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DESTRUCTION OF THE SHARP TRANSITION IN COLLISIONLESS SHOCK WAVES DUE TO HIGH β EFFECTS

by

A. IIYOSHI*

ABSTRACT

In this paper the magnetosonic wave propagating almost perpendicular to the magnetic field is discussed in connection with the destruction of a sharp shock transition at high Alfvén Mach number $(M_A \gtrsim 3)$ observed in collisionless shock experiments.

It is proposed that at a critical Alfvén Mach number $(M_A = 3 \sim 4)$ the values of β and the local magnetosonic Mach number behind the shock become nearly unity. These effects would allow the waves to propagate upstream forming another quasi-shock wave whose dissipative mechanism would be mainly damping of the magnetosonic waves.

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C O N T E N T S

1. INTRODUCTION

Recent experiments on collisionless shock waves propagating perpendicular to the magnetic field have clarified several problems [1-3].

At low Alfvén Mach number $(M_A \lesssim M_{AC})$ a sharp transition of length of the order $10 \frac{c}{\omega_{pe}}$ has been observed [1,4]. When the Alfvén Mach number exceeds a critical value, destruction of the sharp transition starts and a clear double structure with a long foot of length the order of $\frac{c}{\omega_{pi}}$, $(\omega_{pi}^2 = \frac{4\pi n e^2}{M_i})$, is formed. At still higher Mach numbers the broad structure in the front and to the rear of the sharp transition becomes dominant and at $M_A \simeq 6$ the sharp transition disappears.

The sharp transition characterised at $M_A \lesssim M_{AC}$ may be explained qualitatively by 'anomalous' resistive effects due to electron drift instabilities such as two-stream instability and/or ion-acoustic instabilities.

The long foot which appears at higher M_A still does not seem to be identified with any instabilities. One plausible explanation, suggested by Sagdeev [5], is that it is due to an ion counter-streaming instability. This explanation, however, is based on the assumption that large amplitude hydromagnetic waves can build up without changing their properties until 'breaking' occurs. Theoretically [5,6] in cold plasma this breaking occurs at $M_A = 2$. In an actual plasma, even when the plasma is approximately cold (in low β plasma), the development of these waves is suppressed up to $M_{AC} = 3 \sim 4$ by dissipation mechanisms such as anomalous resistivity. When the value of β behind the shock becomes of the order of unity, these waves should be distorted for another reason, i.e. the dispersion and damping of the magnetosonic waves in high β plasma.

It is noted that even when the initial value of β, (β_1), is very small, the value of β behind the shock (β_2) can easily become unity. Hence an Alfvén Mach number which leads to a value of β_2 equal to unity will be called a critical Alfvén Mach number and will be denoted by M_{AC} in this paper.

In Section 2 some dispersive properties of the magnetosonic waves in both low β and high β plasmas are discussed in connection with the formation of collisionless shock waves. In Section 3, the dispersion relation for damped magnetosonic waves propagating almost perpendicular to the magnetic field is described. It is found that in moderately high β_2 plasma a significant damping is possible even for nearly perpendicular propagation $\frac{k_\parallel}{k_\perp} \sim \sqrt{\frac{m_e}{m_i}}$. In Section 4 the destruction of the sharp transition is explained because of the high β effects.

2. PROPERTIES OF MAGNETOSONIC WAVES IN LOW β AND HIGH β PLASMA

Since in collisionless plasma the dissipative effect is mainly due to collective interactions between waves and particles, the dispersive properties of the waves have a very important role to play in the process of shock formation.

In this section we will compare the dispersive properties of a low β plasma with those of a high β plasma.

In a low β plasma the dispersion relation for the magnetosonic waves propagating perpendicular to the magnetic field, derived from the cold plasma equations, is given by

$$\frac{\omega^2}{k^2} = V_A^2 \frac{\frac{\omega_{pe}^2}{c^2}}{k^2 + \frac{\omega_{pe}^2}{c^2}} \quad . \qquad \ldots (1)$$

The magnetosonic wave is non-dispersive for long wavelength regions and the first dispersive effect sets in at very short wavelength of the order of $\frac{\omega_{pe}}{c}$ (see Fig.1(a)). Hence if there is no dissipative effect, the balance between the increase in wave amplitude by the non-linear steepening effect at long wavelength regions and its convection away by dispersive wave propagation at short wavelength regions, allows a steady state non-linear hydromagnetic wave (i.e. solitary wave) of wavelength of the order $\frac{c}{\omega_{pe}}$.[5,6] If there is no dissipation mechanism, this solitary wave forms an oscillatory structure and the wave train is backward.

