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Report

SOME FACTORS INFLUENCING THE CONTAINMENT OF A CUSP PLASMA

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SOME FACTORS INFLUENCING THE CONTAINMENT OF A CUSP PLASMA

by

G.D. HOBBS
I.J. SPALDING

A B S T R A C T

A survey of some of the factors influencing the containment of a plasma in cusp geometry is presented, with particular emphasis on the effects arising from the presence of walls or plasma outside the containment volume. The effects of wall short-circuits on both the collision-free hole size and ion-ion collisional diffusion are considered. Space charge limitations of the electron thermal conduction loss are also discussed. A simple two-fluid MHD model of the SPICE II cusp compression experiment is described. Its predictions are found to agree with observation to within a factor of two, which is within the experimental uncertainty of the initial plasma conditions.

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CONTENTS

1. INTRODUCTION

The object of this report is, firstly, to outline some factors which might influence the containment of field-free ($\beta = 1$) plasma in cusped geometries; secondly to construct and evaluate a theoretical model embodying those features which are particularly relevant to the SPICE II cusp compression experiment[1].

Provided that radiation losses are not dominant, the containment of field free plasma in a cusped magnetic bottle may be characterised by the size of the 'hole' through which particles (and therefore energy) escape. The following questions must then be asked: What determines the effective size of the hole? What are the mechanisms responsible for the transport of energy through it? It can not be assumed that the answers depend on the properties of the confined plasma alone, since the hole is usually connected to the external world by the field lines that define it. If the coupling is strong, the hole size and transport mechanisms may well be dominated by the properties of the walls on which the field lines terminate or, in some experimental situations, by cold plasma and gas just outside the magnetic bottle. For this reason the first part of this report is concerned primarily with end effects.

It should be noted that some of this discussion is also relevant to other open-ended traps (e.g. high β theta-pinches and low β mirrors).

2. HOLE SIZE

THE SMALLEST CONCEIVABLE HOLE

A cusp hole arises from the penetration of the plasma into the confining magnetic field, B_o. Its minimum conceivable size can be estimated by considering the width of the thinnest possible infinite plane sheath separating collision-free $\beta = 1$ plasma from magnetic field.

Grad[2] and Schmidt and Finkelstein[3] have investigated the one-dimensional steady state sheath separating an isotropic, Maxwellian positronium plasma from a uniform magnetic field. They find that the plasma extends asymptotically into the field with a gyro-radius* scale length while the field falls to zero at a finite distance within the plasma. The analogous solution for a plasma whose components have unequal gyro-radii is complicated by the electrostatic field developed perpendicular to B_o. This electric field ensures almost equal penetration of the field by the ions and electrons, the resulting sheath thickness being

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* To avoid semantic difficulties we refer throughout to a gyro radius (r_e or r_i) evaluated at the mean energy of the field free particles in the vacuum field B_o.

in general intermediate between the ion and electron gyro-radii. The simplest sheath of this type is that discussed by Rosenbluth[4] in which thermal motion is neglected. All the ions and electrons start at $-\infty$ with equal velocities, impinge normally on the boundary and return to $-\infty$, i.e. no particles are 'trapped' permanently in the vicinity of the sheath. The width of the boundary layer is found to be c/ω_p, where ω_p is the plasma frequency. Bertotti[5] has generalised this model to include one degree of thermal motion, in the direction of ∇B only. The most general situation without trapped electrons, in which the particles have isotropic velocity distributions about a mean drift velocity perpendicular to B_o, has been studied by Morse[6]. He shows that physically acceptable solutions exist only if the thermal spread is very much less than the drift velocity, the Rosenbluth sheath being the limiting case.

Once trapped electrons are permitted a large degree of arbitrariness enters the problem since their distribution within the sheath is virtually unrestricted and may be dictated by physical intuition or mathematical convenience. Steady state solutions with trapped electrons have been obtained by Hurley[7], Nicholson[8] and Sestero[9]. In order to reduce this arbitrariness Morse[10] has investigated the development of a sheath in time, electrons being trapped in a predictable, although not unique manner by a slowly rising magnetic field. He obtains a solution, starting at $t = 0$ from a Maxwellian plasma with arbitrary electron and ion temperatures, possessing a scale length again of order c/ω_{pe}. This width is almost independent of the ratio T_i/T_e over a wide range of values.

Considerable progress has thus been made in understanding the self-consistent solutions of the infinite plane, i.e. one-dimensional, sheath. Far less information is available on the effects arising from the multi-dimensional nature of cusp geometry. Firsov[11] has assumed the existence of a self-consistent two-dimensional boundary in the absence of trapped electrons and has calculated the steady state loss of plasma from a picket fence configuration. The contained, field-free, plasma is assumed to have an isotropic Maxwellian velocity distribution with $T_e = T_i$ and the effects of an electrostatic field, related to the magnetic field by the condition of charge neutrality, are taken into account. The width of the slit through which plasma escapes is shown to be $\sim 4\ r_e$ and is the same for both ions and electrons. Since the particle flux through the slit is proportional to the mean thermal velocity and $T_e = T_i$, electrons attempt to escape at a greater rate than ions. This must result, in order to maintain charge neutrality, in electric fields being developed parallel to B. In a three dimensional (spindle) cusp configuration allowance must also be made for the divergence of the field lines between the line and point cusps. This general problem of cusp containment has been discussed by Berkowitz, et al[12] and Grad[13] and they conclude (without detailed justification) that the width of the line cusp will still lie somewhere between the ion and electron gyro-diameters and that the loss through the two point cusps will equal that through the line.

