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MICROWAVE EMISSION FROM PLASMAS AT HARMONICS OF THE ELECTRON CYCLOTRON FREQUENCY

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MICROWAVE EMISSION FROM PLASMAS AT HARMONICS OF THE ELECTRON CYCLOTRON FREQUENCY

by

T.J.M. BOYD

A B S T R A C T

A mechanism is proposed to account for the anomalously large microwave emission from certain plasmas in the neighbourhood of harmonics of the electron cyclotron frequency, found in recent experiments. This mechanism involves the scattering of electron plasma waves propagating almost orthogonally to the magnetic field by fluctuations in the ion plasma density; a wave- wave interaction leads to the conversion from one wave type (primarily longitudinal) to another (transverse). A kinetic theory calculation based on this model gives an emission spectrum which may be compared with the observations.

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CONTENTS

1. INTRODUCTION

Many recent experiments[(1-6)] have shown the presence of an anomalous emission and absorption at harmonics of the electron cyclotron frequency in a variety of discharges over a wide range of pressure and gas species. The principal results of the experiments carried out so far will be summarized here, but this is not in any sense intended as a review of the experimental work and is presented merely to provide some picture of the phenomena to be interpreted in terms of the model proposed in this calculation.

The first reported experiment was that of Landauer who observed anomalous behaviour in the emission from a PIG discharge in helium and counted up to 25 harmonics; in this experiment the emission was detected at a fixed frequency and the magnetic field varied to produce the harmonics. The surprising result of this work was that the power radiated in the harmonics is a very slowly varying function of harmonic number. In addition it was found that the intensity decreased rapidly as the gas pressure was increased and a significant difference in the intensity of harmonics for polarization $\underline{E} \perp \underline{B}$ and $\underline{E} \parallel \underline{B}$ was reported, the amplitude of the harmonics for $\underline{E} \parallel \underline{B}$ being much reduced. Landauer also examined the appearance of harmonics as a function of the discharge current.

Bekefi et al.[(4)] have reported results of measurements on both absorption and emission from the positive column of a hot-cathode arc in various rare gases in magnetic fields i.e. in a discharge which might be expected to be more quiescent than the PIG type of Landauer's experiments. A number of harmonics (up to 8) was observed. In these experiments the plasma frequency, $\omega_p \geqslant \omega$, the microwave radian frequency, and collisional effects in the plasma could be neglected to a good approximation since $10^{-4} \leqslant \nu/\omega \leqslant 10^{-1}$ where ν is the collision frequency. The change in the microwave power absorption as a function of the discharge current was investigated and their results are represented qualitatively in Fig. 1. When $\omega_p/\omega \ll 1$ absorption occurs mainly at $\omega = \Omega$, the electron cyclotron frequency, and the line shape is approximately Lorentzian. As the discharge current is increased and ω_p increases, but still $\omega_p/\omega < 1$, the line (a) becomes asymmetrically broadened and the peak is shifted to lower values of Ω, i.e. of B_o, the static magnetic field. As Bekefi et al. point out, this is what theory predicts should happen; the uniform plasma theory predicts resonant absorption at $\frac{\Omega}{\omega} = (1 + \frac{\omega_p^2}{\Omega^2})^{-\frac{1}{2}}$. Now, since there is certainly a density gradient across the discharge tube i.e. $\omega_p = \omega_p(r)$, where r is the plasma column radius, one would expect to find the observed asymmetric broadening of the line as shown by (b). As $\omega_p/\omega \to 1$, the line in (b) becomes broader as shown in (c). Now, however, additional features appear in the neighbourhood of harmonics of the electron cyclotron frequency. As in the previous experiment the amplitudes of these harmonics are of the same order of magnitude. Bekefi et al. also studied the relation of the amplitude of a given harmonic to the discharge current and remark on the inversion of the line as shown in Fig. 1. All the above discussion refers to absorption. When the emission spectrum is studied similar effects appear.

More detailed studies of emission from plasmas in magnetic fields have been made by Tanaka and Kubo(6); their plasma was the positive column of an arc discharge, and measurements were carried out over a wide pressure range. Again the receiver was tuned to a fixed frequency and harmonics were observed by varying the magnetic field. Harmonic features appeared irrespective of the gas used in the discharge tube. Up to about 15 harmonics were detected (in a Hg vapour discharge at pressures of $\sim 2.6 \times 10^{-3}$ Torr). Their principal findings can be summarized as follows:

(i) the amplitudes of successive harmonics are roughly comparable, decreasing slowly with increase in harmonic number.

(ii) the amplitudes of the harmonic features increase as the discharge current, I_d, increases up to ~ 800 mA (after onset at ~ 500 mA), above which the amplitudes decrease.

(iii) the widths of the lines decrease as the harmonic number increases; however, since the lines are superposed on background radiation their profiles cannot be known exactly.

(iv) interesting differences were seen in the emission spectra according to whether the part of the positive column under observation was near to the anode or the cathode. In general the emission was much more intense from the cathode side of the positive column.

(v) the number of harmonics observed increases as the gas pressure is decreased.

(vi) interesting fine structure is observed near the second and third harmonics. This feature has been found in absorption by Buchsbaum and Hasegawa(7), and is discussed briefly in Appendix D.

As has already been hinted, one of the basic differences between the experiments of Tanaka and his group and those of Landauer, is the P I G discharge used by the latter. A well-known characteristic of PIG discharges is the presence of electron streams with energies of the order of the discharge voltage and these may be capable of producing growing waves via a beam-plasma interaction. Some recent work(8,9) deals more specifically with cyclotron radiation from plasmas generated by electron beams. Bekefi and Hooper(8) have generated a plasma by an electron beam in Hg vapour and obtained an emission spectrum consisting of a series of doublet lines near the harmonics of the electron cyclotron frequency. The components of the doublet show distinctly different behaviour in that one is Doppler-shifted (i.e. it originates from the beam electrons) while the other, although not Doppler-shifted, is displaced from the cyclotron harmonic frequency to a higher frequency; the displacement is proportional to the electron beam density. Unlike the experiments of Tanaka, Landauer and others , the relative intensities of neighbouring harmonics falls off rapidly as the harmonic number increases, but the line intensities are much greater than the intensities of radiation produced by single (i.e. uncorrelated) particles. These authors conclude that emission depends on the collective motions of the electrons. In the work of Gruber et al.[9] in a beam-generated He plasma, up to 15 harmonics are obtained with the peak always on the high frequency side of the cyclotron harmonic frequency. They find evidence of electrostatic waves growing spatially along the beam and decaying rapidly outside the beam. In this experiment they conclude that the observed emission intensities and the evidence of spatially growing waves suggest that these phenomena have their origin in instabilities.