In an actual plasma, when the amplitude of this wave becomes large enough the current induced by the wave excites other types of waves. This current instability could provide the main dissipation mechanism responsible for turbulent collisionless shock waves in low β plasma[1-4], although there is still no convincing theory for the current instability with a magnetic field and with gradients of the magnetic field and density.

Due to this so-called 'anomalous resistivity' the solitary wave is distorted and the thickness of the shock front observed experimentally is of the order of $10 \frac{c}{\omega_{pe}}$.

We do not know much about this turbulent heating mechanism but recent electron temperature measurements[7] by Thomson scattering show that the electron temperature increases through this turbulent heating. This electron heating, as we show later, seems to be important for the destruction of the sharp transition at $M_A \gtrsim M_{AC}$.

In moderately high β plasma, the full dispersion relation is extremely complex, but some of the main features of these waves can be stated. Under the assumption that $\omega \ll \Omega_i$ (ion gyro frequency) the phase velocity is given by equation (6) in Section 3.

$$\frac{\omega^2}{k^2} = V_A^2 (1 + \beta) = V_A^2 + v_i^2 \left(1 + \frac{T_e}{T_i}\right). \qquad \ldots (2)$$

From equation (2) it can be seen that $\gamma_e = \gamma_i = 2$, i.e. when $\omega \ll \Omega_i$ both electron and ion gas are coupled with the magnetic field. Equation (2) also shows that the magnetosonic waves are still non-dispersive in the long wavelength regions. In this case, however, the finite Larmor radius effect will have a dispersive effect at wavelengths of the order of Larmor radius of the ion, R_i. It is very difficult to state exactly the characteristics of the magnetosonic waves near $\omega \sim \Omega_i$. However, as Sagdeev[8] and Kennel and Sagdeev[9] described, when the wavelength is much smaller than ion gyro-radius but still larger than electron gyro-radius, the ion but not the electron motion will be decoupled from the magnetic field. Thus the ion motion will be one dimensional with $\gamma_i = 3$ while two dimensional electrons still have $\gamma_e = 2$. This suggests that the phase velocity in the short wavelength region is faster than in the long wavelength region. Thus, the dispersion curve would have different characteristics from that for low β plasma (see Fig.1(b)).

The phase velocity in the long wavelength region is slower than in the short wavelength region in this case. It is also easily seen that the group velocity is greater than the phase velocity. It is, therefore, reasonable to expect that when β behind the shock increases up to the order of unity, this new dispersive effect, which is related to the ion Larmor radius, appears in the long wavelength region and this effect will influence the sharp front which is related to the electron-inertia dispersive effect.

Another major difference between low β and high β plasmas concerns the damping of the waves. The magnetosonic wave propagating exactly perpendicular to the magnetic field is undamped. However, recently Barnes[10] studied the damping of hydromagnetic waves and found that, expect for the Alfvén mode, hydromagnetic waves are strongly damped in moderately high β (greater than about half) plasma for most angles of propagation, especially for values of θ (the angle between the wave vector and the magnetic field) of $20^\circ \sim 40^\circ$ and $85^\circ \sim 90^\circ$. This suggests that we can expect strong damping even for almost perpendicular propagation.

In the next section we describe the magnetosonic waves for almost perpendicular propagation.

3. DISPERSION RELATION OF MAGNETOSONIC WAVES PROPAGATING ALMOST PERPENDICULAR TO THE MAGNETIC FIELD

The dispersion relation for damped magnetosonic waves propagating almost perpendicular to the magnetic field is given, as a special case of equation (9-38) of reference[11], by

$$\frac{k_\perp^2}{\omega^2} = \frac{1}{V_A^2\,(1+\beta)} + i\,\frac{\sqrt{\pi}}{1+\beta}\,\beta_e\,\frac{k_\perp^2 c^2}{\omega^2}\,\alpha_e\,\exp\left(-\alpha_e^2\right) \qquad \ldots (3)$$

subject to the conditions $\omega^2 \ll \Omega_i^2 = \frac{e^2B^2}{m_i^2c^2}$, $V_A^2 \ll c^2$ and $\beta_\perp = \beta_\parallel$, and the inequality

$$\beta_e \ll \sqrt{\frac{m_e}{m_i}}\,\frac{k_\perp}{k_\parallel}\,, \qquad \ldots (4)$$

where

$$\beta = \beta_i + \beta_e, \quad \beta = \frac{8\pi n\kappa T}{B_0^2} \quad \text{and} \quad \alpha_e = \frac{\omega}{k_\parallel v_e}\,. \qquad \ldots (5)$$

Since $|\alpha_i| \gg |\alpha_e| \gg |$ when $k_\parallel$ is small, the ion Landau damping term is neglected in equation (3).