None of the equilibrium solutions discussed above has yet been subjected to a stability analysis.

In this section the hole size has been discussed only in terms of internal plasma properties. In the following section complications arising from features external to the contained plasma are considered.

SHORT-CIRCUIT EFFECTS

(i) In the collision free limit

The smallest conceivable hole for a hydrogen plasma discussed in the preceding paragraph is characterised by the existence of an electric field perpendicular to B_o. If this field were short-circuited the electrons and ions would be free to escape through holes of order r_e and r_i respectively, i.e. the particle loss rate would increase by an order of magnitude (assuming $T_e \sim T_i$). The form of the short-circuit must be such that electrons appear, in order to neutralise the ion space charge, at distances from the boundary large compared to r_e; consequently an adequate source of electrons must be available outside the cusp.

If the walls are both conducting and electron emitting the problem is apparently simple. If the walls are made of insulating material, secondary and photo-electric emission may still provide a source of electrons but the rate at which they can be supplied may be severely restricted by the presence of electrostatic sheaths at the wall[23]. An alternative source may be provided experimentally by a background of neutral gas in the equipment (in the SPICE II experiment the background pressure was approximately 1μ and was too low to be of importance). Neutrals can also be produced by sputtering and desorption at the wall. For example, a desorbed thermal monolayer of hydrogen would have a density in excess of $10^{17} cm^{-3}$ after 1 μsec and would become highly ionized by a plasma flux of 10^{22} particles cm^{-2} sec^{-1}, which is typical of wall conditions in high β experiments such as SPICE II. That such short-circuit effects are possible has been demonstrated by Ashby[14] who has measured the current flowing to a glass wall in a low density injection experiment. It may be remarked, finally, that the speed with which a short-circuit can propagate from the wall is determined by the inductance of the sheath. Ashby (private communication) has shown that for high β plasmas this speed is approximately the ion sound speed.

(ii) On collisional diffusion

Classical (resistive) diffusion due to electron-ion collisions can significantly enhance the hole size at low electron temperatures and has been discussed fully elsewhere[15,16]. In SPICE II[1] the resistive skin depth (on an ion transit time scale) is numerically of order $r_i/3$. In a high β reactor (cf Appendix) it is of order $2r_e$. Thus in both these situations collisional diffusion does not determine the hole size, although any enhanced diffusion would soon be important.

Simon[17] has shown that if short-circuiting can prevent the appearance of an ambipolar electric field, and if the scale length ℓ for density variation is comparable with r_i then the diffusion flux due to ion-ion collisions can exceed that due to electron-ion collisions by a factor of order

$$\left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{r_i}{\ell}\right)^2 . \qquad \ldots (1)$$

In the absence of any short-circuit Kaufman[18] has shown that the factor (1) must be replaced by one of order

$$\left(\frac{r_e}{\ell}\right)^2 , \qquad \ldots (2)$$

i.e. the effects of like particle collisions become negligible.

Summarising: in both the collision free and collision dominated regimes there are 'classical' mechanisms that could result in a significant increase in hole size. For them to be effective however, it is a necessary condition that any ambipolar electric fields appearing perpendicular to the magnetic field should be short-circuited. Whether or not this will happen in practice depends critically on the conditions prevailing outside the cusp and in particular at the walls upon which the field lines terminate.

ANOMALOUS DIFFUSION

The plasma field boundary layer departs from local thermodynamic equilibrium in two ways:

(i) there is a non-Maxwellian velocity distribution;

(ii) there is a density gradient.

Thus there are at least two reservoirs of energy available to drive an (unstable) boundary to a more stable equilibrium, provided that a suitable instability mechanism exists.

The electron drift velocity required to maintain pressure balance between a field-free plasma and a confining magnetic field is roughly $\bar{v}_i$ (or $\bar{v}_e$) when the boundary layer is r_i (or r_e) thick. One might ask whether the two-stream instability could broaden this layer. Stringer[19] has shown that for a uniform, field-free plasma the critical drift velocity must exceed $\bar{v}_e$ (or $\bar{v}_i$) for ion-to-electron temperature ratio $T_i/T_e = 1$ (or 0.1). These limits also appear to hold for electrons drifting through ions across a magnetic field, although the magnetic field increases the growth time of the instability[20]. The growth time of the instability is then of the order of the ion-plasma period, i.e. it is shorter than the ion gyro-period and it is therefore free to grow in the boundary layer. It therefore appears that when $T_i = T_e$ a sheath thinner than r_i should be possible.

