundary conditions) and equation (21) (conducting wall boundary conditions) for the parameters used in the CYREX experiment.

From now on we restrict the discussion to using equation (21) for determination of λ (conducting boundaries) in conjunction with using the distribution function given by equation (24) in the dispersion relation (equation (22)). The main reason for doing this is that some analytical results may be readily deduced for the above set up. The dispersion relation (equation (22)) with distribution functions of the form of equation (24) may be written

$$\begin{aligned} &\lambda^2 \left[1 + \frac{\omega_{be}^2}{\omega_{ce}^2 - (\omega - \ell\, \omega_{ce} - iK_z V_{\theta b})^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2 - (\omega - iK_z V_{\theta p})^2} \right] \\ &= - K_z^2 \left[1 - \frac{\omega_{be}^2}{(\omega - \ell\, \omega_{ce} - iK_z V_{\theta b})^2} - \frac{\omega_{pe}^2}{(\omega - iK_z V_{\theta p})^2} \right] , \end{aligned} \qquad \text{... (25)}$$

where $\lambda = n\pi/a$ [see equation (21)].

Instability can be due to either the second term of the LHS of equation (25) ($K_\perp$ type instability) or the second term on the RHS of equation (25) ($K_\parallel$ type instability). For $\omega_{be} \ll \omega_{pe}$ (and this is usually the case we are interested in) the two types of instability occur in different frequency ranges. The $K_\perp$ instability occurs in a small frequency range about $(\ell - 1)\, \omega_{ce}$ whereas the $K_\parallel$ instability occurs near $\ell\, \omega_{ce}$. In the remainder of this section we will deal with the $K_\parallel$ instability leaving the $K_\perp$ instability to the next section.

Approximate conditions for the onset of the $K_\parallel$ instability for a zero temperature beam and plasma may be derived as follows. For a zero temperature plasma and beam relation (25) simplifies to

$$\lambda^2 \left[1 + \frac{\omega_{be}^2}{\omega_{ce}^2 - (\omega - \ell\, \omega_{ce})^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2} \right] = - K_z^2 \left[1 - \frac{\omega_{be}^2}{(\omega - \ell\, \omega_{ce})^2} - \frac{\omega_{pe}^2}{\omega^2} \right] \quad \ldots\ldots (26)$$

For $\omega_{be} \ll \omega_{pe}$ the $K_{\parallel}$ instability occurs at frequency close to $\ell\, \omega_{ce}$. Hence we make the substitution

$$\omega = \ell\, \omega_{ce} + x$$

where

$$x = O(\omega_{be})$$

Then, neglecting tens of $O\left(\frac{\omega_{be}}{\omega_{ce}}\right)$, equation (26) gives

$$\frac{x^2}{\omega_{be}^2} = \left\{ \frac{\lambda^2}{K^2}\left[1 - \frac{\omega_{pe}^2}{\omega_{ce}^2(\ell^2 - 1)}\right] + 1 - \frac{\omega_{pe}^2}{\ell^2\omega_{ce}^2} \right\}^{-1},$$

hence for instability we must have

$$\omega_{pe}^2 > \ell^2\omega_{ce}^2 + \frac{\lambda^2}{K^2}\,\ell^2 \left[\omega_{ce}^2 - \frac{\ell^2\omega_{pe}^2}{\ell^2 - 1}\right]$$

provided $\ell \neq 1$.

For the special case of $\ell = 1$ we must use numerical methods unless $\lambda \ll K_z$ in which case we find that approximately.

$$\omega_{pe} > \frac{\ell\, \omega_{ce}}{\left[1 + \left(\frac{\omega_{be}}{\omega_{pe}}\right)^{2/3}\right]^{3/2}}$$

is the requirement for unstable solutions.

To include finite thermal velocities one must resort to numerical methods unless approximations of the form $\lambda \ll K_z$ are used (see Harris[2]). In the remainder of this section a summary of the numerical results for the $K_{\parallel}$ type instability will be given.

In Fig.2 a series of K_z real contours are plotted in the omega plane ($\omega_i < 0$ denotes instability) for different values of ω_{pe}. We see that the plasma first becomes unstable when $\omega_{pe} = 0.9\ \omega_{ce}$ and becomes stable again when $\omega_{pe} = 2.1\ \omega_{ce}$. Plotting similar K_z real curves for various values of $V_{\theta p}$ and $V_{\theta b}$, we find that variations in $V_{\theta b}$ effect the growth rate and the onset of instability to a greater extent than do variations in $V_{\theta p}$. Thus one may conclude that increasing the beam temperature would be more effective in quenching the electron cyclotron resonance instability than increasing the temperature of the plasma electrons. Fig.3 illustrates the effect of Landau damping for increasing beam temperature.

