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Report

A STUDY OF THE END LOSSES FROM A THETA PINCH USING PIEZOELECTRIC PRESSURE PROBES

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A STUDY OF THE END LOSSES FROM A THETA PINCH USING PIEZOELECTRIC PRESSURE PROBES

by

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A B S T R A C T

A 1 mm diameter piezoelectric pressure probe with a response time of 0.5 μsec has been developed and used successfully to study the pressure distribution of the plasma escaping from the end of a theta pinch.

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CONTENTS

1. INTRODUCTION

This paper describes an investigation of the end losses from a theta pinch using a fast piezoelectric pressure probe. The type of probe selected for this work incorporates a thin piezoelectric crystal mounted in a quartz rod, one end of which projects axially into the plasma. Piezoelectric probes have been used to study shock waves and to make measurements in high temperature plasmas by several authors(1-5) but no observations on theta pinches have been reported. The rise time of the pressure pulses in the present experiments is 0.5 μsec, compared with more than 1 μsec in previous work, and pressure probes with a suitably high frequency response had to be developed. The first part of this paper gives a short account of the design and construction of the probes and discusses briefly the factors which determine their performance.

The second part of the paper describes the study of end losses from a theta pinch. The problem of containment in a theta pinch has received considerable attention(6-12) and measurements have been carried out using, for example, interferometry(7,8), optical measurements on the emitted radiation(9), diamagnetic loops(10) or scattered laser radiation(12). Pressure probes have been used to measure the particle losses from a cusp(13) - a situation which is similar to that in a theta pinch. The purpose of the present experiment is to investigate the plasma escaping axially from the end of a theta pinch coil; measurements with pressure probes, diamagnetic loops and high-speed photography are described and compared for both parallel and anti-parallel initial trapped magnetic field. The results are discussed in terms of various theoretical models.

2. DESIGN AND CONSTRUCTION OF PRESSURE PROBE

DESIGN CONSIDERATIONS

The pressure probe was required to record pulses in the range 1 to 100 megadynes per sq. cm. and with rise times of 0.5 μsec. These requirements depend primarily on the dimensions of the crystal and associated quartz acoustic line and a compromise has to be made to achieve adequate output and a sufficiently short rise time. The rise time can be limited by the rise time of the crystal itself, the time constant of the electrical circuit or by dispersion in the quartz rod, which is the most important factor for the type of probe used in this work. Dispersion in the rod becomes important at frequencies for which the axial wavelength, λ, in the rod becomes comparable to its diameter, a; Rayleigh surface waves are excited which travel more slowly than longitudinal waves and the pulse becomes dispersed. Theoretical dispersion curves for the propagation of longitudinal waves along bars were given by Bancroft[14,15], who plotted v/v_o (v is the phase velocity and v_o is the rod velocity, defined as $\frac{E}{\rho}$ where E is Young's modulus and ρ the density of the rod) against λ/a, for different values of the Poisson ratio. For quartz a rod diameter of 1 mm is suitable for a pulse with a rise time of 0.5 µsec. The length of the crystal, x, was chosen as 0.25 mm to give adequate signal and a crystal rise time, $\frac{x}{v_o}$, of 10^{-7} second, much smaller than the dispersion limitation.

CONSTRUCTION, CALIBRATION AND PERFORMANCE OF 1 mm PROBES

The essential features of the 1 mm probe are shown in the schematic diagram in Fig.1. The general concept of the probe is similar to that used by Stern and Dacus[2]. The piezoelectric crystal is mounted in a 40 cms long acoustic line made from two 1 mm diameter

