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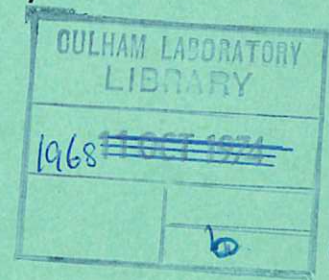
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United Kingdom Atomic Energy Authority

RESEARCH GROUP

Report



THE CALCULATION OF THE DISTRIBUTION OF
SOLAR AND EARTH HEAT INPUT TO A
NON-SPINNING SATELLITE IN A
CIRCULAR EARTH ORBIT

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1968

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THE CALCULATION OF THE DISTRIBUTION OF SOLAR AND EARTH HEAT INPUT
TO A NON-SPINNING SATELLITE IN A CIRCULAR EARTH ORBIT

by

H.J. CRAWLEY

A B S T R A C T

The successful operation of the equipment carried by most artificial satellites often depends critically on the temperature distribution and fluctuations being kept within close limits. Before the temperatures can be calculated it is necessary to determine the distribution of power input from the sun and earth to the external surfaces of the satellite. This report explains the basis of these power input calculations and describes a computer program which provides results for any given satellite attitude and orbit in a form suitable for the subsequent temperature calculations.

This report was compiled under the contract (ESTEC Contract No. 235/66) between the European Space Research Organisation and the United Kingdom Atomic Energy Authority

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January, 1968

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NOMENCLATURE

A_i	Area of surface represented by the i^{th} node (cm^2)
C_{ij}	Thermal conductance between the i^{th} and j^{th} nodes ($\text{watts}/^{\circ}\text{K}$)
E_i	Emissivity of i^{th} node
E_n	Unit vector normal to surface of earth at (φ, ψ)
e	Surface emissivity
R_{ij}	Radiation transfer coefficient between the i^{th} and j^{th} nodes ($\text{watts}/^{\circ}\text{K}^4$)
R_i	Radiation space loss coefficient of the i^{th} node ($\text{watts}/^{\circ}\text{K}^4$)
H	Height of orbit above the earth's surface (kilometers)
L	Distance from satellite to the point (φ, ψ) on the earth's surface (kilometres)
L_n	Unit vector in the direction of L
P_i	Internal power developed in i^{th} node (watts)
P_n	Unit vector in direction of normal to nodal-surface
q_{al}	Incident heat flux from earth's solar 'albedo' heat (watts/cm^2)
q_{es}	Incident heat flux from earth's 'black body' heat (watts/cm^2)
QAL_i	Earth's Albedo heat absorbed by i^{th} node (watts)
QES_i	Earth's black body heat absorbed by i^{th} node (watts)
QS_i	Solar heat absorbed by i^{th} node (watts)
$QTOT_i$	Total external heat absorbed by i^{th} node (watts)
Q_i	$QTOT_i + P_i$ (watts)
R	Earth's radius (kilometres)
S	Solar constant ($0.14 \text{ watts}/\text{cm}^2$)
S_n	Unit vector in direction of sun
S_{al}	Amount of solar radiation reflected by earth (about $S/3$)
T_i	Absolute temperature of i^{th} node ($^{\circ}\text{K}$)
T_e	Effective 'black body' temperature of earth ($^{\circ}\text{K}$)
W_i	Thermal capacity of i^{th} node ($\text{joules}/^{\circ}\text{K}$)
(X, Y, Z)	Stellar co-ordinates, orientation fixed relative to stars and moving with satellite (X, Y in plane of orbit)
(X_s, Y_s, Z_s)	Satellite co-ordinates, fixed in satellite, Y_s along axis of symmetry
α	Angular position of satellite in orbit (Fig.1b)
α_s	Mean surface absorbtivity for solar radiation
β, γ	Angular co-ordinates of vector P_n in stellar frame of reference (Fig.1a)
θ	Angle between OA and E_n (Fig.1b)
λ, ν	Angular co-ordinates of S_n in stellar frame of reference (Fig.1a)
σ	Stefan's radiation constant ($5.67 \times 10^{12} \text{ watts}/\text{cm}^2/^{\circ}\text{K}^4$)
φ, ψ	Angular co-ordinates of L_n (Fig.1b)

The few remaining variables not listed above are defined in the text when they arise

1. INTRODUCTION

The scientific equipment carried by artificial satellites is designed to carry out, remotely and unattended, accurate physical measurements which are very often difficult to make, even in the laboratory. This is particularly true of the proposed orbiting astrophysical observatories of N.A.S.A. and E.S.R.O.^(1,2) where the scientific aims involve high resolution ultra-violet spectroscopy. In these systems the required optical resolution and efficiency put very exacting limits on the thermal design of the satellite and of the structural and optical elements of the instrument itself; temperature fluctuations and gradients must be kept very small whilst the operating temperature of certain critical components (e.g. UV detectors) must be kept below some fixed value.

In the vacuum of space, radiation interchange is the only significant heat transfer process between a satellite and its environs, although thermal conduction takes place within the satellite, between one component and another.

If the satellite is considered as a simple single orbiting mass, its temperature would exhibit a periodicity as it travelled through the alternating dark and sunlit parts of the orbit, some shift in phase being evident due to the thermal lag of the system. The temperature T of this simple system would satisfy the following heat balance equation:

$$cM \frac{dT}{dt} = Q_{\text{input}} - \epsilon \sigma AT^4 \quad \dots (1)$$

Where the left-hand side represents the rate of change of sensible heat whilst the right-hand side represents the difference between the total instantaneous power input and the radiative power loss to space (see Nomenclature). Clearly before this equation can be solved for the temperature, the variation of the power input term, with time, as the satellite follows its orbit, must be known.

A single mass approximation to the satellite is not very informative from the point of view of thermal design. Obviously nothing can be predicted regarding geometrical temperature gradients within the structure or of the history of fluctuations of internal temperatures. To accomplish this the system must be represented as a number of isothermal masses (referred to in this report as thermal nodes) each having its own heat balance equation of the same form as shown above, with the addition of new terms accounting for the radiative and conductive coupling with the other masses of the system. Since these new terms involve the temperatures of the other masses (nodes) the heat balance equations comprise a set of first order non-linear simultaneous equations in the temperatures of the following form

$$W \frac{dT_i}{dt} = Q_i + \sum C_{ij}(T_j - T_i) + \sum R_{ij}(T_j^4 - T_i^4) - R_i T_i^4, \quad i = 1, n \quad \dots (2)$$

Usually the number of individual nodes in a full satellite representation is large, maybe one hundred or more, and digital computers are required in order to obtain numerical solutions to specific problems. The number of equations is usually limited by the store size of the computer although the necessary preparation of a large amount of basic input data also constitutes an inhibiting factor.

It is not the purpose of this present report to discuss the derivation or solution of these equations any further, but to describe the method for calculating the power input

terms Q_i . Some nodes in the representation will be identified with external surfaces of the satellite and the power input will include a fraction due to solar and earth radiation. The power input will depend not only on the satellite position in orbit, (as has already been discussed for a single mass) but also on the orientation of each surface with respect to the sun and earth. When the attitude of the satellite with respect to the sun and earth is changed, the amount of external heat incident on the external surface associated with each node is also changed, and each of the power input terms Q_i acquires a new set of values over the orbit. For a given position in the orbit the complete set of the functions Q_i describes the distribution of power amongst the nodes and hence, approximately, the distribution of power throughout the satellite. The distribution and fluctuations of the node temperatures will follow in some complicated manner the variations in these input terms. In addition to the external variations there may be significant variations of internal power, due to intermittent operation of electronic equipment, with corresponding temperature effects.

This report describes the basic calculations required to determine the external power inputs to a simple flat surface element on a non-spinning satellite. Also given are the operating details of a computer programme which will calculate the power input to a set of prescribed satellite surfaces where these are referred to axes fixed in the satellite system.

2. CALCULATION OF POWER INPUT TERMS (Q_i)

Many of the nodes (masses) referred to in the introduction will have one or more radiating surfaces whilst others will transfer heat by conduction alone. Of the radiating surfaces, some will be enclosed by the system and will 'see' only other internal surfaces whereas the remaining external surfaces will receive radiation from the sun and the earth. It is this latter radiation, together with any electronic heat P_i developed in the node, which is eventually treated as a combined internal power-source Q_i in the temperature analysis.

For a given external surface element the incident solar radiation can be either direct, its intensity depending only on the surface inclination to the sun, or indirect by reflection from the earth's surface and atmosphere (referred to as 'Albedo' radiation). The magnitude of the Albedo fraction will depend upon the 'area view factor' between the surface and the sunlit side of the earth. The fraction of this incident radiation which is absorbed by the surface is determined by the mean absorbtivity of the surface for solar radiation. The earth itself emits radiation in the infra red wavelengths and some fraction of this will also be absorbed by the surface element.

Thus each surface associated with a node (a nodal-surface, for want of a better term) will receive the sum of thermal powers given by

$$Q_i = P_i + QS_i + QAL_i + QES_i \quad \dots (3)$$

$$\text{i.e. Total Power} = \left(\begin{array}{c} \text{Internal} \\ \text{Power} \end{array} \right) + \left(\begin{array}{c} \text{Solar} \\ \text{Heat} \end{array} \right) + \left(\begin{array}{c} \text{Albedo} \\ \text{Heat} \end{array} \right) + \left(\begin{array}{c} \text{Earth} \\ \text{Heat} \end{array} \right)$$

The analytical problem is to determine the three external components of instantaneous thermal power Q_i to each nodal-surface for a given position in the required orbit and

and for a given satellite attitude. Assuming that all the nodal-surfaces can be considered sensibly flat, within the accuracy of the problem, it is sufficient to determine the various heat components to a set of single orbiting plates for given attitudes with respect to the sun and earth.

THERMAL POWER ABSORBED BY AN ORBITING PLATE

We require to calculate the amount of solar and earth radiation incident onto and absorbed by one side of an orbiting plate.

DIRECT SOLAR HEAT INPUT

The incident solar heat flux q_s depends only on the inclination of the plate to the sun-plate line, that is:

$$q_s = S(P_n \cdot S_n) \dots (4)$$

Where the unit vectors P_n and S_n are defined in Fig.1 and S is the solar thermal constant, (the scalar product $(P_n \cdot S_n)$, represents the projected unit area in the direction of the sun, i.e. the cosine of the angle between the two unit vectors). The absorbed solar energy over area A is given by:

$$Q_s = \alpha A q_s \dots (5)$$

Q_s will be zero when the plate enters the earth's shadow.