Other types of wave can grow in low β plasmas. In particular, 'universal' instabilities driven by plasma density gradients can often produce 'anomalous' diffusion at rates approaching that given by the (empirical) Bohm coefficient[21]

$$D_B \sim r_e^2 \ \nu_{ce}/3 \ ,$$

where ν_{ce} is the electron cyclotron frequency.

The ratio of the Bohm to the classical diffusion coefficient is

$$\frac{D_B}{D_C} = \frac{\nu_{ce}}{\nu_{ei}} \sim \frac{3 \times 10^4 \ T^{3/2}}{n_e} \ B \quad \text{(in units of } {}^{o}\text{K, gauss, cms)},$$

where ν_{ei} is the electron-ion collision frequency, and is therefore of order 30 in SPICE II and 10^5 in a 'reactor' (cf. Appendix). The corresponding Bohm skin depths (evaluated on an ion-transit time scale) are 5 mm and 1 cm respectively, compared to an ion gyro radius $r_i = 2$ mm for both SPICE II and a 'reactor'. It is clear that anomalous diffusion at the Bohm rate could account for the magnitude of the hole measured in SPICE II and would significantly enhance the losses from a $\beta = 1$ open-ended trap having $n_e \sim 5 \times 10^{16}$ cm^{-3} and $T \sim 10^8$ ^{o}K. However, at the time of writing there has been no rigorous investigation of universal instabilities in conditions appropriate to a bounded, high-β, cusp plasma having $T_i = T_e$, and where the ion cyclotron frequency is ill-defined.

3. ENERGY TRANSPORT

ENERGY LOSS PROCESSES

It is convenient to distinguish three regimes of energy loss associated with particle motion along the field lines[16]. Each can be characterised by a different range of values of the parameter λ/L, where λ (= $\lambda_{ee} = \lambda_{ii}$, when $T_e = T_i$) is the mean free path and L is a typical dimension of the contained plasma. Each is dominated by a different energy transport mechanism.

(a) Sonic Flow

The total energy flux (ergs cm^{-2} sec^{-1}) arising from adiabatic flow ($\Upsilon = 5/3$) at sonic velocities:

$$Q_1 \simeq \frac{n_i v_i}{4} \frac{5}{2} k(T_e + T_i) \ .$$

(b) Electron Thermal Conduction

$$Q_2 \simeq (\tfrac{1}{2} n_e k v_e \lambda_{ee}) \frac{dT_e}{dz} \simeq \frac{n_e v_e}{4} 2kT_e \left(\frac{\lambda}{L}\right) .$$

(c) Free Electron Flow

$$Q_3 \simeq \frac{n_e v_e}{4} \frac{5}{2} kT_e \ .$$

In order to obtain Q_2 a one dimensional steady state situation was assumed. The thermal conduction coefficient in parentheses is written in a form familiar from elementary kinetic theory. It demonstrates immediately that ion thermal conduction along the field lines is less than that due to electrons by a factor $v_i/v_e \sim (m_e/m_i)^{\frac{1}{2}}$. This gas kinetic equation for thermal conduction should be multiplied by a factor 2.7 in order to reproduce the result given by Spitzer[22] for a real hydrogen plasma including thermo-electric effects.

The flux Q_3 represents that due to electrons effusing freely through an electron gyro-radius hole. It should be regarded as an upper limit which need not necessarily be reached because of the requirements of charge neutrality within the plasma.

The characteristic values of λ/L are as follows:

(a) If $\lambda/L < (m_e/m_i)^{\frac{1}{2}}$, the electron-ion equipartition time is less than the sound transit time ($\sim L/v_i$) hence $T_e \sim T_i$ and $Q_1 > Q_2$, i.e. sonic flow is the dominant transport mechanism.

(b) If $1 > \lambda/L > (m_e/m_i)^{\frac{1}{2}}$, thermal conduction is a valid concept and will be the dominant energy transport process. It can only be responsible for an energy loss from the plasma however if an adequate heat sink is available externally.

(c) If $\lambda/L > 1$, thermal conduction is no longer a valid concept. The electron motion is unimpeded by collisions but is strongly influenced by space-charge effects.

Thermal conduction losses across the field also occur, but detailed calculation of their importance is complicated by the strong gradients involved (cf. the discussion on ion-ion collisional diffusion on page 4). However, their importance should decrease with increasing temperature.

THE IMPORTANCE OF AN ENERGY SINK

If a temperature gradient exists between the centre and the 'boundary' of a plasma, thermal conduction will result in a flow of energy towards the boundary at a rate given initially by Q_2. If there is an adequate heat sink at the boundary, i.e. if energy can be dissipated there at a rate comparable to or greater than Q_2, the flow will persist and the energy loss will continue to be dominated by thermal conduction. If, on the other hand, the heat sink is inadequate, i.e. it is incapable of dissipating energy at a rate comparable to Q_2, then the temperature gradient will be reduced to a value such that energy flows to the 'boundary' no faster than the sink can dispose of it. The energy loss rate will thus be dominated by the properties of the 'boundary' and may be significantly less than that given by Q_2.

