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THE EFFECT OF COULOMB COLLISIONS ON THE SCATTERING OF LIGHT BY PLASMAS

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THE EFFECT OF COULOMB COLLISIONS ON THE SCATTERING OF LIGHT BY PLASMAS

by

T.J.M. BOYD

A B S T R A C T

The effect of Coulomb collisions on the low frequency ion acoustic resonance is estimated using a simple hydrodynamic approach and the line profile obtained is compared with that found by DuBois and Gilinsky in the limit of collision-dominated plasmas. In the neighbourhood of the ion acoustic resonance the results from the two approaches are identical but the line shape in the region of zero frequency predicted by the DuBois-Gilinsky calculation is distinct from that found here. The main result is that the low-frequency ion feature narrows significantly in collision dominated plasmas. In situations intermediate between the collisionless plasma and the collision-dominated plasma the determination of the line profile has not yet been carried out.

The contribution of Coulomb collisions to the broadening of the feature at the electron plasma frequency (sometimes called the 'plasma line' or 'satellite feature') has also been estimated and compared, over a range of parameters, with the contribution due to the Landau damping in a collision-free plasma.

It is shown that over the range of parameters covered by the Culham experiments, collision effects are not important.

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$$\tau = \frac{3}{\sqrt{2\pi}} \frac{\sqrt{m}(k_B T)^{\frac{3}{2}}}{ne^4\psi} \qquad \ldots (10)$$

$$= \log \left\{ 1 + \left(\frac{4k_B T\lambda_D}{l^2}\right)^2 \right\} \qquad \ldots (11)$$

›oadened line has been given by DuBois and Gilinsky(10) ınd DuBois(11) on the low frequency conductivity from a ıted the plasma conductivity at low frequencies and wave- coupled kinetic equations for ions and electrons. These rd type and may be solved by using a modification of the :ase of $\omega \ll \Gamma_{ii}$, where Γ_{ii} is the ion-ion collision way and the scattered line shape found near the acoustic ınalytic results are obtained in two cases: for α where $\alpha^2 = m/M$. The first case corresponds to adia- sothermal behaviour of the electrons, and the damping is and thermal conductivity of the ions as in ordinary sound on in which both ions and electrons behave adiabatically etermined by the thermal conductivity of the electrons. es, Kivelson and DuBois have not been able to find a mping.

α one finds for the damping

$$\gamma_L = \frac{\omega_L^2}{4\alpha^2 \nu_{ee}} \qquad \ldots (12)$$

$$2k^2c_i^2; \quad c_s^2 = \frac{5}{3}\frac{k_B T}{M_s}, \quad s = i, e.$$

$$\nu_{ee} = \frac{15}{4} \cdot \frac{c_e^2 n}{\kappa_e} . \qquad \ldots (13)$$

.y of the electrons. The line profile is simply

$$n\, \frac{r_0^2}{4\pi}\; \frac{3}{10}\; \frac{\gamma_L}{(\omega-\omega_L)^2 + \frac{\gamma_L^2}{4}}\; \frac{(1+\cos^2\theta)}{2} \qquad \ldots (14)$$

OMPARISON OF LINE SHAPES

ɔuld be the same and in this section we check that this is ·-widths of the lines at the ion acoustic resonance is

namic fluctuations (of olecular orientation. vely broad feature which ture due to the thermo- the size of the relaxa- set of τ_j (characterising uld be smaller than the computed the total in- $_{\ell m} = \delta\, \varepsilon\, \delta_{\ell m}$ and

tistically independent

$$\ldots (2)$$

at the time dependence of . Thermodynamic fluctua- ch propagate at the speed do not propagate relative result from pressure the speed of sound in of scattered radiation, and $\omega_o \approx \omega_s$. So the a Brillouin doublet, the e entropy fluctuations d result in a central

ıctuations; these pro- re γ is some coefficient]). The intensity dis-

$$\ldots (3)$$

simply (from (7), (12))

$$\frac{2\Upsilon}{\Upsilon_L} = \frac{2\kappa}{\kappa_e} = 1 \qquad \ldots (15)$$

since the κ_e used by KD turns out to be twice the κ defined by Marshall[17].