In addition to the work on emission and absorption at the cyclotron harmonic frequencies transmission measurements have been carried out by Crawford, Kino and Weiss[10] between probes set in the positive column of an Hg vapour discharge, when up to 10 harmonics were counted. This work confirms in the laboratory an effect which appeared earlier in data taken by the top-side sounder satellite, Alouette[11]. In the ionospheric experiments resonances at, or very close to, the cyclotron harmonic frequencies, up to the 6th, were detected. The satellite carried a transmitter producing a pulse of variable frequency; at the end of the pulse, ringing of the ionosphere at the cyclotron harmonic frequencies was observed. No such effects are observed in ground-based ionosonde experiments.

In addition to the extensive experimental work on cyclotron harmonic phenomena in plasmas there has been a considerable theoretical effort to try to understand and interpret the observed effects and the principal models so far proposed will be discussed in some detail in Section 4 . When the present calculation was undertaken, two possible explanations of the observations had been proposed: the first, due to Canobbio and Croci[12], put forward at the VIth Conference on Ionized Gases in 1963, sought to interpret Landauer's observations of emission from a PIG discharge in terms of a mechanism involving quasi-electrostatic waves propagating almost perpendicularly to the magnetic field (the so-called Bernstein modes). These waves are excited by a flux of suprathermal electrons - a feature characteristic of PIG discharges - and propagate only mechanical energy. Canobbio and Croci explain the observed emission in terms of the coupling of the Bernstein modes to the radiation field via a (postulated) sharp density gradient at the edge of the plasma. The unchanged i.e. Rayleigh scattering. It should be appreciated that other processes may give rise to emission of radiation; one such is the scattering of incident electron plasma oscillations by electron density fluctuations to produce longitudinal waves with frequency close to twice the plasma frequency which in turn may be scattered by electron density fluctuations and converted into transverse waves with frequencies close to ω_p and $3\omega_p$. However such double scattering processes are normally very much weaker than single scattering events and may be safely ignored.

We start with a spectrum of quasi-longitudinal plasma waves without inquiring into what generates these waves. In a P I G discharge, of course, it is easily argued that the substantial flux of suprathermal electrons will strongly excite plasma waves. The present calculation, however, is carried out assuming only a thermal spectrum of quasi-longitudinal plasma waves and ion density fluctuations; a later paper[23] will consider an extension of the work described here to include plasmas containing fluxes of energetic particles. Indeed, as one might predict, it turns out that the emissivity of the plasma for the thermal equilibrium situation is several orders of magnitude too small and so the mechanism proposed here is really only viable in non-equilibrium plasmas. Nevertheless certain results of the experiments of Tanaka and his associates do agree with the predictions of this calculation.

Some fairly drastic approximations must be made at the outset to enable us to find an expression for the plasma emissivity. We shall ignore entirely the effects of the boundary sheaths on the emitted radiation and, further, suppose that the plasma is optically thin.

The kinetics of the process may be described in terms of the Vlasov equation. On our model a wave of frequency ω_1, wave-vector $\underline{k}_1$ (quasi-electrostatic wave) is converted into a wave of frequency ω_3, wave-vector $\underline{k}_3$ (radiation) through its interaction with a wave of frequency ω_2, wave-vector $\underline{k}_2$ (ion acoustic wave). Conservation of energy and momentum prescribe that

$$\omega_3 = \omega_1 - \omega_2 \quad ; \quad \underline{k}_3 = \underline{k}_1 - \underline{k}_2 \qquad \dots (1)$$

We represent the distribution function $f(\underline{r}, \underline{v}, t)$ in the form

$$f(\underline{r}, \underline{v}, t) = f_o(\underline{v}) + f_1(\underline{r}, \underline{v}, t) + f_2(\underline{r}, \underline{v}, t) + f_3(\underline{r}, \underline{v}, t) \qquad \dots (2)$$

where the terms on R.H.S. represent successively the equilibrium distribution function, perturbations due to the field of the incident quasi-electrostatic wave, the ion density fluctuations and the wave-wave interaction. Likewise we set

$$\begin{aligned} \underline{E}(\underline{r}, t) &= \underline{E}_1(\underline{r}, t) + \underline{E}_2(\underline{r}, t) + \underline{E}_3(\underline{r}, t) \\ \underline{B}(\underline{r}, t) &= \underline{B}_o + \underline{B}_1(\underline{r}, t) + \underline{B}_2(\underline{r}, t) + \underline{B}_3(r, t) \end{aligned} \qquad \dots (3)$$

and again the perturbations derive from the same sources as described for the distribution function. $\underline{B}_o$ is the static magnetic field (note that in most of the experiments B_o is not constant but is varied to produce the harmonic resonances, while the frequency of the receiver is held constant). At this point we introduce an ordering into our perturbation scheme and suppose that

$$f_3 \ll f_1 \quad , \quad f_2 \quad \text{and} \quad |E_3| \ll |E_1| \quad , \quad |E_2| \; .$$

The following hierarchy of equations then results from this perturbation scheme on the Vlasov equations:

$$\frac{\partial f_j}{\partial t} + \underline{v} \cdot \frac{\partial f_j}{\partial r} + \frac{e}{m}\left[\underline{E}_j + \frac{\underline{v} \wedge \underline{B}_j}{c}\right] \cdot \frac{\partial f_o}{\partial \underline{v}} + \frac{e}{m_e}\frac{\underline{v} \wedge \underline{B}_o}{c} \cdot \frac{\partial f_j}{\partial \underline{v}} = 0 \qquad j = 1,2 \ldots \qquad \ldots (4)$$

$$\frac{\partial f_3}{\partial t} + \underline{v} \cdot \frac{\partial f_3}{\partial \underline{r}} + \frac{e}{m_e}\left[(\underline{E}_3 + \frac{\underline{v} \wedge \underline{B}_3}{c}) \cdot \frac{\partial f_o}{\partial \underline{v}} + \frac{\underline{v} \wedge \underline{B}_o}{c} \cdot \frac{\partial f_3}{\partial \underline{v}} + \sum_{j=1}^{2} (\underline{E}_j + \frac{\underline{v} \wedge \underline{B}_j}{c}) \cdot \frac{\partial f_{3-j}}{\partial \underline{v}}\right] = 0 \qquad \ldots (5)$$

Examining (4), (5) we see at once that a number of terms drop out, while in (5) certain terms have not been included, i.e. those like

$$\underline{E}_1 \cdot \frac{\partial f_1}{\partial \underline{v}}$$

which represents the non-linear effect of the interaction of the wave with its own field and make no contribution to the plasma emissivity[24]. The incident wave is quasi-electrostatic and, we suppose, propagates almost orthogonally to $\underline{B}_o$. Tanaka and Kubo[6] and Canobbio and Croci[12] first suggested that such waves were important in this context. These modes have been discussed most clearly by Bernstein[25] (see also Stix[26]); they are quasi-electrostatic waves which propagate almost orthogonally to $\underline{B}_o$ and are usually confined to narrow passbands just above the cyclotron harmonic frequencies. As Bernstein shows these electrostatic modes do not propagate exactly at the cyclotron harmonics since, for directions orthogonal to the magnetic field, the cyclotron harmonic frequencies are not solutions of the dispersion equation; for non-orthogonal propagation the waves are severely damped. However propagation at frequencies within the passband can occur in nearly orthogonal directions such that the waves suffer only slight Landau damping. Propagation at an angle to the magnetic field will be severely Landau damped unless one satisfies the condition

$$\frac{\omega_1 - m\Omega}{k_1^{\parallel}} \gg \frac{2\kappa T_{\parallel}}{m_e}$$

where Ω is the electron cyclotron frequency, m the harmonic number, κ Boltzmann's constant, $T_{\parallel}$ the temperature parallel to $\underline{B}_o$ and m_e the mass of the electron.

