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Report

THE INTERNAL FIELDS OF m = ±1 HELICONS IN INDIUM

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THE INTERNAL FIELDS OF m = ± 1 HELICONS IN INDIUM

by

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A B S T R A C T

Measurements were made of the radial electric field patterns of standing helicon waves in short cylinders of pure indium at 4.2°K. Agreement with the calculated field patterns was obtained for the modes (m = 1, n = 2) and (m = -1, n = 2).

It was confirmed that for m = -1, the simplest mode propagated is that having n = 2, and hence that there is a discrepancy between the theoretical and experimental dispersion relations for this mode.

A further investigation of the dispersion relation was made by studying the resonant frequencies of the modes (m = 1, n = 2) and (m = -1, n = 2) in short cylinders.

A comparison of the end effects necessary to obtain agreement between the observed resonances and those calculated on the basis of the theoretical dispersion relations emphasises the above discrepancy.

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CONTENTS

1. INTRODUCTION

Helicons are circularly polarised electromagnetic waves which may be propagated in suitable conductors when a strong magnetic field is applied. Suitable conductors are those which satisfy the conditions that (i) the wave frequency is less than the electron cyclotron frequency, and (ii) the collision frequency is less than the electron cyclotron frequency. These conditions can be satisfied in the laboratory in very pure metals at low temperatures.

The existence of helicon waves, and their properties, was first predicted by Aigran in 1960[1]. Since then, much work has been carried out in investigating the nature of the waves propagating in various media e.g. semi-conductors[2], pure sodium and other metals[3,4] and plasmas[5]. Helicons have also been used to determine the Hall coefficients of several metals[6].

A theoretical investigation of the dispersion and attenuation of helicon waves in a bounded cylindrical medium has been carried out by Klozenberg, McNamara and Thonemann[7] and, in parallel with this, a practical determination of the damping characteristics and dispersion relation by a method involving direct wavelength measurement was undertaken by Harding and Thonemann[8] at the same laboratory, using pure indium at 4.2^{O}K.

These experiments gave results which agreed well with theoretical predictions for the m = 0 and m = +1 modes, but revealed a marked discrepancy in the dispersion relation for the m = -1 mode. The present investigation was undertaken in order to obtain a better understanding of the m = -1 mode and to see whether the discrepancy could be resolved.

2. ELEMENTARY THEORY OF HELICONS IN A CYLINDRICAL MEDIUM

A detailed study of the dispersion and attenuation of helicon waves in a bounded cylindrical plasma is given in reference (7). Only a brief account of the elementary theory will be given here.

If $\underset{\sim}{E}$ is the electric field and $\underset{\sim}{J}$ the current density due to the wave, then

$$\underset{\sim}{E} - \frac{\underset{\sim}{J} \times \underset{\sim}{B}}{N.e} - \eta \underset{\sim}{J} = 0 \qquad \ldots (1)$$

where $\underset{\sim}{B}$ is the applied steady magnetic field, η the resistivity of the plasma, N the electron number density, and e the electronic charge. The above equation is written for a conductor in which electrons are the current carriers. In fact, in indium, the current is carried by positive holes, and this requires that the second term in equation (1) is positive. However, it is convenient to consider electrons, the only difference for holes being that in the case of $m = 1$ either the magnetic field or the sense of rotation must be reversed. The second term in equation (1) is the electric field due to the Hall effect, and the third term, that due to the ohmic resistance of the medium.

By neglecting motion of the crystal lattice, and assuming the resistivity to be isotropic, equation (1) may be combined with Maxwell's equation to give a wave equation:

$$\frac{\partial^2 b}{\partial z^2} = \frac{(B_z)^2}{(Ne\mu_0)^2} \frac{\partial^2}{\partial t^2} (\nabla^2 b)$$

to which we may assume a solution for the wave magnetic field, in cylindrical geometry, of the form

$$b = \hat{b}(r) \exp i(\omega t - kz + m\theta). \quad \ldots (2)$$

The dispersion relation in bounded geometry, $F(\omega,k) = 0$, is found by solving the wave equation with the application of appropriate boundary conditions and depends on m, the azimuthal wave number. Moreover, for each value of m, there exist an infinite set of solutions corresponding to the roots ($n = 1,2,3 \ldots$) of the Bessel functions in the dispersion relation. Thus, n determines the number of radial nodes in the field pattern, $n = 2$ being appropriate to a wave having one radial node.