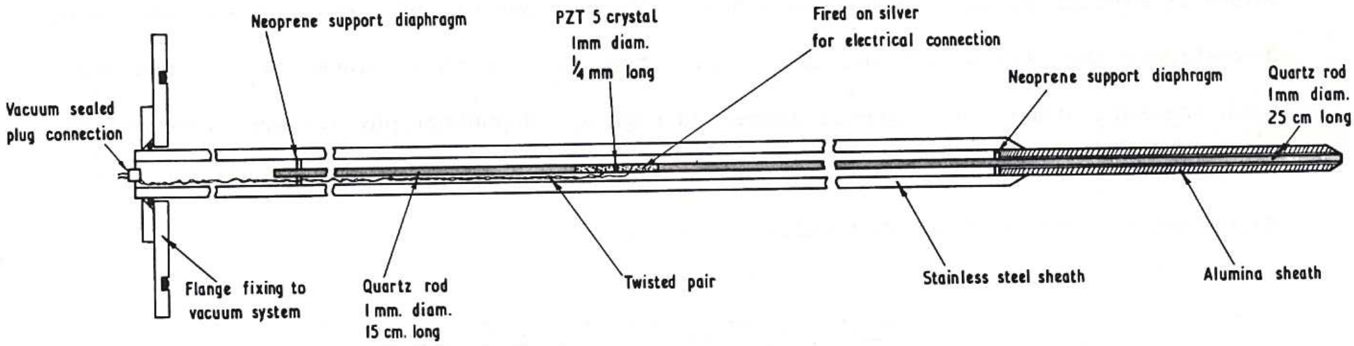


Fig.1 Schematic diagram of 1 mm pressure probe (CLM-R70)

quartz rods 15 and 25 cm long. The longer rod allows the crystal to be outside the discharge region, and delays the arrival of the pressure pulse until after any electrical interference, which is often serious because of the high input impedance (6MΩ) of the circuit. The shorter rod provides an acoustic match at the rear face of the crystal, and introduces a delay in the arrival of the reflected pulse until the incident pressure pulse is over. The crystal, of diameter 1 mm and thickness ¼ mm, is of lead zirconate titanate (PZT.5). The rod assembly is mounted inside a 1 cm diameter stainless steel tube by thin neoprene diaphragms; the end 10 cm of the rod projecting into the plasma is surrounded by a 3 mm diameter alumina sheath which is fixed into the steel tube at one end and tapered at the other, which is flush with the end of the rod.

The probes were calibrated by dropping a 1/32" diameter steel ball onto the face from a height of 4 cm. The pressure pulse was recorded and its time integral equated to the calculated loss of momentum of the ball. This method gives highly reproducible results and it has been shown(1) to agree with the shock wave method. The calibration constant was 0.061 ± 0.003 volts/megadyne/cm^2.

3. MEASUREMENTS ON A THETA PINCH

EXPERIMENTAL CONDITIONS

These experiments were carried out on a 15 kV, 22.5 kJ capacitor bank(16). The coil was 21 cm long and 5 cm in diameter over the middle 15 cm; at each end the diameter was reduced over 3 cm to give a geometrical mirror ratio of 1.1. A quartz tube with a uniform bore of 4.6 cm was used. The peak magnetic field of 75 kG was reached in 7.5 μsec, and the measurements were made during the second current half cycle; the trapped magnetic flux was controlled by varying the initial deuterium pressure, as described in earlier work(17). The measurements reported in this paper relate to one of two conditions - a reversed trapped field discharge at 200 mtorr and a parallel trapped field discharge at 50 mtorr. The initial trapped flux was 35 milomaxwells in each case. The Hain-Roberts(18) code was run for both conditions; this code does not include the effect of end losses or axial contraction, both major effects in short coils. However, it does give a guide to the probable temperature which were, for both ions and electrons at 3 μsec, 100 eV for parallel flux and 120 eV for reversed flux.

The pressure probe was inserted axially into the tube, its tip extending to the midplane of the mirror region, 1½ cm from the end plane of the coil. The discharge was also studied by means of double magnetic pickup loops(10) and a radial S.T.L. image converter camera.

