CLM - P 578

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Preprint

DIRECTIONAL DEPENDENCE AND NON-UNIFORMITY OF JOULE HEATING IN NATURAL CONVECTION EXPERIMENTS

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1979

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ABSTRACT

Volumetric heating of a confined horizontal layer of fluid to produce natural convection is often carried out using Joule heating to give a uniform heat source density. However if the mean electric current flows horizontally, the heat source density is the deficient in both upper and lower boundary layers; if the current flows vertically the heat source density is enhanced in these boundary layers.

At low Rayleigh number when the convection induced is weak the consequence of this inhomogeneity can be calculated easily and allowed for. At high Rayleigh number this effect can be ignored in the thin upper boundary layer. In the lower boundary layer, which is thicker and has a much smaller Nusselt number, these departures of the heat source density from uniformity can significantly distort the dependence of the Nusselt number on the Rayleigh number. Appropriate correction formulae are developed and applied. To first order, the downward heat flux density differs from its value for uniform heating by a term proportional to the square of the downward flux fraction.

(Paper to be presented at 18th ASME Heat Transfer Conference, San Diego, California, August 1979).

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INTRODUCTION

In the study of natural convection in a horizontal layer of fluid containing heat sources, attention has been mainly focused on a uniform distribution of heat sources throughout the fluid [1-7]. The physical properties - thermal conductivity, viscosity, density, expansion coefficient-are usually taken to be constant except for the buoyant density variation which in the Boussinesq approximation drives the natural circulation and enhances the heat transfer coefficients over the value appropriate for just conduction. To some extent the variations of these physical properties from strict uniformity have been examined [8-9]. The dependence of the heat source function on temperature has received less attention[10] in this context.

The most convenient and practical way of generating volumetric heating of a fluid in the laboratory is to make the fluid an electrically conducting one and to pass an electric current through it. If η is the electrical resistivity of the fluid and j the electric current density, then the heat source density h is ηj^2. If η and j have the same value throughout the fluid, then the heat source density is uniform; but if η is a function of temperature, significant departures from uniformity may occur.

There are two common ways of arranging the configuration in laboratory experiments:- (i) to place electrodes at either end of the convection cell and pass electric current horizontally [1, 5, 11, 12] and (ii) to use the upper and lower plates of the convection cell as electrodes and to pass current vertically [11, 13]. These can produce different results (see below). For weak aqueous solutions, the electrical conductivity σ can be written as [12]

$$\sigma = \sigma_o \, (1 + \gamma[\theta-\theta_o]) \qquad (1)$$

If θ_o is $\sim$ 300K then γ has the value of $\sim 0.02\text{K}^{-1}$. Clearly if temperature variations of 50 K are present in the convection cell then σ may vary by a factor of two. The sensible approach is naturally to restrict the temperature difference to be as small as possible. If $\theta-\theta_o \sim 2$ K then σ is almost constant. However, for high Rayleigh number studies in the strongly turbulent regime, the thermal fluctuations can be large, and to obtain as much resolution as possible from the instrumentation it is desirable to allow as large a temperature variation as possible without vitiating the experiment.

QUALITATIVE DISCUSSION

Consider, to be specific, natural convection in a volumetrically heated liquid contained between two horizontal plates both maintained at a uniform temperature θ_o. (see fig 1). The equations for such a system are the Navier-Stokes equation:

$$\partial \underline{u}/\partial t + (\underline{u}.\nabla)\underline{u} + \rho_o^{-1}\nabla p = \nu\nabla^2\underline{u} + \rho g/\rho_o \qquad (2)$$

and the advective Fourier equation:

$$\partial T/\partial t + (\underline{u}.\nabla)T = \kappa\nabla^2 T + h/(\rho_o c) \qquad (3)$$

where the Boussinesq equation of state is

$$\rho = \rho_o(1 - \alpha T) \qquad (4)$$

The thermophysical properties λ, c, κ are taken

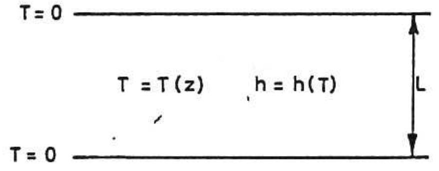

Fig.1 A horizontal layer of fluid containing distributed heat sources of strength h(T).

be constant and the temperature dependence of the density is only retained in the buoyancy term $\rho g/\rho_o$ which drives convection once the critical Rayleigh number is exceeded. The relative temperature $T=\theta-\theta_o$ where θ is the absolute temperature. The reference density ρ_o is the density of the liquid when T = 0. The heat source density h=h(T) is considered as temperature dependent only via the temperature dependence of the electrical conductivity.

To see qualitatively the difference between passing current horizontally and vertically it is convenient to consider a high Rayleigh number regime in which the time averaged temperature profile is essentially vertically stratified. For these boundary conditions this profile can be roughly represented by fig 2(a). It is in essence a uniform temperature save for a thin boundary layer of thickness δ_U at the upper surface and a thicker boundary layer of thickness δ_L at the lower surface. Superimposed on this are turbulent fluctuations in temperature, with intermittent plumes and falling sheets of cold liquid. These are not however germane to this discussion.