The sign of the number, m, in equation (2) determines the direction of rotation of the wave pattern about the z axis. This rotation is distinct from the wave polarisation, which is a rotation of the magnetic field vector about an axis parallel to the z axis. Positive m numbers correspond to right-handed rotation, and negative m numbers to left-handed rotation. The polarisation is right-handed on the axis for all waves. For the $m = -1$ wave, therefore, the direction of rotation of the field pattern and the field polarisation at the axis, are in opposite senses.

3. EXPERIMENTAL ARRANGEMENT

Three separate experiments were carried out:

(i) a straightforward repetition of Harding and Thonemann's[8] determination of the dispersion relations for $m = \pm 1$ waves

(ii) investigation of internal electric fields in short cylinders at resonance

(iii) determination of end-effects in short cylinders at resonance.

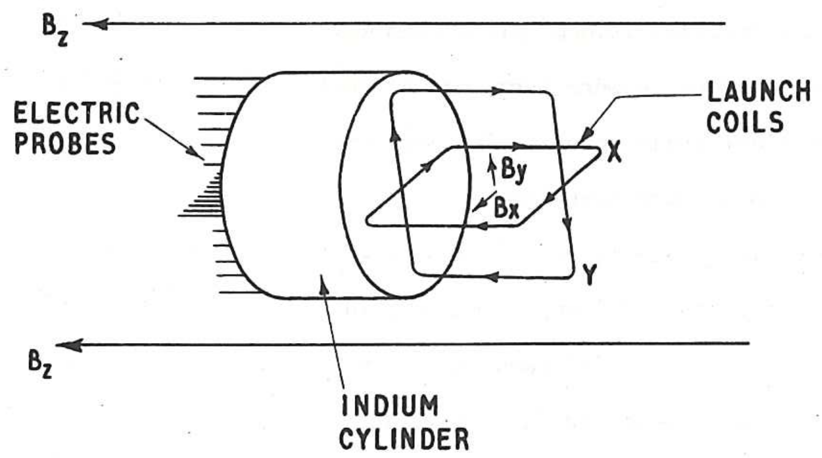

Fig. 1 Experimental arrangement (CLM-R66)

The experimental arrangement for (i) has already been described[8]. That for (ii) and (iii) is shown diagrammatically in Fig. 1.

A standing wave was set up in a short cylinder of indium by means of a system of launch coils at one end. The electric field pattern of the wave at resonance could then be investigated, with an arrangement of electrical probes held in contact with the other face of the cylinder. The signals from these probes were also used to determine at what frequencies the wave was resonant within the cylinder.

In order to propagate the $m = \pm 1$ waves, a steady magnetic field was applied parallel to the axis of the indium cylinder and two coils, mutually perpendicular, were mounted at one end such that their axes were perpendicular to the axis of the sample (see Fig. 1). The two coils were fed with sinusoidal currents having a $\pm\pi/2$ phase difference between them, so that the resulting magnetic field modulation was made to rotate one way or the other, thus energising the radial and azimuthal components of the vacuum magnetic field. In Fig. 1 the current in X leading on Y by $\pi/2$ will yield the $m = +1$ wave, and reversing the magnetic field or the phase difference will yield the $m = -1$ wave.

The driving coils were fed with equal currents from audio-frequency amplifiers, which were driven from a common oscillator providing both 0° and 90° phase shifted signals. The oscillator was calibrated against a valve-maintained tuning fork, and the phase shift was correct to within $\pm 1^{\circ}$. The amplifiers introduced no change in the relative phase of the two signals.

In order to propagate the $n = 2$ modes, for experiment (iii), a modified launch coil system was used in which ten small coils were arranged as shown in Fig. 2(i). The three coils labelled 'a' were connected in series, taking care that they were all wound in the same sense. Similarly for the coils labelled 'b'. The two coils 'c' were connected in series and then in series with 'a' such that each was wound in the opposite sense to 'a'. Similarly for coils 'd' and 'b'.

If signals $\pi/2$ out of phase are fed to the series arrangements of a with c and b with d, the resulting field will exhibit a nodal ring and will rotate one way or the other according to the phase lag or lead. The relative phase of the currents in each of the coils is shown in Fig. 2(ii).