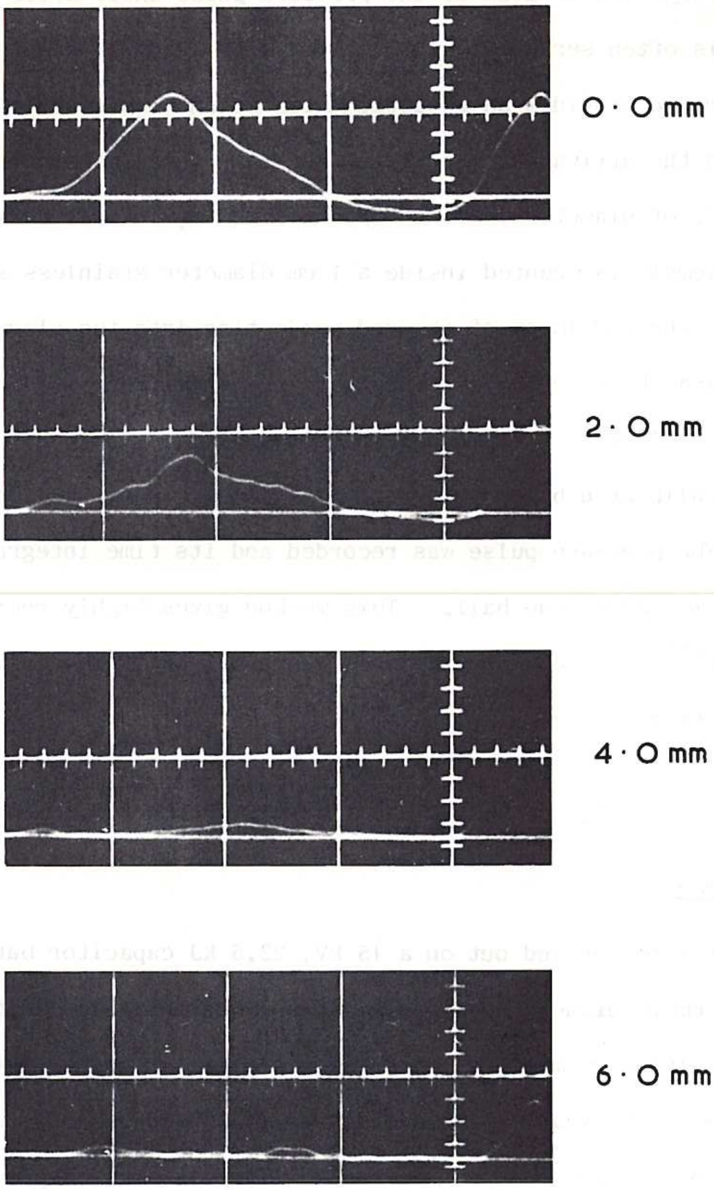


Fig.2 (CLM-R70)

Pressure pulses at different distances from the tube axis. Reversed trapped field, 2 μsec/division

RESULTS

Spatial Pressure Variation

Horizontal and vertical scans of the axial pressure were carried out; about five shots were recorded at each position and after some 10 to 20 conditioning discharges the pressure pulses became reproducible - the standard deviation at maximum amplitude was 5%. Fig.2 shows, for the reversed field discharge, typical pressure pulses at different distances from the tube axis (which corresponds with the coil axis to within ± ½ mm). At peak magnetic field the pressure pulse reached about 30 megadynes/cm^2 on the axis, falling sharply as the probe was moved radially outwards.