Let us suppose that the experiment consists of a rectangular tank with horizontal (x and y) dimensions of X and Y. When electric current is passed horizontally, the planes x=0 and x=X are the electrodes. When current is passed vertically the planes z=0 and z=L are the electrodes. The side walls of the tank are assumed to be sufficiently separated for the assumption that the mean fluid properties are independent of x and y to be true over a substantial fraction of the tank.

The quantities which can be controlled by the experimenter are V, the potential drop between the electrodes, and J the total current flowing. The local

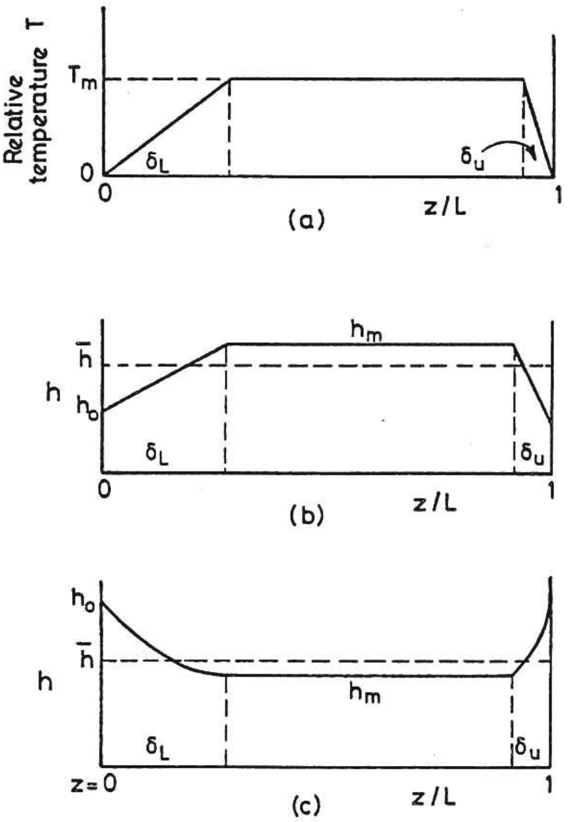

Fig.2 Schematic vertical profiles at high Rayleigh number for the temperature (a) and the corresponding heat source density h for lateral (b) and vertical (c) current flows.

current density $\underline{j}$ and electric field $\underline{E}$ cannot be controlled absolutely. The total power input is $P \equiv \int \sigma E^2 \, dxdydz = \int \eta j^2 \, dxdydz$ from Ohm's Law $\underline{j} = \sigma \underline{E}$. When the current is applied horizontally, the same potential drop V exists between all pairs on points on the planes x=0 and x=X respectively. Moreover the electric field in the tank is horizontal of constant magnitude V/X independent of z. However the density of the horizontally flowing current varies as a function of height. The local heat source density $h = \sigma \underline{E}^2$ varies with height only through σ. Hence

$$h = h_o \ (1 + \gamma T) \qquad (5)$$

where h_o $(=\sigma_o V^2/X^2)$ is a constant and σ_o is the electrical conductivity at θ_o. This shows that h increases with increasing T.

On the other hand where the current is applied vertically the potential drop exists between the top and bottom plates, but the magnitude of the vertical electric field is no longer independent of height. However the density of the vertically flowing electric current is independent of height = J/XY. The local heat source density $h = j^2/\sigma$ which varies with height only through σ. Hence

$$h = h_o/(1 + \gamma T) \qquad (6)$$

where h_o $(=J^2/(\sigma_o X^2 Y^2))$ is a constant. This shows that h decreases with increasing T. The difference between the vertical and horizontal current cases is essentially the difference between resistances in series and in parallel.

If the result of a convection experiment is a profile such as fig 2(a), then it follows that in the case of a horizontal current, the heat source density is not uniform, but as illustrated in fig 2(b). The heat source density in the bulk of the fluid is $h_m = h_o \ (1 + \gamma T_m)$. The average heat source density $\bar{h}$ is $h_m - \frac{1}{2}(\delta_U + \delta_L)(h_m - h_o)$. This indicates that in the main part of the fluid the heat source density h_m is slightly in excess of the mean value $\bar{h}$, whereas in the boundary layers there is a heat source density deficit compared with the mean.

Correspondingly for the vertical current if the same profile were achieved, the heat source density is as illustrated in fig 2(c). The average heat source density $\bar{h}$ is $h_m + (\delta_U + \delta_L)[h_o \ln(1+\gamma T_m)/\gamma T_m - h_o]$ where here $h_m = h_o/(1+\gamma T_m)$. This indicates that in the main part of the fluid the heat source density h_m is slightly below the mean value $\bar{h}$, whereas in the boundary layers the heat source density may be considerably enhanced over the mean value.

This preliminary discussion indicates the qualitative differences between uniform and temperature dependent heat sources; the following sections put the analysis on a qualitative basis.