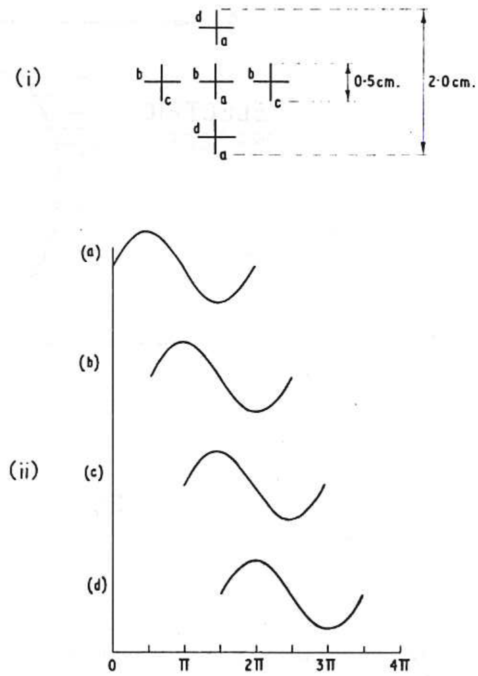

Fig. 2 (CLM-R66)
(i) Sectional diagram of coils for propagating waves having one radial node, i.e. $n = 2$; (ii) relative phase of the currents in each coil

The arrangement of probes was held against the face of the indium cylinder by means of small compression springs so that they remained in electrical contact during and after the contraction of the indium as it was cooled to 4.2°K. The probes were made of beryllium copper 0.05 cm in diameter and had pointed ends. In all, 25 probes were used; one in the centre, and six equispaced along each of the four radii.

The signals from the probes were amplified and displayed on a calibrated double beam oscilloscope, in order that their phase could be compared with that of one of the driving currents displayed on the other trace. The indium sample was 2 cm in diameter and 1 cm long, and the whole assembly was suspended in a cryostat between the poles of an iron-cored magnet, so that the axis of the indium was parallel to the field. The magnet provided a magnetic field up to about 1 Wb m^{-2} (10 kG) and was calibrated by nuclear magnetic resonance.

For experiment (iii), in order to obtain more resonances in the longer wavelength region of the dispersion curve, a second cylinder of indium 3 cm long and 2 cm diameter was used in addition to the 1 cm long sample used previously. Due to limitations of space, the longer sample necessitated the use of a superconducting solenoid to replace the iron cored magnet. The superconducting solenoid was 9.6 cm long, had a core diameter of 2.1 cm and gave a field of 3.5 Wb m^{-2} (35 kG). The indium was insulated from the copper former of the solenoid. The field at the centre of the solenoid and at a distance of 1.5 cm away from the centre along the axis (corresponding to the ends of the indium sample) was measured by nuclear magnetic resonance using the F19 nuclei in a sample of coloured fluorite. The field at the ends of the sample was 1.7% lower than at the centre.

4. MEASUREMENTS AND RESULTS

REPETITION OF HARDING AND THONEMANN'S TRAVELLING WAVE EXPERIMENTS

The dispersion curves obtained by Harding and Thonemann[8] for the m = + 1 and m = - 1 waves, were confirmed using the same apparatus and taking measurements in a similar manner (Fig.3).

