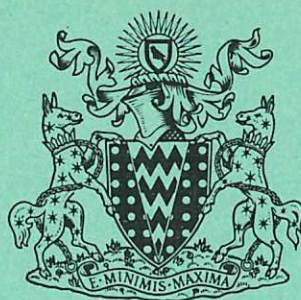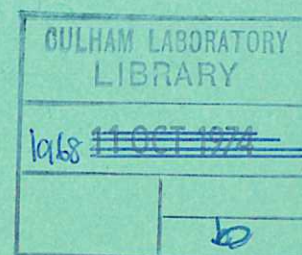CLM - R 78

United Kingdom Atomic Energy Authority

RESEARCH GROUP

Report

A METHOD FOR THE MICROWAVE HEATING
OF PLASMA ELECTRONS

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ABSTRACT

This report suggests a new method of heating the electronic component of an arbitrarily dense, magnetised plasma. Examination of the Appleton-Hartree equation reveals the existence of a wave which propagates in the plasma and is subject both to strong collisional absorption and Landau damping, and which could thus be used to heat plasma electrons.

The wave may be launched by coupling from a microwave slow-wave structure outside the plasma, although the launching efficiency is poor if a conventional TM structure is used. If a TE structure can be developed, the coupling to the plasma would be greatly improved.

A free-space treatment of the oupling is supported by an alternative treatment which regards plasma and slow-wave structure as an integrated whole; this allows a rigorous analysis of the effect of the plasma boundaries and permits the inclusion of non-uniform plasma, but cannot give the efficiency of the coupling between structure and plasma.

It is concluded that provided an efficient TE slow wave structure can be developed, a plasma electron heating method of wide application has been found.

C O N T E N T S

1. INTRODUCTION

The ability to control the electron temperature of a plasma is advantageous for the investigation of some forms of instability. The application of microwave power to the plasma is not only technically convenient, but also leads to the least gross disturbance of the plasma, and this report describes a method of microwave electron heating which is of wide applicability.

Five features are required of a heating system:

(a) It should be largely independent of the plasma parameters.

(b) It should be applicable to plasma confined by closed magnetic lines of force.

(c) It should not require the insertion of material probes into the plasma.

(d) Heating should occur evenly throughout the bulk of the plasma.

(e) The electron energy distribution should remain Maxwellian.

Condition (a) excludes resonance methods, to which condition (e) may also be relevant, and condition (b) excludes the launching of whistlers into the end of a plasma column. Condition (d) excludes the irradiation of a dense plasma by microwaves, as it may be shown[1] that heating is both inefficient and confined to a thin layer near the plasma boundary.

In this report we consider electron heating under the conditions in which the Appleton-Hartree equation[2] holds (the method is thus applicable in principle to the vast majority of laboratory plasmas), which implies that collisional absorption of energy is the governing process. The extension of the method to non-collisional heating is indicated, and we make mention of the possibility of ion heating.

We use the Appleton-Hartree equation to decide upon a suitable highly absorbed wave, and then consider the problem of launching this wave. Two complementary approaches have been found necessary: the first treats the plasma and launching structure as weakly coupled, whilst the second treats them as an integrated system. The second treatment describes the mode pattern of the waves rigorously, but only the first treatment give the coupling from the launching structure to the plasma: the launching process is essentially asymmetric in that power is applied in the first place to the launching structure alone.

2. THE APPLETON-HARTREE EQUATION

The Appleton-Hartree equation gives the refractive index μ for a wave of frequency ω travelling in a magnetised plasma with the wave normal inclined at angle θ to the static magnetic field

$$\mu^2 = 1 - \frac{X}{1 - iZ - \frac{Y^2 \sin^2\theta}{2(1-X-iZ)} \pm \left\{ \frac{Y^4 \sin^4\theta}{4(1-X-iZ)^2} + Y^2 \cos^2\theta \right\}^{\frac{1}{2}}}$$

where

$X = \omega_p^2/\omega^2$ (normalised plasma density)

$Y = \omega_0/\omega$ (normalised electron cyclotron frequency)

$Z = \nu/\omega$ (normalised collision frequency).

Here

$$\omega_p = \left[\frac{4\pi\,ne^2}{\varepsilon_o m} \right]^{\frac{1}{2}} ,$$ the critical (plasma) frequency for plasma of electron number density n.

Note that, through the terms in Z, μ is complex, implying absorption of the wave by electron collisions, but that for plasma of thermonuclear interest Z is small in the microwave frequency range and for the purposes of defining a wave may be neglected.

From physical considerations it is evident that waves subject to a high absorption will be found near a resonance[3] where μ becomes infinite (for Z = 0). Such a resonance may be seen to occur when

$$X = \frac{Y^2 - 1}{Y^2 \cos^2\theta - 1} \quad .$$

As both X and Y are greater than one, this resonance will occur in plasma of thermonuclear interest at sufficiently low microwave frequencies. This region (X > 1, Y > 1) permits the propagation of whistlers[4], a term generally confined to waves whose θ-value is small. There is a continuous transition in θ from the whistler wave at $\theta = 0$, through waves with intermediate θ-values (sometimes called 'unducted whistlers'), and finally to the θ-value for resonance.

It is not possible for a wave to propagate into a resonance unless that resonance occurs at $\theta = \pi/2$, for Snell's law, which requires $\mu \cos \theta$ to be constant, is obviously not obeyed for an infinite μ and a non-zero $\cos \theta$. Resonance at $\theta = \pi/2$ occurs when $X = 1 - Y^2$ (the Upper Hybrid resonance), and as both X and Y are less than unity this resonance is of little immediate interest. The accessibility of this resonance is discussed elsewhere[2,3].

Although the resonance cannot be attained, waves with a large value of refractive index will propagate at angles close to the resonant angle

$$\theta_{res} = \cos^{-1} \left[\frac{X + Y^2 - 1}{X\,Y^2} \right]$$

and it may be verified from the Appleton-Hartree equation that these waves are subject to high collisional attenuation. However, as these waves have $\mu \cos \theta$ greater than one it is evident that they cannot be launched by coupling from a wave propagating at a real angle in free space outside the plasma ($\mu = 1$): the wave needs to be slowed so that the

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phase propagation velocity measured along the plasma surface is $c/\mu \cos\theta$. Maxwell's equations show that this wave will of necessity have an exponential decay of field components in the direction perpendicular to the plasma boundary, and may indeed be regarded formally as a wave propagating at an imaginary angle such that $\cos\theta$ is real but greater than one.

The factor by which an electromagnetic wave may be slowed is set by technical considerations, which will be described later. This factor, which we will denote by S, will be equal to $\mu \cos\theta$ for the wave in the plasma, and it will be found that S will be about 10. We thus require to examine waves in the plasma with a given value of $\mu \cos\theta = S$.

Consider a sharply bounded plasma, with a static magnetic field lying in the plane of the boundary. Take the x-direction to be perpendicular to the boundary, the positive z-direction to be that of the static magnetic field, and restrict consideration to waves whose wave-normal lies in the x-z plane: we assume no variation in the y-direction.