LOW INTERNAL RAYLEIGH NUMBER

In a horizontal layer of depth L and uniform heat source density h between horizontal plates of equal temperature, the critical value R_c of the internal Rayleigh number $R \equiv \alpha g h L^5/(\nu\kappa\lambda)$ is $2^6 x 560$ [4]. For $R/R_c < 1$, the liquid is at rest and heat is transferred only by conduction. For $1 \leq R/R_c \leq 1.25$, the amplitude of the convection which occurs is such that the temperature distribution differs little from the stratified parabolic profile [4]. We consider here what effect the temperature dependence of the heat source density has on such a profile.

Lateral Current Flow

If the electrodes are at the sides of the convection cell, then the heat source density has the form given by (5). The heat conduction equation can then be written (neglecting the advective terms) as

$$\lambda \frac{d^2T}{dz^2} + h_o(1 + \gamma T) = 0 \qquad (7)$$

with boundary conditions T=0 on z=0 and on z=2a (it is convenient for later analysis to take the layer depth to be 2a here). The configuration is thus symmetric about the plane z=a. This has the solution

$$T/T_1 = (2\phi^2)^{-1}(\cos(2\phi[z-a]/a) - \cos 2\phi) \ / \ (\cos 2\phi) \qquad (8)$$

where $$T_1 = h_o \ a^2/(2\lambda) \qquad (9)$$

is the maximum temperature difference in the layer when γ=0 and

$$\phi^2 = \gamma T_1/2 \qquad (10)$$

The maximum temperature difference T_m, when $\gamma \neq 0$, occurs when z=a, and is given by

$$T_m/T_1 = (\sin\phi/\phi)^2/(\cos 2\phi) \qquad (11)$$

The heat Q_L generated in the lower half of the layer is given by

$$Q_L = h_o a(\tan 2\phi/2\phi) \qquad (12)$$

so that the total heat generated in the layer Q is $2Q_L$.

The modified Nusselt number for downwards heat transfer from the layer is defined by

$$N_L' = Q_L a/(\lambda T_m) \qquad (13)$$

Hence

$$N_L' = 2(T_1/T_m)(\tan 2\phi/2\phi)$$

$$= 2\phi \cot\phi \qquad (14)$$

The downward heat flux density $\dot{q}_L$ is equal to Q_L in this case and can be written as

$$\dot{q}_L = \sqrt{2\lambda T_m h_o}\{\cos^2\phi/\cos 2\phi\}^{\frac{1}{2}} \qquad (15)$$

Obviously as $\gamma \to 0$ then $\phi \to 0$ and so $T_m \to T_1$, $Q_L \to h_o a$ $N_L \to 2$ and $\dot{q}_L \to \sqrt{2\lambda T_1 h_o}$ which are the appropriate values for uniform heating.

Vertical Current Flow

If the electrodes are above and below the convection cell, then the heat source density has the form given by (6). The heat conduction equation can then be written as

$$\lambda(d^2T/dz^2) + h_o(1 + \gamma T)^{-1} = 0 \qquad (16)$$

If $\gamma T \ll 1$, this can be linearized to

$$\lambda(d^2T/dz^2) + h_o(1 - \gamma T) = 0 \qquad (17)$$

The solution is then straight forward:

$$T/T_1 = (2\phi^2)^{-1}(\cosh 2\phi - \cosh(2\phi[z-a]/a))/(\cosh 2\phi) \qquad (18)$$

and equations (11) to (15) are modified only in as much as each circular trigonometric function is replaced by its hyperbolic equivalent ($\cos\phi \to \cosh\phi$ etc).

Aliter, if we remain with (16) and define

$$s = 1 + \gamma T \qquad (19)$$

then (16) can be written as

$$s(d^2z/dz^2) + 4(\phi/a)^2 = 0 \qquad (20)$$

This has a first integral

$$(ds/dz)^2 + 8(\phi/a)^2 \ln(s/s_m) = 0 \qquad (21)$$

where $s = s_m$ when $(ds/dz) = 0$ so that

$$s_m = 1 + \gamma T_m \qquad (22)$$

Further integration gives

$$|1-(z/a)| = \text{erf}([\ln(s_m/s)]^{\frac{1}{2}})/\text{erf}([\ln s_m]^{\frac{1}{2}}) \qquad (23)$$

where s_m satisfies

$$\sqrt{2}\phi = s_m(\sqrt{\pi}/2)\text{erf}([\ln s_m]^{\frac{1}{2}}) \qquad (24)$$

which is thus also an implicit equation for T_m. (Erf x is the error function.)

Also

$$Q_L = h_o a[(\ln s_m)/(2\phi^2)]^{\frac{1}{2}} \qquad (25)$$

and

$$N_L' = 2(\sqrt{2}\phi)(\ln s_m)^{\frac{1}{2}}/(s_m - 1) \qquad (26)$$

Now $\pi/4\ (\text{erf}[\ln(1 + x)]^{\frac{1}{2}})^2 = x - 7x^2/6 + O(x^3)$ when $x \ll 1$. Hence to first order in γT_m

$$\left.\begin{aligned} T_1/T_m &= 1 + 5\gamma T_m/6 \\ Q_L/(h_o a) &= 1 - 2\gamma T_m/3 \\ N_L'/2 &= 1 - \gamma T_m/6 \\ \dot{q}_L/\sqrt{2\lambda T_m h_o} &= 1 - \gamma T_m/4 \end{aligned}\right\} \qquad (27)$$