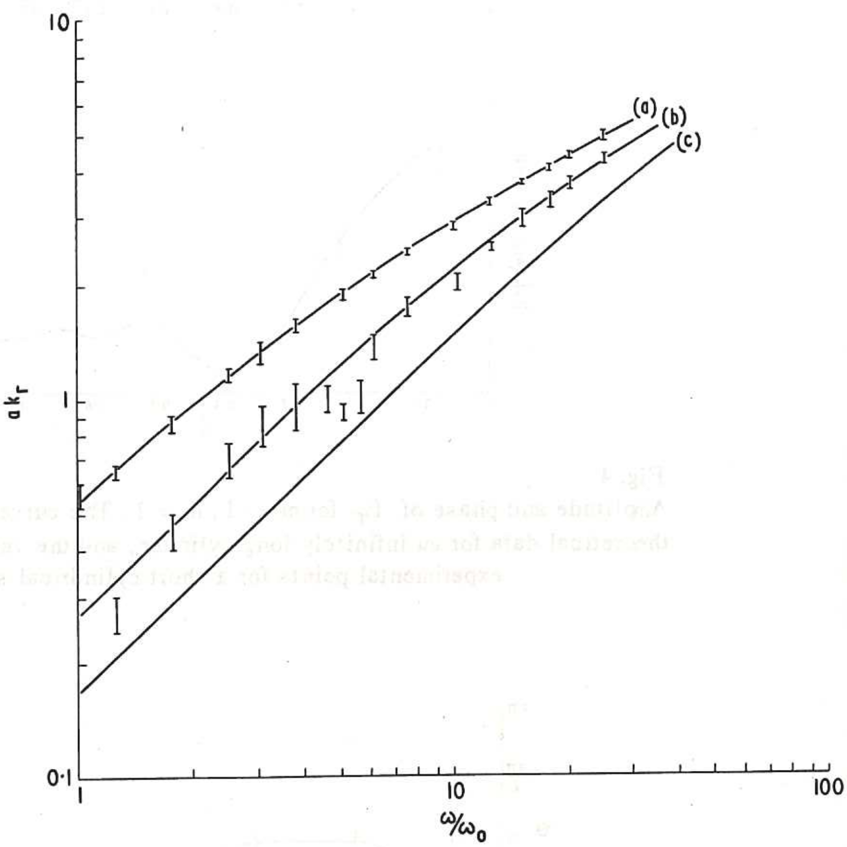

Fig. 3 (CLM-R66)
(a) Theoretical dispersion curve for m = +1, n = 1 with experimental points obtained from the travelling wave experiment shown as bars; (b) line of best fit through the experimental points for the m = -1 travelling wave; (c) computed dispersion curve for m = -1, n = 2 travelling wave

INTERNAL FIELD PATTERNS

In order to establish whether the wave being propagated was m = + 1 or m = - 1, measurements were made of the relative phases of each of the signals obtained between the centre probe and the four adjacent probes selected in turn. The direction of rotation of the pattern can then be found from the relative phases at the four angles, and from this and the direction of the magnetic field, it can be established whether the wave is m = + 1 or m = - 1.

Each wave was propagated in turn at a resonant frequency, and the signal between two adjacent probes displayed on a calibrated double beam oscilloscope. In this way, the amplitude and phase of each of the signals obtained between probes 0,1; 1,2; 2,3; 3,4; etc. were measured. (The centre probe being designated 0.) This was repeated along each of the four radii. The readings of the phase angle were all adjusted to be relative to the signal between probes 0,1 i.e. the signal from 0,1 is attributed zero phase angle.

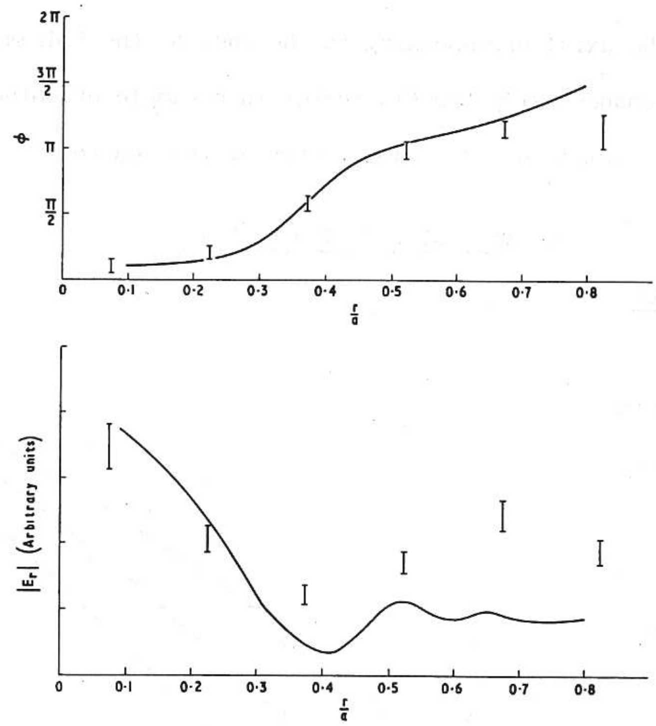

Fig. 4 (CLM-R66)
Amplitude and phase of E_r for m = -1, n = 2. The curves are computed from theoretical data for an infinitely long cylinder, and the vertical bars show the experimental points for a short cylindrical sample

