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# RESONANCE ABSORPTION IN NON-PLANAR PLASMAS

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## ABSTRACT

We describe a numerical routine for the calculation of the absorption coefficient due to resonance absorption for plasmas with non-planar iso-density surfaces. The method used relies on the fact that in the linear approximation for resonance absorption, in plane-parallel plasmas, the absorption coefficient depends exclusively upon the distance from the turning point to the critical surface, $d_t$. For more complex geometries, we assume that the region between the turning point and the critical surface can still be considered as plane-parallel, in which case the absorption coefficient remains a sole function of $d_t$. The determination of $d_t$ is accomplished using a ray tracing routine. Our calculations and assumptions are valid within the limits of the geometrical optics (WKB) approximation.

We illustrate this method by investigating the absorption and reflection of laser light by plasmas with a rippled critical surface.

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A closed analytic expression for the absorption coefficient due to the linear resonance absorption mechanism in plane-parallel geometry is not available at present, although it is possible to calculate the absorption coefficient for a particular angle of incidence by analytically solving the wave equation (Pert, 1978). The absorption coefficient for general angles of incidence has been determined numerically for plane-parallel geometry (Forslund et al, 1975; Estabrook et al, 1975), and recently for spherical geometry (Thomson et al, 1976). The effect of more complex geometries on the absorption coefficient is still a matter for debate.

Recent experiments with plane targets have measured the absorption coefficient as a function of angle of incidence (Manes et al, 1977) and the experimental result shows the general characteristic shape predicted by the linear resonance absorption theory.

However, two discrepancies were noted - (a) there is a finite absorption for normal incidence, and (b) the peak is broader than theoretically predicted. Part of the absorption observed at normal incidence may be due to subsidiary non-linear absorption processes, such as two-plasmon decay (Langdon et al, 1973), extraordinary mode conversion due to self-generated magnetic fields (Kruer and Estabrook, 1977), ion density fluctuations (Faehl and Kruer, 1977; Manheimer et al, 1977; Forslund et al, 1977), and parametric instabilities at the critical surface (Kruer and Dawson, 1972). The unaccounted for remainder, together with the broadening of the peak has been tentatively explained by invoking a rippled critical surface (Thomson et al, 1978).

In the absence of a planar or spherical critical surface, the absorption coefficient must be found through the two-dimensional wave equation for the fields and this has never been done either analytically or numerically as far as we know. Attempts have been made to model resonance absorption including a rippled critical surface (Thomson et al, 1978; Cairns, 1978). In the model of Thomson et al (1978), the theoretical absorption curve for plane-parallel geometry was taken and an average made about particular angles of incidence. The range of angles over which the average was performed is specified by the amplitude and frequency of the rippling.

Besides rippling, it is important to understand the modification to the absorption coefficient introduced by a general curvature of the critical surface. For example, in experiments on plane massive targets performed at Culham, preliminary results suggest that the density contours can be better represented by elliptical rather than planar geometry.

The above then, is the motivation for a study of resonance absorption in non-planar geometry. In what follows we describe an alternative method of calculating the absorption coefficient. We use the geometrical optics (WKB) approximation throughout and, therefore, have to ensure that the plasma inhomogeneity scale lengths are much larger than the wavelength of the incident radiation ie

$$\left| \frac{\nabla N}{N} \right| \ll k_o \qquad \ldots(1)$$

where N is the number density. The plasma density decays exponentially with a given scale length and no self-consistent profile modification is considered, in agreement with our linear approach to the problem.

In the linear resonance absorption theory for plane-parallel plasmas, the absorption coefficient is an exclusive function of the distance from the turning point to the critical surface, $d_t$. This no longer holds for non-planar plasmas because we no longer know, a priori, how the tunnelling mechanism is affected by the geometry of the plasma in this region. However, because we use the WKB approximation we can assume, without introducing large errors, that the region between the turning point and the critical surface is well represented by a plane-parallel geometry. Thus the problem reduces again to a determination of $d_t$. We need therefore a method of finding $d_t$ in non-planar plasmas and in our case this is accomplished by means of an improved version of the ray tracing routine described by Hubbard and Montes (1978) and briefly outlined below.

The ray tracing routine is based on the numerical solution of the general equation for the ray path (Landau and Lifshitz, 1960) viz:

$$\frac{d\underline{l}}{dl} = \frac{1}{n}\left\{\underline{\nabla}n - \underline{l}\,(\underline{l}.\underline{\nabla}n)\right\} \qquad \ldots(2)$$

where n is the refractive index and $\underline{l}(l)$ is a unit vector tangential to the ray at a position l along the ray. Given a point on the ray and the tangent to the ray at that point, the routine calculates the next point using eqn.(2) through a Taylor expansion and also the tangent to the ray at this new point by the same method. Thus, given the refractive index and its gradient at each point on the ray, and the co-ordinates and tangent vector of an initial point on the ray, we can use this method to calculate the co-ordinate points of the ray path. The refractive index is determined through the dispersion relation for the plasma under consideration by inserting in it the analytic expression for the density contours specified by the geometry of the problem.