To analyse wave propagation we use a modified form of the Appleton-Hartree equation, termed the Booker Quartic[2]:

$$q^2 = 1 - S^2 - \cfrac{X}{1 - \cfrac{Y^2 S^2}{2(1-X)} \pm \left\{ \cfrac{Y^4 S^4}{4(1-X)^2} + \cfrac{Y^2 S^2}{1-X} \right\}^{\frac{1}{2}}}$$

where $q = \mu \sin\theta$ and $S = \mu \cos\theta$. Having selected S, we may find q for chosen values of X and Y, and hence obtain $\mu = (q^2 + S^2)^{\frac{1}{2}}$ and $\theta = \tan^{-1} q/S$. For the moment we take $Z = 0$.

As a possible experimental example, to which we shall adhere throughout the bulk of this report, take $S = 10$, $X = 10$, $Y = 3$, which corresponds to a plasma density of 10^{12} cm^{-3} and a static magnetic field of 3000 G with an operating frequency close to 3000 MHz (the microwave S-band). Solution of the Booker Quartic shows that the matching plasma wave has $\mu = 22$ and $\theta = 63^{\circ}$ when the $+$ve sign in the denominator is taken, and $\mu = 1{\cdot}05$, $\theta = \cos^{-1}(9{\cdot}52)$ for the $-$ve sign. The latter wave is an evanescent wave, confined closely to the plasma surface, and is for the moment of no interest.

Having found the μ and θ values of the matching wave, we may insert the values of X, Y and θ into the Appleton-Hartree equation, together now with a suitably chosen Z-value, thus determining the imaginary part of μ and hence the attenuation coefficient. If with our previous values we take $Z = 10^{-5}$, corresponding to an electron temperature of about 40 eV[5], we find $R(\mu)$ is unchanged at $22{\cdot}04$ and $\Im(\mu)$ is $5{\cdot}1 \times 10^{-3}$, corresponding to an attenuation length for wave power of some 145 cm.

This wave, if it can be launched efficiently, is evidently suitable for plasma heating according to the criteria set in the introduction. For convenience we term this the Quasi- Resonant Wave, recognising it as an extreme example of the unducted whistler.

3. THE QUASI-RESONANT WAVE

The analysis so far only gives the refractive index and propagation direction of the wave; to understand the structure of the wave as revealed by the field components we refer to the polarization equation[2]

$$\rho = \frac{1 + X/(\mu^2 - 1)}{i \, Y \cos \theta} \quad .$$

Here ρ is the wave polarization, defined as the ratio of the components of the electric field perpendicular to the propagation direction

$$\rho = E_y/E_{||}$$

where $E_{||}$ lies in the x-z plane.

For our specific example ($S = 10$, $X = 10$, $Y = 3$, $Z = 0$) we find $\rho = -0 \cdot 75i$ so that the wave is right-hand elliptically polarized.

This wave also has a field component E_L in the propagation direction (it is not a TEM wave)

$$E_L/E_{||} = - \frac{i \, \rho \, Y \sin\theta \, (\mu^2 - 1)}{1 - X} = 108 \cdot 0 \quad .$$

The predominant electric field is thus longitudinal, and at the resonance the ratio $E_L/E_{||}$ becomes infinite.

Similarly we may obtain the components of the Poynting vector

$$P_{||} = \frac{E_{||}^2}{Z_0} \cdot \frac{i \, \rho \, \mu \, (\mu^2 - 1) \, Y \sin\theta}{2(1 - X)} = 1190 \, \frac{E_{||}^2}{Z_0}$$

$$P_y = 0$$

$$P_L = \frac{E_{||}^2}{Z_0} \cdot \mu(1 - \rho^2) = 17 \cdot 2 \, \frac{E_{||}^2}{Z_0}$$

where Z_0 is the impedance of free space. Note that the resultant Poynting vector makes and angle $\tan^{-1} (1190/17 \cdot 2) \doteqdot 89 \cdot 2^O$ with the wave normal, so that the energy flow direction is almost at right angles to the wave normal.

It is of practical interest to express the z-component of the Poynting vector in terms of the z-component of the wave electric field: $P_z = 0 \cdot 46 \, \frac{E_z^2}{Z_0}$. Thus a z-directed power flux

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of 1 watt cm^{-2} requires a z-directed electric field of 28·5 volts cm^{-1}. This flux would cause a temperature rise of about 3×10^4 eV sec^{-1} in plasma of density 10^{12} cm^{-3} and electron temperature 40 eV.

So far the wave propagation has been considered in an infinite plasma. In practice these waves would be launched on some sort of plasma column, so that we should discuss the effects of boundaries in cylindrical geometry. However, we will restrict discussion to a plane infinite slab of plasma, justifying the considerable mathematical simplification thus obtained by remarking that in this, as in many other bounded-wave propagation problems, the wavelength is much smaller than the column diameter, so that the Bessel functions arising from a cylindrical solution are well approximated by the trigonometrical functions appropriate to a plane slab.

At any point in the slab we may superimpose two waves, travelling at $\pm\theta$ to the magnetic field. As before, the static field defines the z-direction, the slab surfaces lie in the y-z plane and the wave normal in the x-z plane. The two waves give a guided wave mode, having field zeros separated by $\frac{\lambda_0}{2\mu \sin\theta}$ in the x-direction, and we define a mode number for a slab of thickness t as $\frac{2\mu\, t\, \sin\theta}{\lambda_0}$. Thus for our previous example the mode number for a 10 cm thick slab is about 39 - it would be exactly 39 if a conducting wall were inserted at the appropriate field zero planes, but the free plasma boundary affects the mode pattern to give a non-integral mode number as defined. The various modes in the slab travel at different velocities, and as we require a velocity match between waves in plasma and outside the boundary, it may be necessary to ensure that the velocity chosen matches one of the plasma modes. We return to this topic later.

The attenuation length for the wave measured along the z-axis is less than for a wave in infinite plasma by a factor $\cos\theta$; thus the attenuation length for our numerical example now becomes 66 cm.

So far the plasma has been taken to be of uniform density; however, the quasi-resonant wave can exist for all plasma densities greater than the critical density, i.e. for all X greater than unity:

TABLE I

X	μ	θ (degrees
1·1	10·4	16·4
1·5	11·9	32·8
2·0	13·4	41·6
5·0	18·5	57·2
10·0	22·0	63·0
50·0	26·1	67·5
S = 10·0		

Thus there should be no qualitative difference if the waves are launched on a non-uniform column, if the plasma boundary is taken to be the X = 1 position. For X < 1 the wave with S = 10 is evanescent.
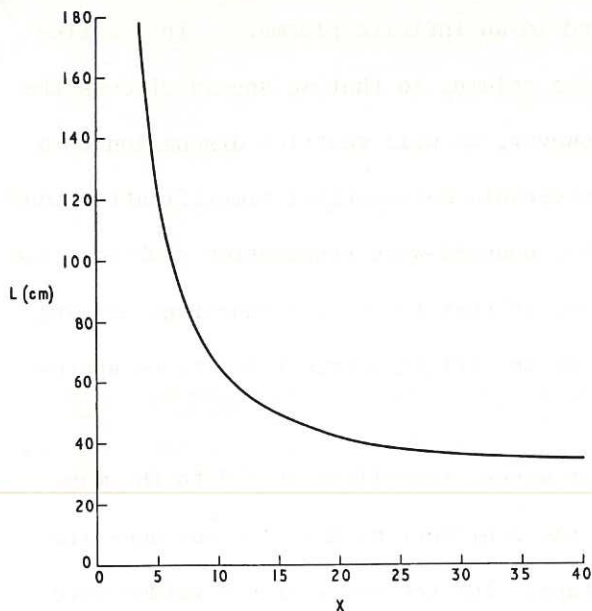