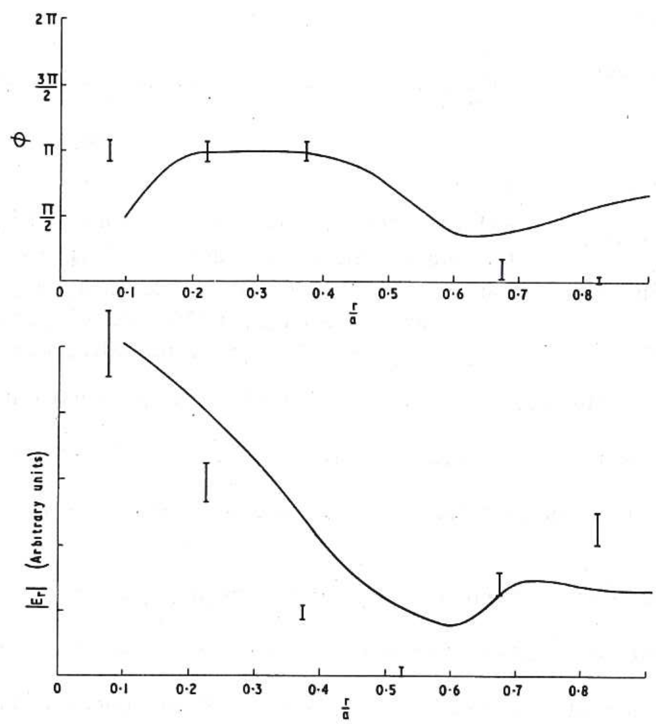

Fig. 5 (CLM-R66)
Amplitude and phase of E_r for m = +1, n = 2. The curves are computed from theoretical data for an infinitely long cylinder, and the vertical bars show the experimental points for a short cylindrical sample

The mean values of the readings from all four radii, giving the amplitude and phase for each pair of probes were then plotted against their radial distances expressed as fractions of the radius of the indium sample.

The graphs of $|E_r|$ and phase angle φ versus radius are shown for the m = -1 wave in Fig.4, together with the theoretical curves calculated from Reference(7).

The experimental points for the m = +1, n = 2 wave are shown in Fig.5, together with the computed curves. Although the agreement is not complete, these results demonstrate the existence of a nodal ring when the n = 2 launching coils were used.

The experimental points for the m = 1, n = 1 wave are also given, in Fig.6, for comparison with the n = 2 curves. Theoretical data for this mode was not readily available.