Discussion

Fig 3 shows $Q_L/(h_o a)$, T_m/T_1, N_L' and $(\dot{q}_L/\sqrt{2\lambda T_m h_o})$ for both the lateral and vertical current flow cases. As the earlier discussion indicated, T_m and Q_L are enhanced in the lateral current flow case and diminished in the vertical current flow case. An important feature of the lateral current flow solution is the fact that both Q_L and T_m tend to infinity as ϕ tends to $\pi/4$. This 'explosive' behaviour corresponds to $\gamma T_1 = 1.25$, a value one might have thought tolerable. The Nusselt number remains finite despite the divergence of Q_L and T_m and only differs from its expected value of 2 by 20% for $\phi \leq \pi/4$ in both cases. For the lateral current flow case, $\sqrt{2\lambda T_m h_o}$ rapidly becomes a poor approximation to $\dot{q}_L$ as γ increases. These results show that even for quite small values of γT_1, it is necessary to compensate for the temperature dependence of the heat source density in Joule heated experiments. In this simple conductive problem, which can be solved completely, knowledge of h_o, γ and λ is sufficient for correction factors to be calculated. The evaluation of the critical Rayleigh number experimentally will obviously be sensitive to γ, see [5].

HIGH INTERNAL RAYLEIGH NUMBER

Baker, Faw and Kulacki [14] have examined the experimental results which they and their co-workers have obtained for thermal convection in a horizontal layer driven by uniform internal heating and cooled by horizontal bounding surfaces maintained at the same uniform temperature. They find to a good approximation that the horizontal layer of depth L may be considered as composed of two sub-layers. The lower sub-layer of depth a is essentially stagnant and has a parabolic profile upto a temperature maximum T_m; the heat transferred downwards is equal to the heat generated in this sub-layer, and so is equal to a/L of the total heat generated in the whole layer. The upper sub-layer, of depth b is convecting vigorously, and at higher Rayleigh numbers is turbulent. As the Rayleigh number increases the downward flux fraction μ decreases.

It was found by them that the sub-layer Nusselt number for downward heat transfer N_L' remains close to 2, the conductive value, for internal Rayleigh numbers in the range $10^3 < R/64 < 10^4$. This implies $\dot{q}_L = h_o a = \sqrt{2\lambda T_m h_o}$. The sub-layer Nusselt number for upward heat transfer N_U' was found to depend on R* as

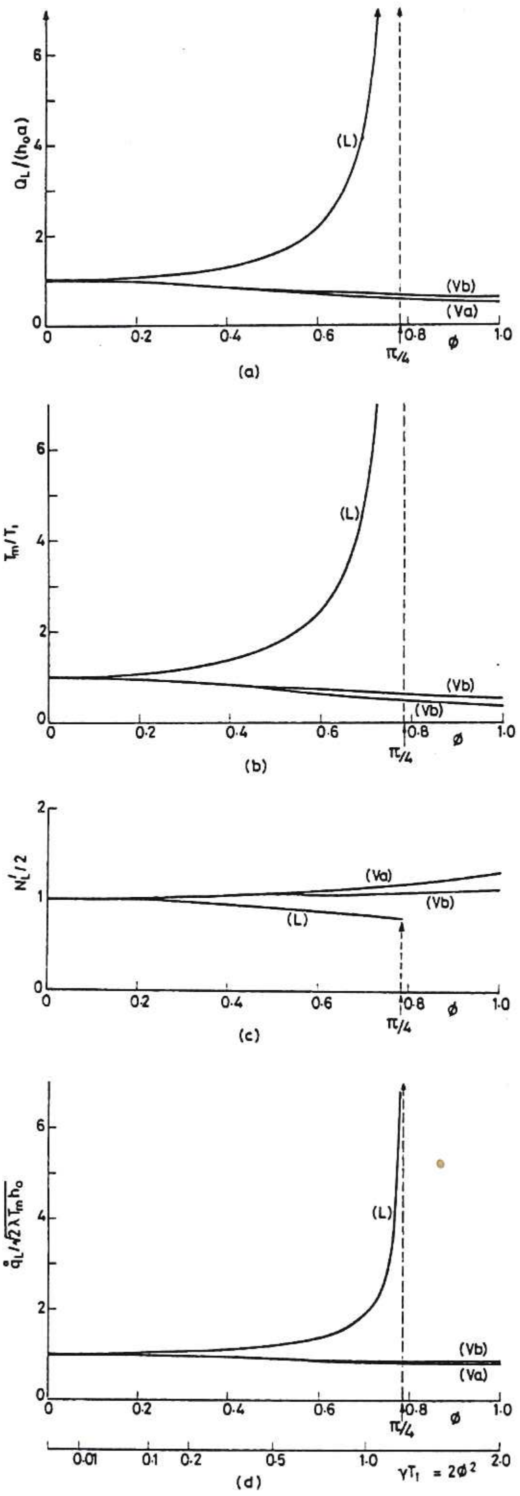

Fig.3 The dependence on ϕ ($\equiv\sqrt{\gamma T_1/2}$) of (a) $Q_L/(h_o a)$, (b) T_m/T_1, (c) $N_L'/2$ and (d) $\dot{q}_L/\sqrt{2\lambda h_o T_m}$. The lateral current case is denoted by (L); the vertical current case based on (17) is denoted by (Va), and that based on (16) by (Vb). The scale for γT_1 is also shown.

$$N_U' = \varepsilon(R^*)^n \tag{28}$$

where Kulacki and Emara [15] obtained ε=0.20, n=0.226 in their experiments. The modified internal Rayleigh number R* is based on the upper sub-layer depth b and so is equal to $(b/L)^5R$.