A further simplification stems from the fact that the quasi-electrostatic wave is scattered by a low-frequency mode, which allows us to ignore the spatial dispersion term in (5). Following Bass and Blank[27] the wave-wave interaction term f_3 in the distribution function may be written $f_3 = f_3^o + f_3^1$ where f_3^o is proportional to the field of the scattered wave $\underline{E}_3$ and f_3^1 is determined by the last contribution in brackets of the source term in (5). The current producing the converted wave is determined by f_3^1.

Finally we shall discard yet one more term in (5) and, in the last bracket, retain only the $\underline{E}_2 \cdot \frac{\partial f_1}{\partial v}$ contribution. We may reject the remainder of the source term since, in the present work, we are interested solely in the plasma emissivity in the neighbourhood of the harmonics of the electron cyclotron frequency and the contribution to this quantity from the $\underline{E}_1 \cdot \frac{\partial f_2}{\partial v}$ term which is not resonant in these frequency ranges, is negligible.

Equation (5) is then reduced to simply

$$\frac{\partial f_3^1}{\partial t} + \frac{e}{m_e} \frac{\underline{v} \wedge \underline{B}_o}{c} \cdot \frac{\partial f_3^1}{\partial \underline{v}} = - \frac{e}{m_e} \underline{E}_2 \cdot \frac{\partial f_1}{\partial \underline{v}} \qquad \ldots (6)$$

the solution of which is

$$f_3^1 = \frac{ie}{m_e \Omega} \int_{\pm\infty}^{\varphi} \exp\left[- \frac{i\omega_3}{\Omega} (\varphi - \varphi^1) \right] (\underline{E}_2 \cdot \frac{\partial f_1}{\partial \underline{v}}) \, d\varphi^1 \qquad \ldots (7)$$

in which f_1 is determined from (4); i.e.

$$f_1 = \frac{4\pi e^2}{m_e k_1^2} \delta N_e (\underline{k}_1, \omega_1) \left[k_1^{\perp} \frac{\partial f_o}{\partial v_{\perp}} + k_1^{\parallel} \frac{\partial f_o}{\partial v_{\parallel}} \right] \cdot$$

$$\cdot \sum_{m=-\infty}^{\infty} \frac{J_m\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \exp\left[im\varphi - i \frac{k_1^{\perp} v_{\perp}}{\Omega} \sin\varphi \right]$$

Inserting the expression found for f_3^1 into

$$\underline{j}_3 = e \int f_3^1 \, \underline{v} \, d\underline{v} \qquad \ldots (8)$$

one arrives at the result for the current associated with the converted wave. For details of the calculation the reader is referred to Appendix A.

We now wish to compute the intensity of the emission from the plasma and to do this we are obliged to make some further approximations. We shall ignore entirely the effects of the sheaths at the walls of the discharge on the emitted radiation and, further, suppose that the plasma is optically thin. The emission/unit volume per unit solid angle in the direction $\hat{\underline{k}}$/unit frequency interval is then

$$\frac{1}{V}\frac{dI}{d\omega d\sigma} = \frac{(2\pi)^5}{4}\frac{\omega^2}{c^3}\left\langle j_{3\alpha} j_{3\beta}^{*}\right\rangle_{\underline{k},\,\omega} \qquad \ldots (9)$$

where $\langle j_{3\alpha}\, j_{3\beta}^{*}\rangle_{\underline{k},\,\omega}$ is the correlation function of the current spectral density. If we choose Ox such that the incident quasi-electrostatic wave propagates almost parallel to Ox the quantity of interest is $\langle j_{3x}\, j_{3x}^{*}\rangle_{\underline{k},\,\omega}$ which has been computed in Appendix A:

$$\left\langle j_{3x}\, j_{3x}^{*}\right\rangle_{\underline{k}\,\omega} \simeq 4e^2\left(\frac{4\pi e^2}{m}\right)^4 \int \frac{d\underline{k}_1}{k_1^4\,|\underline{k}-\underline{k}_1|^2}\cdot\frac{\omega_1^2+\Omega^2}{(\omega_1^2-\Omega^2)^2}$$

$$\cdot\left|\sum_{m=-\infty}^{\infty} m\Omega\int dv_\perp\int dv_\parallel\,\frac{J_m^2\left(\frac{k_1^\perp v_\perp}{\Omega}\right)\frac{\partial f_o}{\partial v_\perp}}{\omega_1+k_1^\parallel v_\parallel+m\Omega}\right|^2$$

$$\cdot\left\langle \delta N_e\,\delta N_e^{*}\right\rangle_{\underline{k}_1,\,\omega_1}\left\langle \delta N_i\,\delta N_i^{*}\right\rangle_{\underline{k}-\underline{k}_1} \qquad \ldots (10)$$

We wish to note here that the remark in the Belgrade conference paper[28] that (10) represents an overestimate of the current spectral density correlation function which has to be corrected for the lower harmonics is incorrect; the contribution to (10) from the neglected terms is negligible even for $m = 2$.

3. EMISSION FROM PLASMAS IN THERMAL EQUILIBRIUM

One sees at once from (10) that the plasma emissivity will be peaked in the neighbourhood of the cyclotron harmonic frequencies. To extract further information it is necessary to insert an explicit form for the equilibrium distribution function $f_o(v_\perp, v_\parallel)$ in (10), and one must also specify the correlation functions of the electron and ion density fluctuations. It is clear from (A7), (A8) that the emission is strongest in directions closely perpendicular to $\underline{B}_o$ and we confine our attention to these directions in the following discussion. A more general discussion, including plasmas with velocity distributions differing from a Maxwellian due to the presence of a flux of suprathermal electrons will be given in a later paper[23]. Thus, with

$$f_o(v_\perp\ v_\parallel) = N\left(\frac{m_e}{2\pi\kappa T_\perp}\right)\left(\frac{m_e}{2\pi\kappa T_\parallel}\right)^{1/2}\exp\left\{-\frac{1}{2}\frac{m_e v_\perp^2}{\kappa T_\perp}-\frac{1}{2}\frac{m_e v_\parallel^2}{\kappa T_\parallel}\right\}$$

one may compute the intensity of emission from the plasma in the neighbourhood of the cyclotron harmonic frequencies and determine the ratio of the intensities of neighbouring lines. One is obliged to put into (10) simply the thermal equilibrium values for the density fluctuation correlation functions although, as stressed earlier, the mechanism proposed here is really only viable for strongly non-equilibrium fluctuations. So using