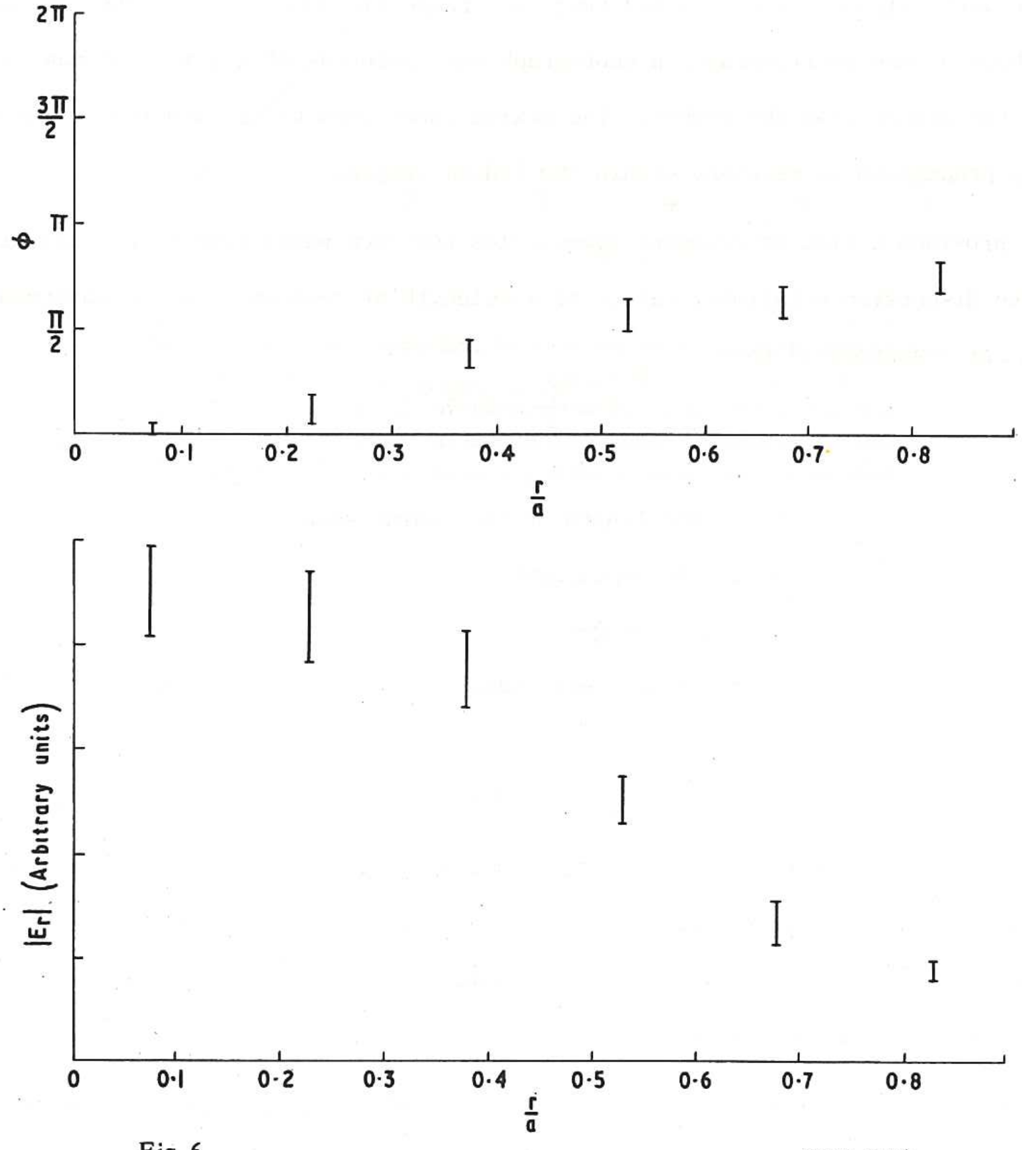

Fig. 6 (CLM-R66)

Experimental results of measurements of amplitude and phase of E_r for m = +1, n = 1 in a short cylindrical sample

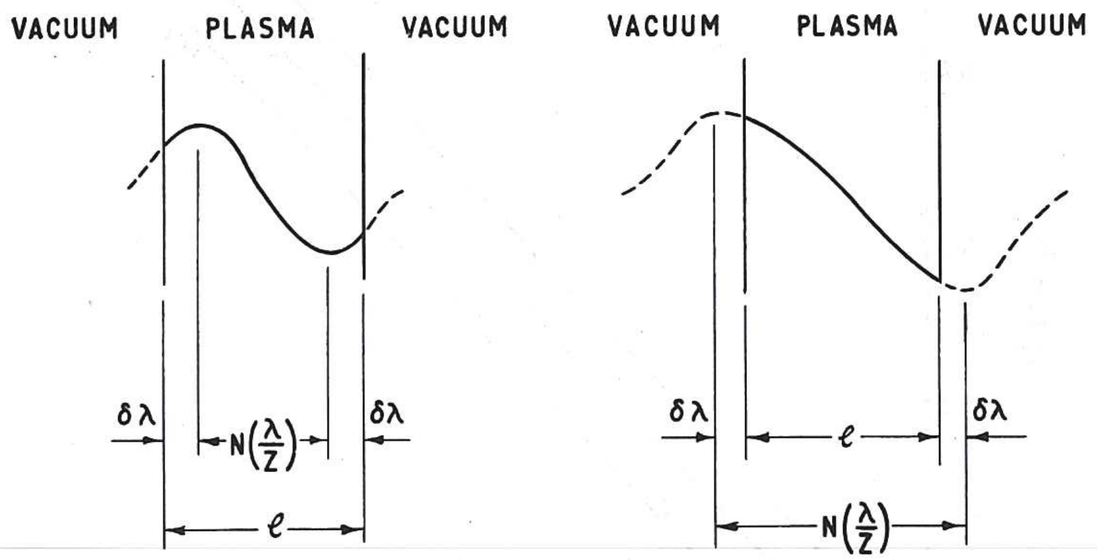

Fig. 8 (CLM-R 66)
Illustrating end correction for resonant standing waves in short cylinders of indium

a positive intercept. The two possibilities for the sign of the end correction are illustrated in Fig.8 (i) and (ii).