A trivial example of an 'inadequate' heat sink is provided by an (idealised) experiment in which the plasma is separated initially from the wall by a vacuum. Significant amounts of energy cannot be lost from the plasma in a time rather less than the sound transit time to the wall, although thermal conduction may bring about a large change in the 'internal' temperature distribution. If however the vacuum of the preceding example were replaced by a mass of cold, very dense, impure gas capable of radiating strongly, then the other extreme of an almost perfect energy sink could be envisaged.

SPACE CHARGE EFFECTS

Even when a plasma is in contact with a wall it is not always obvious that heat can flow to it at a rate as large as Q_2. In the simplest situation, when the wall is perfectly absorbing and non-emitting, (even under plasma bombardment), the requirement of charge neutrality for the contained plasma leads to the formation of the classical Langmuir sheath and the bulk of the plasma electrons never collide with the wall. Thus the sheath provides thermal insulation between the plasma and the wall. It can be shown(23) that the resultant energy flux to the wall is

$$Q = \left(\frac{2\pi m_e}{m_i}\right)^{\frac{1}{2}} \cdot \frac{1}{4} \; n \, \bar{v}_e \left[\frac{5}{2} kT_e + e\varphi \right] ,$$

where $e\varphi \approx 3\, kT_e$ is the negative sheath potential. For deuterium this flux is an order of magnitude less than that due to free electron flow.

In a practical situation the wall will not behave this simply. Effects such as secondary and photoelectric emission, gas desorption and plasma recombination will have to be taken into account. They will be considered in more detail elsewhere(23).

4. MODEL OF A CUSP COMPRESSION EXPERIMENT

Taking the discussion of the preceding paragraphs as a guide, a theoretical model for the SPICE II compression experiment will now be constructed. The model is similar to that discussed by Bickerton[16] to study the SPICE I[15] experiment, with the following differences:

(i) In the SPICE I experiment the electron and ion temperatures were low and approximately equal ($\lesssim$ 12 eV). A one fluid model was therefore sufficient to describe its behaviour. The SPICE II plasma is formed by the collision of two plasma blobs ejected from opposing conical theta-pinch guns. The initial electron temperature (~ 15 eV) is very much less than the ion temperature, the latter arising from a rapid thermalisation of the directed motion of the ions (100-250 eV). Under these conditions the ion-electron equipartition time is relatively long and it is therefore necessary to consider a two-fluid model of the plasma, T_e and T_i being determined by separate energy equations.

(ii) The time taken to compress the SPICE I plasma was comparable to the time required for a sound wave to cross it; inertial effects were therefore important. In SPICE II the reverse is true ($t_{comp} \sim$ 5-10 μsec, $L/V_s \sim \frac{1}{2}$ μsec). Equilibrium can therefore be assumed to exist, at all times, between the uniform plasma pressure and the compressing magnetic field.

(iii) In SPICE I, due to the low electron temperature, the mechanism predominantly responsible for the size of the hole was resistive diffusion. As already pointed out (page 4) this is not the case in SPICE II unless ion-ion collisions are important.

The equations describing the model are derived in the following paragraphs. They have been solved numerically and the results are compared with experiment in Section 5.

GEOMETRY

The plasma volume is approximated by a solid of revolution having a volume

$$V = \frac{32\pi}{105} R^3 ,$$

where R(t) is the radius (at time t) in the median plane beyond which there is assumed to be a negligible plasma pressure (Fig.1). V is the volume generated by revolving the hypocycloid

$$x^{2/3} + y^{2/3} = R^{2/3}$$

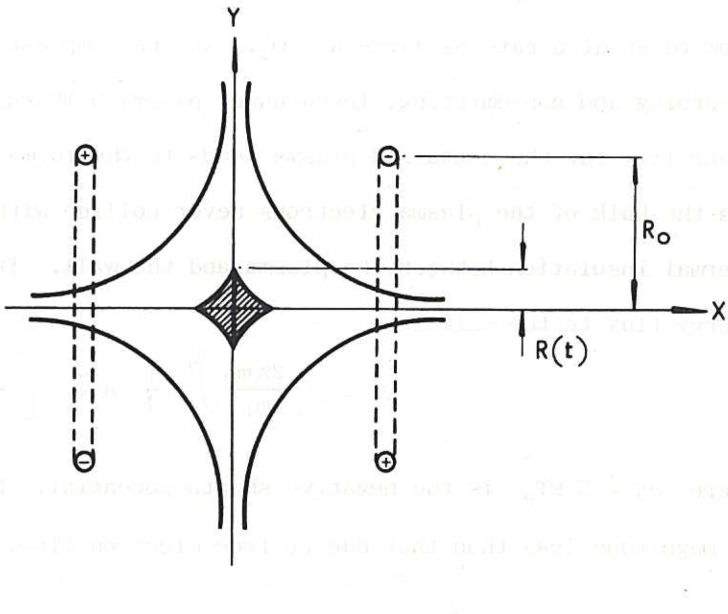

FIG. 1.

about the x axis; this hypocycloid represents approximately the free surface of a two-dimensional $\beta = 1$ plasma confined by the field due to a linear quadrapole. The magnetic field is produced by coils of radius R_o placed at equal distances R_o from the median plane. At the instant of confinement $(t = o)$ $R = R_o$.