Fig. 1   (CLM-R78)
The variation of attenuation length L with normalised plasma density X. Electron temperature 40 eV, wave phase velocity c/10.
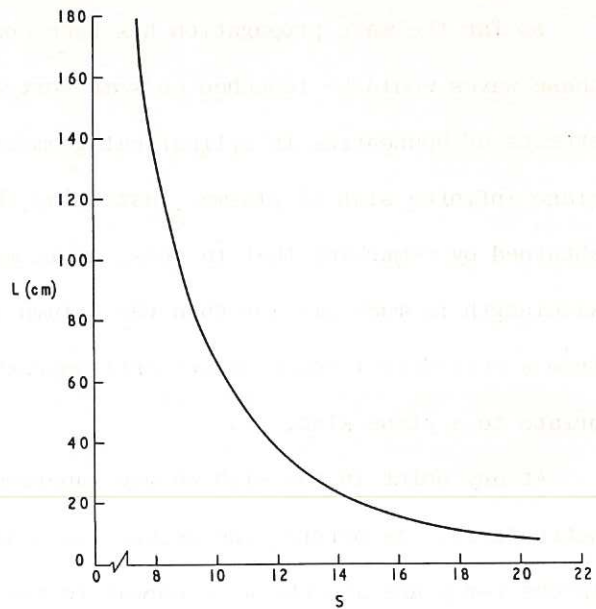

Fig. 2   (CLM-R78)
The variation of attenuation length L with wave slowing factor S (wave phase velocity c/S). Electron temperature 40 eV, normalised plasma density 10.

The variation of the attenuation length with density for a plasma with constant electron temperature (so that $Z \propto X$) is shown in Fig.1 for a wave with S = 10: evidently X should exceed about 8 for effective absorption to occur. As the attenuation length is proportional to Z, it will vary (to a close approximation) as $T_e^{3/2}$, becoming for example 1000 cm for X = 10, S = 10, Y = 3 when T_e becomes about 250 eV.

Similarly, the variation of attenuation length with phase velocity is shown in Fig.2: since the length varies approximately as S^{-3} a worthwhile increase in the power absorption can be obtained for a modest decrease in phase velocity.

4.   LAUNCHING QUASI-RESONANT WAVES

The Quasi-Resonant wave in the plasma may be launched by coupling from a slow electromagnetic wave outside the plasma. Such a wave is necessarily a form of surface wave, confined more or less closely to a slow-wave structure (discussed in more detail later). The fields of the slow wave fall off exponentially in the direction perpendicular to the structure, and there is no power flow in this direction if the structure is in isolation. However, the proximity of a dielectric which can support a propagating wave causes energy to be coupled from the structure by the well known 'tunnelling' effect through the evanescent wave gap. Provided that the coupling from the structure is weak the effect upon the

structure of the reflected evanescent wave generated by mismatch at the dielectric boundary may be ignored. We take this approach in the following two sections, but give an alternative treatment allowing strong coupling in a later section.

As the plasma can support two waves, corresponding to the alternative signs in the Booker Quartic, and either wave has all three orthogonal electric field components, the coupling into the plasma from a given electric field outside the boundary is a matching problem of some complexity. However, the methods used to treat problems of wave reflection from the ionosphere[2] may be adapted by treating the incident (evanescent) wave as if it had an imaginary angle of incidence.

After considerable algebraic effort we find the ratio of the z-directed component of the quasi-resonant wave electric field to the z-directed launching field just outside the plasma boundary, obtaining for X = 10, Y = 3, Z = 0, S = 10 (uniform plasma):

$$|Q_{E_z}|/|L_{E_z}| = 1 \cdot 8 \times 10^{-2}$$

(Q indicates quasi-resonant, L indicates launching). The coupling is evidently weak. It is also found that the ratio of the other, surface (s), wave to the wanted wave is large (expressed in terms of the y-field components):

$$|S_{E_y}|/|Q_{E_y}| = 4250 \quad .$$

The surface wave may be regarded as the incident wave field, slightly modified by the dielectric properties of the plasma. The quasi-resonant wave then arises as a consequence of the rather small mismatch at the plasma boundary.

If a purely y-directed launching field were used, the coupling to the quasi-resonant wave is stronger:

$$|Q_{E_z}|/|L_{E_y}| = 64 \cdot 3$$

and the proportion of surface wave falls:

$$|S_{E_y}|/|Q_{E_y}| = 116 \quad .$$

Coupling from waves with z-directed and y-directed electric fields may be regarded as 'electric' and 'magnetic' coupling respectively.

Conventional microwave slow-wave structures do not generate y-directed (transverse) electric fields, and are thus termed transverse magnetic or TM. It is uncertain whether a structure which has y-directed fields (in the limit, a TE structure) can be developed without its having an unacceptably high attenuation, so as an alternative we consider coupling from a TM structure with the propagation direction skew to the static magnetic field in the plasma. For a plasma column, this corresponds to a helical component in the static field.
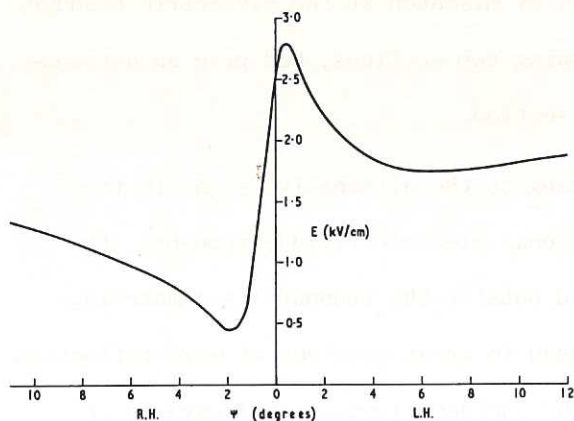
Fig. 3   (CLM-R 78)
Electric field E required at plasma surface
to launch a quasi-resonant power flow of
1 watt cm^{-2}, as a function of angle ψ between
static magnetic field and propagation direc-
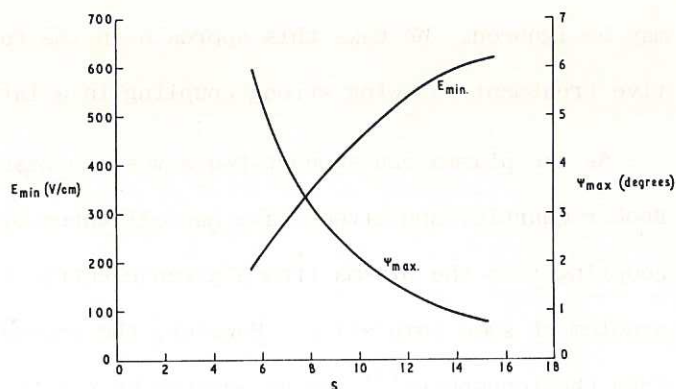tion. X = 10, Y = 3, Z = 0, S = 10.