We use the ray tracing routine described above to determine the minimum value of the refractive index $n_t$ obtained on the ray path. This effectively determines the distance from the turning point to the critical surface $d_t$. $n_t$ is then used in the determination of the parameter $\tau$, where $\tau$ is defined in the usual manner (Ginzburg, 1970) as follows:

$$\tau = (\kappa_o L)^{\frac{1}{3}} \sin \theta_{eff}. \qquad \ldots(3)$$

$\theta_{eff}$ is given by

$$\sin \theta_{eff} = n_t, \qquad \ldots(4)$$

and eqn.(4) is validated by our assumption that the region between the turning point and the critical surface can be described by a plane-parallel geometry. Once we know the value of $\tau$, the absorption coefficient for this ray is calculated from an approximate expression for the linear absorption coefficient (see, eg. Ahlstron, 1977)

$$A(\tau) \simeq \frac{\Phi^2(\tau)}{\pi} \qquad \ldots(5)$$

$\Phi(\tau)$ is the well known Ginzburg function (Ginzburg, 1970), for which we use the following expression

$$\Phi(\tau) \simeq 2.24\, \tau \exp\left\{-\frac{2}{3}\tau^3\right\} \qquad \ldots(6)$$

Thus as a ray is traced through the plasma we can ascribe to it a particular absorption coefficient $A_r$ calculated as described above. We approximate the laser beam by a large number of rays, tracing each ray through the plasma, and attributing to each an absorption coefficient $A_{ri}$, where i is the index of the ray. If we assume that all rays have the same intensity, the overall absorption coefficient for the beam is given by

$$A = 100\left\{\frac{1}{N}\sum_{i=1}^{N} A_{ri}\right\}\% \qquad \ldots(7)$$

where N is the number of rays in the beam.

In the following calculations, the wavelength of the incident beam is 10.6 μm, the beam width is chosen as 200 μm, as this corresponds to a typical focal spot diameter calculated for the optics used in $CO_2$ laser-target interaction experiments at Culham, and a collimated incident beam is used, as this approximates to the situation at the Gaussian focus.

In Fig.1 we show a comparison of our numerical results for a plane-parallel plasma with the theoretical curve as given by eqn.(5). Also on this figure we have indicated the absorption curve obtained by Pert (1978) and Forslund et al (1975). These curves show good agreement between the values generated by our routine and those from eqn.(5). The error introduced by the use of this approximation is apparent from the figure.

We now apply our method to the determination of the absorption coefficien as a function of the angle of incidence for plasmas with elliptical iso-density contours. The refractive index in this case is given by

$$n(x,y) = \sqrt{1 - \exp\left\{\frac{a_{crit} - a}{L}\right\}} \qquad \ldots(8)$$

where

$$a = \sqrt{x^2 + \left\{\frac{y}{\epsilon}\right\}^2} \qquad \ldots(9)$$

$\epsilon$ is the eccentricity, L is the scale length of the density gradient in the x direction, and $a_{crit}$ is the semi-minor radius of the ellipse representin the critical surface.

Fig.2 shows the ray paths for a spherical plasma, ie we have set $\epsilon$ equal to 1 in eqn.(8). It shows the large angular scatter expected for this case. Due to the symmetry of the problem there is no variation in absorption coefficient with angle of incidence provided the central ray in the beam is directed towards the origin. The calculated absorption coefficient for this case is 30%.

Fig.3 shows the absorption coefficient versus angle of incidence for $\epsilon = 3$ resulting from our model. The curves shown are for three values of the scale length, namely L = 25 μm, L = 50 μm and L = 200 μm. The theoretical position of the peak absorption in plane-parallel plasmas is given by

$$\sin \theta_{opt} \simeq \frac{0.8}{(k_o L)^{\frac{1}{3}}} \qquad \ldots(10)$$

This gives values of $\theta_{opt} = 19^o$, $15^o$ and $9^o$ for L = 25 μm, 50 μm and 200 μm respectively. Comparing these values with our results shown in Fig.3, we notice that there is no variation for L = 200 μm, and a slight shift for L = 50 μm and L = 25 μm. In all cases we observe a non-zero absorption coefficient for normal incidence and a lower value of the absorption coefficient at optimum angle of incidence than theoretically expected for plane-parallel plasmas.