In the absence of plasma the magnetic field is given approximately at points in the median plane by*

$$B(R) = B_o \frac{R}{R_o} \quad .$$

B_o is the field at the radius R_o and is given by

$$B_o(t) = B_{max} \sin (wt + \varphi) ,$$

where w is a characteristic of the external circuit and φ is a parameter introduced to ensure pressure balance at $t = o$ (cf. page 10).

In the presence of the plasma the magnetic field is assumed to equal the vacuum value outside and to be zero inside the plasma. Moreover, it is assumed to take the same value at all points on the plasma surface.

Thus the magnetic pressure compressing the plasma in this model arises from a field of strength

$$B(t) = B_o(t) \frac{R(t)}{R_o} \quad .$$

CONTINUITY EQUATIONS

(i) Ions

It is postulated that ions are lost by a process of effusion through the 'holes' formed at the cusps. The circumferential hole is taken to have an area $2\pi RKr_i$ where

$$r_i = \left(\frac{3kT_i}{m_i} \right)^{1/2} \frac{m_i c}{eB} ,$$

the ion gyro-radius, is evaluated for a mean energy $\frac{1}{2} m_i v_i^2 = \frac{3}{2} kT_i$ and with $B = B(R)$. The factor K is a constant of order unity and is found experimentally to be ~ 2. Reasons why this might be so have been discussed in section 2. The loss of ions through the spindle cusps is found experimentally to be some 50% of that through the line cusp and can be taken into account by a suitable adjustment to the value of K. Thus the total rate of loss of ions is given by

$$\frac{dN_i}{dt} = - \frac{n_i}{4} \left(\frac{3kT_i}{m_i} \right)^{1/2} 2\pi R K r_i ,$$

---

* Verified experimentally.

where n_i is the ion density and $N_i = Vn_i$, the total number of ions present at any instant. This equation can be rewritten in the form

$$\frac{dN_i}{dt} = -\frac{N_i}{\tau_{ci}} ,$$

where the ion containment time is given by

$$\tau_{ci}(t) = \left(\frac{64e}{315cK}\right) \frac{B(t)\ R^2(t)}{kT_i(t)} .$$

Ions leaving the cusp at a radius R(t) will not necessarily be lost permanently from the plasma. They will be moving adiabatically into a region of higher magnetic field and, provided collisions are infrequent, i.e. provided $\lambda/R_o > 1$, many of them will be reflected back towards the cusp by the mirror effect. This has been taken into account in an approximate manner by assuming that only those ions escape which leave the 'hole' at angles lying within the loss cone of the 'mirror'. The result is a multiplication of the containment time by the mirror ratio $B_o(t)/B(t) = R_o/R(t)$, i.e.

$$\tau_{ci}^{(m)} = \left(\frac{64e}{315cK}\right) \frac{B(t)\ R_oR(t)}{kT_i(t)} . \qquad \ldots (3)$$

Equation (3) shows that the effect of the mirror confinement is to cause the ion containment time to decrease more slowly during the plasma compression ($\propto R_oR(t)$) than it would have done otherwise ($\propto R^2(t)$). In a later paragraph it will be seen that the neglect of this effect can give rise at high compression ratios to a catastrophic loss of plasma.

(ii) Electrons

No separate continuity equation has been used for the electrons. It is assumed, in order to preserve charge neutrality within the plasma, that electrons are lost at the same rate as ions. Thus, at all times,

$$N_e = N_i = N ,$$

$$n_e = n_i = n .$$

MOMENTUM EQUATIONS

For the reasons already outlined it is assumed that at all times there is a balance between the kinetic pressure of the plasma and the magnetic field pressure, i.e.

$$nk\ (T_e + T_i) = \frac{B^2(t)}{8\pi} .$$

ENERGY EQUATIONS

(i) Ions

Three contributions to the energy of the ions are considered important. Firstly there is that due to adiabatic compression, secondly that due to the energy carried away by the ions lost through the hole, and, lastly, there is the interchange of energy due to collisions with electrons. If the ions are effusing in such a way that each ion sees a hole size characteristic of its own gyro-radius, then it can be shown that the mean energy carried out per ion is $5/2\ kT_i$ (not $2kT_i$ as for a hole of fixed dimensions). Thus

$$\frac{3}{2}\frac{d}{dt}(NkT_i) = -(nkT_i)\frac{dV}{dt} + \left(\frac{5}{2}kT_i\right)\frac{dN}{dt} + \frac{3}{2}N\frac{k(T_e - T_i)}{\tau_{eq}}$$

where[22]

$$\tau_{eq} = \frac{3m_i}{8\sqrt{2\pi}\, e^4\, m_e^{\frac{1}{2}}\, \mathrm{Log}\,\Lambda}\ \frac{(kT_e)^{3/2}}{n}$$