Fig. 4   (CLM-R 78)
Angle ψ for optimum coupling and electric
field E for 1 watt cm^{-2} power flow, as a func-
tion of slowing factor S. X = 10, Y = 3,
Z = 0.

A full analysis of the wave propagation on a column with a helical magnetic field has
not been attempted, and we once again consider the boundary conditions appropriate to a
slab of plasma. Using the method of matching fields at the plasma boundary, it is found
that the coupling to the quasi-resonant wave is quite strongly dependent on the angle ψ
between magnetic field and propagation direction, i.e. upon the pitch of the field helix
if we transform to a cylindrical column. This is illustrated in Fig.3, which shows the
field required at the plasma boundary to give a power flux of 1 watt cm^{-2} along the plasma
column, as a function of the angle ψ. Note that the coupling depends upon the sign of
ψ, as would be expected from the non-reciprocal properties of the magneto-plasma: coupling
is better with a right-handed field helix, looking along the propagation direction. In
Fig.4 we show the variation of the angle which gives optimum coupling (ψ_{max}), and the elec-
tric field required to give 1 watt cm^{-2} at that angle, with the phase velocity factor S.
We also find that ψ_{max} varies roughly as Y^{-1}.

An electric field of 450 volts cm^{-1} at the plasma boundary, giving a quasi-resonant
wave power flux of 1 watt cm^{-2}, also gives a power flow in the unwanted surface wave of
155 watts per cm width of surface. Evidently this heating method becomes more efficient
as the plasma column becomes larger in diameter.

5. SLOW-WAVE STRUCTURES

It may be shown from Maxwell's equations that if it is required to guide a slow elec-
tromagnetic wave along a surface, then that surface must be reactive.

We take rectangular axes x, y, z, let the surface x = 0 be a reactive sheet, and con-
sider waves propagating in the z-direction in the positive x half space, with no y

variation. The slow surface wave has the form:

$$e^{-\alpha x}\, e^{i(\omega t - \Upsilon z)}$$

where α is the reciprocal of the e-folding length for the evanescence in the x direction, and Υ is the propagation constant. From Maxwell's equations we have $\Upsilon^2 - \alpha^2 = \beta_0^2$ where β_0 is the free-space propagation constant.

Two types of wave may occur:

(a) The TE wave: $E_x = E_z = H_y = 0$

Surface reactance $- E_y/H_z = - i\omega\mu_0/\alpha$

(Capacitative)

(b) The TM wave: $H_x = H_z = E_y = 0$

Surface reactance $E_z/H_y = + i\alpha/\omega\varepsilon$

(Inductive)

If we let the surface $x = 0$ have a reactive impedance $+ i\sigma Z_0$, then it is found that $\alpha = \sigma\beta_0$ and $\Upsilon = \beta_0 \sqrt{1 + \sigma^2}$, so that the slowing factor S becomes

$$S = \Upsilon/\beta_0 = \sqrt{1 + \sigma^2}\,.$$

The wave field components have the form

$$E_z = E_0 \cdot e^{-\alpha x} \cdot e^{i(\omega t - \Upsilon z)}$$

$$E_x = - i\,\frac{\sqrt{1 + \sigma^2}}{\sigma^2} \cdot E_z$$

$$H_y = - \frac{i}{\sigma Z_0} \cdot E_z\,.$$

Furthermore, if P is the power flow per unit width of the surface, we find

$$E_0^2 = \frac{4\pi Z_0 \sigma^2 P}{\lambda_0 \sqrt{1 + \sigma^2}}\,.$$

Thus, for example, if we require an electric field of 100 volts cm^{-1} at a distance of 0·5 cm from a TM structure with $S = 10$ at 3000 Mc/s, the field at the structure surface would be 4640 volts cm^{-1}, and the power flow 1120 watts/cm width.

If now we place another reactive surface at $x = d$, we have a reasonably tractable analogy to a reactive surface and a plasma boundary: the coupling between surfaces will be qualitatively similar to the coupling between surface and plasma.

If the surfaces have impedances $+ i\sigma_1 Z_0$ and $+ i\sigma_2 Z_0$ it may be shown that there are two independent slow waves which become the individual unperturbed waves associated

with each surface as the separation d becomes large, for we find as the propagation

condition:

$$\tanh \alpha d = \frac{\alpha \sigma_1 \sigma_2 \beta_0}{\alpha^2 + \sigma_1 \sigma_2 \beta_0^2} .$$

We assume αd is large, and also take $\sigma_1 = \sigma_2$. It may now be shown from standard

coupled mode theory[6] that if a wave is launched from some point on one structure alone,

the power will transfer completely to the other structure in a distance Λ:

$$\Lambda \approx \frac{\lambda_s}{8} \exp\left(\frac{2\pi d}{\lambda_s} \right)$$

where $\lambda_s = 2\pi/\gamma$ is the slow wave wavelength. Thus if $d = \frac{1}{2} \lambda_s$, power transfer occurs

in about 3 slow wavelengths, whereas if $d = \lambda_s$, some 70 wavelengths are required.

   This result applies only if the structures are both lossless (the surfaces are purely

reactive), and if the phase velocities of the waves associated with each surface are equal

(equal impedances). If the structures are lossy the overall picture becomes more compli-

cated, and the transfer of power is not complete. Fig.5 shows the power flow on the driven

line (the structure which receives power by coupling from the driving line), and Fig.6 the

total power dissipated, for various values of the relevant parameters, assuming a perfect
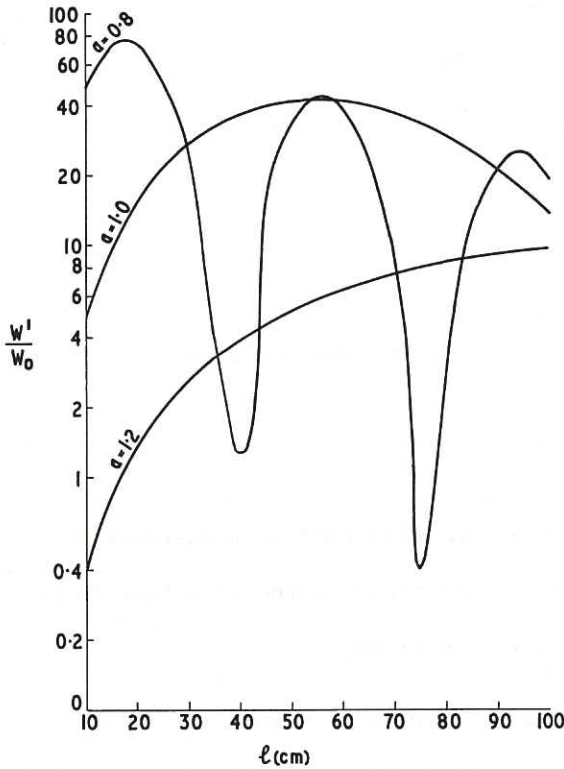
velocity match.