Further comparing the heights of the resonances we have

(i) DuBois - Gilinsky result

$$\begin{aligned} \frac{d\sigma}{d\omega d\Omega} &= \frac{nr_o^2}{\pi} \frac{3}{10\Upsilon_L} \frac{1}{2} (1 + \cos^2\theta) \frac{\omega_2}{\omega_1} \\ &\simeq \frac{nr_o^2}{\pi} \frac{3}{10\Upsilon_L} \end{aligned} \qquad \ldots (16)$$

(ii) Hydrodynamic result

From (2), (3) the height of the line is

$$\begin{aligned} \frac{dI}{d\Delta\omega} &= \frac{3}{4\pi} \frac{\omega^4}{6\pi c^4} \frac{T\rho}{\Upsilon} \left(\frac{\partial\rho}{\partial p}\right)_T \left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2 \\ &= \frac{3}{4\pi} \frac{r_o^2}{6\pi} \frac{(4\pi)^2 N^2}{\Upsilon} \frac{T}{\rho} \left(\frac{\partial\rho}{\partial p}\right)_T \end{aligned} \qquad \ldots (17)$$

and

$$\frac{1}{\rho} \left(\frac{\partial\rho}{\partial p}\right)_T = \frac{3}{10NT}$$

$$\therefore \quad \frac{dI}{d\Delta\omega d\Omega} = \frac{nr_o^2}{\pi} \frac{3}{20\Upsilon} \qquad \ldots (18)$$

which checks with (16).

18. DG have also discussed the line shape near zero ω, given in equation (4.11) in their paper. If one drops the viscosity terms in this expression and neglects ν_{ii} compared with ν_{ee} and, further, multiplies by $k_D^2\omega_p^2$ (since they have used normalised ω,k) one finds (correcting an erroneous k^{-2} in their equation (4.11))

$$\lim_{\omega\to o} \frac{d\sigma(k,\omega)}{d\omega d\Omega} = \frac{nr_o^2}{\pi k^2} \frac{1}{25} \nu_{ee} \qquad \ldots (19)$$

And from para 1 and above

$$\lim_{\Delta\omega\to o} \frac{dI}{d\Delta\omega d\Omega} = \frac{nr_o^2}{\pi} \frac{3}{20\Upsilon_c} \qquad \ldots (20)$$

i.e. ratio of intensities of the scattered light at zero frequency, $R \equiv (19)/(20)$.

$$\begin{aligned} R &= \frac{\nu_{ee}}{25k^2} \frac{20\Upsilon_c}{3} \\ &= \frac{2}{3} \frac{\kappa}{\kappa_e} \\ &= \frac{1}{3} \end{aligned} \qquad \ldots (21)$$

i.e. the central part of the DG spectrum displays a considerably weaker resonance than that derived from the classical theory of light scattering by isotropic fluids. The main result however, is that the ion acoustic resonance in the central feature of the scattered light spectrum becomes much sharper in the collision dominated plasma. The scattered light in this region has been represented by a Lorentz line profile; to determine the line shape away from $\omega = \pm kc_s$ one may make use of a general expression in the work of DuBois and Gilinsky (ref. equation 3.16).

19. It would be interesting to look for this difference from the line profile of the central feature in the <u>collisionless</u> plasma experimentally. Unfortunately it seems (cf. Appendix) that for typical plasma densities and temperatures one would be forced to look very close to the forward direction. Situations intermediate between the collisionless and the collision-dominated cases have not yet been discussed.

LINE AT THE PLASMA FREQUENCY

20. The work of Salpeter, Dougherty and Farley and others in their calculations based on the random phase approximation gave the scattering cross section in terms of the spectrum of electron density fluctuations; this quantity was then connected[4] via a generalised Nyquist (fluctuation-dissipation) theorem to the conductivity as computed from the Vlasov equation. Some authors[14,15] have determined the effect of close Coulomb collisions on the expression for the conductivity; the correction introduced is of order k_D^3/N where k_D is the Debye wave number, N the electron density. We make use of this correction term to calculate the collision broadening of the line at the plasma frequency.