(ii) Electrons

A similar equation has been adopted for the electron energy:

$$\frac{3}{2}\frac{d}{dt}(NkT_e) = -(nkT_e)\frac{dV}{dt} + \left(\frac{5}{2}kT_e\right)\frac{dN}{dt} - \frac{3}{2}N\frac{k(T_e - T_i)}{\tau_{eq}}$$

It should be noted that, in this case, there is less justification for assuming the energy lost per electron to be $5/2\ kT_e$ since the actual loss process has not been specified. The following picture is consistent although it may not be realistic. If electrons are effusing through an electron gyro-radius hole then (a) they are being lost at the same rate as ions (if $T_e = T_i$) and (b) they will carry out $5/2\ kT_e$ per particle. In the cases of interest (cf. Section 5) $\lambda/R \lesssim (m_e/m_i)^{\frac{1}{2}}$ at all times, hence the energy loss due to thermal conduction is negligible compared to that due to particle effusion (cf. page 6).

5. A COMPARISON OF THEORY AND EXPERIMENT

EXPERIMENTAL RESULTS

The full details of the experiments conducted on SPICE II have been reported elsewhere[1]. The following is a summary.

The initial electron and ion temperatures are believed to be ~ 15 eV and ~ 100 eV respectively, and the initial density 2-3 × 10^{15} cm^{-3}. The peak temperatures inferred from (a) O VI and Ne VIII line ratio measurements in the vacuum ultra-violet, (b) soft x-ray absorber plots and (c) ion energy analysis are of the order of 70-80 eV. The peak electron density, estimated from absolute measurements of the optical continuum intensity at 4010 Å and from emission in the far infra-red (60-600 μ) is approximately 10^{16} cm^{-3}. The temperatures and densities all peak at about the same time, namely 5-6 μsec after energizing the cusp coils. The width of the line cusp (measured with multiple pressure probes) is of the order of 1-2 ion gyro-radii. (Some reasons for this have been discussed in Section 2; another possibility which has not been completely discounted experimentally is the presence of a small amount of magnetic field trapped in the plasma during the initial stages of the compression when Rayleigh-Taylor instabilities could occur.)