INDIRECT SOLAR HEAT INPUT (EARTH'S ALBEDO)

There are two components of heat input from the earth, a fraction of the earth's own infra-red emission (earth shine) and a fraction of the solar heat reflected by the earth ('Albedo' heat). A plate in a near-earth orbit will 'see' a limited spherical area of the earth's surface although the effective solid angle subtended by the plate at the earth will be large (Fig.1). Heat will not be received uniformly from all parts of the area seen and the total heat input must be found by summing incremental values over the whole surface.

Assuming that the local value on the earth of the reflected solar intensity is proportional to the cosine of the local angle of incidence of the sun's rays, then the 'Albedo' component of radiation flux incident on the surface of the plate is found by summing over the whole earth cap and is given by:

$$q_{al} = \frac{S_{al}}{\pi} \iint (E_n \cdot S_n) (P_n \cdot L_n) \frac{(E_n \cdot L_n)}{L^2} dA \dots (6)$$

where it can be seen that the earthshine integrand (see below) is multiplied by the scalar product of the unit vectors E_n and S_n .

The unit vectors are defined together with L in Fig.1 and dA is an elementary area of the earth cap.

EARTH SHINE INPUT

Assuming the earth to be a classical black body radiating diffusely the total infra-red heating incident on the plate from the earth cap is:

$$q_{es} = \frac{\sigma T_e^4}{\pi} \iint (P_n \cdot L_n) \frac{(E_n \cdot L_n)}{L^2} dA .$$

Where T_e is the effective 'black body' temperature of the earth (about 250 °K) the total energy absorbed at the surface of the plate is then given by

$$Q_{Tot} = Q_s + Q_{AL} + Q_{ES} = A \alpha_s (q_s + q_{al}) + A \epsilon q_{es} \quad \dots (8)$$

EVALUATION OF INTEGRALS

For the numerical or analytical evaluation of the above integrals it is convenient to choose a spherical co-ordinate system (L, ϕ, ψ) whose origin lies in the plate (which is assumed small) (Fig.1).

In this system it can easily be shown that

$$\frac{E_n \cdot L_n}{L^2} dA = \sin \psi d\psi d\phi \quad \dots (9)$$

Geometrically this is the projection of the element of area of the earth cap onto a unit sphere surrounding the plate⁽³⁾.

The two integrals now reduce to

$$q_{es} = \frac{\sigma T_e^4}{\pi} \iint \cos \mu \sin \psi d\psi d\phi \quad \dots (10)$$

and

$$q_{al} = \frac{S_{al}}{\pi} \iint \cos \epsilon \cos \mu \sin \psi d\psi d\phi \quad \dots (11)$$

where

$$\begin{aligned} \cos \mu &= P_n \cdot L_n \\ \cos \epsilon &= E_n \cdot S_n \end{aligned}$$

Both μ and ϵ vary over the area of the integrations.

Only the thermal radiation incident on one side of the plate is to be determined, so that, when the side considered sees only part of the earth cap, the limits of integration will be functions of ϕ and ψ .

In the case of the earth shine integral q_{es} it is possible to obtain an exact solution for any plate inclination to the earth's surface (see Appendix 1), although the parallel problem of the Albedo integral has proved to be intractable. As the numerical procedure for the Albedo integration also produces the earth shine value, the only value of the exact solution is to provide a guide to the accuracy of the numerical method, (that is unless some approximation for the Albedo component can be used, for example a mean or central value for $\cos \epsilon$ in the range of integration).

The integrals can be evaluated numerically by calculating values of the integrands at the intersections of a square mesh extending over the whole range of the variables (ϕ, ψ) (i.e. $2\pi > \phi > 0, \psi_{max} > \psi > 0$) and summing the results using a two-dimensional form of Simpson's rule. However the values of $\cos \mu$ and $\cos \epsilon$ must be tested at each mesh point and if either is negative the contribution to the integral is counted zero; $\cos \mu$ negative implies that the back of the plate is 'seen' by the mesh point, whilst $\cos \epsilon$ negative implies that the mesh point lies in the earth's shadow.

The values of $\cos \mu$ and $\cos \epsilon$ can be calculated from the components of the unit vectors P_n, S_n using the inertial or stellar frame of reference (X,Y,Z) shown in Fig.1. Where the Z axis is perpendicular to the plane of the orbit and the instantaneous value of the angle between the X axis and the OA is α . Thus

$$\cos \mu = P_n \cdot L_n = P_x L_x + P_y L_y + P_z L_z$$

$$\cos \varepsilon = E_n \cdot S_n = E_x S_x + E_y S_y + E_z S_z$$

where

$$P_n = \begin{vmatrix} \sin \gamma \cos \beta, \\ \sin \gamma \sin \beta, \\ \cos \gamma, \end{vmatrix} \quad S_n = \begin{vmatrix} \sin \lambda \cos \nu, \\ \sin \lambda \sin \nu, \\ \cos \lambda, \end{vmatrix}$$

$$L_n = \begin{vmatrix} \sin \psi \cos \phi \sin \alpha + \cos \psi \cos \alpha, \\ \cos \psi \sin \alpha + \sin \psi \cos \phi \cos \alpha, \\ \sin \psi \sin \phi, \end{vmatrix}$$

$$E_n = \begin{vmatrix} \cos \theta \cos \alpha + \sin \theta \cos \phi \sin \alpha, \\ \cos \theta \sin \alpha - \sin \theta \cos \phi \cos \alpha, \\ \sin \theta \sin \phi, \end{vmatrix}$$

3. THE POWER-INPUT COMPUTER PROGRAMME

The analysis described above has dealt with a single plate. In an actual satellite problem the number of nodal-surfaces to be considered may be quite large and several different satellite attitudes may have to be considered. Further, if temperature transient conditions around the orbit are eventually to be considered then values of the power input to each surface are required for a series of different positions in the orbit. This constitutes a tremendous amount of numerical work if each plate is to be treated separately.

To eliminate this a computer programme has been developed (primarily for work on an orbiting telescope) in which the co-ordinates of all the various surfaces of the satellite nodal-representation are required, these being referred to axes fixed in the satellite itself. The programme assumes a circular orbit and allows for any attitude of orbit plane with respect to the sun to be set in the input data. The satellite attitude is set in the data by two angular co-ordinates defining the pointing direction of the axis of symmetry and, once this has been done, the attitude of the individual surfaces with respect to the sun and earth is calculated within the programme. The power input values are then calculated for all the nodal surfaces at discrete positions around the orbit for the given satellite attitude.

An additional sub-programme is included which calculates the external heat absorbed by surface elements on the walls of a cylindrical cavity concentric with the satellite axis of symmetry. The criteria used in this programme is described and established in Appendix 3.

The programme produces a punched card and magnetic tape output which is compatible with a further programme designed to calculate the temperature solution of the heat balance equations referred to in the Introduction. This programme will be described in a future report.

The operating details of the power programme are given in Appendix 2.

4. ACKNOWLEDGEMENTS

The author would like to acknowledge the assistance of Mr Barry Brunning in the early stages of this work. The published work⁽⁴⁾ and programmes of the Goddard Space Flight Centre have also provided valuable guide lines for the development of the present programme.

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APPENDIX 1

EVALUATION OF THE EARTH SHINE INTEGRAL FOR A FLAT PLATE

We require the value of the following integral derived in section 2

$$I = \frac{1}{\pi} \iint \cos \mu \sin \psi \, d\psi \, d\phi \quad \dots (A-1)$$

The integral being expressed in terms of the co-ordinate system shown in Fig.1b. Where μ is the angle between the plate normal and the plate-earth element line OB, and B has the co-ordinates ψ, ϕ as shown.

If the plane of the plate is extended it will divide the earth spherical cap subtended by O into two sections. We require the integral I to be evaluated over the section 'seen' by the positive side of the plate.

The angle μ between the plate normal and the line OB can be obtained from

$$\cos \mu = \cos \gamma \sin \psi \cos \phi - \sin \gamma \cos \psi \quad \dots (A-2)$$

where

$$\gamma = \delta - \frac{\pi}{2} \quad \dots (A-3)$$

and δ is the angle between the plate normal and the earth-plate axis of symmetry OA.

When $\mu = \frac{\pi}{2}$ the line OB lies in the plane of the plate, defining the lower limits of the integral. Thus

$$I = \frac{2}{\pi} \int_0^{\phi_{\max}} \int_{\psi_{\min}}^{\psi_{\max}} (\cos \gamma \sin \psi \cos \phi - \sin \gamma \cos \psi) \, d\psi \, d\phi \quad \dots (A-4)$$

where

$$\sin(\psi_{\max}) = \frac{R}{R + H} \quad \dots (A-5)$$

$$\tan(\psi_{\min}) = \frac{\tan \gamma}{\cos \phi} \quad \dots (A-6)$$

$$\cos(\phi_{\max}) = \frac{\tan \gamma}{\tan \psi_{\max}} \quad \dots (A-7)$$

R is the earth radius; H is the orbit length.