From a series of numerical results similar to those in Fig.2 for various $V_{\theta b}$ and ℓ we may construct a map of the unstable regions for a given ω_{pb}, $V_{\theta p}$ and a. The unstable regions for the $\ell = 1,2$ and 3 wave are shown in Fig.4. We can see that to quench the instability the thermal velocity of the beam must increase as the density of the plasma increases.

We may however deduce the stability condition for $\omega_{pe} \gg \ell\omega_{ce}$ and $\ell \neq 1$ analytically; by expanding the dispersion relation in powers of ω_{be}/ω_{pe}. We find that for stability we must have

$$V_{\theta b} > \frac{(\ell^2 - 1)^{\frac{1}{2}} \omega_{ce} \omega_{be}}{\lambda \{ \omega_{pe}^2 - (\ell^2 - 1) \omega_{ce}^2 \}^{\frac{1}{2}}} .$$

By solving the dispersion relation for K_z in terms of ω and using the methods of Briggs[7] it was shown that in general the $K_{\parallel}$ instability was of the non-convective type.

5. K PERPENDICULAR TYPE INSTABILITIES

The $K_{\perp}$ type instabilities are usually only important when $K_z \ll \lambda$. This means that we may neglect the terms in the second bracket of equation (25). The resulting dispersion relation

$$\frac{1}{\omega_{pe}^2} + \frac{\omega_{be}^2/\omega_{pe}^2}{\omega_{ce}^2 - (\omega - \ell\omega_{ce} - iK_z V_{\theta b})^2} + \frac{1}{\omega_{ce}^2 - (\omega - iK_z V_{\theta p})^2} = 0 \qquad \ldots (27)$$

may then be dealt with analytically. If we put the thermal spreads of the beam and plasma equal to zero, we may write the dispersion relation in the following form

$$F(\omega) = \frac{\omega_{be}^2/\omega_{pe}^2}{\omega_{ce}^2 - (\omega - \ell\omega_{ce})^2} + \frac{1}{\omega_{ce}^2 - \omega^2} = \frac{-1}{\omega_{pe}^2} .$$

By investigating the form of $F(\omega)$ near $\omega = (\ell - 1)\omega_{ce}$ one can show that a necessary condition for instability is $\ell \geqslant 3$.

With finite thermal velocities one may solve dispersion relation (27) for the case of $\omega_{be} \ll \omega_{pe}$. We take for simplicity the case of equal thermal velocities $(V_{\theta p} = V_{\theta b})$; other cases may be treated in a similar way but the analysis is slightly longer. We make the substitution

$$\omega = iK_z V_{\theta b} + (\ell - 1)\, \omega_{ce} + x'$$

for ω in equation (27) and assume x' is small compared with ω_{ce}. Neglecting terms of $O(x'^3)$ we find the approximate solution of (27) is

$$r' = \omega - (\ell-1)\omega_{ce} - iK_z V_{\theta b} = \frac{\omega_{ce}}{4(\ell-1)} \left[1 - \frac{\omega_{ce}^2}{\omega_{pc}^2}\{(\ell-1)^2-1\} - i\sqrt{\frac{4\omega_{be}^2(\ell-1)}{\omega_{pe}^2} - \left[\frac{\omega_{ce}^2\{(\ell-1)^2-1\}}{\omega_{pe}^2} - 1\right]^2} \right] \quad \ldots (28)$$

Thus for instability we must have

$$16\,(\ell-1)^2\,K_z^2\,V_{\theta b}^2 < \omega_{ce}^2 \left[\frac{4\omega_{be}^2(\ell-1)}{\omega_{pe}^2} - \left\{ \frac{\omega_{ce}^2\{(\ell-1)^2-1\}}{\omega_{pe}^2} - 1 \right\}^2 \right]$$

Fig.5 shows the region of instability. The growth rate may be easily deduced from equation (28)

6. SUMMARY

Both types of electron cyclotron resonance occur in bands surrounding multiples of the cyclotron frequency. However the range of ω_{pe} for which the $K_\perp$ type instability is important is very short indeed compared with the range of ω_{pe} for which the $K_\parallel$ type instability is important (compare Figs.4 and 5). Thus it would appear that the $K_\parallel$ instability is the more serious one.

It was found that in general the $K_\parallel$ type instability was non-convective so any hope of the instability being damped by unstable waves being convected into a region of stability can be discounted. To quench the $K_\parallel$ instability it was found that the thermal spread in the plasma beam has to increase as the plasma frequency increases. This latter remark suggests that at higher ω_{pe} than have been obtained so far, cyclotron resonance instabilities will become more important.