The boundary conditions for a slab in a plane perpendicular to the magnetic field have been discussed by Legéndy[9] who shows that, in the case of a thin slab, the end correction is positive. Although a solution for the present geometry has not yet been obtained analytically, it is reasonable to assume that the end correction would also be positive in this case. Further weight is given to the assumption of a positive end-correction by the fact that the modes ($m = 0$, $n = 1$) and ($m = 1$, $n = 1$) give positive intercepts[10].

5. EXPERIMENTAL ERRORS

The errors involved in the measurements of the amplitude and phase of the radial electric field varied considerably, depending in the amplitude of the signal, and the signal to noise ratio.

The smallest detectable phase difference, under ideal conditions, was about 5^o, but as the amplitude of the signal decreased (near the nodal ring) the probable error increased to an estimated $20^o - 30^o$. The amplitude of the smallest detectable signal was set by the noise level, which was of the order of 0.05 μV. The maximum amplitudes measured were approximately 1 μV.

The estimated errors in the measurement of $|E_r|$ and φ, then are ± 5% and ± 5^o respectively at best, and increasing as the amplitude of the signal decreases.

In the determination of the end-corrections, the accuracy with which a resonant frequency could be measured, depended upon the accuracy of measuring the peak on the photograph. Generally this could be done to within 1 mm, on a trace 10 cm long, which corresponded to a frequency range of 10 c/s. A resonant frequency, therefore, could be measured to within ± 0.1 c/s. This error is typical, but in practice the errors were less than this at low frequencies (1-20 c/s) and greater at higher frequencies (60-100 c/s).

The magnetic field was measured to within ± 0.2%, and the dimensions of the sample to within ± 0.3%. Thus for a typical frequency of, say, 20 c/s the experimental error is ± 1%.

6. DISCUSSION

The repeat of the travelling wave experiments of Harding and Thonemann[8] has confirmed the existence of a discrepancy between the experimentally determined dispersion relation for the m = -1 wave, and that computed from the theory on the basis that m = -1, and n = 2. The theory predicted that the simplest mode in which the m = -1 wave would propagate was that for which n = 2, and all computations were carried out on this basis.

The agreement between the measurements of $|E_r|$ and φ and the values computed from the theory verifies that the observed m = -1 wave has n = 2, and hence that the computations were performed for the relevant mode.

The measurements of end-corrections throw some doubt on the theoretical dispersion relation for the mode (m = -1, n = 2). For the m = +1, n = 2 wave, the end correction was found to be positive (assuming the dispersion relation computed from theory). This conclusion is supported by considerations of the matching of the plasma and vacuum waves, Fig.8. Accepting that the end correction for the (m = -1, n = 2) mode is also positive would favour the experimental dispersion relation, since this gives a positive intercept on the graph of reciprocal λ verus N. Assumption of the theoretical dispersion relation, on the other hand, yields a negative result for the end-correction which is much more difficult to explain in terms of the matching of the wave and vacuum fields.

7. CONCLUSIONS

The main conclusion to be drawn is that the agreement between the measured values of the radial electric field and those computed from the theory confirms the theoretical prediction that for the $m = -1$ wave, $n \geqslant 2$.

It thus seems safe to conclude that the experimentally determined dispersion relation for the $m = -1$ travelling wave refers to the mode ($m = -1$, $n = 2$), and that a real discrepancy exists between the experimental and the computed dispersion relations. This conclusion is supported by the fact that the experimental dispersion relation account for the observed end effects more satisfactorily than does that computed from the theory.

The fact that the end corrections for the $m = +1$, $n = 2$ mode can be satisfactorily accounted for using the computed dispersion relation gives confidence in the computation for this mode and hence in the compilation of the programme for the $m = -1$, $n = 2$ wave; the same programme was used for each, the only difference being in the sign of the data input.

It is therefore not easy at the present time to see how the discrepancy arises.

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