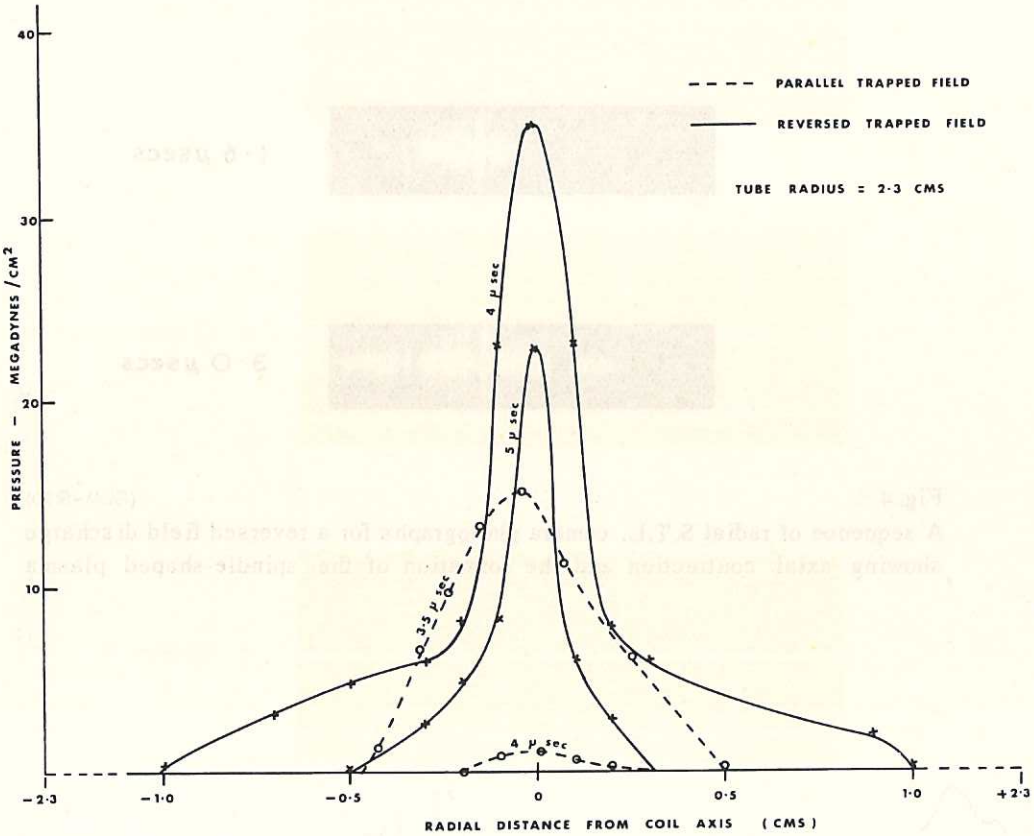


Fig. 3 (CLM-R70)
Pressure profiles for parallel and reversed field discharges

Fig.3 shows pressure profiles for parallel and reversed trapped flux at two different times. After the radial implosion the pressure always peaked strongly within ± 1 mm of the coil axis. The central region of the profile for reversed trapped flux was always narrower than for parallel trapped flux and the minimum radius of the loss hole Δ, (defined as the half width of the profile at half height) is 0.75 ± 0.25 mm and occurred at about 5 μsec, after the formation of the spindle-shaped plasma[16]. The radial framing camera photographs for reversed trapped flux in Fig.4 show axial contraction leading to the formation of the spindle plasma. The radius of the luminous column in the mirror region is approximately equal to Δ.

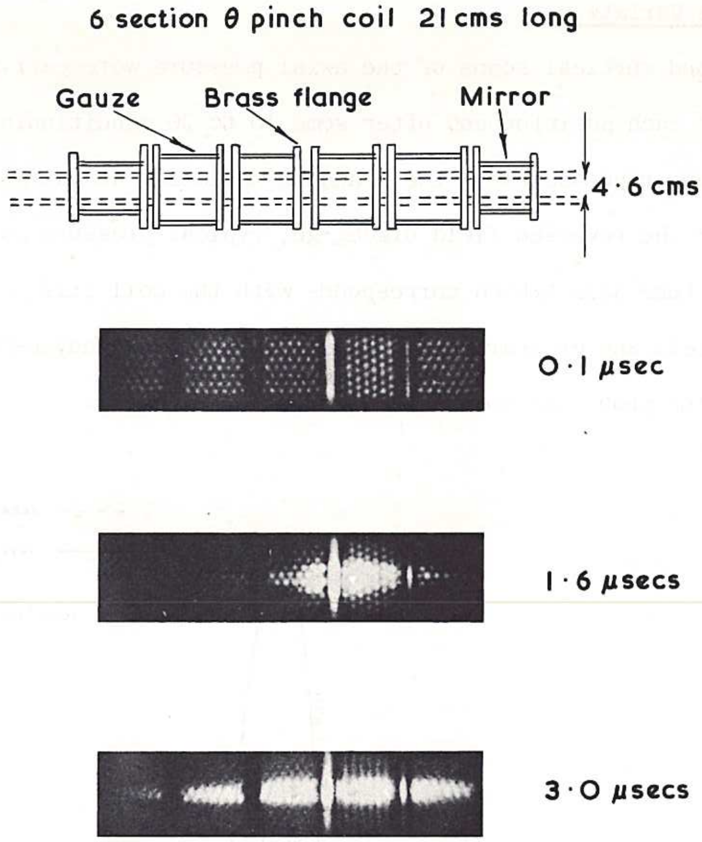
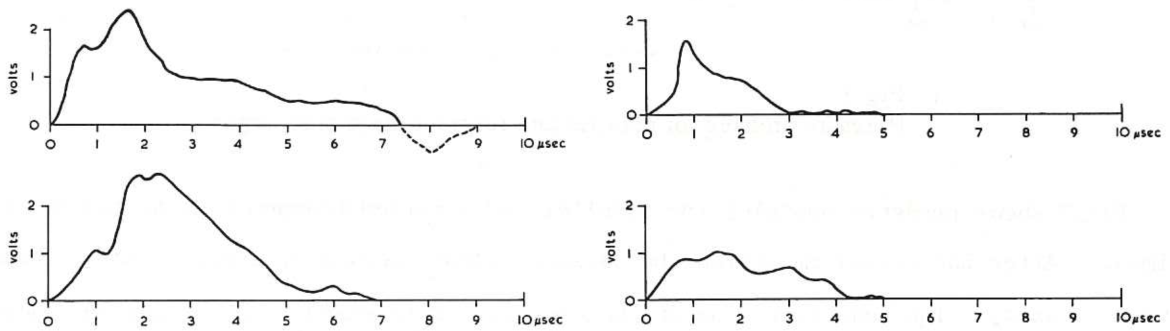