We now consider how these results would be modified if the electrical conductivity in the aqueous solutions used for these experiments is taken into account. At high Rayleigh number the average temperature profile is as shown in fig 4. This profile represents the mean vertical profile when the horizontal variations and the temporal turbulent fluctuations have been averaged out.

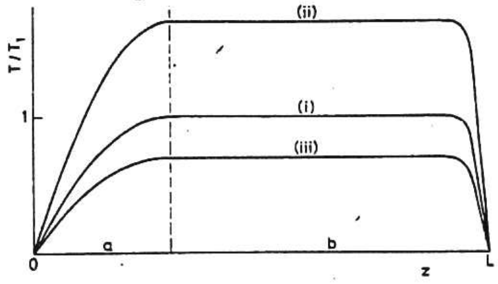

Fig.4 Mean temperature profiles at high Rayleigh number for a given thickness a of the lower sub layer (i) uniform heating (ii) lateral current with $\gamma T_1 = \frac{1}{2}$ (iii) vertical current with $\gamma T_1 = \frac{1}{2}$.

Lateral Current Flow

For the 'stagnant' lower sub-layer, the analysis of the previous section for low Rayleigh number stands unchanged (though b is here the lower sub-layer depth rather than half the full layer depth).

The upper sub-layer at high Rayleigh number is essentially at a uniform temperature T_m (apart from turbulent fluctuations) except for a boundary layer adjacent to the upper cooling plate. An assumption well justified from experimental results [4, 11] is that the heat source density in the upper boundary layer makes a negligible contribution to $\dot{q}_U$. Since the layer is thin for high Rayleigh number then provided $|h_m-h_o| \leqslant h_o$ this holds good. This implies that a small deficit in the heating in the upper boundary layer does not affect $\dot{q}_U$, at least to first order.

A comparison of the relationship between R* and N_U^1 for the case of convection with equal top and bottom temperatures and that for the case with insulated lower boundary shows that the lower boundary has little influence on the (N_U'-R*) relationship for sufficiently high Rayleigh number. The main body of fluid has a heat source density h_m corresponding to the electrical conductivity σ_m for T_m. Consequently the flux at the upper surface and the experimental Nusselt number N_U are those appropriate to a Rayleigh number $R^*_m \equiv \alpha g h_m b^5/(\nu\kappa\lambda)$.

The heat generated in the upper sub-layer is given by

$$Q_U = h_m b \tag{29}$$

where $h_m = h_o(1 + \gamma T_m)$ and T_m is given by (11). Hence

$$Q_U = h_o b/(\cos 2\phi) \quad (30)$$

neglecting the heat source deficit in the upper boundary layer. Hence the Nusselt number for upwards heat transfer from the sub-layer is

$$N_U' = 2(b\phi/a \sin \phi)^2 \quad (31)$$

and the downward flux fraction μ is given by

$$\mu = a \sin 2\phi/(a \sin 2\phi + b.2\phi) \quad (32)$$

As $\gamma \to 0$, $\sin 2\phi \to 2\phi$ and $\mu \to a/L$, the value appropriate to uniform heating.

Vertical Current Flow

As for lateral current flow, the low Rayleigh number analysis can be applied without change to the 'stagnant' lower sub-layer. The general discussion in the previous sub-section is also applicable with the main body of fluid at temperature T_m with corresponding heat source density h_m.

If the heat source density is given by (6), then

$$Q_U = h_o b/s_m \quad (33)$$

$$N_U' = (\pi/2)(b/a)^2 (s_m/[s_m - 1])(\text{erf } \beta_m)^2 \quad (34)$$

and

$$\mu = a\beta_m/[a\beta_m + b(\sqrt{\pi}/2)\text{erf } \beta_m] \quad (35)$$

where $s_m = 1 + \gamma T_m$ and $\beta_m{}^2 = \ell n\ s_m$

If on the other hand (6) is linearized to

$$h = h_o(1 - \gamma T) \quad (36)$$

then one obtains

$$Q_U = h_o b/(\cosh 2\phi) \quad (37)$$

$$N_U' = 2(b\phi/a \sinh \phi)^2 \quad (38)$$

and

$$\mu = a \sinh 2\phi/[a \sinh 2\phi + b.2\phi] \quad (39)$$

which again $\to a/L$ as $\phi \to 0$.

Discussion

Fig 3 has already shown how Q_L and N_L' vary with φ for given sub-layer depth a. Fig 5 shows how $(Q_U/h_o b)$ and N_U' depend on φ. As $\phi \to \pi/4$, the heat generated in the sub-layer becomes infinite for the lateral case, whereas the Nusselt number does not diverge until $\phi \to \pi$.