$$\langle \delta N_i \; \delta N_i^* \rangle_{\underline{k}-\underline{k}_1} = \frac{N}{(2\pi)^6} \frac{|\underline{k}-\underline{k}_1|^2 + \kappa_D^2}{|\underline{k}-\underline{k}\;|^2 + 2\kappa_D^2}$$

where κ_D is the Debye wave number, one finds (Appendix B)

$$\langle j_{3x} \; j_{3x}^* \rangle_{\underline{k},\,\omega} \simeq \frac{e^2}{(2\pi)^{12}} \; \omega_p^8 \left(\frac{m_e}{\kappa T_\perp} \right)^2 \left(\frac{m_e}{\kappa T_\parallel} \right) \frac{\omega_1^2 + \Omega^2}{(\omega_1^2 - \Omega^2)^2}$$

$$\cdot \int_0^\infty \!\! \int_0^\infty \frac{dk_1^\perp dk_1^\parallel}{k^{\perp 5}} \left| \sum_{m=0}^{\infty} \frac{m\Omega}{k_1^\parallel} \exp\left(-\frac{k_1^{\perp 2}}{2\Omega^2} \frac{\kappa T_\perp}{m_e} \right) \cdot I_m \left(\frac{k_1^{\perp 2}}{2\Omega^2} \frac{\kappa T_\perp}{m_e} \right) Z_m \left(\frac{\omega_1 - m\Omega}{k_1^\parallel \sqrt{\frac{2\kappa T_\parallel}{m_e}}} \right) \right|^2 \qquad \ldots (11)$$

where $I_m(\zeta)$ is the modified Bessel function of the first kind and $Z_m(z)$ is the plasma dispersion function.

In Appendix B we have computed the intensity of emission from the plasma using the above expression, and inserting values of the parameters which correspond approximately to the experiments of Tanaka et al. We find intensities of the order of 10^{-22}watts-cycle^{-1} sec. which is three orders of magnitude smaller than typical observed intensities. However, one expects that for non-equilibrium plasmas this intensity could be significantly enhanced and the mechanism rendered viable.

While it is not possible to make other than qualitative remarks about the absolute intensities at the cyclotron harmonics, one may estimate the ratio of the intensities of neighbouring harmonics which one might expect would not depend very strongly on the actual form of the correlation functions of the density fluctuations. This has been done numerically (see Appendix C) using parameters which correspond fairly closely to the experiments of Tanaka and one finds the ratios shown in Table 1, in which $\rho_{p,p+1}$ represents the ratio of the intensity of the p^{th} harmonic to that of the $(p+1)^{th}$.

TABLE 1

p	2	3	4	5	6	7
theor. $\rho_{p,p+1}$	1.0	1.4	1.4	1.4	1.4	1.4
exp $\rho_{p,p+1}$	1.2	1.3	1.3	1.3	1.2	1.2

From Table 1 it is apparent that the intensity of emission falls off rather slowly with increase in harmonic number, which is one of the principal features of the observations.

4. DISCUSSION

It remains to set the calculation reported here in the context of other work on this problem. With one exception all the mechanisms proposed involve the conversion of energy in electron plasma waves into radiation. The exception is the model proposed by Simon and Rosenbluth[13] who observed that electrons reflected by sheaths at the walls of the discharge tube produce radiation at harmonics of the electron cyclotron frequency. The electron Larmor orbits are distorted by the sheath electric fields and so emit radiation at the cyclotron harmonic frequencies. Simon and Rosenbluth found that the intensities of the emitted radiation at the harmonic frequencies were roughly comparable. However they find that the harmonic lines are narrow and centred at the harmonic frequency only in the weak field case (i.e. large sheath thickness) and in this situation the line intensities are several orders of magnitude down on observed values. To get the observed intensities means having a strong field and in this case the line shape is very different from the observed spectra. The optimum fit with experimental data occurs for sheath thicknesses of the order of a Debye length. However for typical sheath thicknesses found in practice the SR model predicts line profiles and intensities considerably at variance with those observed. These authors remark that to some extent their model is capable of being tested and suggested varying the sheath thickness and observing the intensity variation and the predicted shift of the emission lines from the exact harmonic positions. Buchsbaum[29] has carried out an experiment (in absorption) in an effort to check the SR theory; he attempted to distinguish between absorption occurring in the sheath region and that occurring in the volume of the plasma by varying the diameter of the plasma column and "shaping" the radial dependence of the microwave field in the TE_{011}- mode of the cylindrical cavity. Unfortunately the results of this experiment were inconclusive; but Buchsbaum[30] has remarked that while absorption at harmonic frequencies does occur at the periphery of the plasma column it does not wholly originate in the sheath regions, and infers that although the SR theory may account for some of the absorption or emission at the cyclotron harmonics it is not the whole story.

The other mechanisms[12,14] both involve the quasi-electrostatic waves which appear in the present calculation. The work of Canobbio and Croci suggested that the quasi-electrostatic waves propagating almost orthogonally to the magnetic field are coupled to the radiation field at the sharp density discontinuity at the plasma surface. Against this one may raise the objection levelled at the SR model i.e. that it appears from experiments that the phenomena in emission and absorption are not simply surface effects. Secondly, in estimating the 'transmission coefficient' coupling the longitudinal waves with the vacuum electromagnetic waves Canobbio and Croci have used an expression which appears to give a significantly larger value for this quantity than that found by Tidman and Boyd[31]. It was at this point that the work described in this report was undertaken in an attempt to avoid invoking coupling mechanisms which operated only at the edges of the plasma. Since then, Canobbio and Croci[32] have modified their original work to relax the requirement that coupling to the radiation field takes place at the sharp discontinuity at the plasma boundary. They now envisage coupling taking place in the neighbourhood of the resonances of the index of refraction for the longitudinal waves and claim that the value of the coupling coefficient is comparable to that found in their earlier paper.

Finally there is the mechanism due to Kuckes and Dawson[14] in which energetic electrons gyrating in the region of plasma hybrid resonance give rise to cyclotron harmonic emission. In this model there are two distinct electron energy distributions - one cold and high density plus one of low density hot electrons. They remark that if the plasma has a density gradient there exists a plane of hybrid resonance $[\omega^2 = \omega_p^2 + \Omega^2]$ for extraordinary mode propagation and this region of hybrid resonance can have dimensions much less than a cold electron Larmor radius. Then, as the fast electrons cross this singularity plasma oscillations get excited and these are coupled to the vacuum radiation field via an evanescent region. They find agreement with Landauer's observations if they assume a density of hot electrons of 2×10^9 electrons/cm^3 at 400 eV; in the PIG discharges of Landauer's experiments such a concentration of suprathermal electrons is not unlikely but, for the more quiescent discharges in the work of Tanaka and Kubo for example, these figures would not be realistic. This mechanism, together with a modification proposed by Stix[33], is discussed more fully elsewhere[23].