Integrating (A-4) with respect to ψ

$$\pi I = 2 \int_0^{\phi_{\max}} \left[\frac{\cos \gamma \cos \phi}{2} \left(\psi - \frac{\sin 2\psi}{2} \right) + \frac{\sin \gamma}{4} \cos 2\psi \right]_{\psi_{\min}}^{\psi_{\max}} \, d\phi \quad \dots (A-8)$$

The integral with respect to ϕ of the upper limit is

$$\pi I_{\text{upper}} = \cos \gamma \sin \phi_{\max} \left(\psi_{\max} - \frac{\sin 2\psi_{\max}}{2} \right) + \phi_{\max} \frac{\sin \gamma}{2} \cos 2\psi_{\max} \quad \dots (A-9)$$

For the lower limit we substitute for ψ_{\min} from (A-6) and integrate with respect to ϕ to obtain

$$\pi I_{\text{lower}} = \psi_{\max} \cos \gamma \sin \phi_{\max} + \frac{\phi_{\max} \sin \gamma}{2} - \tan^{-1} (\tan \phi_{\max} \sin \gamma) \quad \dots (A-10)$$

subtracting (A-10) from (A-9)

$$\pi I = \tan^{-1} (\tan \phi_{\max} \sin \gamma) - \phi_{\max} \sin \gamma \sin^2 \psi_{\max} - \cos \gamma \sin \phi_{\max} \frac{\sin \psi_{\max}}{2} \quad \dots (A-11)$$

Using equation (A-7)

$$\varphi_{\max} = \cos^{-1} \left\{ \frac{\tan \gamma}{\tan \varphi_{\max}} \right\}$$

so that

$$\sin \gamma \tan \varphi_{\max} = \sqrt{\cos^2 \gamma \tan^2 \psi_{\max} - \sin^2 \gamma} \equiv U \quad (\text{say})$$

substituting in (A-11)

$$I = \frac{1}{\pi} \left[\tan^{-1} U - U \cos^2 \psi_{\max} - \varphi_{\max} \sin^2 \psi_{\max} \sin \gamma \right] \quad \dots (A-12)$$

Equation (A-12) is applicable in the range

$$-\psi_{\max} \geq \gamma \leq \psi_{\max}$$

when only part of the earth cap is 'seen' by one side of the plate. When the whole cap is visible the limits of integration are simplified, (i.e. when $\gamma \leq -\psi_{\max}$) then

$$\begin{aligned} \psi_{\max} &\geq \psi \geq 0 \\ 2\pi &\geq \varphi \geq 0 \end{aligned}$$

and the integration yields

$$\begin{aligned} I &= -\pi \sin \gamma \sin^2 \psi_{\max} \\ &= -\pi \left(\frac{R}{R+H} \right)^2 \sin \gamma \end{aligned} \quad \dots (A-13)$$

The incident Earthshine power intensity to any plate can be calculated using the view-factor equations 12 or 13 with the corresponding constant, that is $q_{es} = \sigma T_e^4 I$.

A range of values of I for various values of δ (i.e. $\gamma + \frac{\pi}{2}$) is given in Fig.2.

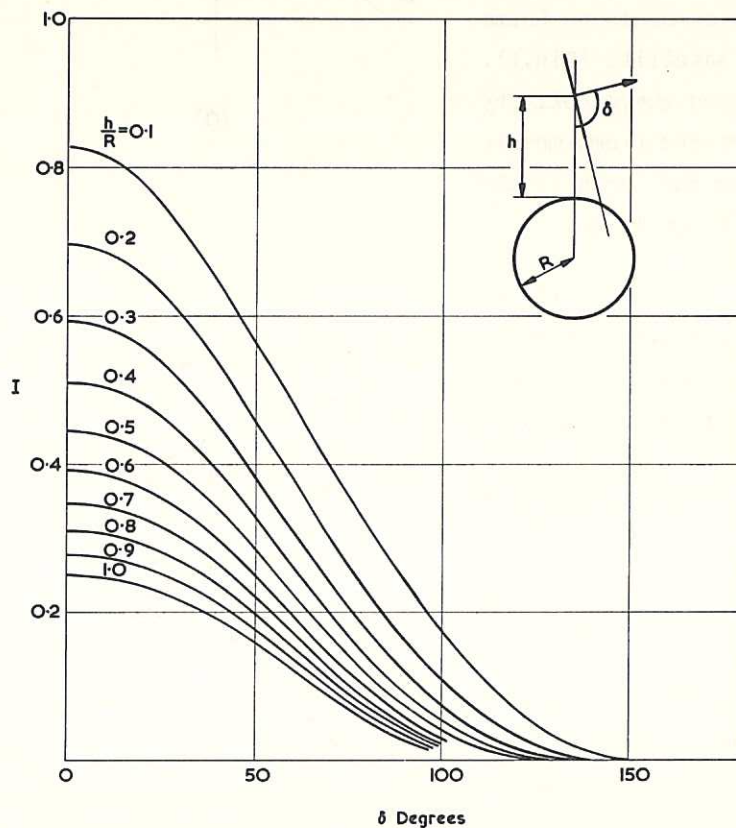


Fig.2 (CLM-R79)
Graphs of radiation view factor between a small plate and a sphere

APPENDIX 2

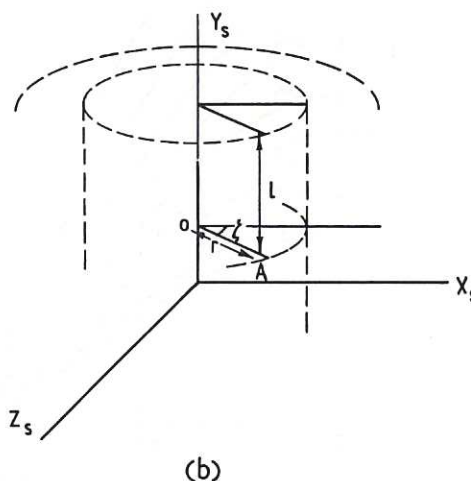
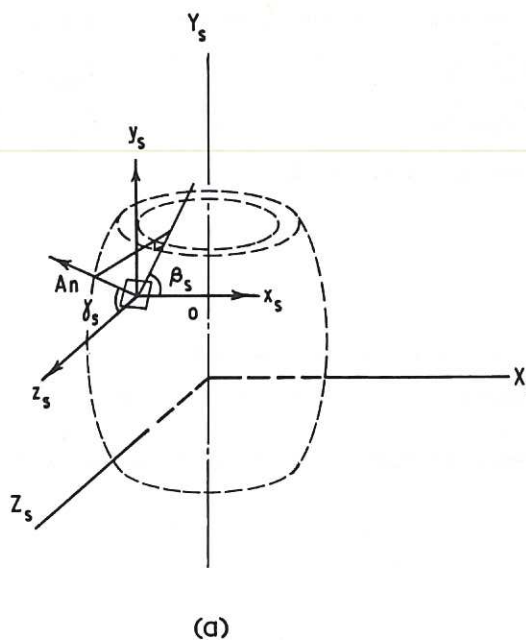
SATELLITE 'NODE-POWER' COMPUTER PROGRAMME

The programme calculates the 'Solar', 'Earthshine' and 'Albedo' power absorbed by the external surface elements of a satellite for a given attitude and circular orbit in space, the surface elements being the 'nodal-surfaces' of a multi-mass (i.e. finite-difference network) thermal representation of the satellite system. These various power components, together with the internal power developed in the nodes are summed to give the power source terms Q_i of the multi-node heat balance equations for any desired number of positions in the satellite orbit (see Introduction, equation 2). Results can be supplied on magnetic tape and on punched cards for use with a subsequent programme which calculates the temperature solutions of the equations.

Before the programme can be used it is necessary to decide on the distribution of nodes in the satellite, and to determine the angular co-ordinates (γ_s, β_s) of the normal to each surface element represented by a node with respect to axes (X_s, Y_s, Z_s) fixed in the satellite (Fig.3a). An identification number must be assigned to each node. The computations are based on the analysis described in the main text where the radiation view factor integrals of each node are evaluated in a co-ordinate frame (ϕ, ψ) moving with the satellite (Fig.1). The integrals are calculated numerically by summing the heat contributions from a mosaic of surface elements over the area of the earth cap visible from the satellite.

The power absorbed by nodal-surfaces on the walls of a cylindrical cavity coaxial with the satellite can also be determined but in, this case, the position (r, ξ, ℓ) of the node in terms of the satellite co-ordinates shown in Fig.3b must also be given together with the radius of the cavity.

The satellite orbit is defined by its height above the earth's surface and by its inclination to the sun vector. The orbit inclination and the satellite attitude are both established by angular co-ordinates referred to a frame of reference (X, Y, Z) fixed relative to the stars and moving with the satellite (Fig.1). All these data are defined in section 1 of



(CLM-R79)
Fig.3 (a) Co-ordinate system fixed in a satellite (external surface). (b) Additional 'cavity' co-ordinates fixed in satellite.

this Appendix. A brief outline of the programme logic is given in section 2 of this Appendix whilst the full programme 'lists' are given in Appendix 4.

1. DEFINITIONS OF PROGRAMME INPUT DATA

The programme data definitions are given below, and also in List 17 together with the input formats.

Node Data

The rectangular co-ordinate system (X_S, Y_S, Z_S) fixed in the satellite, to which the node positions are referred, is shown in Fig.3; the Y_S axis being coincident with the axis of the cylindrical cavity in the centre of the satellite. The position of the other two axis is arbitrary.

The required angular co-ordinates of the normal A_n to each nodal-surface are as follows (the programme variable names are given in brackets):

- (1) The angle γ_S (SGAMMA) between the normal and the Z_S axis in degrees.
- (2) The angle β_S (SBETA) between the projection of the normal on the $X_S Y_S$ plane and the X_S axis, in degrees.

For nodal-surfaces within the cavity the additional co-ordinates required are:

- (3) The axial length ℓ (LENGTH) from the end of the cylinder to the centre A of the node (Fig.3b) in cm.
- (4) The radial distance OA (RAD) of the node centre from the Y_S axis in cm.
- (5) The angle of rotation ζ (ZETA) of the node centre about Y_S in the direction of Z_S in degrees.

(N.B. For nodes outside the telescope the programme requires these data to be set at zero, except RAD which is set negative).

In addition to the above co-ordinates the following are also required for all nodes:

- (6) The surface Infra Red emissivity (EMI)
- (7) The absorbtivity for solar radiation (ALPHA)
- (8) The node area (AREA), sq. cm.

The above data is supplied for all external nodes which could conceivably 'see' the sun or earth. However in order to calculate the output array (POWIN) giving the total value of the source term Q_i for every node in the system, it is necessary to supply the power dissipation (P_i) for all nodes having attached heat sources (i.e. electronics etc.)

In addition to the node number, provision is made in the programme for giving each node an identifying name of not more than sixteen characters. These names appear on the printed output.

1.2 Orbit Data

The inclination of the orbit is established by reference to the direction of the sun vector (SN) using a stellar co-ordinate system (X, Y, Z) moving with the satellite (Fig.1) where the Z axis is normal to the orbit plane and the axes remain fixed with respect to the stars. Thus the orbit is defined by:

1. The height (H) of the satellite above the earth's surface, in kilometres.
2. The angle λ (LAMBDA) between the sun vector and the Z axis, in degrees.
3. The angle ν (NU) between the projection of the vector on the XY plane and the X axis, in degrees.