We next consider a plasma with rippled isodensity contours. Rippling is indicated both experimentally and theoretically. In experiments it was inferred (Thomson et al, 1978) from the low backscattering at normal incidence reported by Manes et al (1977) and Ripin (1977). Theoretically, computer simulations (Estabrook, 1976; Valeo and Estabrook, 1975; Forslund et al, 1977), have indicated that instabilities of the critical surface may develop, resulting in a density modulation in a direction perpendicular to the initial density gradient. It has been suggested that these modulations can also be produced by hot spots in the incident laser beam through the ponderomotive force (Thomson et al, 1978).

We introduce rippling into our model by superimposing a cosinusoidal density modulation on planar isodensity contours thus

$$n(x,y) = \sqrt{1 - \exp\left[-\frac{1}{L}\left\{x - x_{crit} + a\cos(by)\right\}\right]} \qquad \ldots(11)$$

where a and b are the amplitude and frequency respectively, of the rippling, and $x_{crit}$, the mean distance of the critical surface from the y axis (here representing the target surface) measured along the x axis. From eqn.(11) we see that the critical surface is defined by

$$x = x_{crit} - a\cos(by) \qquad \ldots(12)$$

Fig.4 shows two examples of absorption curves obtained for a rippled critical surface. The values of a and b shown on the figure satisfy eqn.(1), and have been chosen to give an average angle of illumination $\bar{\varphi}$ equal to $10^o$ at normal incidence. We note that this particular value of $\bar{\varphi}$ can be obtained from an infinite number of combinations of a and b. The figure shows that a different absorption curve results for each particular combination. For this reason we characterise the critical surface by a and b rather than the single parameter $\bar{\varphi}$ of Thomson et al (1978). As in the previous case (Fig.3), comparing our results to that of plane-parallel plasmas, we observe a non-zero absorption coefficient at normal incidence and a broader peak with a lower value for the maximum absorption; although for this case we notice a slight shift from the position of peak absorption ($\theta_{opt} \simeq 19^o$), given by eqn.(10) with L = 25 μm. But contrary to the previous case and to the result of Thomson et al (1978), our results show an accentuated dip for angles of incidence around $\bar{\varphi}$. The origin of this dip becomes clear when one looks at the ray paths for these angles of incidence. A large fraction of the rays are backscattered due to the fact that for angles of incidence around $\bar{\varphi}$, refraction brings most of the rays into near normal illumination of the critical surface. Necessarily, the contribution of these rays to the absorption coefficient is decreased. This effect is enhanced by the tendency of the rays to refract into regions of lower density causing the kind of beam filamentation shown in Fig.5. We emphasise at this stage, however, that the above effect is unlikely to be seen as clearly in experiments as there is no reason to believe that the critical surface will maintain the same shape for all angles of incidence.

In Fig.6 we compare the absorption curves from Thomson et al (1978) with a curve generated by our routine for the same $\bar{\varphi}$, namely $10^o$. We wish to point out here that although our results are for L = 25 μm and $\lambda_o$ = 10.6 μm, the absorption coefficient is a function of $\tau$, and therefore scales as $\kappa_o L$. Thus we are able to assume that our results are equivalent to a situation in which L = 2.5 μm and $\lambda_o$ = 1.06 μm. The parameters a and b were chosen to give the best fit at the peaks. The discrepancies between the curves are obvious but we note that the refraction implicit in Thomson et al (1978) is that of a plane-parallel geometry while in our case we consider the effects on the refraction due to rippling, this could be the reason for the absence of the dip in their result. We display also in Fig.6 the experimental results for the absorption coefficient given by Manes et al (1977). This shows that when one takes refraction into account using a plasma with rippled isodensity contours, the agreement between theory and experiment is no longer satisfactory. However, we now compare, in Fig.7, the experimental results obtained by Manes et al (1977), with an absorption curve generated by our routine using a plasma with elliptical isodensity contours as a model. The width and magnitude of the peak are seen to be in good agreement with the experimental results. The absorption at near-normal incidence is much lower than measured but we note that our program does not take into account the subsidiary absorption processes mentioned above, which are likely to be responsible for part of the absorption at small angles of incidence.

In conclusion we have developed a method that enables us to calculate the absorption coefficient due to resonance absorption for non-planar plasmas. It is based on a compact ray-tracing routine and is fairly precise as shown by the comparison of our results for plane-parallel geometry with the theoretical curve as in Fig.1. Its simplicity stems from the fact that we do not need to solve a two-dimensional wave equation to obtain the absorption coefficient. We believe that the results obtained with the method are only as good as the modelling of the isodensity contours used, and it is only through this modelling that self-consistency can be introduced in the problem. As an example of the application of the routine we have tried to reproduce some experimental results. We note that there is a significant discrepancy between our results and the experimental data when we model the plasma by fixed rippled isodensity contours. On the other hand, good agreement is obtained just by considering an elliptical plasma geometry. These results indicate that resonance absorption can be responsible for a large fraction of the absorption observed at large angle of incidence even in the absence of plasma rippling.