Fig. 4 (CLM-R70)
A sequence of radial S.T.L. camera photographs for a reversed field discharge showing axial contraction and the formation of the spindle-shaped plasma



Double loop signal (above) and pressure pulse. 200 μ Hg. Reversed trapped field.

Double loop signal (above) and pressure pulse. 50 μ Hg Parallel trapped field.

Fig. 5 (CLM-R70)
Pressure pulses and double loop signals for parallel and reversed field discharges

Time Variation of Axial Pressure

Fig.5 shows integrated double magnetic loop waveforms and plasma pressure pulses taken on the axis for the parallel and reversed trapped flux discharges. The loop records the plasma diamagnetism, which gives a measure of the plasma energy[10], predominantly lost by particle flow in short coils. For parallel trapped flux the plasma diamagnetism falls rapidly, reaching about 5% of its peak value at 3 μsec indicating that most of the plasma has escaped; the corresponding pressure pulse also decays, reaching zero soon after 4 μsec. For reversed trapped flux both the diamagnetic signal and the pressure pulse remain for two or three μsec longer.

4. DISCUSSION

It was seen in Fig.5 that for parallel trapped flux both the diamagnetism and pressure pulses decayed in about 4 μsec suggesting that a large fraction of the plasma was rapidly lost. This fall of diamagnetism has been discussed elsewhere[11] and the new data from the pressure probe indicates directly that when the diamagnetism decays the axial pressure falls to a small value and there is no further flow of plasma from the end. Since the profiles are wide, slow escape from a small loss hole is excluded. For reversed trapped flux both the diamagnetism and the flow of plasma remain for longer, showing that the axial contraction when the spindle-shaped plasma has been formed the axial loss of plasma is reduced. In this case the profiles are narrow indicating the plasma is escaping slowly through a small loss aperture. For reversed trapped flux the radius of the loss hole is always substantially smaller than for parallel trapped flux and the smallest value is two or three ion Larmor radii (R_{Li} = 0.3 mm for a 100 eV ion in the external magnetic field at the mirror). The parallel field discharge has a loss hole with a diameter only slightly smaller than the plasma diameter in the midplane.

Studies of plasma containment in the theta pinch have shown that when the plasma contains trapped field the radius of the loss aperture, which determines the containment time on both steady state[19-21] and non steady state[22] theories, is controlled by the magnitude of the trapped flux, most readily expressed in terms of β. ($\beta = \frac{\text{plasma pressure}}{\text{external magnetic field pressure}}$). When trapped flux is excluded from the plasma i.e. $\beta = 1$, the size of the loss hole is determined by the trapped flux in the sheath. In the present experiments the radius of the loss hole in the parallel field discharge is determined by the trapped flux model. For the reversed field discharge, as first pointed out by Phillips[19], the formation of closed magnetic field loops followed by axial contraction means that no flux passes out through the ends of the plasma, which therefore behaves as $\beta = 1$ from the point of view of containment.