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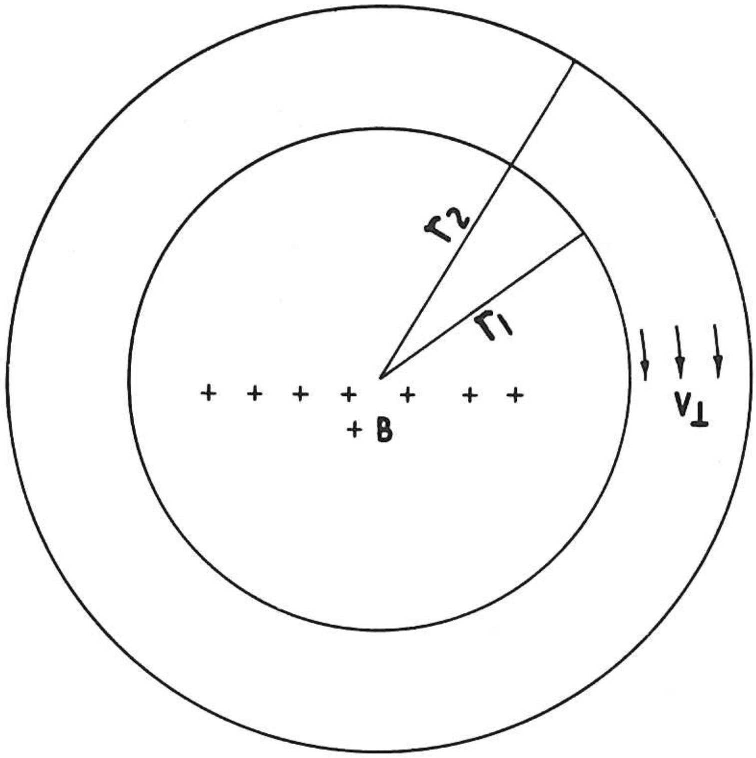

Fig. 1 (CLM-R 44)
Initial plasma configuration. The magnetic field B is directed into the paper

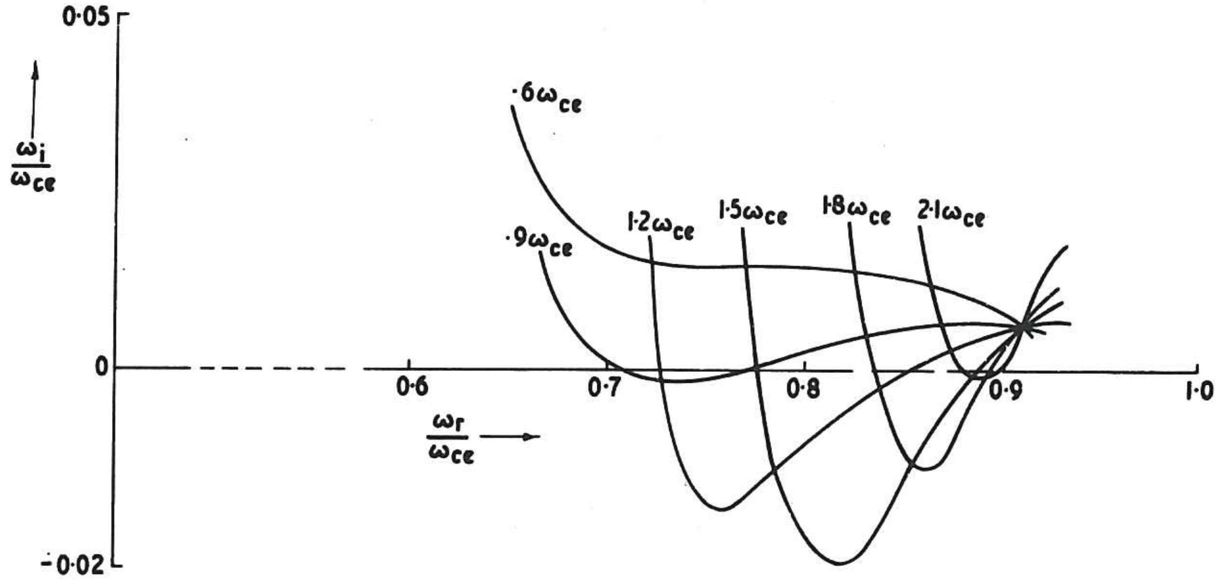

Fig. 2 (CLM-R 44)
K_z real curves plotted in ω/ω_{ce} plane for different values of ω_{pe} ; $\omega_{pb} = 0.2\ \omega_{ce}$; $v_{\theta b} = 0.03\ \omega_{ce} a$; $v_{\theta p} = 0.1\ \omega_{ce}$ and $\lambda = \pi/a$