1.3 Satellite Attitude Data

The satellite attitude is fixed by the relative position of the satellite co-ordinate axes (X_S, Y_S, Z_S) with respect to the stellar co-ordinate axes (X, Y, Z). In the interests of simplicity, and without serious loss of generality, the number of degrees of freedom between these two co-ordinate systems has been reduced to two by constraining the X_S axis to lie in the XZ plane. The two angular co-ordinates required are:

- (1) The angle YGAMMA between the Y_S and Y axes.
- (2) The angle OMEGA between the X_S and X axes measured in the direction of z.

The additional degree of freedom could be restored by providing a rotational co-ordinate about the Y_S axis of the satellite frame, but for a non-spinning axially symmetric satellite this would have little application.

1.4 Programme Constants

The following programme constants must be supplied with the input data:

1. Stefans radiation constant σ (SIGMA) (5.67×10^{-12} watts/cm²/sec).
2. The mean 'black body' temperature of the earth (TE) (about 250 °K)
3. The mean radius of the earth (R) in kilometres.
4. The radius of the telescope cavity (RADIUS) in cm.
5. The total number of nodes (M) in the satellite multi-node thermal model.
6. The number of stations around the orbit (NORBIT) at which the power inputs are to be calculated.
7. The number of subdivisions (NFI, NPSI) of the co-ordinates (ϕ, ψ) (Fig.1) to be used in the numerical integration. (NFI and NPSI must be even numbers). The corresponding accuracy of the earthshine results can be checked with the exact integral given in Appendix 1.

1.5 Control Data

There are two output control variables whose values must be provided:

(1) Punch Control Variable (NK)

NK = 2 gives punched card output of the average values of Q_i round the orbit which can be used as input data for the temperature programme

NK = 1 suppresses this output.

(2) Magnetic Tape Control Variable (IRITE)

IRITE = 2 produces a tape record of the array POWIN giving the values of Q_i at each prescribed station round the orbit suitable for 'input' to the temperature programme for transient and periodic steady state calculations. The date, time, case title and orbit period (TAU) are also recorded at the beginning of the tape

IRITE = 1 suppresses this output.

A summary of these variable definitions, together with the data list shown in List 17, is printed for reference at the head of the printed output.

2. PROGRAM STRUCTURE

The general structure and execution of the program are described briefly below. These are shown fully in the sub-routine lists referred to under each heading (Appendix 4).

Prelude (See List 1)

A local program segment particular to the Egdon system of the KDF 9 computer which arranges the storage sequence of 'common' arrays, some with variable 'dimensions', the required values of the latter being obtained from the input data.

Chain 1 - The Main Program (List 2)

The main program 'reads' and prints the program constants, the orbit data, the node data (in satellite co-ordinates) and the satellite attitude data. It also prints the definitions of the input and output variables and the data list from subroutines SYMBL 2, and SYMBL 3 and reads and prints the case title using subroutine TITLE (see list 11).

The program proceeds in the following order, the main values calculated being:

- (1) The orbit period TAU
- (2) The direction cosines of the sun vector (Sn) in stellar co-ordinates from the angular co-ordinates given in the data.
- (3) The transformation matrix F from the satellite frame of reference to the stellar frame, using the attitude angles YGAMMA and OMEGA, and calculates the inverse matrix FT using subroutine TRANPO (see list 9).
- (4) The direction cosines of the normal AS to the nodal-surface under consideration, in satellite co-ordinates, using the input angles SGAMMA and SBETA and transforms these to the stellar frame using matrix F and the subroutine COMPO (see list 10)
- (5) The power absorbed by the node at each specified station in the orbit using subroutine TRAP 4 (and its subroutines), (see list 3).

The nodes are treated in turn in the order in which they are submitted in the data (arbitrary) so that the programme cycles between 4 and 5 until all the data has been used for the first attitude prescribed.

Punched card and magnetic tape output are produced (if NK = 2 and IRITE = 2) before returning to the next pair of attitude co-ordinates (YGAMMA and OMEGA) to repeat the whole cycle.

Subroutine TRAP 4 (List 3)

The subroutine is 'called' for each node in turn by the main program. It executes the following operations:

- (1) Tests if the current node lies within or outside the telescope by checking the sign of the variable RAD (RAD > 0 implies a telescope cavity node) and sets a corresponding control variable MISS.
- (2) Calculates the incident solar energy for each orbit station, testing whether the satellite lies in the earth's shadow and, if not, whether the sun sees

the positive side of the nodal-surface. In the case of a telescope-node a test is made to see if solar radiation can reach the node through the mouth of the telescope cavity, using subroutine MASK (List 5). (The routine sets the appropriate value of the control variable NMASK).

- (3) Calculates the 'Earthshine' and 'Albedo' radiation intensity incident on the nodal surface for the specified stations in the orbit using subroutine VUFAC (List 4) and the power absorbed using subroutine POWER (List 7). The latter routine also prints the current values through OUT 5 (List 16) and calculates the orbit averages of the various heat components.

Subroutine VUFAC (List 4)

VUFAC calculates the radiation view factor between the required nodal-surface and the earth by numerical integration over the surface of the earth cap visible from the satellite station; the cap being divided into $NFI \times NPSI$ elements (NFI and NPSI are even numbers provided in the input data).

When the node lies within the telescope a check is made on the line-of-sight from each earth element to the node to see if this can enter the mouth of the telescope. This is done by transforming the stellar direction-cosines of the line-of-sight vector to the satellite frame of reference using COMPO (List 10) and checking the vector direction relative to the satellite system in subroutine MASK (List 5).

The elements of the Earthshine and Albedo viewfactors are stored in the two dimensional arrays DVFES and DVFAL respectively.

The earthshine and Albedo integrals are found by summing the above arrays, using a two-dimensional Simpsons rule, in function subroutine SUM (List 6).

GENERAL PURPOSE ROUTINES

A number of general purpose subroutines are included with the program which deal with the 'input' and 'output' of data or carry out simple matrix or vector operations. Some of these have already been mentioned and the structure of all is sufficiently clear from the program listings.

APPENDIX 3

CRITERION FOR EXTERNAL HEAT INPUT TO THE INNER WALLS
OF A CYLINDER OPEN AT ONE END

The methods described in this report have been used for the thermal analysis of a proposed orbiting astronomical telescope. The telescope would be housed in a central cylindrical cavity, coaxial with the satellite body, and it was necessary to calculate the amount of external earth and solar heat which would fall on the inner surfaces of this cavity.

In principle the view factor integrals derived in the text still apply for any given surface element on the cylinder wall, except that the range of integration is limited to the portion of the earth cap 'seen' by the element through the open end of the cavity. (The solar heat input also must enter the open end.) The problem of determining the limits of integration is too difficult even for a numerical integration so that the same numerical procedure used for the Albedo integral has been employed; that is all values are calculated over a finite mesh covering the whole earth cap subtended by the satellite. But, before each mesh contribution is calculated, a test is made to see if the telescope wall cuts the line of sight from the earth mesh point to the telescope surface element considered. If this happens, the contributions to the integrals at this point are set to zero. The integrals are then formed by summing all values over the whole earth cap.

The procedure adopted is to calculate the direction-cosines of the unit vector L_n (defining the direction of L Fig.1) in the co-ordinate system (X_s, Y_s, Z_s) fixed in the satellite (Fig.3)

and to determine whether this vector, extended from the surface element under consideration, would pass through the circular opening at the end of the cylinder.

DERIVATION OF TEST CRITERIA

Let the components of the unit direction vector L_n in satellite co-ordinates be

$$L_x, L_y, L_z$$

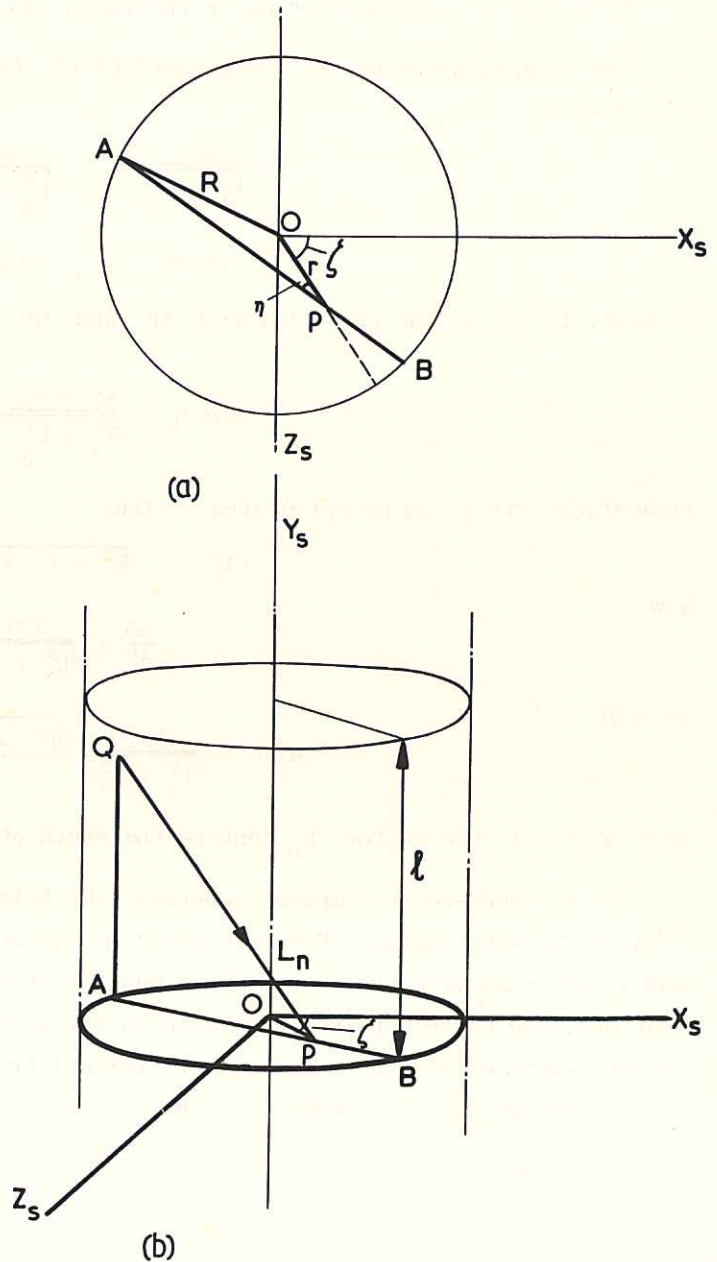


Fig.4 (CLM-R79)
Diagram of telescope cavity (see Appendix 3)

For the vector to enter the cylinder L_y must be negative; this is the first criterion.