Assuming $|R_e\omega| \gg |I_m\omega|$ i.e. weak damping, we find

$$(R_e\omega)^2 = k_\perp^2\,V_A^2\,(1+\beta)\,, \qquad \ldots (6)$$

$$|I_m\omega| = \sqrt{\pi}\,\frac{k_\perp^2\,V_A^2}{R_e\omega}\,\beta_e\alpha_e\,\exp(-\alpha_e^2)\,. \qquad \ldots (7)$$

Using equation (6), we obtain

$$\alpha_e^2 = \frac{1+\beta}{\beta_e}\,\frac{m_e}{m_i}\left(\frac{k_\perp}{k_\parallel}\right)^2. \qquad \ldots (8)$$

Under the condition for almost perpendicular propagation, i.e. $\frac{k_\parallel}{k_\perp} \sim \sqrt{\frac{m_e}{m_i}}$, we can write $\alpha_e^2 \sim \frac{1+\beta}{\beta_e}$. For $\beta_e \ll 1$ the exponential term $\sim \exp\left(-\frac{1}{\beta_e}\right)$ is very small and damping is negligible. When β_e increases towards unity the exponential term becomes important even for almost perpendicular propagation. The imaginary part of ω is given approximately by

$$|I_m\omega| \simeq \sqrt{\pi}\,\beta_e^{1/2}\,.\,R_e\omega\,.\,\exp\left(-1/\beta_e\right) \qquad \ldots (9)$$

Although equation (9) is derived assuming $\omega^2 \ll \Omega_i^2$ and $|R_e\omega| \gg |I_m\omega|$, the imaginary part of ω near $\omega \sim \Omega_i$ may be approximated by

$$|I_m\omega| \simeq \beta_e^{1/2}\,\Omega_i\,.\,\exp\left(-1/\beta_e\right) \qquad \ldots (10)$$

In Fig.2 $\left|\frac{I_m\omega}{\Omega_i}\right|$ is plotted against β_e. For values of β_e of the order of unity or larger the assumption of weak damping is not valid and equations (6) and (7) do not apply.

We need more complex calculation, but we may expect strong damping at least up to values of β_e of the order of unity. It is noted that since the damping is a maximum when $\alpha_e \sim 1$, i.e. $\frac{R_e \omega}{k_{\parallel}}$ is of the order of the electron thermal velocity, the electrons but not the ions are heated. This electron heating mechanism is called 'transit time damping' by STIX due to its magnetic analogue with Landau damping.

It is also worthwhile noting that this magnetosonic wave damping is significant when β_e but not β_i is large. This condition seems to be fulfilled in the plasma behind the sharp shock front due to electron heating.

4. INTERPRETATION OF THE DESTRUCTION OF THE SHARP TRANSITION

All the experiments so far have been aimed at studying collisionless shock waves in low β plasma. However even though the initial β is small, the value of β behind the shock may easily become unity. For example, using generalized Rankine-Hugoniot relations with $\gamma = \frac{5}{3}$; for $\beta_1 = 0{\cdot}1$, $\beta_2 \cong 1$ when $M_A \simeq 3{\cdot}5$ and for $\beta_1 = 0{\cdot}5$, $\beta_2 \cong 1$ when $M_A \simeq 3{\cdot}2$. This means that when the Alfvén Mach number exceeds M_{AC} ($= 3 \sim 4$ depend upon β_1), we have to take into account high β effects behind the shock.

At first, the phase velocity of the magnetosonic waves, v_{ph}, increases as V_A and β increases. The local magnetosonic Mach number

$$M_m = \frac{u}{v_{ph}} , \qquad (u = \text{flow velocity}),$$

is reduced behind the shock. The finite Larmor radius effect increases the phase velocity. It is reasonable to expect that the local magnetosonic Mach number will be reduced to unity. Under these conditions the local flow is nearly sonic, and small changes in flow can allow the waves to begin to propagate upstream. This suggests the possibility of producing another shock if there is any collisionless dissipative mechanism. One possible mechanism might be the decay type instability proposed by Kennel and Sagdeev[9] for weak shocks in high β plasma. However, since the magnetosonic waves are highly damped in moderately high β_e plasma, as described in Section 3, it is more likely that the disturbances produced behind the sharp front would be damped out in short distances (but larger than the sharp front) forming a 'quasi-shock'. It is noted that since the group velocity is faster than the phase velocity in high β plasma (see Fig.1(b)) the wave energy is carried upstream in this case. In the actual situation the incoming plasma is still low β plasma and the phase velocity of incoming waves becomes slow again. It is, therefore, reasonable to expect that the low β plasma in front would play a role like an evanescence region and that the precursor waves propagating upstream through the sharp front would damp a part of their energy and would then reflect back towards the high β_e plasma, damping the rest of their energy. Thus the damping region ahead of the sharp front must be less than one wavelength $\sim R_i$. This result seems to be consistent with the experimental results.