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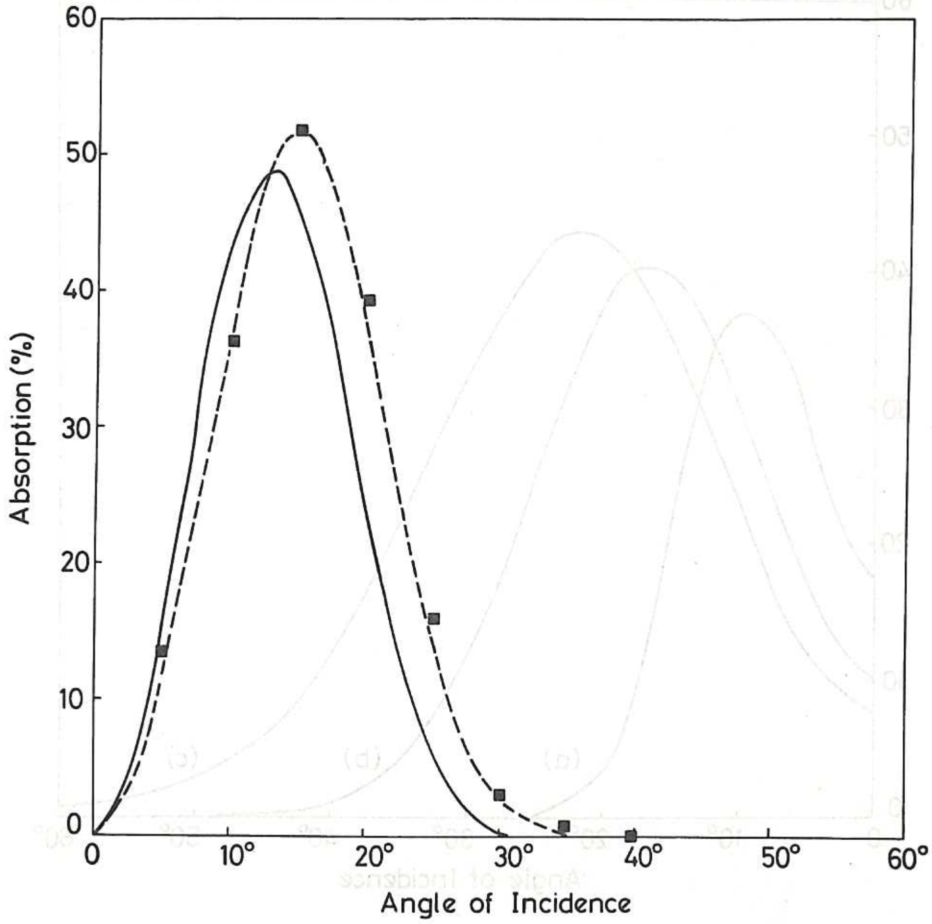


**Fig.1** Absorption coefficient versus angle of incidence. $k_0 L = 30$. The solid line is as given in Pert (1978) and Forslund et al (1975). The broken line was obtained from eqns. (5,6). The points (■) are the results from our program.