Let the point under consideration be, $P(r, \xi, \ell)$ (Fig.4) and let Q be the point where the vector L_n , extended from P , cuts the cylinder wall.

Let ABQ lie in the plane of L_n parallel to the Y_S axis. Then L_n extended from P , will pass through the circular end of the cylinder if AQ , the extension of L_y is greater than ℓ .

Therefore we require to find if the ratio AQ/ℓ is greater or less than unity.

The direction-cosines of the projection of L_n on the line AB and of OP are respectively

$$\frac{L_x}{\sqrt{L_x^2 + L_y^2}}, \quad \frac{L_y}{\sqrt{L_x^2 + L_y^2}}, \quad 0$$

and

$$\cos \zeta, \quad \sin \zeta, \quad 0$$

so that, if η is the angle between AB and OP (Fig.4a), then

$$\cos \eta = \frac{L_x \cos \zeta}{\sqrt{L_x^2 + L_y^2}} + \frac{L_y \sin \zeta}{\sqrt{L_x^2 + L_y^2}}$$

from which $\sin \eta$ can be calculated. Thus

$$AP = \sqrt{R^2 - r^2 \sin^2 \eta} + r \cos \eta$$

Now

$$\frac{AQ}{AP} = \frac{|L_y|}{\sqrt{L_x^2 + L_y^2}}$$

so that

$$AQ/\ell = \frac{|L_y|}{\sqrt{L_x^2 + L_y^2}} \left[\frac{\sqrt{R^2 - r^2 \sin^2 \eta} + r \cos \eta}{\ell} \right]$$

for $AQ/\ell > 1$ the vector L_n enters the mouth of the cavity.

In the node-power computer programme the telescope cavity surface nodes are identified in the input data. For such nodes the subroutine 'MASK' is employed to calculate and test the above expression at each mesh point of the integral. When the ratio is less than one, the corresponding mesh value is set to zero. The components of the vector L_n are transformed from the inertial (or stellar) frame to satellite co-ordinates by the subroutine COMPO before MASK is used.

APPENDIX 4

SUBROUTINE 'LISTS' OF THE NODE POWER PROGRAMME

The programme has been developed using the KDF 9 Egdon system installed at Culham Laboratory. All subroutines are coded in a Fortran dialect which embodies most of the standards of ASA Fortran (Communications of the A.C.M., vol.7, pp.590-625, 1964) with the exception of one 'user-code' subroutine which provides the current date and time for the problem output.

The full programme comprises the following subroutines, the lists for which are on the following pages

<u>ROUTINE</u>	<u>LIST NO</u>
Prelude	1
CHAIN 1	
MAIN	2
TRAP 4	3
VUFAC	4
MASK	5
SUM	6
POWER	7
DOTPRO	8
TRANPO	9
COMPO	0
TITLE	11
INPUT	13
RITE 1	14
OUT 2	14
OUT 5	16
SYMBL 1	17 (This list shows the <u>output</u> of these subroutines)
SYMBL 2	
SYMBL 3	

LIST 2 (Continued)

```

C          COMPONENTS OF SUN VECTOR
C          IN STELLAR COORDINATES
P          SN(1)=SINF(LAMDA)*COSF(NU)
P          SN(2)=SINF(LAMDA)*SINF(NU)
P          SN(3)=COSF(LAMDA)
          SN(1)=-SN(1)
          SN(2)=-SN(2)
          SN(3)=-SN(3)
DO20 I=1,M
P(1)=0.0
C          READ VALUES OF INTERNAL POWER DISSIPATIONS
102 PRINT/02
          FORMAT(/)
          CALL INPUT(P, BHP )
C
C
C          READ NODES
C          READ NODE DATA IN SATELLITE COORDINATES
          K=0.0
7 READ10,I,A1,A2,A3,A4,A5,A6,A7,A8,A9,A10

10 FORMAT(14, 2A8, 60Z )
IF(I) 24,24,25
25 CONTINUE
          K=K+1
          NODE(K)=I
          NAME1(I)=A1
          NAME2(I)=A2
          SGAMMA(I)=A3 *X
          SBETA(I)=A4 *X
          EM(I)=A5
          ALPHA(I)=A6
          AREA(I)=A7
          RAD(I)=A8*0.01
          ZETA(I)=A9*X
          LENGTH(I)=A10*0.01
          GO TO 7
24 CONTINUE
40 CONTINUE
C          CLEAR ARRAYS
DO21 I=1,M
          APOWIN(I)=P(I)
DO21 J=1,NORBIT
          POWIN(I,J)=P(I)
C
C          READ COORDINATES OF SATELLITE AXES REFERRED TO STELLAR FRAME
READ1, YGAMMA, OMEGA
1 FORMAT(80Z)
IF(YGAMMA) 41,42,42
42 CONTINUE
          PRINT/01
101 FORMAT( 5X, 13HATTITUDE DATA )
          PRINT12, YGAMMA, OMEGA
12 FORMAT(/5X, 7HYGAMMA= F10.4, 2X, 7HDEGREES ,/5X,6HOMEGA= F10.4,
1 2X, 7HDEGREES )
          YGAMMA=YGAMMA*X
          OMEGA=OMEGA*X
C
C          CALCULATE TRANSFORMATION MATRIX FROM SATELLITE FRAME TO STELLAR FRAME
P          F(1,1)= COSF(OMEGA)
P          F(2,1)=0.0
P          F(3,1)=SINF(OMEGA)
P          F(1,2)= -SINF(YGAMMA)*SINF(OMEGA)
P          F(2,2)= COSF(YGAMMA)
P          F(3,2)=SINF(YGAMMA)*COSF(OMEGA)
P          F(1,3)= -COSF(YGAMMA)*SINF(OMEGA)
P          F(2,3)= -SINF(YGAMMA)
P          F(3,3)= COSF(YGAMMA)*COSF(OMEGA)
C          CALCULATE INVERSE OF TRANSFORMATION MATRIX
          CALL TRANPO (F,FT)
C
DO50 J=1,K
          I=NODE(J)
          A1=SGAMMA(I)/X
          A2=SBETA(I)/X
C          PRINT HEADINGS FOR ITH NNODE
PRINT5, I, NAME1(I), NAME2(I), A1, A2
8 FORMAT(14H, //2X, 4HNODE 14, 2X, 2A8, 2X, 7HSGAMMA, F10.2, 2X,
1 6HSBETA= F10.2 )
          PRINT22, EM(I), ALPHA(I), AREA(I)
22 FORMAT(/2X, 11HEMISSIVITY= F10.4, 2X, 13HABSORBTIVITY= F10.4,
1 2X, 5HAREA= F10.4 )
          IF( RAD(I) ) 52,53,53
52 PRINT54
54 FORMAT( 10X, 13HEXTERNAL NODE )
          GO TO 55
53 A3=ZETA(I)/X
          PRINT56, RAD(I), A3, LENGTH(I)
56 FORMAT(10X, 33HRADIAL COORDINATE OF NODE CENTRE= F6.3,
1 2X, 6HMETRES ,/10X, 35HANGULAR COORDINATE OF NODE CENTRE=
2 F6.2, 2X, 7HDEGREES ,/10X, 53HAXIAL DISTANCE OF NODE CENTRE FROM
3 END OF TELESCOPE= , F6.3, 2X, 6HMETRES )
55 CONTINUE
C          CALCULATE COMPONENTS OF SURFACE UNIT NORMAL VECTOR IN SATELLITE
C          COORDINATES
P          AS(1)=SINF(SGAMMA(I))*COSF(SBETA(I))
P          AS(2)=SINF(SGAMMA(I))*SINF(SBETA(I))
P          AS(3)=COSF(SGAMMA(I))
C          TRANSFORM COMPONENTS OF NORMAL VECTOR TO STELLAR COORDINATES
          CALL COMPO ( F, AS, PN )
C          CALL TRAP4 TO COMPUTE HEAT ABSORBED BY NODE
          CALL TRAP4(I)
C          REPEAT FOR NEXT NODE
50 CONTINUE
C
C
C          PUNCH AND OR PRINT AVERAGE VALUES OF HEAT ABSORBED BY NODES ROUND ORBIT
          CALL OUT2(APOWIN,8HAPOWIN ,M,HK )
C          IF IRITE=2 WRITE OUTPUT TAPE OF POWER ABSORBED BY EACH NODE FOR EACH
C          POSITION ROUND ORBIT
          GO TO(31,32), IRITE

32 CALL RITE1( CASE,TAU,POWIN,M,NORBIT )
31 CONTINUE
30 CONTINUE
          GO TO 40
41 CONTINUE
          CALL EXIT
          YYYYYY
          END