An important quantity in industrial applications of results from studies of thermal convection driven by internal heat sources is the ratio of heat transferred downwards to that transferred upwards viz. $\mu/(1-\mu)$. For uniform heating this ratio is by definition a/b. Fig 6 shows the variation of this ratio with φ for lateral and vertical current cases. For $\phi \sim \pi/4$, μ can differ by upto 30% from the uniform heating value, with lateral and vertical current cases producing deviations of opposite signs.

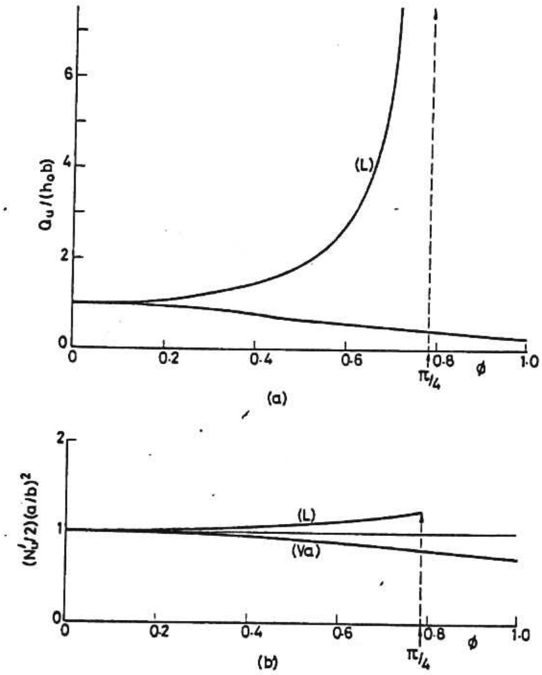

Fig.5 The dependence on φ ($\equiv \sqrt{\gamma T_I/2}$) of (a) $Q_U/(h_o b)$, and (b) $(N_U'/2)(a^2/b^2)$. The lateral current case is denoted by (L); the vertical current case based on (36) is denoted by (V).

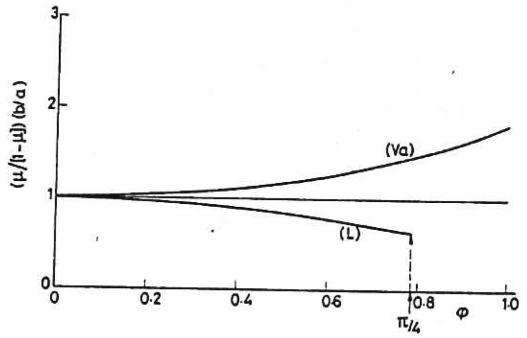

Fig. 6 The dependence on φ of the flux ratio $(\mu/(1-\mu))$. (b/a) for both lateral (L) and vertical (V) current cases.

IMPLICATIONS AND CORRECTION FORMULAE

The main implications of the analysis of the earlier sections are that even a weak temperature dependence of the electrical conductivity can significantly modify the results obtained from those which would be obtained if the heating were truly uniform, and that lateral current flow and vertical current flow produce deviations with different signs. The standard theory for internally heated natural convection assumes uniform heating. The point of this section is to provide a procedure for calculating first order corrections to the experimental data obtained using (non-uniform) Joule heating to obtain the true dependence of Nusselt numbers and the flux fraction as a function of the Rayleigh number for uniform internal heating.

If the temperature dependence of the heat source density is completely neglected, the procedure for reducing the data from the experiments is relatively straightforward. The measured quantities are the maximum temperature difference in the layer T_m, the downward flux density $\dot{q}_L$, the upward flux density $\dot{q}_U$ and the power dissipated P. The mean heat source density $\bar{h}$ can be simply calculated as P/(XYL). If the insulation of the sidewalls is satisfactory then $\bar{h}.L$ should equal $\dot{q}_L + \dot{q}_U \equiv Q$. The Rayleigh number $\bar{R}$, the customary Nusselt numbers N_L and N_U, and the downward flux fraction μ can be calculated from their definitions. Thus the relationships (μ-$\bar{R}$), (N_U-$\bar{R}$) and (N_L-$\bar{R}$) can be plotted for a number of data points, and correlations sort the usual way.

These results do indeed characterise the system, but it is difficult to apply correction formulae to results presented in this fashion.

For the lateral current flow case we now consider which quantities need to be measured and how the correction factors can be applied. The heat source density is given by (5), and this can be calculated as a function of the vertical co-ordinate. The electrical conductivity σ_o, the temperature coefficient γ and the cell width X are constants which can be determined simply, the potential drop V across the cell can be measured for each experiment, and the temperature profile T(z) is also measurable. Hence h(z) is known. At high Rayleigh number, h will be equal to h_m (corresponding to T_m) for much of the cell depth. To correct the results obtained to those for a uniform heat source density it is simply necessary to add the deficits in heat production in the appropriate boundary layers to the upward and downward fluxes. Thus $\dot{q}_L$ is replaced by $\tilde{\dot{q}}_L = \dot{q}_L + \Delta\dot{q}_L$ and $\dot{q}_U$ by $\tilde{\dot{q}}_U = \dot{q}_U + \Delta\dot{q}_U$. Also the corrected downward flux fraction $\tilde{\mu}$ is $\tilde{\dot{q}}_L/(\tilde{\dot{q}}_L + \tilde{\dot{q}}_U)$. The appropriate Rayleigh number is R_m, based on h_m. The relationships ($\tilde{\mu} - R_m$), ($\tilde{N}_U - R_m$), ($\tilde{N}_L - R_m$) based on the corrected fluxes can then be plotted, and these will be the ones required. Moreover $\tilde{\mu}$ should now equal a/L, which enables a to be evaluated; the sub-layer Nusselt numbers $\tilde{N}_L'$ and $\tilde{N}_U'$ can then be calculated and compared with the digest of previous experimental results in [14].