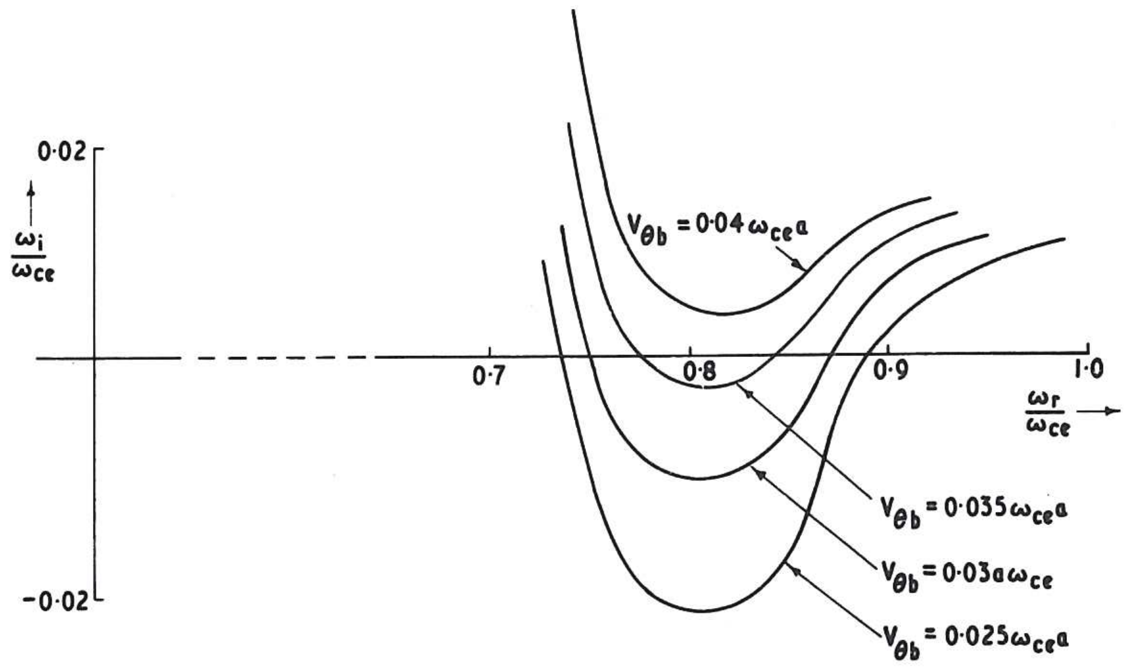

Fig. 3 (CLM-R 44)

K_z real curves plotted in ω/ω_{ce} plane for different values of $v_{\theta b}$; $\omega_{pb} = 0.3\ \omega_{ce}$; $\omega_{pe} = 1.5\ \omega_{ce}$; $v_{\theta p} = 0.1\ a\,\omega_{ce}$ and $\lambda = \pi/a$