5. CONCLUSION

In conclusion one may say that the model chosen here is capable of explaining the appearance of lines in the emission spectrum near the harmonics of the electron cyclotron frequency and predicts the observed slow fall off in intensity with increase in harmonic number. The calculation as carried out here for a plasma in thermal equilibrium gives values of the emissivity that are several orders of magnitude too small, but, qualitatively, one expects that a departure from thermal equilibrium could give rise to much larger values for the emissivity.

6. ACKNOWLEDGEMENTS

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APPENDIX A

From (7), using the expression for f_1 determined from (4) one has:-

$$f_3^1 = - i \left(\frac{4\pi e^2}{m_e}\right)^2 \frac{1}{\Omega k_1^2 k_2^2} \int_{-\infty}^{\varphi} d\varphi^1 \, \delta N_e (\underline{k}_1, \omega_1) \left[\delta N_i (\underline{k}_2, \omega_2) - \delta N_e (\underline{k}_2, \omega_2) \right]$$

$$\cdot \underline{k}_2 \cdot \frac{\partial}{\partial \underline{v}} \left\{ (k_1^\perp \frac{\partial f_o}{\partial v_\perp} + k_1^\parallel \frac{\partial f_o}{\partial v_\parallel}) \sum_{m=-\infty}^{\infty} \frac{J_m (\frac{k_1^\perp v_\perp}{\Omega})}{\omega_1 + k_1^\parallel v_\parallel + m\Omega} \right. \cdot$$

$$\left. \cdot \exp \left[- \frac{i k_1^\perp v_\perp}{\Omega} \sin \varphi^1 + im \varphi^1 - \frac{i\omega_3}{\Omega} (\varphi - \varphi^1) \right] \right\} \qquad \text{... (A1)}$$

In our chosen coordinate system

$$\underline{v} = v_\perp \cos \varphi \, \hat{\underline{x}} + v_\perp \sin \varphi \, \hat{\underline{v}} + v_\parallel \, \hat{\underline{z}} \cdot$$

and

$$\underline{k}_{1,2} = k_{1,2}^\perp \, \hat{\underline{x}} + k_{1,2}^\parallel \, \hat{\underline{z}} \quad .$$

Because this is Rayleigh scattering we can replace ω_3 in (A1) by ω_1 ; substituting in (8) gives

$$j_3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\frac{4\pi e^2}{m_e}\right)^2 \frac{e}{\Omega} \frac{\delta N_e (\underline{k}_1, \omega_1) \left[\delta N_i (\underline{k}_2, \omega_2) - \delta N_e (\underline{k}_2, \omega_2) \right]}{k_1^2 k_2^2}$$

$$\int_0^\infty dv_\perp \, v_\perp \int_{-\infty}^{\infty} dv_\parallel \left\{ k_2^\perp \frac{\partial}{\partial v_\perp} + k_2^\parallel \frac{\partial}{\partial v_\parallel} \right\} \left\{ (k_1^\perp \frac{\partial f_o}{\partial v_\perp} + k_1^\parallel \frac{\partial f_o}{\partial v_\parallel}) \right. \cdot$$

$$\left. \cdot \sum_m \sum_\mu \frac{J_m (\frac{k_1^\perp v_\perp}{\Omega}) \, J_\mu (\frac{k_1^\perp v_\perp}{\Omega})}{\omega_1 + k_1^\parallel v_\parallel + m\Omega} \, \frac{1}{m - \mu + \frac{\omega_1}{\Omega}} \right. \begin{bmatrix} \frac{v_\perp}{2} [\delta_{m,\mu-1} + \delta_{m,\mu+1}] \\ \frac{v_\perp}{2i} [\delta_{m,\mu-1} - \delta_{m,\mu+1}] \\ v_\parallel \, \delta_{m,\mu} \end{bmatrix} \qquad \text{... (A2)}$$

Making use of the approximation

$$k_1^\perp \gg k_1^\parallel \quad , \quad k_2^\perp \gg k_2^\parallel$$

(A2) becomes

$$j_{3x}(\underline{k}, \omega) \simeq - e \left(\frac{4\pi e^2}{m_e}\right)^2 \int d\underline{k}_1 \frac{k_1^{\perp}}{2\Omega k_1^2 |k - k_1|}$$

$$\cdot \left\{ \delta N_e (\underline{k}_1, \omega_1) \left[\delta N_i (\underline{k} - \underline{k}_1, \omega - \omega_1) - \delta N_e (\underline{k} - \underline{k}_1, \omega - \omega_1) \right] \right. \qquad \ldots \text{(A3)}$$

$$\cdot \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\perp} \frac{\partial f_o}{\partial v_{\perp}} \sum_{m=-\infty}^{\infty} \frac{J_m\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \left[\frac{J_{m+1}\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\frac{\omega_1}{\Omega} - 1} + \frac{J_{m-1}\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\frac{\omega_1}{\Omega} + 1} \right]$$

$j_{3y}(\underline{k}, \omega)$ is identical to (A3) except for a factor (x i) and the last bracket is replaced by

$$\left[\frac{J_{m+1}\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\frac{\omega_1}{\Omega} - 1} - \frac{J_{m-1}\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\frac{\omega_1}{\Omega} + 1} \right]$$

$$j_{3z}(\underline{k}, \omega) \simeq - e \left(\frac{4\pi e^2}{m_e}\right)^2 \frac{k_1^{\parallel}}{\Omega k_1^2 |\underline{k} - \underline{k}_1|} \delta N_e (\underline{k}_1, \omega_1) \left[\delta N_i (\underline{k} - \underline{k}_1, \omega\ \omega_1) - \delta N_e (\underline{k} - \underline{k}_1, \omega - \omega_1) \right]$$

$$\cdot \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp} v_{\parallel} \frac{\partial f_o}{\partial v_{\parallel}} \sum_m \frac{J_m^2 \left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \frac{\Omega}{\omega_1} \qquad \ldots \text{(A4)}$$

We may now compute the quantity appearing in (9), i.e. the correlation function of the current density fluctuations defined by

$$\langle j_{3\alpha} j_{3\beta}^* \rangle_{\underline{k}, \omega} \equiv \iint \langle j_{3\alpha}(\underline{r}, t) \, j_{3\beta}^*(\underline{r} + \underline{\rho}, t + \tau) \rangle \cdot \exp \left[- i (\underline{k} \cdot \underline{\rho} - \omega \tau) \right] d\underline{\rho} d\tau$$

If we choose Ox such that the incident quasi-electrostatic wave propagates almost parallel to Ox the quantity of interest is $\langle j_{3x} j_{3x}^* \rangle_{\underline{k}, \omega}$.