21. For the plasmas of interest, collisions between charged particles and neutrals are unimportant. The scattering cross section σ, derived by Dougherty and Farley, is given by

$$\sigma \, d\Omega \, d\omega = r_o^2 \left[1 - \sin^2 \alpha \cos^2 (\varphi - \varphi_o) \right] S \, (\underline{k} - \underline{k}_o, \omega - \omega_o) \, d\Omega d\omega \qquad \ldots (22)$$

where

$$r_o = \frac{e^2}{mc^2} = \text{classical electron radius.}$$

α = angle between $\underline{k}$, $\underline{k}_o$; φ = azimuthal angle for $\underline{k}$.

and

$$S \, (\underline{k}, \omega) = L^3 \left\langle |\Delta N_e \, (\underline{k}, \omega)|^2 \right\rangle \qquad \ldots (23)$$

Their calculation gives

$$\left\langle |\Delta N_e \, (\underline{k}, \omega)|^2 \right\rangle d\omega = \frac{\kappa \, T \, k^2}{e^2 \, \pi \, \omega^2 \, L^3} \; \mathrm{Re} \left\{ \sigma_L \right\} d\omega \qquad \ldots (24)$$

where σ_L is the longitudinal part of the conductivity tensor $\sigma_{\alpha\beta}$. Rewriting (24) in terms of the complex impedance in the notation used by Dawson and Oberman $Z = R + iX$, (23) becomes

$$S(\underline{k}, \omega) = \frac{\kappa T k^2}{e^2 \pi \omega^2} \frac{R}{R^2 + X^2} \qquad \ldots (25)$$

We are interested in the frequency region $\omega - \omega_o \sim \omega_p$ where ω_p is the electron plasma frequency, i.e. for wave numbers $|\underline{k} - \underline{k}_o| \ll k_D$. The impedance Z as used by Dawson and Oberman relates the current in the plasma to an external electric field; denoting this as Z_{ext}, it is related to the internal impedance by

$$Z_{ext} = Z_{int} + \frac{4\pi i}{\omega} \qquad \text{i.e.} \quad X_{ext} = X_{int} + \frac{4\pi}{\omega} .$$

Relating X_{int} to the longitudinal part of the conductivity tensor, $X_{int} = - |Z|^2 \operatorname{Im} \sigma_L$ and using the relation between the local conductivity tensor and the dielectric tensor i.e.

$$\varepsilon_L = 1 + \frac{4\pi i}{\omega} \sigma_L \qquad \ldots (26)$$

where

$$\varepsilon_{\alpha\beta}(\underline{k}, \omega) = \frac{k_\alpha k_\beta}{k^2} \varepsilon^L (\underline{k}, \omega) + (\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}) \varepsilon^T (\underline{k}, \omega)$$

we find

$$\operatorname{Re}\left\{\varepsilon_L\right\} = 1 - \frac{4\pi}{\omega} \frac{1}{|Z|^2} \left[\frac{4\pi}{\omega} - X_{ext}\right] \qquad \ldots (27)$$

$$\text{i.e.} \quad 1 - \frac{\omega_p^2}{\omega^2} (1 + \frac{3\kappa^2}{k_D^2}) = 1 - \frac{4\pi}{\omega} \frac{1}{|Z|^2} \left[\frac{4\pi}{\omega} - X_{ext}\right]$$

$$\therefore X_{ext} = \frac{4\pi}{\omega}\left[1 - \frac{\omega^2}{\omega_p^2} + \frac{3k^2}{k_D^2} + \text{higher order terms}\right] \qquad \ldots (28)$$

For

$$\omega - \omega_o \sim \omega_r , \omega_r^2 = \omega_p^2 (1 + \frac{3k^2}{k_D^2})$$

$$X(\omega) = X(\omega_r) + (\omega - \omega_r) \left.\frac{\partial X}{\partial \omega}\right|_{\omega = \omega_r} + \ldots$$