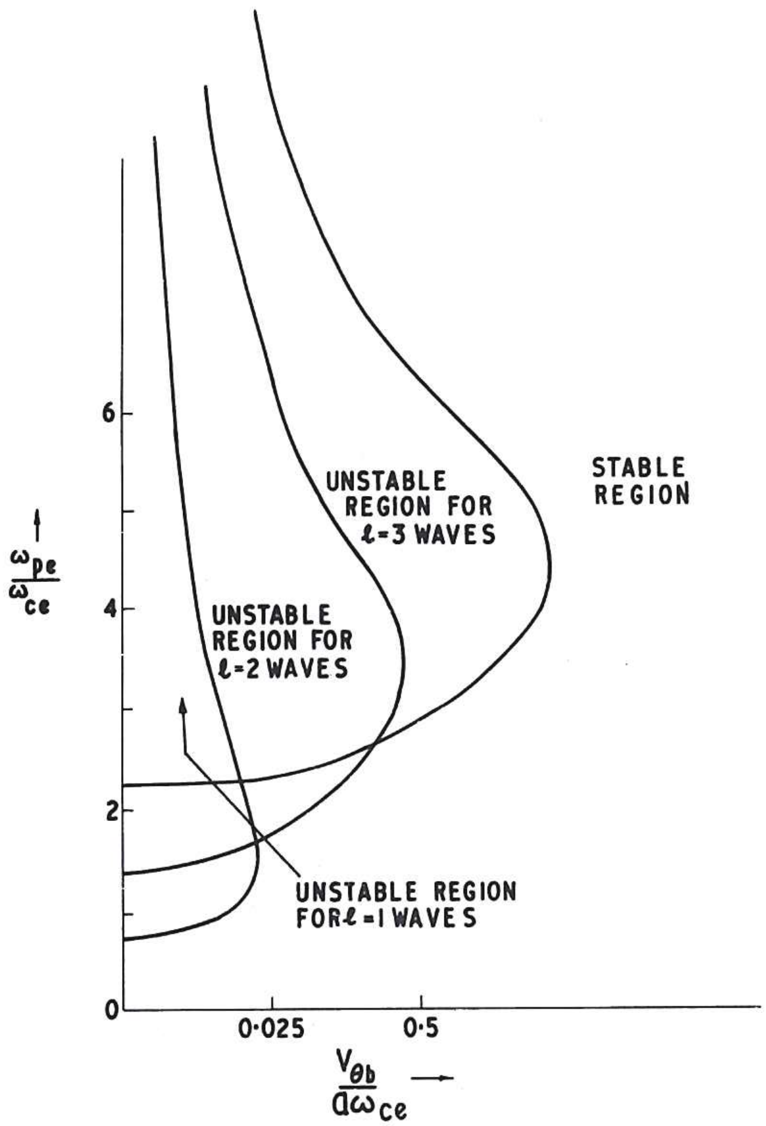

Fig. 4 (CLM-R 44)

Unstable regions $\ell = 1, 2$ and 3 waves; $v_{\theta p} = 0.1\ a\,\omega_{ce}$; $\omega_{pb} = 0.2\,\omega_{ce}$ and $\lambda = \pi/a$

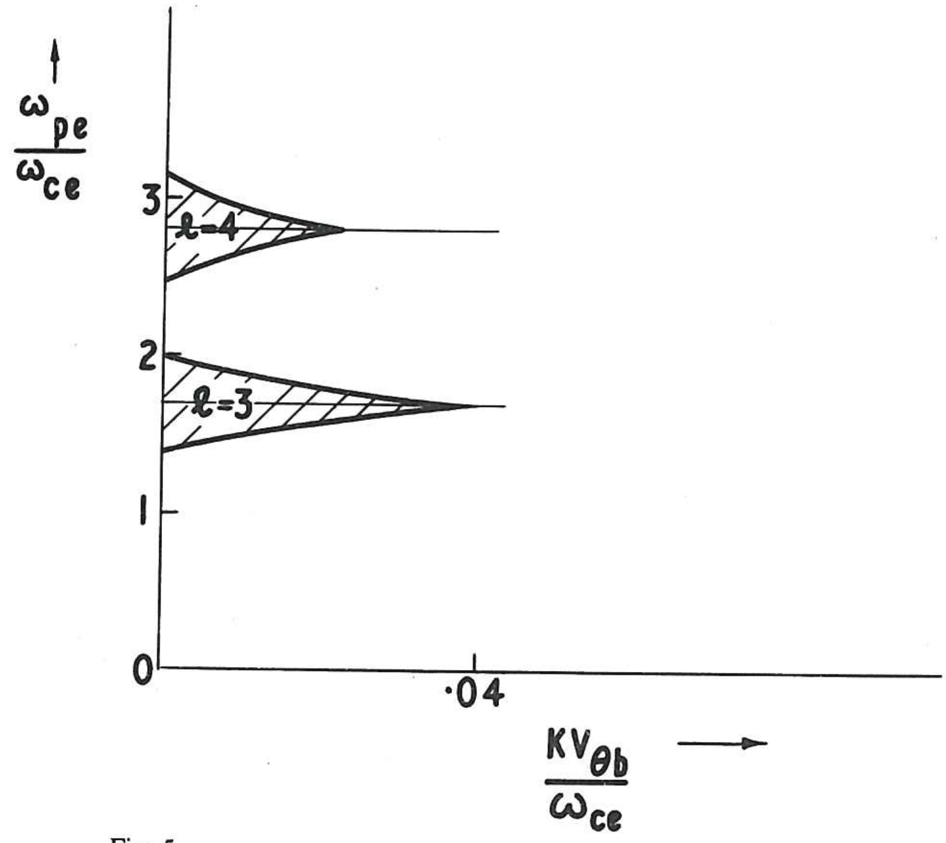

Fig. 5 (CLM-R 44)
The shaded areas depict the regions of unstable solutions for the $k_\perp$ type instability; $v_{\theta p} = 0.1\, a\omega_{ce}$, $\omega_{pb} = 0.2\, \omega_{ce}$

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