TRAPPED FLUX MODELS

The presence of magnetic field lines passing through the loss hole determines its size; in the original steady state model[19] it was assumed that in the "neck" (region of minimum plasma diameter) the transverse plasma pressure drops to zero. The area, A_E, of the loss aperture is then given by

$$A_E = \frac{1}{B_e} \int_0^{r_m} 2\pi r \, B_{iM} \, dr \qquad \ldots (1)$$

where B_e is the external field at the end, B_{iM} the trapped field where the plasma column has its maximum radius r_m, usually at the midplane of the coil. For the case of a uniform mixture of plasma and trapped field, equation (1) reduces to the well-known one

$$A_E = A_M \sqrt{1 - \beta} \qquad \ldots (2)$$

Taylor and Wesson[19] have considered this model in terms of fluid flow and calculated the value of the pressure in the neck; this approach leads to an expression similar to equation (2) above. Wesson[21] has considered the non-steady state case, which is more relevant to the present work, in terms of rarefaction waves moving inwards along the plasma column from the ends with a velocity $V = C_s \sqrt{1 - \beta}$, where C_s is the sound speed. Again the exclusion of flux, i.e. a high β, gives a small velocity and good confinement. Equation (2) is replaced by the expression

$$A_E = A_M \left(\frac{1 - \beta}{\gamma} \right)^{1/3} \qquad \ldots (3)$$

where γ is the ratio of specific heats.

In the parallel field discharge the experimental area ratio A_M/A_E corresponds to a value of β less than 0.3, and the plasma flows out freely at approximately the sound speed. For anti-parallel trapped field the area ratio lies between 10 and 20, and equation (3) gives β very close to unity; in this case the flux model is no longer valid.

SHEATH MODELS

These have been discussed by several authors[19,23] who show that the width of the loss aperture depends on the sheath thickness. At high temperatures and in the absence of instabilities in the sheath, its thickness is determined by the electron or ion Larmor radius.

The radius of the loss hole can approach its minimum possible value[23] of several electron Larmor radii only if the radial electric field in the plasma is not short circuited; if the radial field is short circuited the minimum radius of the loss hole is approximately an ion Larmor radius, since an ion can penetrate a distance of this order into the magnetic field. Taylor[24] shows that with zero radial electric field the radius of the loss hole is given by $\Delta = \left(\frac{3\pi}{2}\right)^{1/4} \left(\bar{R}_{Li} R\right)^{1/2}$ where $\bar{R}_{Li}$ is the r.m.s. ion Larmor radius and R is the maximum plasma radius, usually at the midplane. Apart from small differences in the numerical constant this expression can be derived assuming a sheath thickness equal to the ion Larmor radius at the midplane and flux conservation in the sheath. In the present experiments with reversed field $R_{Li} = 0.3$ mm, $R = 3$ mm and so $\Delta = 1.4$ mm, about twice the observed value of $\Delta = 0.75 \pm 0.25$ mm. It should be noted that the physical size of the probe would not permit substantially smaller holes to be observed. However, other considerations, in particular the observation of plasma rotation[25], suggest that the electric field is small due to short circuiting and the ion Larmor radius model is more likely.

5. CONCLUSION

A 1 mm diameter piezoelectric pressure probe with a rise time of 0.5 μsec has been used successfully to study the pressure distribution of plasma escaping from the ends of a theta pinch. This distribution was measured for discharges trapping parallel and reversed flux. For the former the diamagnetic signal showed that the plasma was lost rapidly from the ends in a time of 3 or 4 μsec and the pressure probe provided direct evidence that when the diamagnetic signal had decayed to zero plasma flow from the ends was very small. For reversed trap flux the diamagnetic signal persisted for about 7 μsec showing improved containment and during this period the pressure probe showed that the flow of plasma from the ends continued at slow rate. The pressure of the escaping plasma always peaked strongly close to the tube axis and half width of the profile for reversed trapped flux was a factor of four smaller than that for parallel trapped flux. The minimum observed value of the radius of the loss hole for reversed trapped field was 0.75 ± 0.25 mm and occurred when the plasma was in the form of a spindle following the axial contraction; the corresponding ion gyro-radius was 0.3 mm. These observations were discussed in terms of the trapped flux and sheath models of plasma confinement. When there is parallel trapped flux within the plasma the radius of the loss hole is primarily determined by the magnitude of this flux, i.e. the ratio β. When no field lines pass out through the ends of the plasma, either because trapped parallel flux has been excluded entirely from the plasma or because the trapped flux is reversed and forms closed loops within the plasma, the loss hole is determined by the trapped flux in the current sheath. In the conditions presumed to exist in the present experiments, namely with the radial electric field in the sheath small due to short circuiting at the ends, theory predicts a loss hole of radius 1.5 mm in reasonable agreement with the experimental value.

6. ACKNOWLEDGEMENTS

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