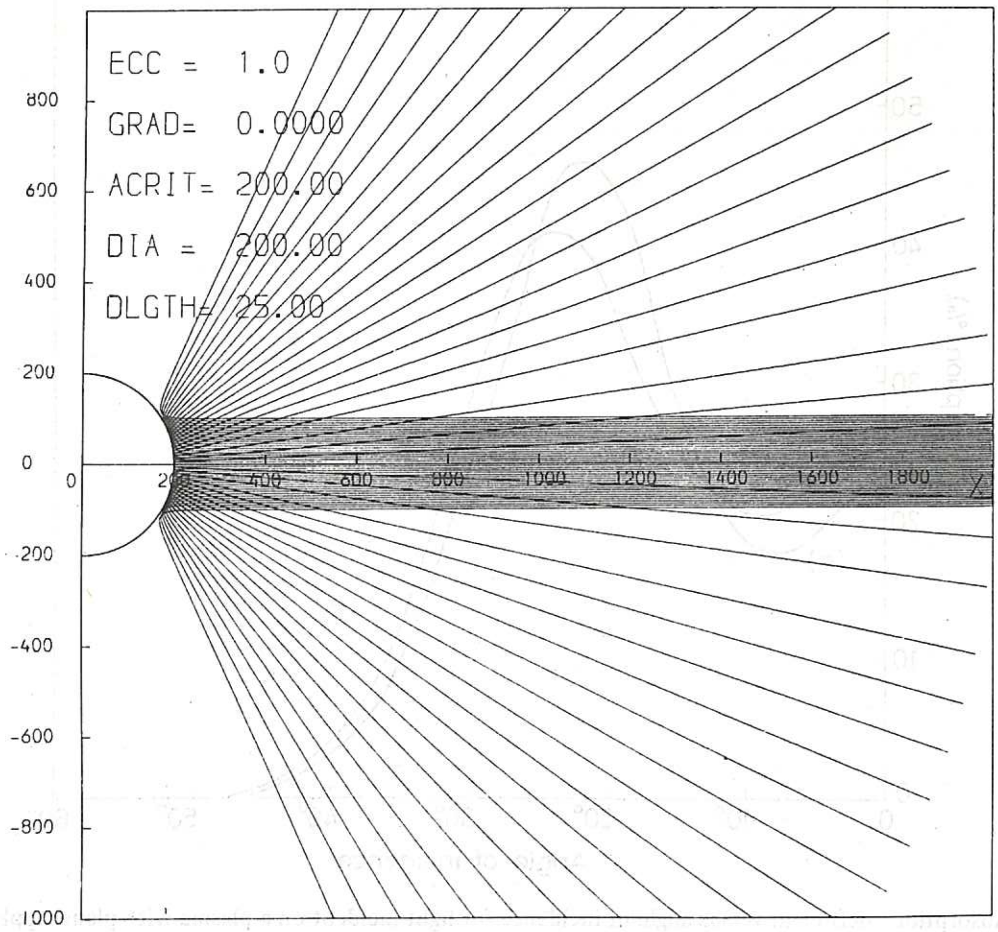


**Fig.2** An example of the output of the ray-tracing routine for a spherical plasma. $k_0 L = 15$. Dimensions are in microns.

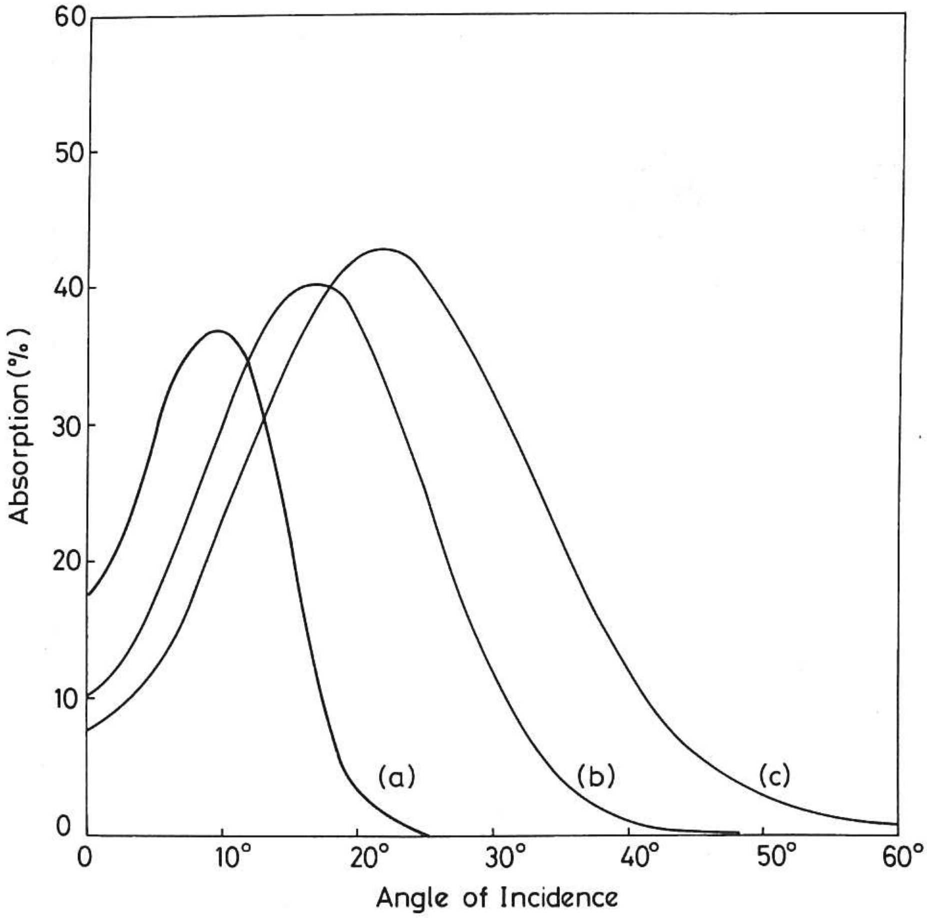


**Fig.3** Absorption coefficient versus angle of incidence for an elliptical plasma with eccentricity equal to 3. Curve (a) $k_0 L = 120$, (b) $k_0 L = 30$, (c) $k_0 L = 15$.