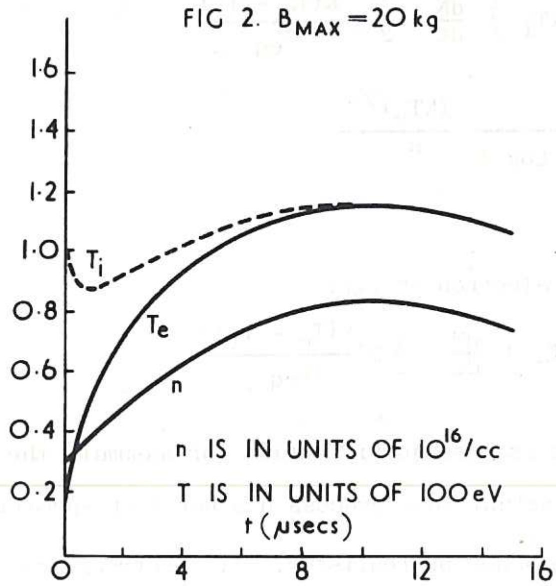

FIG 2. $B_{MAX} = 20$ kg

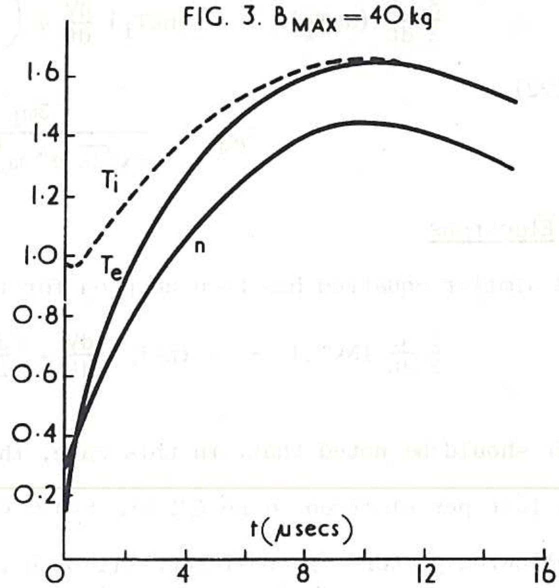

FIG. 3. $B_{MAX} = 40$ kg

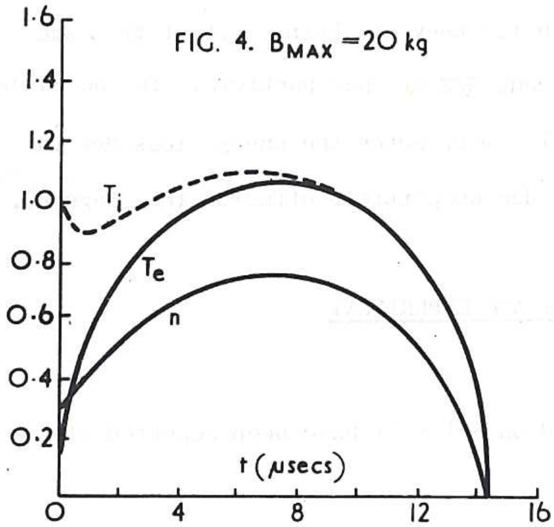

FIG. 4. $B_{MAX} = 20$ kg

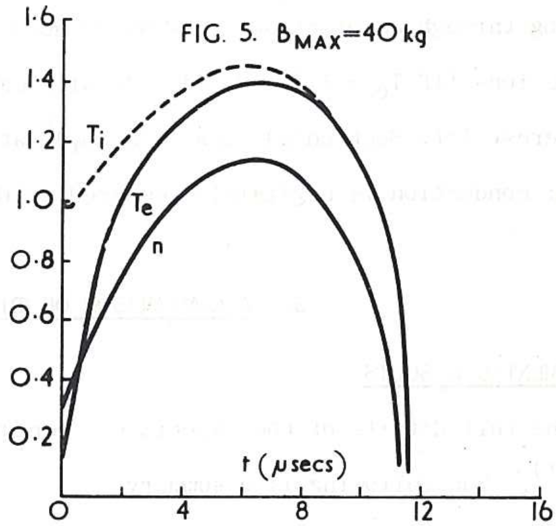

FIG. 5. $B_{MAX} = 40$ kg

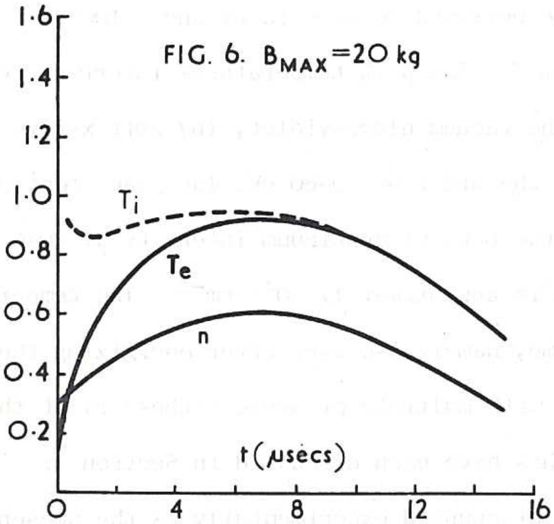

FIG. 6. $B_{MAX} = 20$ kg

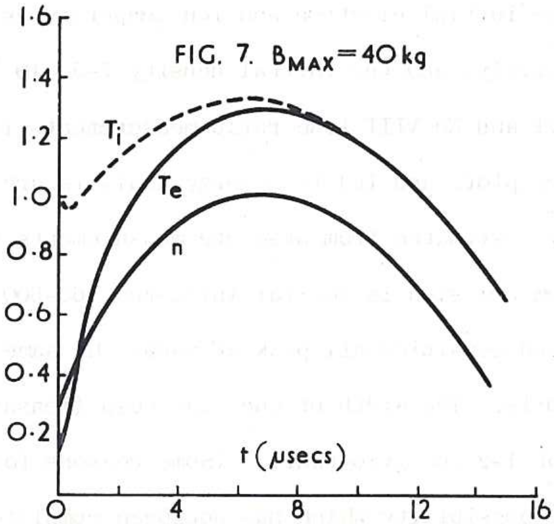

FIG. 7. $B_{MAX} = 40$ kg

THEORETICAL RESULTS

Figs.2–7 show the results of calculations using the model described in Section 4. In all these cases the initial conditions taken were $n_o = 3 \times 10^{15}$ cm^{-3}, $kT_{eo} = 15$ eV, $kT_{io} = 100$ eV, $R_o = 10$ cms and time to peak field = 15 μsec.

Figs.2 and 3 illustrate the time variation of n, T_e and T_i when K = 1 and the mirror effect is included, for two values (20 and 40 kG) of B_{max}. It should be noted that both the density and the temperature reach their maxima at the same time, but significantly earlier than the magnetic field.

Figs.4 and 5 show the consequence of omitting the mirror effect. As would be expected, all the parameters attain lower maximum values earlier. However the most striking feature is the catastrophic collapse of the plasma which occurs when the compression ratio is large. When the mirror effect is omitted the hole increases in area so rapidly with decreasing R that, once beyond a certain point, the density falls to zero at an ever increasing rate which is essentially independent of B_o ($dn/dt \propto n^{-6/5}$). If the mirror effect is included the rate of increase of the hole area is reduced and the density approaches zero at a rate determined by B_o alone ($dn/dt \propto B_o(t)$). Since this type of spontaneous implosion is not observed experimentally it is concluded that the mirror effect is an important feature of the model.