Fig. 5 (CLM-R78)
Power flow W^1 on driven line at end of inter-
action length ℓ, expressed as percentage of
power flow W_0 on driving line at start.
Driving line attenuation 2dB metre^{-1}, driven
line attenuation length 40 cm, slow wave-
length 1 cm. Coupling gap a cm.

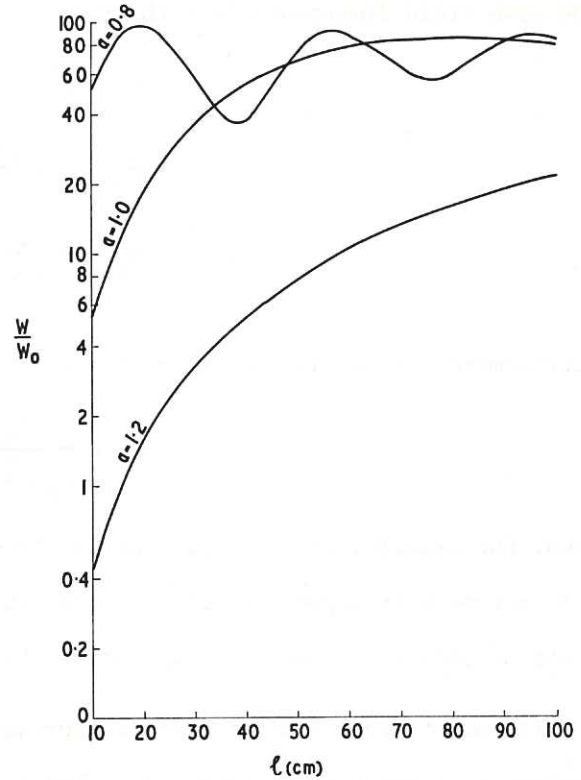Fig. 6 (CLM-R78)
Total power W dissipated on driven line by
the end of the interaction length ℓ, expressed
as a percentage of power flow W_0 on driving
line at start. Parameters as Fig. 5.

Fig.7 shows the effect of a velocity mis-
match, which occurs if the particular mode
excited on the plasma column has a velocity
slightly different from the structure velo-
city. Note that at a mode number of around
40 the greatest possible velocity mismatch to
the nearest mode is 1·25%.

Thus we see that the phenomenon of the
progressive energy transfer between coupled
structures allows the coupling of a substan-
tial fraction of the input power into the
plasma. Some complication is introduced by
the presence of two waves in the plasma, for
the coupling into the wanted wave, measured
in terms of the electric field at the plasma

Fig.7   (CLM-R78)
Fractional power flow W^1/W_O on driven line
at position where this flow is a maximum, as
a function of the percentage velocity mis-
match Δv between driving and driven lines.
Parameters as Fig.5.

boundary is weak. However, the unwanted wave suffers very little collisional attenuation,
so that the weakness of the coupling into the quasi-resonant wave can be regarded as an
apparent increase in the gap between slow wave structure and plasma; thus there is no
qualitative difference between the coupling from structure to structure and that from
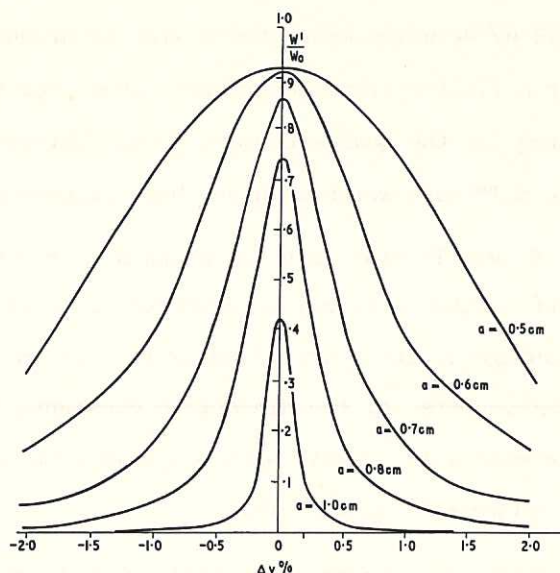structure to plasma.

6. PRACTICAL SLOW-WAVE STRUCTURES

A reactive surface can be realised either by corrugating a conducting sheet at right
angles to the direction of propagation, or by coating a conducting sheet with a dielectric
slab[7].

The corrugated waveguide has been extensively studied, and is widely used in microwave
amplifiers and particle accelerators. In principle it can support a wave whose phase velo-
city may be made arbitrarily small by choosing a frequency sufficiently close to the reso-
nant frequency of the loading slots, but the structure then becomes extremely dispersive.
This has the effect of lowering the wave group velocity, which causes the attenuation of a
real imperfectly conducting structure to become prohibitively high. Thus a structure des-
cribed by Dain et al[8] working at 3000 MHz with a slowing factor of 5 had an attenuation
of 35 dB metre^{-1}, as the group velocity was about c/250.

A more advanced design[9] achieved a loss of only 2 dB metre^{-1} with a slowing factor of 12 by detuning every third slot to produce a substantially non-dispersive characteristic over a limited frequency range: this type of structure would make an acceptable launching system for the quasi-resonant wave. Unfortunately the corrugated waveguide can only support a TM wave which we have shown to produce inherently poor coupling.

A pure TE slow wave can propagate on a conducting surface loaded with a dielectric sheet, although it may be difficult to excite as the TM mode is always dominant. The disadvantage of dielectric loading is that the practical limit to the slowing factor is about $0 \cdot 8 \sqrt{\varepsilon}$, where ε, the dielectric constant, is limited to about 100 in commercially available material. A wave with S = 8 in such a material would have an attentuation of about $2 \cdot 7$ dB metre^{-1} at 3000 MHz.

There is a further possibility of loading a conducting surface by sets of corrugations at right angles, a configuration used in waveguide filter design[10]. Such a structure would support some hybrid EH mode, which might give good coupling to the quasi-resonant wave (itself an EH wave), but the propagation characteristics have not been examined in published work.

Finally, as an alternative to dielectric loading we mention the possibility of loading a conducting surface with some suitable microwave ferro-magnetic material (ferrite). This material has in general a high dielectric constant and a high magnetic permeability, so that a wave propagating in ferrite is considerably slowed. However, the magnetised ferrite is anisotropic and thus gives rise to considerable mathematical complexity when it is considered as a loading material for a slow wave structure; as, furthermore, it tends to be rather lossy we have not studied this possibility further.

It would appear that there is scope for a considerable development of slow wave structure of the TE type.