Now

$$\langle j_{3x} j_{3x}^* \rangle_{\underline{k}^1 \omega} \delta(\underline{k} + \underline{k}^1) = \langle j_{3x} (\underline{k}, \omega) \, j_{3x}^* (\underline{k}^1, \omega) \rangle \qquad \ldots \text{(A5)}$$

Substituting for $j_{3x}(\underline{k}, \omega)$ from (A3), using the fact that δN_e, δN_i are statistically independent and representing the other terms in $\underline{k}_1$ in (A3) symbolically by $M(k_1)$, (A5) becomes

$$\langle j_{3x} j_{3x}^* \rangle_{k^1 \omega} \delta(\underline{k} + \underline{k}^1)$$

$$= \iint d\underline{k} \, d\underline{k}^1 \, M(k_1) M^*(k_1^1) \langle \delta N_e \, \delta N_e^* \rangle_{\underline{k}_1^1} \delta(\underline{k} + \underline{k}_1^1) \cdot \langle \delta N_i \, \delta N_i^* \rangle_{\underline{k}^1 - \underline{k}_1^1} \delta(\underline{k} - \underline{k}_1 + k^1 - \underline{k}_1^1)$$

i.e.

$$\langle j_{3x}\, j^*_{3x} \rangle_{\underline{k},\,\omega} = \int d\underline{k}_1 \; |M(k_1)|^2 \; \langle \delta N_e \; \delta N_e^* \rangle_{\underline{k}_1,\,\omega} \; \langle \delta N_i \; \delta N_i^* \rangle_{\underline{k}-\underline{k}_1} \qquad \text{... (A6)}$$

and

$$M(k_1) = -e \left(\frac{4\pi e^2}{m_e} \right)^2 \frac{1}{k_1^2 \; |\underline{k} - \underline{k}_1|} \; \frac{\Omega}{\omega_1^2 - \Omega^2}$$

$$\sum_{m=-\infty}^{\infty} \cdot \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} dv_{\perp} \; \frac{J_m^2\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right)}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \left\{ m\omega_1 \frac{\partial f_o}{\partial v_{\perp}} + \frac{\Omega}{2} \frac{\partial}{\partial v_{\perp}} \left(v_{\perp} \frac{\partial f_o}{\partial v_{\perp}} \right) \right\}$$

After some algebra one arrives at the following expressions for

$$\langle j_{3x}\, j^*_{3x} \rangle_{\underline{k},\,\omega} \quad , \quad \langle j_{3z}\, j^*_{3z} \rangle_{\underline{k},\,\omega} \quad :$$

$$\langle j_{3x}\, j^*_{3x} \rangle_{\underline{k}\,\omega} \simeq 4e^2 \left(\frac{4\pi e^2}{m}\right)^4 \int \frac{d\underline{k}_1}{k_1^4 \; |\underline{k}-\underline{k}_1|^2} \cdot \frac{\omega_1^2 + \Omega^2}{(\omega^2 - \Omega^2)^2}$$

$$\cdot \left| \sum_{m=-\infty}^{\infty} m\Omega \int dv_{\perp} \int dv_{\parallel} \; \frac{J_m^2\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right) \frac{\partial f_o}{\partial v_{\perp}}}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \right|^2 \cdot \langle \delta N_e \; \delta N_e^* \rangle_{\underline{k}_1,\,\omega_1} \langle \delta N_i \; \delta N_i^* \rangle_{\underline{k}-\underline{k}_1}$$

... (A7)

$$\langle j_{3z}\, j^*_{3z} \rangle_{\underline{k}\,\omega} \simeq 4e^2 \left(\frac{4\pi e^2}{m}\right)^2 \int \frac{d\underline{k}_1}{k_1^4 \; |\underline{k}-\underline{k}_1|^4} \; \frac{k_1^{\parallel 2} \, \Omega^2}{\omega_1^2}$$

$$\cdot \left| \sum_{m=-\infty}^{\infty} \int dv_{\perp} \int dv_{\parallel} \; \frac{v_{\parallel} \, J_m^2\left(\frac{k_1^{\perp} v_{\perp}}{\Omega}\right) \frac{\partial f_o}{\partial v_{\parallel}}}{\omega_1 + k_1^{\parallel} v_{\parallel} + m\Omega} \right|^2 \cdot \langle \delta N_e \; \delta N_e^* \rangle_{\underline{k}_1,\,\omega_1} \; \langle \delta N_i \; \delta N_i^* \rangle_{\underline{k}-\underline{k}_1}$$

... (A8)

APPENDIX B

We wish to use (A7) to estimate the intensity of emission from the plasma at the cyclotron harmonic frequencies. Taking

$$f_o\,(v_\perp\, v_\parallel) = N\,\left(\frac{m_e}{2\pi\kappa T_\perp}\right)\left(\frac{m_e}{2\pi\kappa T_\parallel}\right)^{1/2}\exp\left\{-\frac{1}{2}\frac{m_e v_\perp^2}{\kappa T_\perp}-\frac{1}{2}\frac{m_e v_\parallel^2}{\kappa T_\parallel}\right\} \qquad \ldots \text{(B1)}$$

Substituting in (A7) one has

$$\langle j_{3x}\, j_{3x}^* \rangle_{\underline{k},\,\omega} = N^2 e^2 \left(\frac{4\pi e^2}{m}\right)^4 \left(\frac{m_e}{\kappa T_\perp}\right)^2 \left(\frac{m_e}{2\pi\kappa T_\parallel}\right) \int \frac{d\underline{k}_1}{k_1^4\,|\underline{k}-\underline{k}_1|^2}\cdot$$

$$\cdot\left\{\frac{\omega_1^2+\Omega^2}{(\omega_1^2-\Omega^2)^2}\ \langle \delta N_e\, \delta N_e^* \rangle_{\underline{k}_1,\,\omega_1}\ \langle \delta N_i\, \delta N_i^* \rangle_{\underline{k}-\underline{k}_1}\right.$$

$$\left.\cdot\left|\sum_{m=-\infty}^{\infty} m\Omega \int_{-\infty}^{\infty}\frac{dv_\parallel\,\exp\left(-\frac{mv_\parallel^2}{2\kappa T_\parallel}\right)}{\omega_1+k_1^\parallel v_\parallel + m\Omega}\cdot\int_0^\infty dv_\perp\, v_\perp\,\exp\left(-\frac{mv_\perp^2}{2\kappa T_\perp}\right) J_m^2\left(\frac{k_1^\perp v_\perp}{\Omega}\right)\right|^2\right\} \qquad \ldots \text{(B2)}$$

The integration over $v_\perp$ is simply

$$\int_0^\infty dv_\perp\, v_\perp\,\exp\left(-\frac{m_e v_\perp^2}{2\kappa T_\perp}\right) J_m^2\left(\frac{k_1^\perp v_\perp}{\Omega}\right) = \exp\left(-\frac{k_1^{\perp 2}}{2\Omega^2}\frac{\kappa T_\perp}{m_e}\right) I_m\left(\frac{k_1^{\perp 2}}{2\Omega^2}\frac{\kappa T_\perp}{m_e}\right)$$

where $I_m(z)$ is the modified Bessel function of the first kind.