The deficit can be obtained graphically from the temperature profile since $q_L = a(h_m - \int_o^a h dz) = h_o a\gamma (T_m - \int_o^a T dz)$. However the analysis in the earlier sections enables $\Delta\dot{q}_L$ to be estimated. From (14), $\dot{q}_L$ is calculated to be $(2\phi \cot \phi).(\lambda T_m/a)$ whereas for uniform heating it would be $2(\lambda T_m/a)$. Thus

$$\Delta\dot{q}_L = \dot{q}_L(\tan \phi - \phi)/\phi \qquad (40)$$

for the lateral current case. If

$$\psi^2 \equiv \gamma h_o L^2/2\lambda \equiv \gamma T_2 \qquad (41)$$

so that ψ can be calculated without reference to the temperature profile, then ϕ can be expressed in terms of measured quantities $\dot{q}_U$ and $\dot{q}_L$ by the implicit equation

$$\dot{q}_L/\dot{q}_U = (\psi/2)(1-\psi\phi)^{-1} \sin 2\phi \qquad (42)$$

When this is linearised, one obtains

$$\phi = \mu/\psi \qquad (43)$$

If (40) is expanded in powers of ϕ, then the first order correction term is

$$\Delta\dot{q}_L = \dot{q}_L(\mu^2/3\psi^2) \qquad (44)$$

The correction $\Delta\dot{q}_U$ can be neglected for high Rayleigh numbers.

For vertical current flow similar arguments apply, but with some modification. In this case it is the electric current density which is constant throughout the cell, and this being (J/XY) is easily measured for each experiment. The heat source density h_m, corresponding to T_m, is now the minimum heating rate anywhere in the cell, and consequently to correct the results obtained to those for a uniform heat source density h_m it is necessary to subtract the enhancements in heat production occurring in the appropriate boundary layers; thus $\tilde{\dot{q}}_L = \dot{q}_L - \Delta\dot{q}_L$ etc. If h is given by (6) then

$$\Delta q_L' = q_L(1-\gamma T_m[\ln s_m)/2\phi^2]^{\frac{1}{2}}) \qquad (45)$$

which linearizes to $\Delta\dot{q}_L = \dot{q}_L\phi^2/3$ which is equivalent to (44). If h is given by (36) then

$$\Delta\dot{q}_L = \dot{q}_L(1-\phi^{-1} \tanh \phi) \qquad (46)$$

To first order ϕ again satisfies (43), and so linearization of (46) leads again to (44) as the first order correction term. The correction term can also be obtained easily from the temperature profile if (36) is assumed for h; if (6) is used however $\Delta\dot{q}_L = (h_o a\gamma/s_m) \int_o^a (T_m - T)/s.dz$ which is certainly more tedious to evaluate than (44).

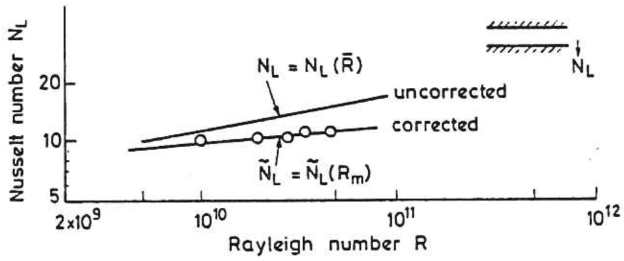

Fig.7 Example of the correction to the lower plate Nusselt number for the vertical current case.

This correction formula has been applied to an experimental investigation [13] using vertical electric current flow to produce the Joule heating. With $\gamma \sim 0.02K^{-1}$, observed temperature differences of up to 12 K were not uncommon, which implies $\gamma T_m \leq 0.25$. The correction term $\Delta\dot{q}_L$ to be subtracted from the measured $\dot{q}_L$ was found to be quite significant. Fig 7 shows the uncorrected curve $N_L = N_L(\bar{R})$ with gradient 0.125 and also the corrected curve $\tilde{N}_L = \tilde{N}_L(R_m)$ with gradient 0.098. This can be compared with a gradient of 0.094 obtained by Kulacki and Goldstein who used much smaller temperature differences in their experiments.

The value of γ for weak aqueous solutions commonly used in Joule heating experiments is of order of a few per cent. For example, $\gamma \approx 0.032$ for common salt solution and $\gamma \approx 0.023$ for H_2SO_4 [16]. Thus the effects of the temperature dependence of the electrical conductivity cannot be avoided, only minimised. The correction formulae presented here are admittedly somewhat crude, but they do indicate the magnitude of the errors which can arise. The analysis also implies that one should be cautious when extrapolating correlations to very high Rayleigh number (particularly for N_L which is sensitive to the spatial distribution of the heat source density).