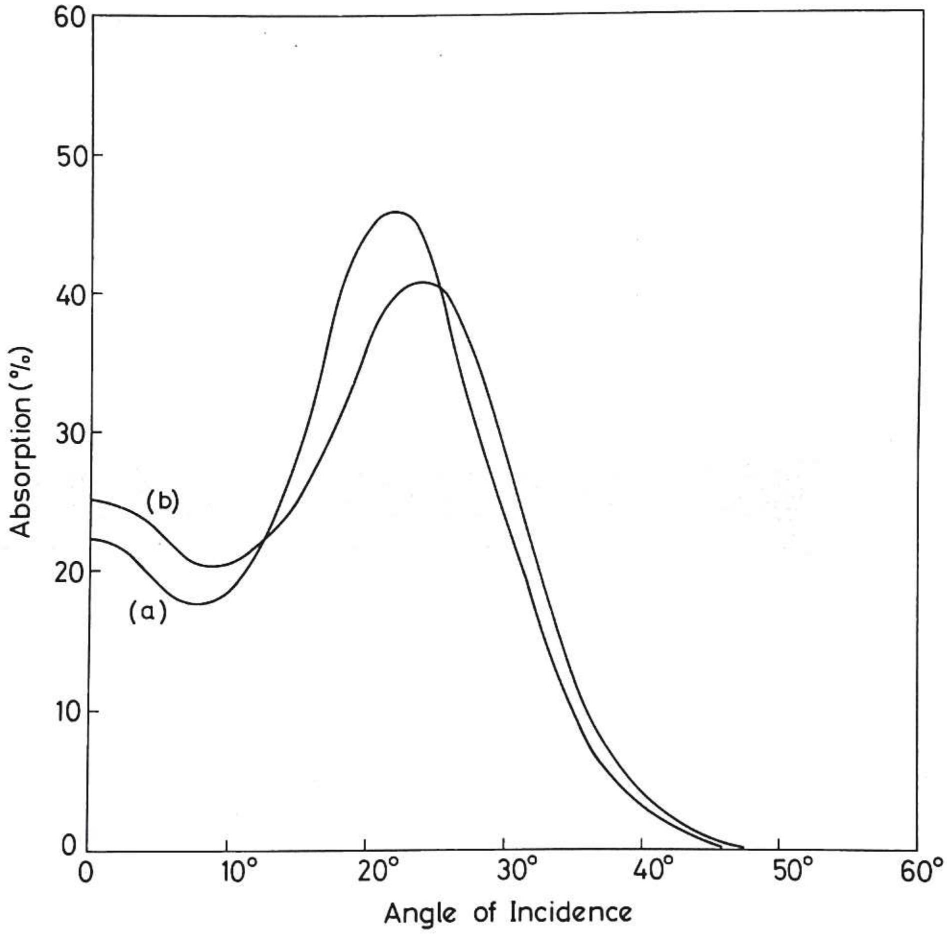


**Fig.4** Absorption coefficient versus angle of incidence for light incident on a plasma with plane rippled isodensity contours. $k_0 L = 15$ and $\bar{\varphi} = 10^\circ$ for both curves. Curve (a) $a = 0.151\ \lambda_0$ and $b = 1.831/\lambda_0$; curve (b) $a = 0.236\ \lambda_0$ and $b = 1.165/\lambda_0$.