The integration over $v_\parallel$ involves the plasma dispersion function $Z_m(\tau)$ (34), defined as

$$Z(\tau) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} dx\cdot\frac{\exp(-x^2)}{x-\tau}$$

for $\mathrm{Im}\,\tau > 0$ and as the analytic continuation of this for $\mathrm{Im}\,\tau \leqslant 0$ i.e. (B2) becomes

$$\langle j_{3x}\, j_{3x}^* \rangle_{\underline{k},\,\omega} = N^2 e^2 \left(\frac{4\pi e^2}{m}\right)^4 \left(\frac{m_e}{\kappa T_\perp}\right)^2 \left(\frac{m_e}{2\pi\kappa T_\parallel}\right)\cdot\frac{\omega_1^2+\Omega^2}{(\omega_1^2-\Omega^2)^2}$$

$$\cdot\int\frac{d\underline{k}_1}{k_1^4\,|\underline{k}-\underline{k}_1|^2}\ \langle \delta N_e\, \delta N_e^* \rangle_{\underline{k}_1,\,\omega_1}\ \langle \delta N_i\, \delta N_i^* \rangle_{\underline{k}-\underline{k}_1}$$

$$\cdot\left|\sum_{m=-\infty}^{\infty}\frac{m\Omega}{k_1^\parallel}\exp\left(-\frac{k_1^{\perp 2}}{2\Omega^2}\frac{\kappa T_\perp}{m_e}\right) I_m\left(\frac{k_1^{\perp 2}}{2\Omega^2}\frac{\kappa T_\perp}{m_e}\right)\cdot Z_m\left(\frac{\omega_1+m\Omega}{k_1^\parallel\sqrt{\frac{2\kappa T^\parallel}{m_e}}}\right)\right|^2 \qquad \ldots \text{(B3)}$$

To proceed with the computation of the plasma emissivity one must now use the thermal equilibrium values for the density fluctuation correlation functions i.e.

$$\langle \delta N \delta N^* \rangle_{\underline{k}} = \frac{N}{(2\pi)^6} \frac{k^2 + \kappa_D^2}{k^2 + 2\kappa_D^2}$$

where κ_D is the Debye wave number. Then (B3) becomes

$$\langle j_{3x} j_{3x}^* \rangle_{\underline{k},\omega} \simeq \frac{e^2}{(2\pi)^{12}} \omega_p^8 \left(\frac{m_e}{\kappa T_\perp}\right)^2 \left(\frac{m_e}{\kappa T_\parallel}\right) \frac{\omega_1^2 + \Omega^2}{(\omega_1^2 - \Omega^2)^2}$$

$$\cdot \int_0^\infty \frac{dk_1^\perp}{k_1^{\perp 5}} \int_{-\infty}^\infty \frac{dk_1^\parallel}{k_1^{\parallel 2}} \left| \sum_{m=-\infty}^{\infty} m\Omega \cdot \right.$$

$$\left. \cdot \exp\left(-\frac{k_1^{\perp 2}}{2\Omega^2} \frac{\kappa T_\perp}{m_e}\right) I_m \left(\frac{k_1^{\perp 2}}{2\Omega^2} \frac{\kappa T_\perp}{m_e}\right) Z_m \left(\frac{\omega_1 + m\Omega}{k_1^\parallel \sqrt{\frac{2\kappa T^\parallel}{m_e}}}\right) \right|^2 \qquad \ldots \text{(B4)}$$

where ω_p is the plasma frequency. Then from (9) we may estimate the intensity of emission from the plasma near to the cyclotron harmonic frequencies; for $m = 2$:-

$$\frac{dI}{d\omega d\tau} \simeq 4.6 \times 10^{-102} \omega^2 \cdot \omega_p^8 T_\perp^{-2} T_\parallel^{-1}$$

$$\cdot \int_0^\infty \frac{dk_1^\perp}{k_1^{\perp 5}} \exp\left(-\frac{k_{1\perp}^2}{\Omega^2} \frac{\kappa T_\perp}{m_e}\right) I_2^2 \left(\frac{k_{1\perp}^2}{2\Omega^2} \frac{\kappa T_\perp}{m_e}\right)$$

$$\cdot \int_0^{\hat{k}_1^\parallel} \frac{dk_1^\parallel}{k_1^{\parallel 2}} \left| Z \left(\frac{\omega - 2\Omega}{k_1^\parallel \sqrt{\frac{2\kappa T^\parallel}{m_e}}}\right) \right|^2 \qquad \ldots \text{(B5)}$$

where $T_\perp$, $T_\parallel$ are measured in electron-volts and $\hat{k}_1^\parallel$ is a maximum value of $k_1^\parallel$ such that

$$\frac{\omega - 2\Omega}{k_1^\parallel \sqrt{\frac{2\kappa T_\parallel}{m_e}}} \gg 1$$

to ensure that cyclotron damping is small.

In computing (B5) we have chosen parameters which correspond fairly closely to the experiments of Tanaka and his associates[(6)]. In this case $\omega \sim 4000$ Mc/s and for the purpose of this estimate it is adequate to use this value for ω_p as well. From (B5) using

$$Z(\zeta) \simeq -\frac{1}{\zeta} \left(1 + \frac{1}{2\zeta^2} + \ldots.\right)$$

$$\frac{dI}{d\omega d\sigma} \simeq 5 \times 10^{-6} T_\perp^{-2} T_\parallel^{-1} \left[\frac{\frac{2\kappa T_\parallel}{m_e} \hat{k}_1^\parallel}{(\omega - 2\Omega)^2}\right]$$

$$\cdot \int_0^\infty \frac{dk_1^\perp}{k_1^{\perp 5}} \exp\left(-\frac{k_{1\perp}^2}{\Omega^2} \frac{\kappa T_\perp}{m_e}\right) I_2^2 \left(\frac{k_{1\perp}^2}{2\Omega^2} \frac{\kappa T_\perp}{m_e}\right)$$

There are no explicit values given in Tanaka's paper for $T_\perp$, $T_\parallel$ so we have assumed $T_\perp \sim 3$ eV, $T_\parallel \sim 1$ eV. Again to arrive at the most optimistic answer for the intensity of emission at the second harmonic we have taken the quantity in square brackets to be $O(10^{-1})$. The integral over $k_1^\perp$ has to be evaluated numerically and for m = 2 this is approximately 0.5×10^{-6}. Then

$$\frac{dI}{d\omega d\sigma} \sim (5 \times 10^{-6}) \times (10^{-2}) \times (0.5 \times 10^{-6}) \text{ ergs.sec}^{-1}\text{-cycle}^{-1}\text{sec.-sterad}^{-1}$$

i.e.

$$\frac{dI}{d\omega d\sigma} \sim 2.5 \times 10^{-21} \text{ watts - cycle}^{-1}\text{sec - sterad}^{-1} .$$