Substituting in (25), the profile of the line in the region of the plasma frequency can be approximated by the Lorentzian shape

$$S(\underline{k} - \underline{k}_o, \omega - \omega_o) \simeq \frac{\kappa T |\underline{k} - \underline{k}_o|^2}{e^2 \pi \omega_p^2} \left(\frac{\omega_p^2}{8\pi}\right) \frac{R}{(\omega - \omega_r)^2 + \frac{\omega_p^2 R^2}{8\pi}} \qquad \ldots (29)$$

To find the width of the line we use the Dawson - Oberman result

$$Z(\omega) \sim \frac{4\pi i \omega}{\omega_p^2} \left\{\left[1 - \frac{e^2\omega_p^2}{6mV^3} (1 - \frac{\pi}{8})\right] - i \sqrt{\frac{2}{\pi}} \frac{e^2\omega_p^2}{6mV^3\omega} \left[\ell n \left(\frac{k_{max}^2}{k_D^2}\right) - 1\right]\right\} \qquad \ldots (30)$$

$$\text{i.e.} \quad R \simeq 4\pi \sqrt{\frac{2}{\pi}} \frac{e^2}{6mV^3} \left[\ell n \left(\frac{k^2_{max}}{k^2_D} \right) - 1 \right] \qquad \ldots (31)$$

where V is a thermal velocity and $k_{max} \sim \frac{mV^2}{e^2}$ the cut-off used in their calculation. Substituting (31) and (29) it follows that the width of the line at the plasma frequency due to collision broadening is

$$\Delta\omega_c \sim \frac{\omega_p^4}{8\pi N} \sqrt{\frac{2}{\pi}} \frac{1}{6V^3} \left[\ell n \left(\frac{k^2_{max}}{k^2_D} \right) - 1 \right]$$

$$\text{i.e.} \quad \Delta\omega_c \sim \frac{\omega_p}{24\pi} \sqrt{\frac{2}{\pi}} \frac{k_D^3}{N} \left[\ell n \left(\frac{k_{max}}{k_D} \right) - 0.5 \right] \qquad \ldots (32)$$

The width calculated by Ron Dawson and Oberman is

$$\Delta\omega_c \sim \frac{\omega_p}{24\pi} \sqrt{\frac{2}{\pi}} \frac{k_D{}^3}{N} \ell n \left(\frac{k_{max}}{2k_D} \right) \qquad \ldots (33)$$

In passing it should be noted that the Dawson - Oberman result, (30), is valid only for frequencies greater than any of the collision frequencies in the plasma.

COMPARISON WITH RESULT OF DUBOIS AND GILINSKY

22. DuBois and Gilinsky[(10)] have made the same calculation though their result is cast in a slightly different form. They find

$$\Delta\omega_c \sim \frac{\omega_p}{6\sqrt{2}\pi^{3/2}} \frac{k_D^3}{N} K_a(\omega_p) \qquad \ldots (34)$$

in which

$$K_a(\omega) = \int_0^\infty dq\, q^3 \exp\left(-\frac{\hbar^2}{8} q^2\right) \exp\left(-\frac{\omega^2}{2q^2}\right) \frac{q^2+1}{q^2+2} \frac{1}{|q^2 + Q^0(\frac{\omega}{q})|^2} \equiv \ell n \left[\frac{C_a(\omega)}{\kappa T\hbar\omega} \right] \qquad \ldots (35)$$

23. In their calculation they have treated electron-ion collisions using the Born approximation; a useful rule of thumb for the validity of the Born approximation is that the kinetic energy should be large compared with bound state energies (i.e. $\sim$ 1 rydberg) - this is what is meant by "fast" collisions. So (35) is really only valid in the limit of high temperatures ($\kappa T \geqslant 100$ eV). In (14) the factor $\exp(-\frac{\hbar^2}{8} q^2)$ cuts off the integration around the reciprocal of the thermal de Broglie wavelength. For lower temperatures DuBois and Gilinsky use the same cut-off as Dawson and Oberman (i.e. classical distance of closest approach, which ignores the possibility of bound states being formed in the electron-ion collision). Their final result is

$$\Delta\omega_c \sim \frac{\omega_p}{6\sqrt{2}\,\pi^{3/2}} \frac{k_D^3}{N} \ell n \left(1.05 \frac{N}{k_D^3}\right) \qquad \ldots (36)$$

as compared with (32) above.