7. PLASMA COLUMN AND SLOW WAVE STRUCTURE AS AN INTEGRATED SYSTEM

As an alternative to the semi-quantitative treatment given so far, we may consider a column of plasma surrounded by a reactive surface, and by solving Maxwell's equations investigate the propagation of waves in such a system. This method is complementary to what has gone before, and has the advantage of resolving the problem of matching at the plasma boundary and the reactive surface without having to assume weak coupling between the two. It may also be extended to the non-uniform plasma column. However, it has the disadvantage of being far removed from the practical aspects of wave launching, and is perhaps to be regarded more as a confirmation of the results obtained earlier.

Inside the plasma the solution requires use of the plasma dielectric tensor[2], which is equivalent to the use of the Appleton-Hartree equation previously, and as before we initially ignore collisions. The problem then becomes one of matching both at the plasma boundary and at the reactive surface. Once more we take a slab model to avoid the mathematical complexity introduced by cylindrical co-ordinates.

The system can support two waves, only one of which penetrates into the bulk of the plasma. These waves correspond to the quasi-resonant wave and the surface wave found earlier, and the relative proportion in which they occur is governed in a rather complicated way by the parameters of the plasma (X and Y), the reactance of the surface $\pm i\sigma Z_o$, and the separation between the plasma and the surface (d). For some specific values of these parameters the surface wave is absent.

The phase velocity of the wave is also related to the various parameters, but if the separation d is large the wave velocity is to a good approximation given by the surface reactance alone. This corresponds to weak coupling between structure and column, and the velocity match between the wave on the structure and the nearest column mode is reflected both by the ratio in which the two plasma waves occur, and by the relative power flow inside and outside of the plasma column.

If now the plasma is assumed to have a finite (small) collision frequency, the propagation constant for the wave becomes complex and the wave is attenuated.

The attenuation length now depends not only on the plasma parameters, but also on the ratio between the quasi-resonant wave and the surface wave amplitudes (for only the quasi-resonant wave suffers appreciable attenuation) and on the ratio between the power flow inside and outside the column. Fig.8 shows the variation of attenuation length L with separation d for the 'standard' plasma conditions (X = 10, Y = 3, Z = 10^{-5}), and the increasing sharpness of the resonances between column and structure as the separation is increased is very striking. For small
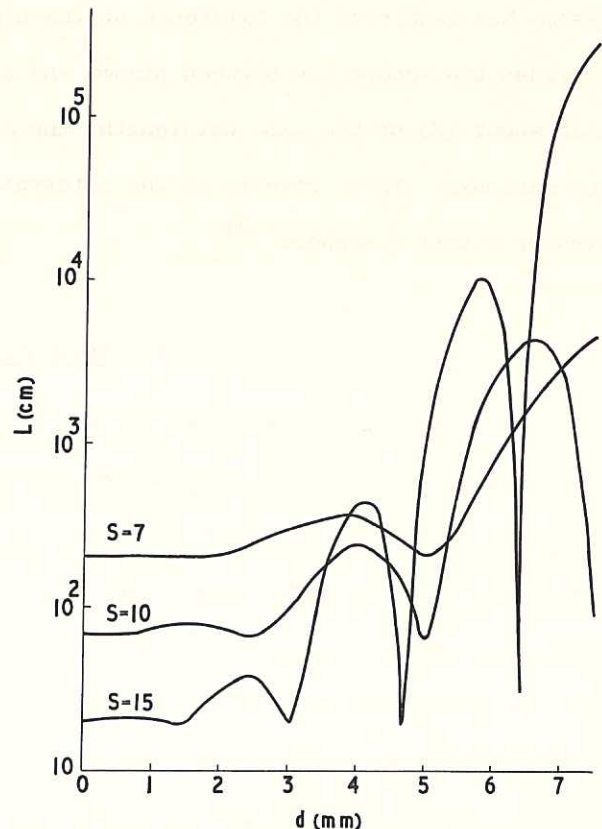
Fig. 8   (CLM-R78)
Variation of attenuation length L in the integrated system with inductive wall (TM) as a function of the coupling gap d, for various values of the slowing factor S. X = 10, Y = 3, Z = 10^{-5}.

separations the column and structure become tightly coupled, and the discrete resonances flatten out. The attenuation lengths found agree with those found previously when due allowance is made for the power flow outside the plasma column (the structures are here taken to be lossless).

If the plasma column is non-uniform, it is necessary to resort to numerical methods of solving the wave equation. The particular interest here lies in the refractive index discontinuity which occurs in a collisionless plasma at the X = 1 region, and it is neces-sary to use a Taylor expansion method about this region to define the appropriate wave equation solutions. Having done this, the problem becomes one of finding eigen-values for the chosen parameters, which now include the shape of the density distribution. The results in general show no qualitative difference between the non-uniform column and the uniform plasma considered so far. It is also possible to show that a suitable skewing of the slow wave structure relative to the static magnetic field allows the system to propa-gate the highly attenuated (quasi-resonant) wave alone; this corresponds to the enhanced coupling found for a correctly chosen obliquity angle mentioned earlier.

Thus the treatment of the plasma column and slow wave structure as an integrated system has confirmed the existence of the highly attenuated wave, and also suggests that provided the separation between plasma and slow wave structure can be kept small (less than about 1/3 of the slow wavelength) the effects of column mode velocity mismatch are not serious. These results on the integrated system will be reported in considerably greater detail elsewhere[12].

8. NON-COLLISIONAL HEATING

So far only electron-ion collisions have been considered as a power-absorbing mecha-nism, giving rise to electron heating, and because of the $T_e^{3/2}$ dependence of the collision frequency upon electron temperature the absorption lengths become impracticably long at temperatures above about 200 eV. However, as the quasi-resonant wave has a slow phase velocity (< c) along the static magnetic field, and also has a large electric field compo-nent in this direction, it will be subject to collisionless Landau damping giving rise to an enhanced power absorption at sufficiently high temperatures.

Using the Landau energy absorption rate for one electron given by Stix[3]:

$$\frac{d}{dt}\left(\tfrac{1}{2}m\sigma^2\right) = \frac{\pi\,\omega e^2E^2}{2m\ k^2}\left(\frac{\partial f(\sigma)}{\partial\sigma}\right)_{\sigma\,=\,\omega/k}$$

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where

$$f(\sigma) = \left(\frac{m}{2\pi\kappa T} \right)^{\frac{1}{2}} \exp \left(-\frac{m\sigma^2}{2\kappa T} \right)$$

(thus assuming a Maxwellian distribution) we find that for a plasma of density 10^{12} cm^{-3}, with $X = 10$, $S = 10$, the Landau damping is negligible if T_e is below 180 eV. However, the damping rate rises very steeply as T_e is increased, and at $T_e = 213$ eV the Landau and collisional damping becomes equal. The Landau damping overwhelms the collisional effects to such an textent that the absorption length at 250 eV is reduced from 1000 cm (collisions alone) to 120 cm (collisions and Landau damping).

The maximum absorption length is about 560 cm, occurring at $T_e = 185$ eV. Thus provided the temperature can be raised to 185 eV by collisional heating, the absorption will begin to increase again and the temperature continue to rise.

We remark that the temperature at which Landau damping takes over (T_L) is governed by the exponential term, so that $T_L \propto S^{-2}$. As the maximum absorption length varies roughly as $T_L^{3/2} \propto S^{-3}$, there is a considerable advantage to be gained by increasing S.