```

LIST 3

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.39.58
SUBROUTINE TRAP4(KK)
PUBLIC H, NORBIT, LAMDA, NU, SN,X,P1,ESCON,ALCON ,DALPHA
PUBLIC PN, FT, ALAMDA,A
PUBLIC APOS,APGAL,APQES,APOWIN
DIMENSION SN(3), PN(3), FT(3,3), SNT(3)
DIMENSION APOWIN(1)
REAL LAMDA, NU
PUBLIC H,ZETA,RAD,XN,ZN
DIMENSION ZETA(M), RAD(M)
C          COMPONENTS OF NODE NORMAL UNIT VECTOR IN STELLAR
C          COORDINATE SYSTEM
PRINT45, PN(1),PN(2), PN(3)
45  FORMAT(/10X,53HDIRECTION COSINES OF PLATE NORMAL (STELLAR COORD
1S ) /10X,4HPNX=F10.4, /10X,4HPNY=F10.4, /10X,4HPNZ=F10.4)
SLAMDA=ALAMDA
P      SGAMMA=SINF(ACOSF(PN(3)))
C          X AND Y COORDINATES OF TELESCOPE NODES IN TELESCOPE
C          COORDINATES
P      XN= COSF(ZETA(KK))
P      ZN=SINF(ZETA(KK))
C
C          CALCULATE SOLAR HEAT INPUT
C          SOLAR HEAT INPUT=QOS
C          TEST IF NODE IS EXTERNAL OR TELESCOPE NODE
C          IF TELESCOPE NODE TRANSFORM COMPONENTS OF SUN VECTOR
C          TO TELESCOPE COORDINATES AND TEST TO SEE IF
C          TELESCOPE WALL OBSTRUCTS SUNLIGHT
C          IF SO PUT INCIDENT SOLAR RADIATION TO ZERO
C          IF NOT CALCULATE INCIDENT SOLAR RADIATION
MISS=1
IF( RAD(KK) )43,41,41
41  MISS=2
GO TO( 43,44), MISS
44  CALL COMPO( FT, SN, SNT )
CALL MASK( SNT, KK, NMASK )
GO TO( 43,14), NMASK
43  CONTINUE
P      QOS=-0.14*( SN(1)*PN(1)+SN(2)*PN(2)+SN(3)*PN(3) )
IF(QOS) 14,15,15
C          IF QOS IS NEGATIVE SUN SHINES ON BACK OF PLATE
C          THEREFORE PUT QOS=0.0
14  QOS=0.0
C
15  CONTINUE
C
C          NSUN=1
C          TEST IF PLATE ENTERS EARTHS SHADOW
C          IF NOT SOLAR HEAT IS CONSTANT
12  IF(SN(3)=A) 12,13,13
NSUN=2
C
13  CONTINUE
APOS=0.0
APGAL=0.0
APQES=0.0
APOWIN(KK)=0.0
ALPHA=0.0
PRINT21
21  FORMAT( 5X, 5HANGLE,7X, 2HP ,7X,
1      2HOS,8X,3HQAL,7X,3HQES,7X,4HGTOT )
C
DO2 I=1,NORBIT
QS=QOS
B=ALPHA-NU
C
C          TEST IF PLATE LIES IN SUNLIGHT OR SHADOW
GO TO(16,17),NSUN
17  IF(COSF(B)*SLAMDA) 30,16,16
30  IF(ABSF(SINF(B))-SORTF(A*A-COSF(B)*COSF(B)*SN(3)*SN(3)))
1  19,19,16
19  QS=0.0
16  CONTINUE
C          SUBROUTINE VUFAC CALCULATES THE INCIDENT EARTHSHINE
C          RADIATION QES AND THE INCIDENT ALBEDO RADIATION GAL
CALL VUFAC( ALPHA, QES, QAL, MISS, KK )
C          SUBROUTINE POWER CALCULATES THE THEEARTHSHINE AND
C          ALBEDO HEAT ABSORBED BY THE NODE
CALL POWER( ALPHA, QS, QAL, QES, KK, I )
P      ALPHA=ALPHA+DALPHA
2  CONTINUE
PRINT31,APOS,APQES,APGAL
31  FORMAT(/10X, 27HAVERAGE SOLAR HEAT INPUT= F8.4,
1 /10X, 27HAVERAGE EARTHSHINE INPUT= F8.4,
1 /10X, 27HAVERAGE ALBEDO HEAT INPUT= F8.4 )
C
10  CONTINUE
RETURN
YYYYYYY
END

```


LIST 4

```

EGTRAM COMPILER          MARK NO. 205          DATE 28/06/67          TIME 11:40:08
C          SUBROUTINE VUFAC( ALPHA, QES, QAL, MISS, KK )
C          SUBROUTINE CALCULATES THE RADIATION VIEWFACTORS BETWEEN THE EARTH AND
C          ONE SIDE OF A PLATE BY NUMERICAL INTEGRATION OVER THE PORTION OF THE
C          SPHERICAL CAP SEEN BY THE PLATE.
C          THE EARTH CAP IS DIVIDED INTO NF,NP ELEMENTS, NF DIVISIONS ALONG THE
C          LATITUDES AND NP UNEQUAL DIVISIONS ALONG THE LONGITUDES, THESE VALUES
C          BEING SUPPLIED BY THE MAIN PROGRAM VIA THE COMMON LIST.
C          THE ELEMENTS OF THE EARTHSHINE AND ALBEDO VIEWFACTORS ARE STORED IN
C          THE ARRAYS DVFES AND DVFAL RESPECTIVELY. THESE ARE SUMMED BY A
C          THREEDIMENSIONAL SIMPSONS RULE BY THE FUNCTIONSUBROUTINE SUM.
C          THE ANGULAR COORDINATE SYSTEM USED FOR THE POSITION OF THE EARTH ELEMENTS
C          IS FIXED IN THE PLATE WITH FI DEFINED AS THE ANGLE OF ROTATION ABOUT THE
C          EARTH-PLATE LINE, CLOCKWISE LOOKING DOWNWARD MEASURED FROM THE ORBIT
C          PLANE. THE ANGLE PSI BETWEEN THE EARTH-PLATE LINE (LENGTH L ) IS DIVIDED
C          INTO NP-1 EQUAL PARTS, CONSEQUENTLY THE CORRESPONDING ARC LENGTHS ON
C          THE EARTH CAP INCREASE WITH PSI.
C          DA IS THE AREA OF THE ELEMENT OF EARTH CAP PROJECTED ONTO A UNIT SPHERE
C          SURROUNDING THE PLATE.
C
C          DIMENSION DVFAL(NP,NF), DVFES(NP,NF), LN(3), LNT(3), EN(3),
1 PN(3), SN(3), FT(3,3)
C          PUBLIC PN, SN, FT, ESCON, ALCON, PI, DFI, DPSI, NPSI ,
1 NFI, NP, NF, G, H, R
C          REAL LNT, LN ,L
C
C          CLAER ARRAYS
C          DO1 I=1,NP
C          DO1 J=1,NF
C          DVFES(I,J)=0.0
C          DVFAL(I,J)=0.0
1 CONTINUE
C
C          CAL= COSF(ALPHA)
C          SAL= SIN(ALPHA)
C          PSI=0.0
C          COMPUTE OVER VALUES OF PSI
C          DO2 I=1,NP
C          SPSI= SIN(PHI)
C          CPSI= COSF(PHI)
C          CALCULATE LENGTH L OF EARTHELEMENT-PLATE LINE
C          AL=(H+R)*CPSI
C          DESC=AL*AL-H*(H+2.0*R)
13 IF(DESC) 12,12,13
C          L=AL-SORTF(DESC)
C
C          GO TO 14
12 L=SORTF(H*(H+2.0*R) )
14 CONTINUE
C
C          CALCULATE THETA
C          THETA=ASINF(L/R*SPSI)
C          STHETA=SINF(THETA)
C          CTHETA= COSF(THETA)
C          CALCULATE ELEMENT OF AREA
C          DA=SPSI*G
C          SELECT ARRAY ELEMENT TO BE CALCULATED
C          NN=2
C          FI=DFI
C          NNF1=NNFI
3 IF( XMODF(1,2) ) 3,3,4
C          NN=1
C          NNF1=NF
C          FI=0.0
4 CONTINUE
C          DO5 J=NN,NNFI,NN
C          SFI= SINF(FI)
C          CFI=COSF(FI)
C          CALCULATE DIRECTION COSINES OF EARTH-NODE LINE
C          IN STELLAR COORDINATES
C          LN(1)= SPSI*CFI*SAL-CPSI*CAL
C          LN(2)= -SPSI*CFI*CAL-CPSI*SAL
C          LN(3)= SPSI*SFI
C          LN(1)=-LN(1)
C          LN(2)=-LN(2)
C          LN(3)=-LN(3)
C          NMASK=1
C          GO TO(6,7), MISS
C          IF PLATE IS PARTIALLY OBSCURRED BY SATELLITE DETERMINE IF EARTHELEMENT
C          IS SEEN BY TRANSFORMING VECTOR LN TO SATELLITE COORDINATES AND TESTING
C          WITH SUBROUTINE MASK. IF OBSCURRED CONTRIBUTION IS PUT ZERO.
7 CALL COMPO( FT, LN, LNT )
C          CALL MASK( LNT, KK, NMASK )
C          GO TO (6,8), NMASK
6 CONTINUE
C          CALCULATE POSITIVE AREA OF PLATE SEEN BY EARTH ELEMENT.
C          PAP=DOTPRO(LN,PN)
C          PAP=-PAP
C          IF PROJECTED AREA PAP IS NEGATIVE (ONLY BACK OF PLATE
C          IS SEEN ) PUT ELEMENT OF VIEWFACTOR TO ZERO
C          IF(PAP<1.0E-10 ) 8,8,10
C
10 CONTINUE
C          IF POSITIVE CALCULATE ELEMENT OF VIEWFACTOR
C          DVFES(I,J)= PAP*DA
C
C          CALCULATE COMPONENTS OF EARTHELEMENT NORMAL IN
C          STELLAR COORDINATES
C          EN(1)=STHETA*CFI*SAL+CTHETA*CAL
C          EN(2)= -STHETA*CFI*CAL+CTHETA*SAL
C          EN(3)= STHETA*SFI
C          TEST IF ELEMENT IS IN SUNLIGHT OR SHADOW
C          IF THE LATTER PUT ELEMENT OF ALBEDO VIEWFACTOR ZERO
C          IF THE FORMER CALCULATE ELEMENT OF ALBEDO FACTOR
C          SNBYEN=-DOTPRO(SN,EN)
C          IF( SNBYEN<1.0E-10 ) 8,8,11
C          CALCULATE CONTRIBUTION OF ALBEDO VIEWFACTOR
11P DVFAL(I,J)= DVFES(I,J)*SNBYEN
C          INCREASE FI
C          FI=FI+NW*DFI
5 CONTINUE
C          INCREASE PSI
C          PSI=PSI+DPSI
2 CONTINUE
C
C          COMPUTE TOTAL EARTHSHINE INCIDENT ON PLATE
C          QES=ESCON*SUM(DVFES,NP,NF,NPSI,NFI )/PI
C          COMPUTE TOTAL ALBEDO INCIDENT ON PLATE
C          QAL=ALCON*SUM(DVFAL,NP,NF,NPSI,NFI )/PI
C          RETURN
C          YYYYYYY
C          END

```

LIST 5