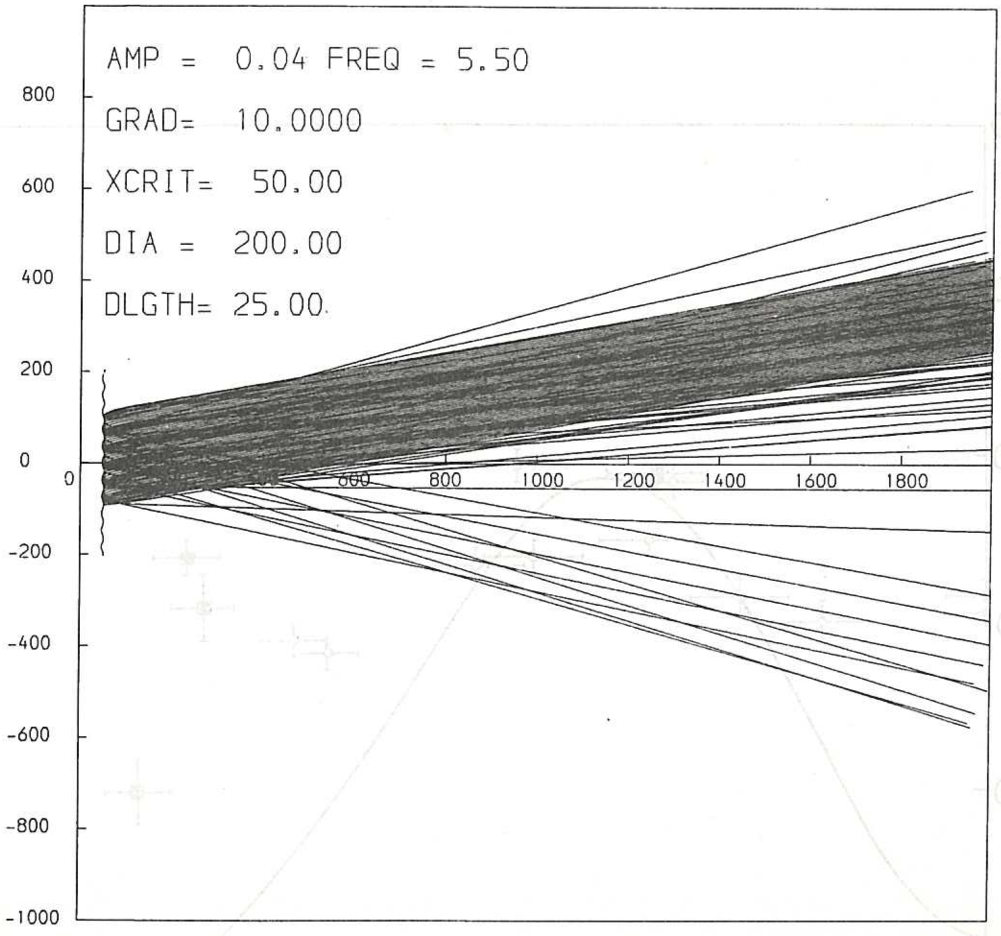


**Fig.5** Ray trace for a plasma with plane rippled isodensity contours. $k_0 L = 15$. The angle of incidence is 10°. $a = 0.151\ \lambda_0$ and $b = 1.831/\lambda_0$ ($\bar{\varphi} = 10°$). Dimensions in microns.

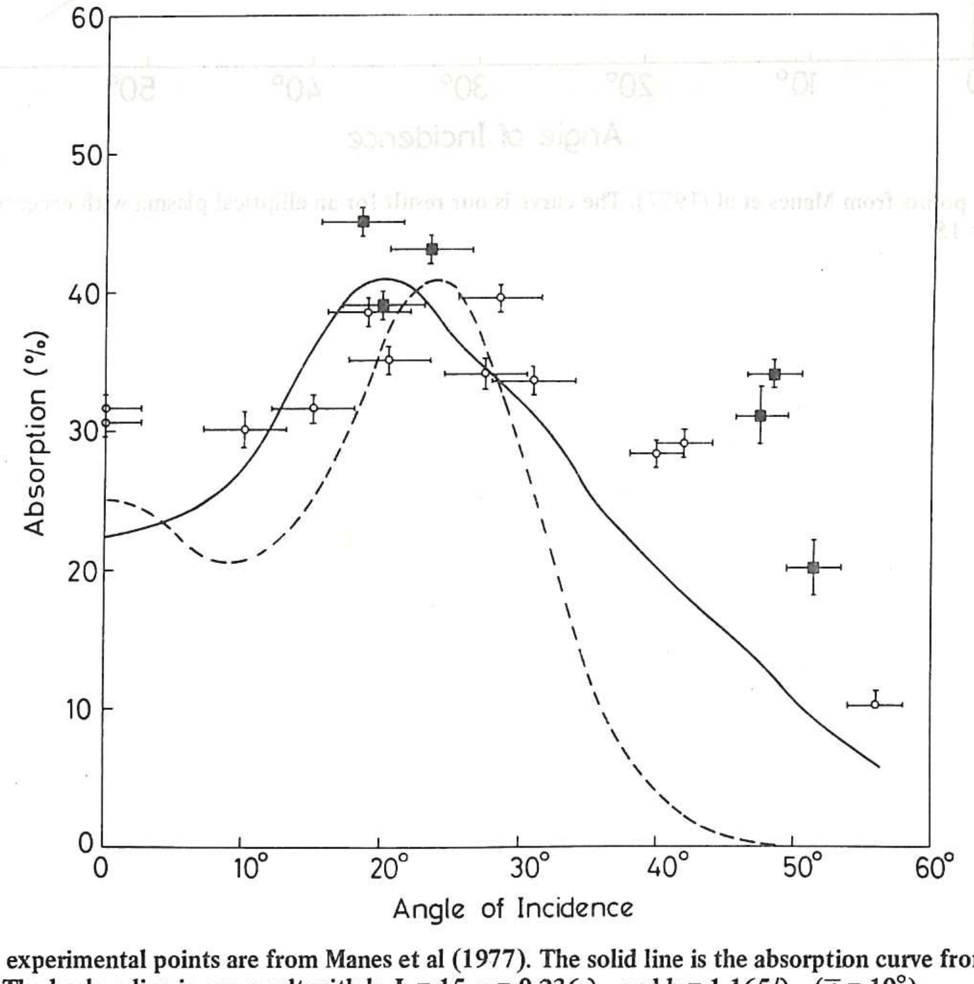


**Fig.6** The experimental points are from Manes et al (1977). The solid line is the absorption curve from Thomson et al (1978). The broken line is our result with $k_0 L = 15$, $a = 0.236\ \lambda_0$ and $b = 1.165/\lambda_0$ ($\bar{\varphi} = 10°$).