If we now use a value of solid angle in keeping with the approximations made above of order $\frac{1}{3} \times 10^{-2}$ one finds for the radiation intensity

$$\frac{dI}{d\omega} \sim 10^{-23} \text{ watts - cycle}^{-1} \text{ sec.}$$

APPENDIX C

We now compute the relative intensities of neighbouring harmonics. We recall that in estimating absolute intensities one had to specify the actual form of the correlation functions of the density fluctuations; we may suppose the relative intensities, however, do not depend very strongly on the actual forms of these correlation functions and so

$$\left.\frac{dI}{d\omega}\right|_{\omega\sim m\Omega} \propto \frac{m^2(m^2+1)}{(m^2-1)^2}\int_{k_1^{\perp}\,min}^{\infty}\frac{dk_1^{\perp}}{k_1^{\perp 3}}\exp\left(-\frac{m^2k_1^{\perp 2}}{\omega^2}\,\frac{\kappa T_{\perp}}{m_e}\right)\cdot I_m^2\left(\frac{m^2k_1^{\perp 2}}{2\omega^2}\cdot\frac{\kappa T_{\perp}}{m_e}\right) \qquad \ldots \text{(C1)}$$

where the lower limit is determined by the dimensions of the discharge tube. Writing

$$x = \frac{m^2k_1^{\perp 2}}{2\omega^2}\,\frac{\kappa T_{\perp}}{m_e}$$

(C1) becomes

$$\left.\frac{dI}{d\omega}\right|_{\omega\sim m\Omega} \propto \frac{m^6(m^2+1)}{(m^2-1)^2}\int_{x_{min}}^{\infty}\frac{dx}{x^3}\exp(-2x)\,I_m^2(x) \qquad \ldots \text{(C2)}$$

and this expression has been evaluated numerically.

APPENDIX D

FINE STRUCTURE IN EMISSION AND ABSORPTION AT LOW HARMONIC NUMBERS

Some of the observations of absorption showed a curious line profile in the region of the cyclotron harmonic frequencies[4]. The main feature was that the central part of the line showed inversion and dipped below the background absorption. This effect has since been examined in some detail by Buchsbaum[30] in absorption and by Tanaka and Kubo[6] in emission. To illustrate the phenomena a typical experimental result is reproduced in Fig.2. One observes a series of peaks in the region of the second and the third harmonics, the magnitudes of which increase with increasing current I_d up to some value (~ 200 mA in emission) while, at the same time, their spacing becomes closer and closer until eventually, they coalesce to form the 'inverted line' reported in the earlier work of Bekefi and others.

These phenomena have been interpreted in a satisfactory way by Buchsbaum and Hasegawa[7]. (In passing, it may be noted that the Simon-Rosenbluth model did predict a line inversion reminiscent of the observed feature; however, the cause of this 'interference dip' is somewhat unclear physically and its relation to the actual inversion seems now to have been coincidental). Buchsbaum and Hasegawa attribute the resonances to the excitation of longitudinal plasma oscillations within narrow pass-bands near each cyclotron harmonic - the so-called Bernstein modes. These are modified distinctly by the presence of density gradients. A good deal of work has appeared lately on the effect of density gradients, in the absence of magnetic fields, on the Tonks-Dattner resonances; see, e.g. the work of Parker, Nickel, and Gould[35] and further papers listed there. The Tonks-Dattner resonances can be excited only at frequencies above the local plasma frequency. In the presence of a magnetic field, however, Bernstein[25] has shown that plasma oscillations may be excited at frequencies lower than the subharmonics of Ω, provided the plasma frequency exceeds some critical value which is a function of B_o. Accordingly these oscillations are confined to a region of the plasma near the axis and are therefore insensitive to conditions in the sheath (whereas, of course, the Tonks-Dattner resonances arise out of the region between the walls and some critical radius given by $r = r_c$, at which $\omega^2 \sim \omega_p^2 (r_c)$ and so depend quite critically on the density profile near the walls). Buchsbaum and Hasegawa claim that these oscillations are responsible for the resonances in their observations. They consider a system with plane geometry, assume a parabolic electron density distribution and show that there exists in the region of the cyclotron harmonics a series of resonances; the amplitudes of these resonances are greatest for $m = 2$, but they persist for $m = 3,4$ as well. Furthermore their spacing shows good agreement with the experimental observations of Buchsbaum and Hasegawa (in cylindrical geometry).

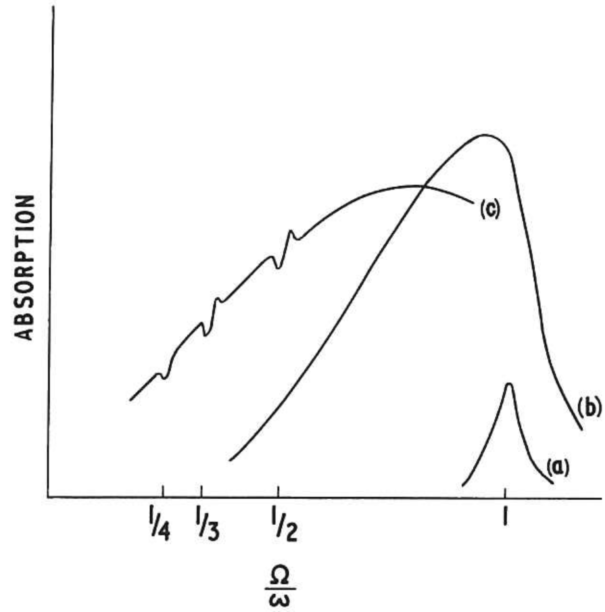

Fig. 1 (CLM-R51)

Schematic plot of absorption when:

(a) $\frac{\omega_p}{\omega} \ll 1$, (b) $\frac{\omega_p}{\omega} \to 1$, (c) $\frac{\omega_p}{\omega} \sim 1$.

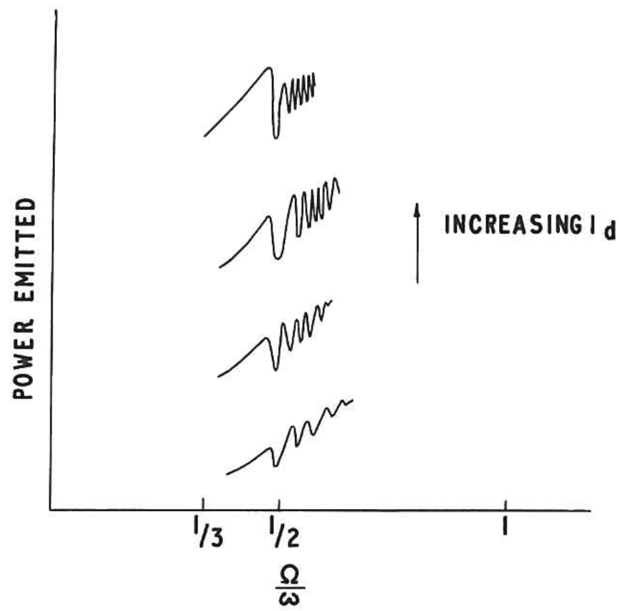

Fig.2 (CLM-R51)

Schematic plot of the fine structure in emission and absorption spectra at low harmonic numbers

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