NUMERICAL RESULTS

24. The 'natural breadth' of the line at the plasma frequency is determined by the Landau damping of the plasma oscillation:-

$$\Delta\omega_L \sim \sqrt{\frac{\pi}{8}}\,\omega_p \left(\frac{k_D}{k}\right)^3 \exp\left[-\frac{k_D^2}{2k^2} - \frac{3}{2}\right] = 0.14\,\omega_p \left(\frac{k_D}{k}\right)^3 \exp\left(-\frac{k_D^2}{2k^2}\right) \qquad \ldots (37)$$

So clearly as $k \to 0$ the width of this feature will be determined by (32) rather than (37).

25. The relative magnitude of the natural and collision breadths has been computed for a range of temperatures and densities corresponding to an arc discharge, to a cool theta-pinch plasma and to a hot theta-pinch. The line-width in wavelength units is

$$\Delta\lambda = 2.52 \quad 10^{-12}\ \Delta\omega\ \text{Å} \qquad \ldots (38)$$

The range of parameters chosen is

(i) arc discharge : $N = 10^{15}$, $T = 2.2$ eV

(ii) cool theta-pinch : $N = 10^{16}$, $T = 10$ eV

(iii) hot theta-pinch : $N = 10^{17}$, $T = 100$ eV

For a ruby laser $\lambda = 6935$ Å, i.e.

$$k = \frac{4\pi}{\lambda} \sin\frac{\theta}{2} = 1.81 \quad 10^5 \sin\frac{\theta}{2} \qquad \ldots (39)$$

The natural width has been computed for $\frac{\theta}{2} = 10^o$, 5^o and 2.5^o. The results appear in the table and are to be compared with the collision breadth.

Plasma	$\Delta\lambda$ in Å			
	$\frac{\theta}{2} = 10^o$	$\frac{\theta}{2} = 5^o$	$\frac{\theta}{2} = 2.5^o$	Collision breadth
Arc discharge	-	0.73	0.06	0.005
Cool theta-pinch	2.1	1.0	2.10^{-4}	0.008
Hot theta-pinch	6.2	3.2	6.10^{-4}	0.003

26. It is seen that the collision breadth is negligible compared to the natural width except for the case of forward scattering at 2.5^o for a theta-pinch. It is seen to be quite negligible for the experiment of DeSilva, Evans and Forrest. In any case the laser line used in this work has a spectral width of 0.05 Å, so the question of collision breadths $\sim$ 0.005 Å is academic; however, recent developments in laser technology suggest that line breadths of $\sim$ 0.001 Å are attainable, so that in the future it may prove possible to resolve even the rather narrow line breadths predicted by this theory.

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REFERENCES

1. BOWLES, K.L. Observation of vertical-incidence scatter from the ionosphere at 41 Mc/s. Phys. Rev. Letters, vol.1, December 15, 1958. pp.454-455.

2. GORDON, W.E. Incoherent scattering of radio waves by free electrons. Proc. I.R.E., vol.46, November, 1958. pp.1824-1829.

3. SALPETER, E.E. Electron density fluctuations in a plasma. Phys. Rev., vol.120, December, 1960. pp.1528-1535.

4. DOUGHERTY, J.P. and FARLEY, D.T. A theory of incoherent scattering of radio waves by a plasma. Proc. Roy. Soc., vol. A259, November, 1960. pp.79-99.