As M_A increases, the broad quasi-shock will develop and the development of the sharp transition will be suppressed due to the increased damping of the magnetosonic waves.

At a certain M_A the sharp transition would disappear and only a broad shock would remain at the steady state. These circumstances might correspond to the experimental results at $M_A \gtrsim 6$[1].

It is worthwhile emphasizing that in the damping of the magnetosonic waves the electrons will be mainly heated. When the front becomes wider, the ordinary resistive effect should also be taken into account.

5. CONCLUSIONS

Theoretically the solitary wave in cold plasma starts to break at $M_A = 2$. In actual low β plasma the anomalous resistivity due to current instabilities would suppress the 'breaking' up to $M_A = M_{AC}$ ($= 3 \sim 4$).

At $M_A \gtrsim M_{AC}$ the dispersion effect and damping of the magnetosonic waves seem to play a significant role and to suppress the development of the current instabilities, forming another quasi-shock wave whose dissipative mechanism is mainly damping of the magnetosonic waves at almost perpendicular propagation. Due to this damping the electrons should be heated. Recent electron temperature measurements[7] seem to support this electron heating even in the long foot. The estimated width of the quasi-shock is of the order of R_i, which seems also to be consistent with the experiments.

There are still no convincing experimental results for $M_A > 6$, and no theoretical evaluation of the damping factor for higher β_e (> 1) but it is reasonable to expect that in the steady state a broad shock whose thickness is of the order of R_i would exist. With this dissipative mechanism due to wave damping, the shock should be laminar in structure at sufficiently high Alfvén Mach number. It is noted that for the broad shock the ordinary resistive dissipation (Spitzer-Härm) may also become important.

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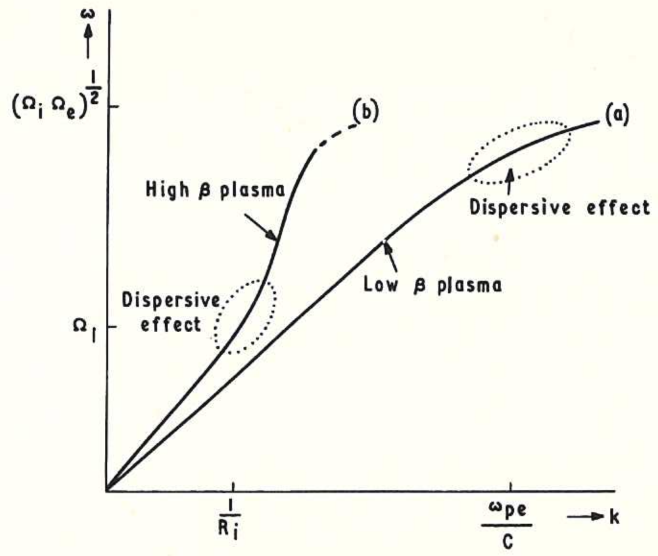

Fig. 1 (CLM-R74)
Characteristics of dispersion relations for magneto-sonic waves in low β and high β plasma.

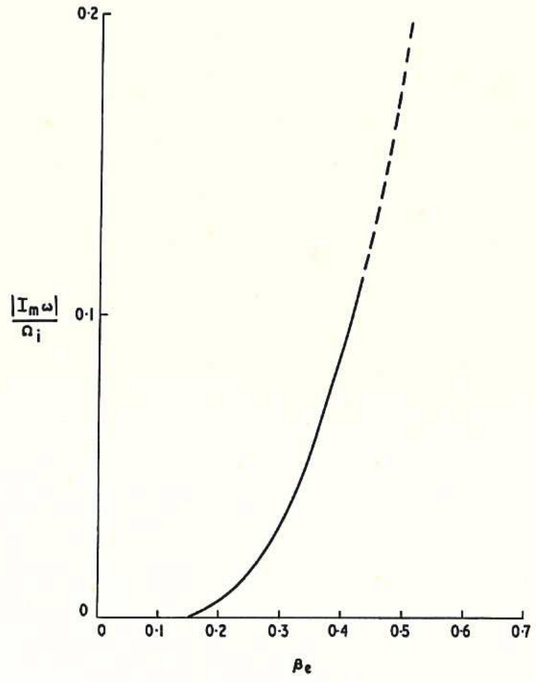

Fig. 2 (CLM-R74)
Damping of magneto-sonic wave against β_e

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