```

EGTRAN COMPILER      MARK NO. 208      DATE 20/06/67      TIME 11:40:22
      SUBROUTINE HASK(A, KK, N )
C   THIS SUBROUTINE IS PARTICULAR TO A SATELLITE WITH A COAXIAL CYLINDRICAL
C   CAVITY OPEN AT ONE END. IT DETERMINES WHETHER THE VECTOR A, PRODUCED
C   FROM A PRESCRIBED POINT WITHIN THE CAVITY, WILL PASS THROUGH THE OPEN
C   END OR WILL CUT THE CAVITY WALL. IN THE FORMER CASE THE VARIABLE N=1
C   IN THE LATTER N=2. N IS USED AS A CONTROL VARIABLE IN THE CALLING
C   ROUTINE.
C   CYLINDRICAL POLAR COORDINATES ARE USED TO DEFINE THE POSITION OF THE
C   PRESCRIBED POINT, THAT IS RAD, ZETA AND LENGTH, WHERE ZETA IS IMPLICIT
C   IN  $XN=COS(ZETA)$  AND  $ZN=SIN(ZETA)$ . LENGTH IS THE DISTANCE TO THE POINT
C   FROM THE MOUTH OF THE CAVITY.
C   NOTE A IS A UNIT VECTOR.
C
      PUBLIC M
      PUBLIC FT, XN, ZN, LENGTH, RADIUS ,RAD
      DIMENSION FT(3,3), A(3), LENGTH(M) ,RAD(M)
      REAL LENGTH
C
      P      N=2
C   TEST IF A(2) IS POSITIVE, IF SO THE VECTOR A CANNOT ENTER OPEN END.
C   THEREFORE RETURN WITH N=2 .
      IF(A(2)) 1,2,2
1     CONTINUE
C
C   CALCULATE THE DIRECTION COSINES OF THE PROJECTION OF A ON THE XZ PLANE.
      U=A(1)*A(1)+A(3)*A(3)
      P      U=SQRT(U)
      A(1)=A(1)/U
      A(3)=A(3)/U
C   CALCULATE THE COSINE OF THE ANGLE BETWEEN THE PROJECTION OF A ON THE XZ
C   PLANE AND THE VECTOR (XN,ZN)
      COSETA=A(1)*XN+A(3)*ZN
      P      SINETA=SINF(ACOSF(COSETA))
C   CALCULATE THE LENGTH OF THE CHORD FORMED BY PRODUCING THE XZ PROJECTION
C   OF A FROM THE TEST POINT TO THE CYLINDER WALL.
      FROM INPUT DATA
      R2=RADIUS*RADIUS
      P      D= RAD(KK)
      P      C=SQRT( R2-SINETA*SINETA*D*D)+D*COSETA
C
      B=LENGTH(KK)
C   EXTEND VECTOR A, IN THE PLANE CONTAINING C AND A, UNTIL THE WALL IS
C   REACHED AND TEST IF A(2) IS GREATER THAN THE VALUE OF LENGTH, IF SO A
C
      PASSES THROUGH THE MOUTH OF THE CAVITY AND N IS SET TO UNITY.
      IF(C/B+U/A(2)) 2,3,3
3P     N=1
2     RETURN
      YYYYYYYY
      END

```

LIST 6

```

EGTRAN COMPILER      MARK NO. 208      DATE 20/06/67      TIME 11:40:27
      FUNCTION SUM(A,M1,HJ,NI,NJ )
C   SUMS THE VALUES OF A OVER A SQUARE MESH USING A TWO
C   DIMENSIONAL SIMPSONS RULE.
      DIMENSION A(M1,NJ)
      ASUM=0.0
      DO1 I=2,M1,2
      DO1 J=2,NJ,2
      ASUM=ASUM+2*A(I,J)+A(I+1,J)+A(I-1,J)+A(I,J+1)+A(I,J-1)
1     CONTINUE
      SUM=ASUM*2.0/3.0
      RETURN
      YYYYYYYY
      END

```

LIST 7

```

EGTRAN COMPILER      MARK NO. 208      DATE 20/06/67      TIME 11:40:29
      SUBROUTINE POWER( ETA, QS, QAL, QES, KK, I )
C   CALCULATES THE INSTANTANEOUS AND AVERAGE SOLAR, ALBEDO
C   AND EARTHSHINE POWER ABSORBED BY NODE USING THE VALUES OF
C   THE INCIDENT FLUXES CALCULATED IN SUBROUTINE VUFAC.
C   AVERAGE VALUES CALCULATED BY SIMPSONS RULE.
C   INSTANTANEOUS VALUES PRINTED BY SUBROUTINE OUTS .
C
      PUBLIC POWIN, EMI, ALPHA, AREA, M, NORBIT, X ,AN ,P
      DIMENSION APOWIN(M), P(M)
      DIMENSION POWIN(M,NORBIT)
      DIMENSION EMI(M), ALPHA(M), AREA(M)
      PUBLIC APOQS, APOAL, APOES, APOWIN
C
      PQS=QS*AREA(KK)*ALPHA(KK)
      POAL=QAL*AREA(KK)*ALPHA(KK)
      PQES=QES*AREA(KK)*EMI(KK)
      PTOT=PQES+POAL+PQS
      POWIN(KK,I)=POWIN(KK,I) +PTOT
C   PRINT OUTPUT
      CALL OUTS( ETA/X, P(KK), PQS, POAL, PQES, POWIN(KK,I) )
C
C
C   ADD VALUES TO COMPUTE AVERAGE VALUES ROUND ORBIT
C   USING SIMPSONS RULE
      AK=(1.0+XNODF(I+1,2)) *AN*2.0/3.0
      APOQS=APQS+AK*PQS
      APOAL=APOAL+AK*POAL
      APOES=APOES+AK*PQES
      APOWIN(KK)=APOWIN(KK)+POWIN(KK,I)*AK
C
      RETURN
      YYYYYYYY
      END

```


LIST 8

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.34
C      FUNCTION DOTPRO (A,B)
      CALCULATES THE SCALAR PRODUCT OF TWO VECTORS
      DIMENSION A(3),B(3)
      DOTPRO =A(1)*B(1)+A(2)*B(2)+A(3)*B(3)
      RETURN
      YYYYYYYY
      END

```

LIST 9

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.36
C      SUBROUTINE TRANPO(F,FT)
      FINDS THE TRANSPOSE FT OF THE MATRIX F.
      DIMENSION F(3,3), FT(3,3)
      DO 1 I=1,3
      DO1 J=1,3
      P      FT(I,J)=F(J,I)
      1 CONTINUE
      RETURN
      YYYYYYYY
      END

```

LIST 10

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.37
C      SUBROUTINE COMPO(F,A,B)
      CALCULATES THE COMPONENTS B OF THE VECTOR A UNDER THE
      TRANSFORMATION DEFINED BY THE MATRIX F.
      DIMENSION F(3,3), A(3), B(3)
      DO1 I=1,3
      SUM=0.0
      DO2 J=1,3
      SUM=SUM+A(J)*F(I,J)
      2 CONTINUE
      P      B(I)=SUM
      1 CONTINUE
      RETURN
      YYYYYYYY
      END

```

LIST 11

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.39
C      SUBROUTINE TITLE(CASE)
      DIMENSION CASE(40)
      REQUIRES SUBROUTINE TIME
      CALL TIME(IDATE,ITIME)
      PRINT4
      4 FORMAT(1H1)
      PRINT1,IDATE,ITIME
      1 FORMAT(/50X,4HDATE ,A8 , 2X, 4HTIME , A8
      READ2,( CASE(I), I=1,40 )
      2 FORMAT(10A8)
      PRINT3,( CASE(I), I=1,40 )
      3 FORMAT( 30X, 10A8 )
      RETURN
      YYYYYYYY
      END

```

LIST 12

```

ENTRYHAMES TIME *
ROUTINE P2,V6 *
#M3,(TIME),
#N4,(DATE),
SET 114,OUT,
#MOM4,(STORES DATE),
#MON3,(STORES TIME),
EXIT 1,
END,

```

LIST 13

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.46
C      SUBROUTINE INPUT (B,NAME)
      READS THE POSITION AND VALUE OF A ONE DIMENSIONAL ARRAY ELEMENT IN
      FORMAT (12,E13.0)
      C      THE ARRAY DATA MUST BE TERMINATED WITH A CARD HAVING A ZERO OR -VE VALUE
      C      IN FORMAT (12)
      C      NAME IS ANY 8 CHARACTER HOLERITH STRING E.G. CALL INPUT (FA,BHALBEDO )
      DIMENSION B(1),NAME(1)
      K=0
      4 READ 1,I,A
      1 FORMAT (12,E13.0)
      IF(1)6,2,3
      3 B(I)=A
      K=K+1
      GO TO 4
      2 PRINT 5,NAME(1),K
      5 FORMAT (10X,17HNO. OF CARDS FOR AB ,1H=14)
      RETURN
      6 CALL EXIT
      YYYYYYYY
      END

```

LIST 14

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.48
SUBROUTINE RITE1( CASE,TAU,POWIN,M,NORBIT )
  DIMENSION CASE(40), POWIN(M,NORBIT)
  CALL TIME( IDATE, ITIME )
  WRITE TAPE1, IDATE, ITIME
  WRITE TAPE1, M, NORBIT, TAU
  WRITE TAPE1, ( CASE(I), I=1,40 )
  WRITE TAPE1, ( POWIN(I,J), I=1,M), J=1,NORBIT )
RETURN
YYYYYYY
END

```

LIST 15

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.50
SUBROUTINE OUT2(A,NAME,M,NK)
  PRINTS THE POSITIVE ELEMENTS OF A ONE DIMENSIONAL ARRAY
  C THE NAME OF THE ARRAY IS TRANSMITTED BY THE TEXT VARIABLE NAME
  C M IS THE DIMENSION OF ARRAY A
  C PUNCHING IS OPTIONAL, SET NK TO 1 IF NOT REQUIRED, 2 IF REQUIRED
  C REQUIRES THE USEPCODE ROUTINE TIME
  DIMENSION A(100), B(100), KI(100), NAME(1)
  K=0
  DO 1 I=1,M
  IF( A(I) - 1.0E-20 ) 1,1,2
  2 K=K+1
  KI(K)=I
  B(K)=A(I)
  N=K
  1 CONTINUE
  IF(K)8,B,11
  11 PRINT 12,NAME(1)
  12 FORMAT (//50X,17HELEMENTS OF ARRAY2X,AB/)
  PRINT10, N, (KI(I), B(I), I=1,N)
  10 FORMAT(10X,14,/(6(2X,1H( 12, 2H)=, E10.3 )))
  GO TO (8,4),NK
  4 CALL TIME( IDATE, ITIME )
  PUNCH 3,N,(KI(1), B(1), IDATE, ITIME, NAME(1), I=1,N)
  3 FORMAT (12/(12,E13.4,25X,AB,8X,AB,8X,AB))
  8 RETURN
  YYYYYYY
  END