Figs.6 and 7 are the result of calculations identical to those of Figs.2 and 3 except that K = 2, i.e. double the hole size. There are no qualitative differences from Figs.2 and 3 but once again the peaks are lower and earlier.

The important results from these calculations are set out, together with the corresponding experimental results, in Table 1. The model giving the best agreement is that retaining the mirror effect and with a line cusp width of 2 ion gyro radii (i.e. the hole size observed experimentally).

TABLE 1

	B_{max} (kg)	Peak Density (10^{16}/cc)	Peak Temp (eV)	Time of Peak (μsecs)
Experimental	30–40	1.0	70–80	5–6
K = 1, With Mirror Effect (Figs. 2 and 3)	20 40	0.85 1.47	116 167	10.5 10.5
K = 1, No Mirror Effect (Figs. 4 and 5)	20 40	0.75 1.1	108 144	7.0 6.4
K = 2, Double Hole Size (with Mirror Effect) (Figs. 6 and 7)	20 40	0.62 1.03	93 130	6.5 6.5

6. CONCLUSIONS

The size of the hole through which particles can escape is critically dependent on the existence of an electrostatic field ($E_\perp$) perpendicular to B. If such a field is permitted the minimum conceivable size is of order r_e. In the collision free limit a short-circuit of $E_\perp$ could result in an increase in size to r_i. In the collision dominated limit ion-ion collisions might enhance the classical diffusion rate by a factor of order $(r_i/\ell)^2(m_i/m_e)^{\frac{1}{2}}$, resulting again in a significant enlargement of the hole. In either situation the plasma in the sheath should rotate.

For a short-circuit to be possible an adequate source of electrons must be available in regions external to the plasma. Electrostatic sheaths, with their electric fields parallel to B, are likely to play an important role in determining the strength of the source. For the short-circuit to be effective within the plasma containment time it must be able to propagate at a sufficiently high speed.

The stability of these collision free boundary layers has not yet been investigated theoretically. However, anomalous diffusion at anything like the Bohm rate would lead to a significantly enhanced hole size. A naive application of two stream instability theory does not preclude a boundary layer thickness of less than r_i.

Energy can be transported through a plasma at a speed in excess of $\bar{v}_i$ by electron thermal conduction or, in the long mean free path limit, by free electron flow. Neither process need necessarily lead to a correspondingly rapid loss of energy from the plasma. The losses due to thermal conduction may be limited by the efficiency of the external heat sink. Both processes can occur only if the build up of positive space charge within the plasma can be neutralised and this in turn is dependent on the availability and mobility of externally supplied electrons.

A simple two-fluid MHD model of the SPICE II experiment has been constructed which possesses a single adjustable parameter K (the ratio of hole width to ion gyro-radius). Agreement with experiment is found to within a factor of two provided (a) K = 2 and (b) the effects are included of the field convergence between the plasma boundary and the magnetic field maximum. The factor of two is typical of the uncertainties in the initial values of T_e and T_i. The value of K required is in agreement with that measured experimentally.

It should be noted finally that some of the problems discussed have direct applications to other open-ended traps, for example, high β theta-pinches and low β mirrors.

7. ACKNOWLEDGEMENTS

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APPENDIX

APPROXIMATE PARAMETERS FOR AN OPEN ENDED $\beta = 1$ REACTOR

1. Particle losses increase with temperature, therefore put $kT_i \sim 10$ keV (for DT reaction ignition temperature neglecting particle losses is ~ 5 keV).

2. B is limited technologically to 2×10^5 gauss, hence for pressure balance $n_i \leqslant 6 \times 10^{16} \mathrm{cm}^{-3}$.

3. Lawson's criterion then requires a containment time of more than 2 msec.

4. Hence:

   Ion-ion collision time $T_{DD} \sim 20$ μsec.

   Equipartition time (if $kT_e \sim 10$ keV) ~ 700 μsec implying $T_e \sim T_i$.

   Then, electron-electron collision time $T_{ee} \sim 0.4$ μsec.

   Thus both ions and electrons have a well defined temperature.

5. $\bar{V}_e = 7 \times 10^9$ cm sec^{-1}

   $\bar{V}_d = 10^8$ cm sec^{-1}

   $\lambda_{ee} = \lambda_{DD} \sim 25$ metres

   Thus the mean free path is comparable to any (prototype) reactor dimension.

6. For the "short cusp" geometry of Fig. 1:

   (a) Classical skin depth (without electron inertia term) on an ion transit time scale $\delta R \sim 2 \times 10^{-3}$ cm

   (b) Electron gyro-radius $r_e \sim 4 \times 10^{-3}$ cm

   (c) Ion gyro radius $r_i \sim 0.2$ cm

   (d) Bohm skin depth on an ion transit time scale $\delta_B \sim 1$ cm.

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