To utilise Landau damping it is necessary that the electron distribution be returned to Maxwellian by electron-electron collisions, otherwise the effect can hardly be called heating, quite apart from the possibility of inducing instabilities by distortion of the electron energy distribution. Furthermore, it would presumably be necessary to launch two quasi-resonant waves in opposite directions along the plasma column, to avoid a net uni-directional momentum transfer to the electrons: otherwise the Landau damping becomes no more than a peculiarly exotic method of inducing a current in the plasma. This bi-directional launching would not appear to present any new difficulties.

Evidently the Appleton-Hartree treatment breaks down if the Landau damping is so high that the properties of the quasi-resonant wave are affected. However, by analogy with collisional damping, it would seem unlikely that severe effects will occur if the damping length exceeds the wavelength by an order of magnitude. For the plasma conditions taken above, this limit occurs at an electron temperature of about 350 eV, and we therefore expect the quasi-resonant wave to exist essentially unchanged up to this temperature.

The surface wave induced in the plasma will also be subject to Landau damping, and there will thus be a tendency for stronger heating to occur near the plasma boundary. This effect is more pronounced if a TM rather than a TE launching structure is used.

9. ION HEATING

This report has so far been concerned solely with electron heating, using electron-ion collisions to randomise the RF electron energy. Stix[11] has suggested the possibility of selectively heating the ions in a plasma by launching a low frequency electromagnetic wave which couples into an ion plasma wave in the region of plasma where the wave frequency is equal to the lower hybrid frequency[3]. The ion plasma wave suffers cyclotron damping, and the net result is ion heating.

At a plasma density of 10^{12} cm^{-3} of deuterium ions, and a static magnetic field of 3000 G, the lower hybrid frequency is 116 MHz (free space wavelength $\lambda_0 = 285$ cm). However, for the resonance to be accessible[3] the wavelength measured along the field (λ_z) must not exceed 130 cm, and an increase in the static magnetic field increases the ratio λ_0/λ_z. Thus the ion-heating waves also require to be launched from some type of slow wave structure.

Detailed analysis of the ion-heating system has not been attempted: it would follow the general scheme presented for electron heating, but the addition of ion motion terms to the components of the dielectric tensor would make the algebra cumbersome. There will again be two waves in the plasma, only one of which will travel to the resonance. (As this resonance occurs at $\theta = \pi/2$ the wave can travel into the resonance region, unlike the electron resonance which cannot be reached as it occurs for $\theta \neq \pi/2$.)

In practical terms the operating frequency of a few hundred MHz is rather inconvenient, lying between the 'conventional technique' and 'microwave' regions. However, there is no difficulty in principle in the design of slow wave structures, and the longer wavelength implies a higher electric field at the plasma surface for a given field on the structure, which may make the coupling from RF generator to plasma wave reasonably strong. Thus there would appear to be no undue technical difficulty in the application of the ion-heating power, but to estimate the effectiveness which ions could be heated requires more detailed analysis which is outside the scope of this report.

10. DISCUSSION AND COMMENT

We have used the Appleton-Hartree equation to find a wave which can propagate in a dense magnetised plasma, but which is subject to strong collisional absorption. This wave if launched in the plasma will therefore produce electron heating. There are several assumptions implicit in the use of the Appleton-Hartree equation, and these must be examined to assure the validity of the treatment.

Firstly, it is assumed that the thermal velocity of the electrons is small compared to the wave velocity. As we have seen, violation of this condition leads to Landau damping and sets an upper limit to the electron temperature of some 350 eV: at higher temperatures the modification of the Appleton-Hartree equation becomes appreciable, but we have not examined the effects in detail.

Secondly, ion-motion terms have been ignored. It is easily seen that these are negligible in the range of frequency and plasma parameters considered here, except, of course, for the ion heating mentioned earlier.

Thirdly, we have assumed full ionization, so that the only collisions suffered by the electrons are with ions. Partial ionization implies a higher collision rate for a given electron density, and hence increased absorption, but makes no qualitative difference to the results given here.

Finally, the Appleton-Hartree equation applies to a uniform plasma only, although the departures observed are negligible provided that the plasma refractive index changes only slightly in a distance equal to the wavelength in the plasma. We have assumed this condition to be met everywhere in the plasma, except for the surface of the uniform column.

Given that the wave exists, the launching problem reduces to one of matching from the fields of a suitable wave outside the plasma boundary, together with the further problem of generating the external wave.

The matching is relatively straightforward, although complicated by the presence of the plasma surface wave. This surface wave is the continuation of the external wave, which as we have seen is itself an evanescent wave, into the plasma: it so happens that the plasma can support such an evanescent wave and presents a refractive index quite close to unity (1·05 for the standard example here). Thus the quasi-resonant wave, which arises as a consequence of the mismatch at the boundary, is only weakly excited. If the external wave is of the TM type, the coupling is so weak that substantial heating of the plasma would require rather extreme levels of microwave power, and would furthermore be very inefficient as most of that power would be dissipated in the structure needed to support the wave. If a TE wave can be supported, the coupling becomes considerably better.

In this report we have not attempted a direct treatment of the coupling from a suitable supporting (slow wave) structure to a plasma column, but have employed an analogy between the column and another slow wave structure. This reveals the progressive coupling effects between such systems having extended interaction, but as the reflected wave generated at the plasma surface has field components which are not present in the incident wave, it

cannot take full account of the interaction of this reflected wave with the launching structure.

The alternative treatment, which regards the plasma and slow wave structure as an integrated system, avoids the difficulties arising from the reflected wave, and gives results which are in broad agreement with the separated system method. However, it has the disadvantage that the launching and propagation of the wave cannot be separated. Thus, there remains some doubt about the absolute value of the coupling between slow wave structure and plasma; this doubt may well require experimental resolution.

The problems connected with the slow wave structure itself are fairly well defined. The wave phase velocity c/S should be as low as possible (for the attenuation length varies as S^{-3}), but the group velocity should be high to give acceptably low attenuation. Structures of the TM type are well documented, but the TE type, which should give better coupling to the plasma, is difficult to construct because the periodic loading in the propagation direction required to slow the wave inevitably gives rise to longitudinal electric fields. The alternative dielectric loading gives only a limited slowing factor, and introduces additional dielectric losses. Nevertheless, it appears essential that some sort of TE structure, or possibly a hybrid EH structure, should be developed if this form of heating is to be efficient.