The effects of the temperature dependence of the heat source density have been calculated for thermal convection driven by internal heat sources between horizontal plates of equal temperature when (i) the internal Rayleigh number is low enough for the temperature profile still to be vertically stratified and (ii) the internal Rayleigh number is high enough for the temperature profile is vertically stratified in the mean. In the intermediate regime when strong laminar convection is occurring the temperature profile has a significant horizontal variation and this analysis is not strictly appropriate. However large values of γT are likely to occur experimentally only for large Rayleigh numbers when one is trying to push as far into the turbulent regime as is practical with the apparatus, and so the errors are less likely to be important in the intermediate regime.

In the high $\bar{R}$ analysis, the heat sources in the upper boundary layer were neglected on the premise that the boundary layer was thick. If N_U is less than ~ 10, this is unreasonable, and then the analysis needs to be extended to take into account the deficits or enhancements in heating in the upper boundary layer. The extension can be carried out on the lines already indicated, and is omitted here since the basic principles can be illustrated without the extra complexity which the extension involves. If both internal heating and applied temperature gradients are present then under certain circumstances, both boundary layers can be quite thick and a full analysis is required to calculate the magnitudes of the correction factors.

CONCLUSIONS

If the electric current flows horizontally through a volume of electrically conducting fluid the heating is proportional to the electrical conductivity which has a positive temperature coefficient, and in the upper and lower boundary layers the heating is less than the mean heat source density. If the electric current flows vertically through a volume fluid which is thermally stratified in the mean, then the heating is proportional to the electric resistivity which has a negative temperature coefficient, and the heating in the boundary layers is greater than the mean heat source density. In both cases, large temperature differences within the layer, coupled with larger temperature coefficients, result in significant but calculable deviations in the Nusselt-Rayleigh relations from their form for uniform heating. This effect can be minimised by keeping the temperature differences within the layer small, but for natural convection experiments involving both internal heating and applied temperature gradients [17] it may be unavoidable. The procedure illustrated in the earlier sections can be carried out to obtain corrections for any configuration where thermal stratification arises in experiments with Joule heating.

ACKNOWLEDGEMENTS

I am grateful to Dr M Bevir for his comments and to the Safety & Reliability Directorate of the UKAEA for supporting this work.

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NOMENCLATURE

a = depth of lower sub-layer, m
b = depth of upper sub-layer, m
c = specific heat, J kg^{-1} K^{-1}
$\underline{E}$ = electric field, V m^{-1}
$\underline{g}$ = gravity, 9.81 ms^{-2}
h = heat source density, W m^{-2}
$\underline{j}$ = electric current density, A m^{-2}
J = total electric current, A
L = total layer depth, m
n = coefficient in (28)
N = Nusselt number $\dot{q}L/\lambda T_m$
N_U' = upper layer Nusselt number, $(b/L)N_U$
N_L' = lower layer Nusselt number, $(a/L)N_L$
p = pressure
P = total power, W
$\dot{q}$ = vertical heat flux density, W m^{-2}
Q = heat generation rate in (sub) layer W m^{-2}
$\overline{R}$ = mean internal Rayleigh number, $\alpha g \overline{h} L^5/(\nu\kappa\lambda)$
R_m = $(h_m/\overline{h})\overline{R}$
R^* = $(b/L)^5\overline{R}$
s = $1 + \gamma T$
t = time, s
T = relative temperature, $\theta - \theta_o$, K
T_1 = $h_o a^2/(2\lambda)$
T_2 = $h_o L^2/(2\lambda)$
u = velocity
V = total potential drop, V
x = horizontal co-ordinate, m
X = horizontal extent in x-direction, m
y = horizontal co-ordinate, m
Y = horizontal extent in y-direction, m
z = vertical co-ordinate (increasing upwards), m
α = coefficient of thermal expansion, K^{-1}
β = $(\ln s)^{\frac{1}{2}}$
γ = temperature coefficient of electrical conductivity, K^{-1}
δ = dimensionless boundary layer thickness
ε = coefficient in (28)
η = electrical resistivity of fluid, m^2 s^{-1}
θ = absolute temperature, K
κ = thermal diffusivity, m^2 s^{-1}
λ = thermal conductivity, W m^{-1} K^{-1}
μ = downward flux fraction, $\dot{q}_L/(\dot{q}_L+\dot{q}_U)$
ν = kinematic viscosity, m^2 s^{-1}
ρ = density, kg m^{-3}
σ = electrical conductivity of fluid, A V^{-1} m^{-1}
ϕ = $(\gamma T_1/2)^{\frac{1}{2}}$
ψ = $(\gamma T_2)^{\frac{1}{2}}$
$\Delta\dot{q}$ = enhancement (or deficit) in $\dot{q}$, W m^{-2}

Subscripts

c = critical
L = lower
m = maximum (corresponding to $T=T_m$)
o = value at boundaries
U = upper

Superscripts

$\sim$ = corrected value
$-$ = (overbar) vertically averaged