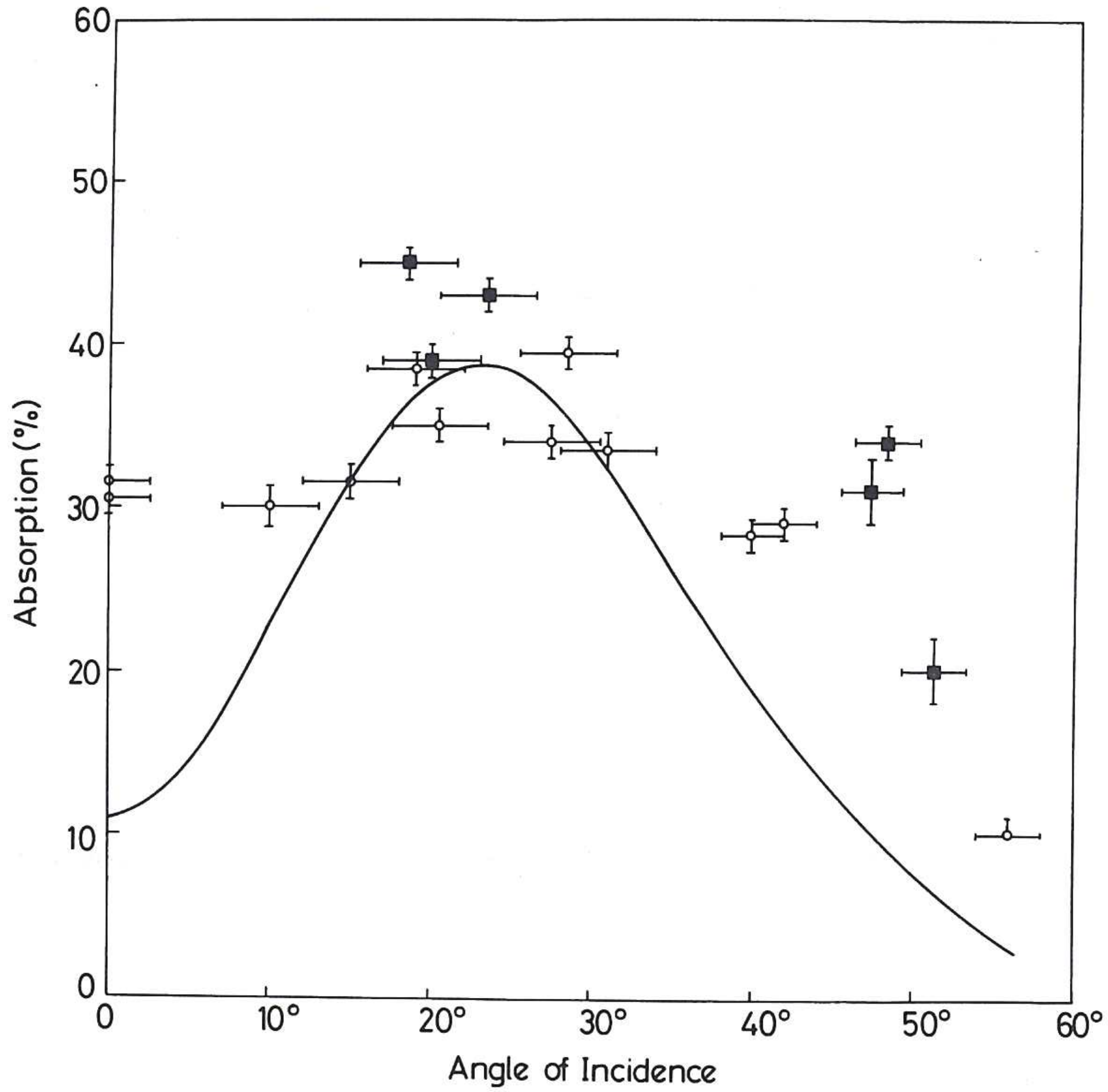


Fig.7 Data points from Manes et al (1977). The curve is our result for an elliptical plasma with eccentricity = 2.5 and $k_0 L = 15$.