5. DeSILVA, A.W., EVANS, D.E. and FORREST, M.J. Observation of Thomson and co-operative scattering of ruby laser light by a plasma. Nature, vol.203, September 26, 1964. pp.1321-1322.

6. FÜNFER, E., KRONAST, B. and KUNZE, H.J. Experimental results on light scattering by a θ-pinch plasma using a ruby laser. Physics Letters, vol.5, no.2, June 15, 1963. pp.125-127.

7. ASCOLI-BARTOTI, A., KATZENSTEIN, J. and LOVISETTO, L. Spectrum of laser light scattered from a single giant pulse in a laboratory plasma. Nature, vol.207, July 3, 1965. pp.63-64.

8. TAYLOR, E.C. and COMISAR, G.G. Frequency spectrum of thermal fluctuations in plasmas. Phys. Rev., vol.132, December, 1963. pp.2379-2384.

9. DUBOIS, D.F., GILINSKY, V. and KIVELSON, M.G. Propagation of electromagnetic waves in plasmas. Phys. Rev., vol.129, March 1963. pp.2376-2397.

10. DUBOIS, D.F. and GILINSKY, V. Incoherent scattering of radiation by plasmas. II. Effect of Coulomb collisions on classical plasmas. Phys. Rev., vol.133, March 1964. pp. A1317-A1322.

11. KIVELSON, M.G. and DUBOIS, D.F. Plasma conductivity at low frequencies and wave numbers. Phys. Fluids, vol.7, October 1964. pp.1578-1589.

12. RON, A., DAWSON, J. and OBERMAN, C. Influence of collisions on scattering of electromagnetic waves by plasma fluctuations. Princeton University. Plasma Physics Laboratory. Annual report, January-December 1963. MATT-Q-21, pp.269-272.

13. WILSON, R. Culham Laboratory. Private communication.

14. DAWSON, J. and OBERMAN, C. High frequency conductivity and the emission and absorption coefficients of a fully ionized plasma. Phys. Fluids, vol.5, May 1962. pp.517-524.

15. PEREL, V.I. and ELIASHBERG, G.M. Absorption of electromagnetic waves in plasmas. Sov. Phys - JETP, vol.14, March 1962. pp.633-637.

16. LANDAU, L.D. and LIFSHITZ, E. Electrodynamics of continuous media. Oxford, Pergamon, 1960.

17. MARSHALL, W. Kinetic theory of an ionized gas, part III. H.M.S.O., 1960. AERE-T/R2419.

APPENDIX

VALIDITY CONDITIONS ON EXPRESSION FOR COLLISION DAMPING

28. We recall that (7), (8) hold provided $\gamma \ll |\Delta\omega_0|$ i.e.

$$\frac{8\kappa}{15nk_B} \frac{\omega^2}{c^2} \sin^2\frac{\theta}{2} \ll \frac{2\omega}{c} \sin\frac{\theta}{2} \sqrt{\frac{10k_BT}{3M}}$$

i.e.

$$\frac{4}{5\sqrt{2\pi}} \sqrt{\frac{3}{10}\frac{M}{m}} \frac{(k_BT)^2}{ne^4\ \psi} \frac{\omega}{c} \sin\frac{\theta}{2} \ll 1. \tag{A1}$$

which gives

$$1.8 \times 10^{14} \frac{T^2}{n\psi} \frac{2\pi}{\lambda} \sin\frac{\theta}{2} \ll 1 \tag{A2}$$

where T is in electron-volts. In the light scattering experiments a ruby laser is used, having $\lambda = 6935$ Å. One then has

$$1.6 \times 10^{19} \frac{T^2}{n\psi} \sin\frac{\theta}{2} \ll 1 \tag{A3}$$

Clearly this condition is not easily satisfied; e.g. if one observes the scattered light at an angle of 10° as DeSilva et al. have done for the collisionless plasma case, this requires (since $\psi \sim O(15)$)

$$10^{17} \frac{T^2}{n} \ll 1 \tag{A4}$$

This condition seems difficult to satisfy as it requires a cold, dense plasma. It could, of course, be met in solid state plasmas.

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