The discussion in this report has taken a microwave frequency of 3000 MHz. Whilst in principle any convenient frequency could be chosen, in practice microwave power and components are available only in limited frequency bands. For example, there is a microwave heating band at 2450 MHz, but the next lower frequency at which high power (megawatts) is available is L-band at 1250 MHz. Likewise, there is a communication band (C-band) at 5500 MHz, but again high power only becomes readily available at 10,000 MHz (X-band). To attempt microwave heating at the higher frequencies would be pointless: not only do the slow wave structures become more difficult to manufacture, but also their losses increase. Furthermore, because of the exponential decay of the evanescent wave field the electric field at a given distance from the structure is strongly dependent on wavelength, and rapidly becomes weaker as the wavelength is decreased. In addition the absorption length in the plasma increases considerably as the operating frequency is raised. It would thus appear beneficial to lower the operating frequency, and it is found that for the same plasma parameters of 10^{12} cm^{-3} and 3000 G an operating frequency of 1000 MHz reduces the electric field outside the plasma required to give a 1 watt cm^{-2} power flow inside to about 15 volts cm^{-1}, and decreases the absorption length to about one-twentieth of the

3000 MHz value. However, the matching now becomes very critically dependent on the angle of the field helicity.

Probably individual circumstances require individual discussion, but it does appear advantageous to lower the operating frequency to the limit set by the size of the slow wave structure (which will be about one quarter of the free space wavelength in thickness).

To estimate the overall efficiency of the electron heating is virtually impossible: too many factors require experimental evaluation. If however we assume that the losses on the slow wave structure can be derived from the resistive loss of copper, that the gap between plasma and structure is adjusted for optimum coupling, and that the coupling treatment given above is strictly accurate, then roughly 1% of the input power at 3000 MHz can be coupled into a plasma with density 10^{12} cm^{-3} and electron temperature 40 eV. This figure assumes that uncoupled power appearing at the end of the slow wave structure remote from the input can be returned again to the input end, and applies to a TM type structure. If this figure of 1% could be attained in practice, the application of 10 KW of microwave power to a 100 cm length of plasma column 10 cm in diameter would raise the electron temperature by about 90 eV in one millisecond, assuming perfect containment of the energy.

11. CONCLUSIONS

Provided that the plasma parameters X (normalised density) and Y (normalised electron gyro-frequency) are greater than one, a quasi-resonant wave may be propagated with arbitrarily slow phase velocity. This wave is subject to strong collisional absorption, and may thus be used to heat the plasma electrons: typical power absorption lengths are a few hundred centimetres at electron temperatures around 100 eV. Furthermore, at higher temperatures absorption by Landau damping occurs, leading to so-called collionless heating.

This wave may be launched by coupling from a structure supporting a slow wave outside the plasma boundary, and although the coupling across the evanescent wave gap is weak, the phenomenon of progressive energy transfer between weakly coupled propagating systems allows the eventual transfer of an appreciable fraction of the slow wave power into the plasma. However, with conventional TM structures only a small fraction of the coupled power appears as the quasi-resonant wave, and it is doubtful if an effective heating system could result from the use of such a structure.
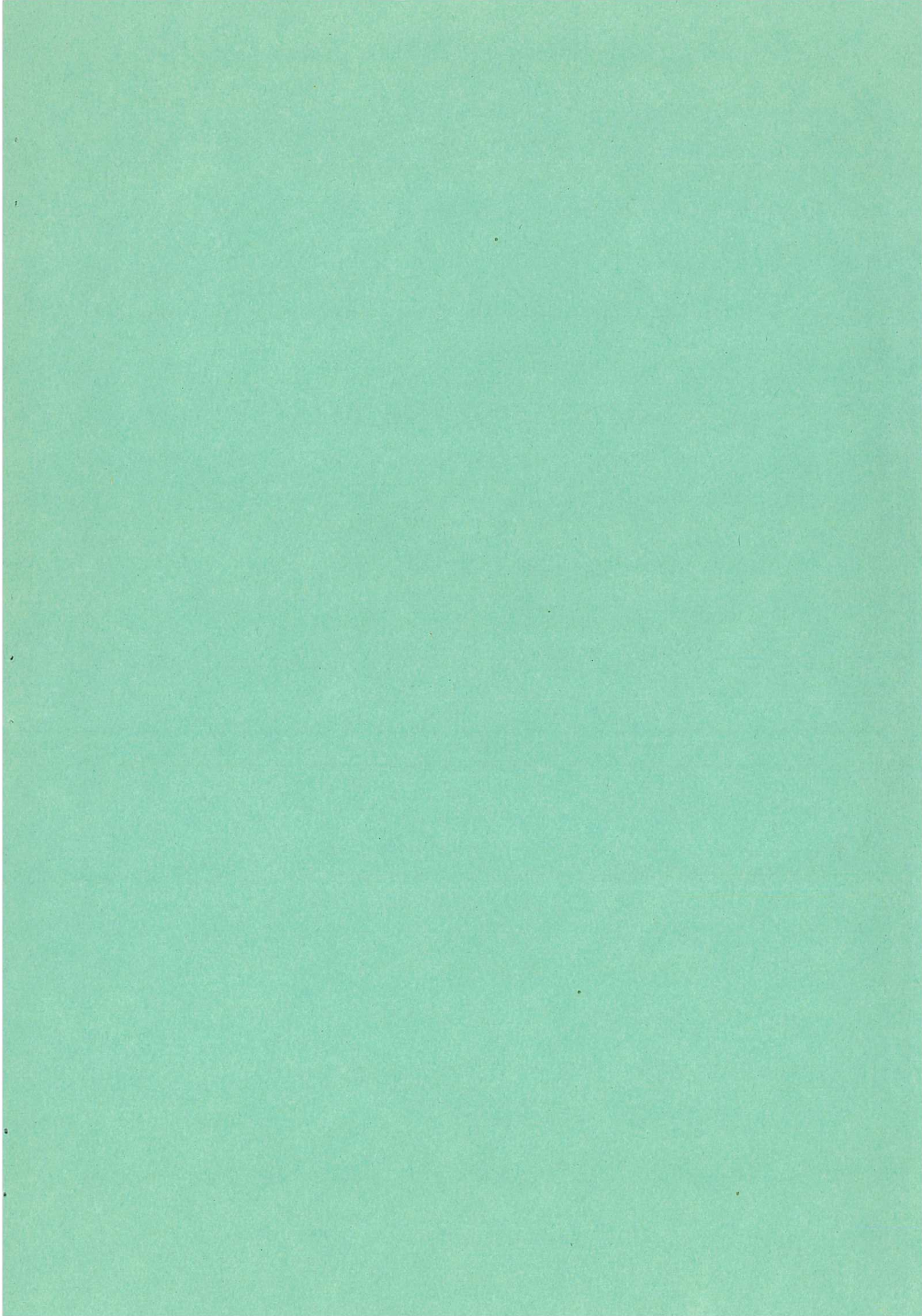
If the slow wave structure could be made to give an appreciable transverse electric field (in the limit, a TE wave structure), the coupling would become very much better. Such a structure has yet to be developed.

The coupling from either structure is considerably enhanced if the static magnetic field is slightly skew to the propagation direction; on a cylindrical column this corresponds to a helicity of a few degrees, comparable to the rotational transform in a stellarator for example. Again the TE structure is both more effective and less sensitive to helicity than the TM structure.

Thus we conclude that the quasi-resonant wave furnishes an effective method for heating plasma electrons, but the difficulties arising from the launching of the wave prevent its immediate application. Although a TM slow wave structure may be used to investigate details of wave propagation, a TE structure requires to be developed before the full possibilities of this method can be realised. The wide applicability and high achievable temperatures should then make this a method of considerable value.

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