```

LIST 16

```

EGTRAN COMPILER      MARK NO. 208      DATE 28/06/67      TIME 11.40.54
SUBROUTINE OUT5(A,B,C,D,E,F)
  PRINT1,A,B,C,D,E,F
  1 FORMAT(2X,F10.1, 5F10.5 )
RETURN
YYYYYYY
END

```



```

C          PROGRAM TITLE NODEPOWER
C          -----

C          DEFINITION OF INPUT VARIABLES IN DEMANDED ORDER

C          M      =NUMBER OF NODES IN THERMAL MODEL
C          NORBIT= NUMBER OF STATIONS AROUND ORBIT FOR WHICH THE NODE POWER IS TO
C                  BE CALCULATED
C          NPSI  = NUMBER OF SUBDIVISIONS OF PSI OVER EARTHCAP FOR NUMERICAL
C                  INTEGRATION
C          NFI   = NUMBER OF SUBDIVISIONS OF FI OVER EARTHCAP FOR INTEGRATION
C                  ( NPSI AND NFI MUST BE EVEN INTEGERS )

C          SIGMA = STEFANS RADIATION CONSTANT
C          TE     = MEAN BLACK-BODY TEMPERATURE OF EARTH DEGREES KELVIN
C          R      = MEAN RADIUS OF EARTH KILOMETRES
C          RADIUS = RADIUS OF TELESCOPE CAVITY CM.

C          NK     = PUNCH CONTROL PARAMETER
C                  NK=2 PRODUCES PUNCHED CARD OUTPUT OF AVERAGE NODE POWERS
C                  NK=1 SUPPRESSES PUNCHED OUTPUT
C          IRITE  =TAPE CONTROL PARAMETER
C                  IRITE=2 PRODUCES OUTPUT TAPE
C                  OF THE NODE POWER VALUES.
C                  IRITE=1 SUPPRESSES PRODUCTION OF
C                  OUTPUT TAPE
C          H      = ORBIT HEIGHT IN KILOMETRES

C          LAMDA = ANGLE BETWEEN Z AXIS AND SUN VECTOR,DEGREES.
C          NU    = ANGLE BETWEEN X AXIS AND PROJECTION
C                  OF SUN VECTOR ON XY PLANE, DEGREES.

C          (LAMDA AND NU ARE IN STELLAR FRAME OF REFERENCE).

C          P      = NODE INTERNAL POWER SOURCE, WATTS.
C          I      = NODE IDENTIFICATION NUMBER ( 1 TO M )
C          A1     = NAME1(I)      NODE DESCRIPTION
C          A2     = NAME2(I)      NODE DESCRIPTION
C                  ( A1 AND A2 TOGETHER ALLOW 16 CHARACTERS FOR EACH NODE TITLE )
C          A3     =SGAMMA(I) = ANGLE (DEGREES) BETWEEN NODAL-SURFACE
C                  NORMAL AND ZS AXIS
C          A4     =SBETA(I) = ANGLE (DEGREES) BETWEEN XS AND
C                  PROJECTION OF NODAL SURFACE NORMAL
C                  ON XS,YS PLANE.
C                  (SGAMMA, AND SBETA ARE IN
C                  COORDINATE SYSTEM FIXED IN SATELLITE)
C          A5     = EMI(I) = NODE INFRARED EMISSIVITY.
C          A6     = ALPHA(I) = NODE ABSORBTIVITY FOR SOLAR RADIATION
C          A7     = AREA(I) = NODE AREA SQR. CM.
C          A8     = RAD(I) = RADIAL DISTANCE OF TELESCOPE NODE FROM TELESCOPE AXIS, CM
C                  (PUT RAD(I) = -1.0 FOR EXTERNAL NODES)
C          A9     = ZETA(I) = ANGLE OF ROTATION (DEGREES) OF
C                  TELESCOPE-NODE-CENTRE ABOUT AXIS OF
C                  TELESCOPE, MEASURED FROM XS TOWARDS ZS.
C          A10    = LENGTH(I) = AXIAL DISTANCE OF TELESCOPE NODE
C                  CENTRE FROM MOUTH OF TELESCOPE, CM.
C                  (THE VARIABLES, RAD, ZETA, LENGTH,
C                  ARE ONLY SIGNIFICANT FOR TELESCOPE NODES.
C                  FOR OTHER EXTERNAL NODES RAD=-1.0, ZETA=0.0, LENGTH=0.0.
C                  FOR THE FREE FORMAT USED THESE VALUES MUST BE SET )

C          YGAMMA = ANGLE (DEGREES) BETWEEN YS(SATELLITE AXIS )
C                  AND Y( STELLAR COORDS )
C          OMEGA  = ANGLE (DEGREES) BETWEEN XS (SATELLITE
C                  COORDINATE AXIS) AND X (STELLAR COORDINATE AXIS)
C                  (THE SATELLITE AXIS XS IS ASSUMED TO
C                  ALWAYS LIE IN THE XZ PLANE OF THE STELLAR FRAME.
C                  THE POSITIVE DIRECTION OF OMEGA IS TOWARDS THE Z AXIS )

C          DATA LIST IN ORDER DEMANDED
C          -----

C          PRELUDE
C          -----
C          READ          FORMAT
C          ----          ----
C          M,NORBIT,NPSI,NFI      (80Z)

C          MAIN
C          ----
C          READ          FORMAT
C          SIGMA,TE,R,RADIUS      (80Z)

C          4 CARDS FOR CASE TITLE MUST BE
C          SUPPLIED EVEN IF BLANK.

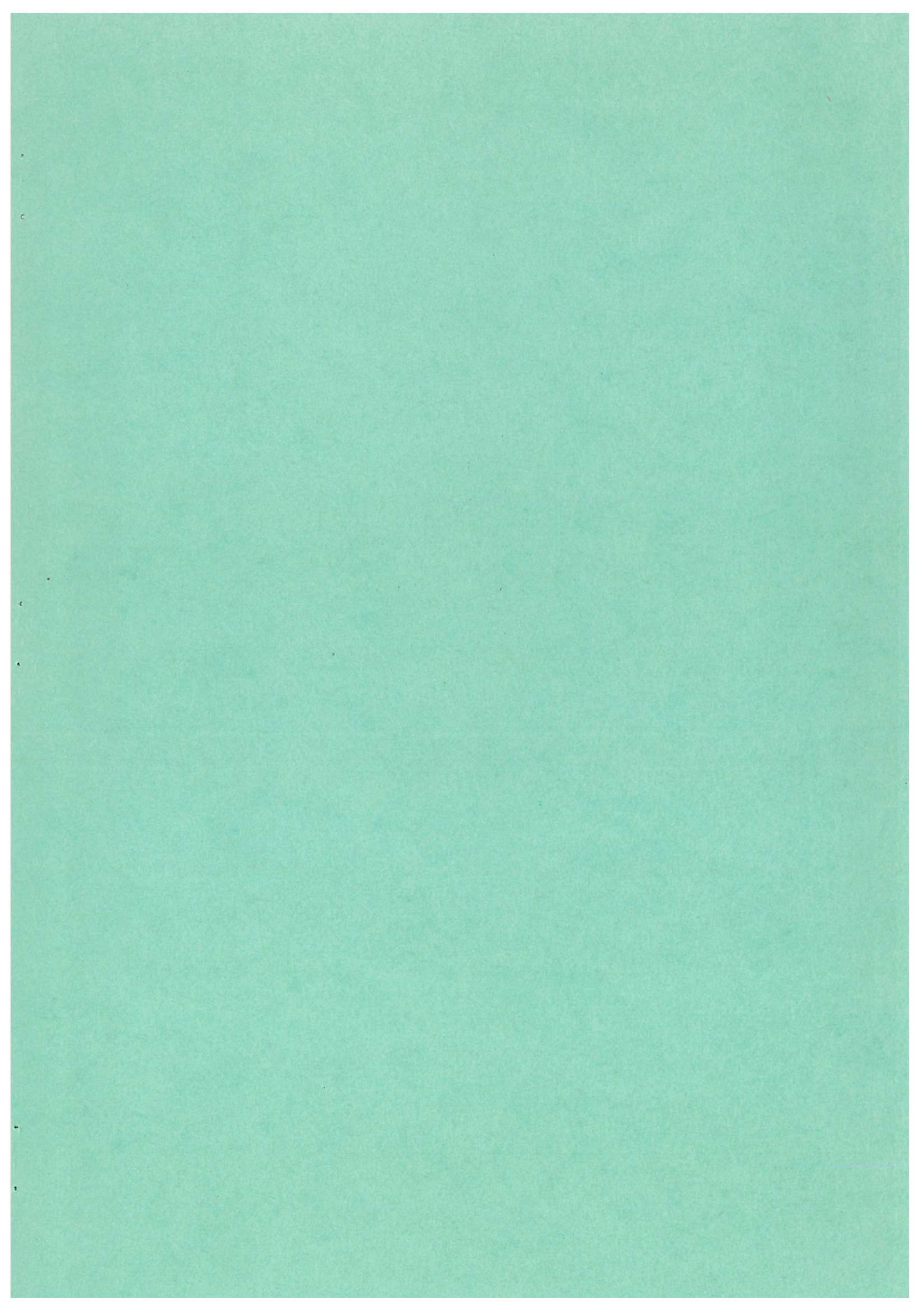
C          NK,IRITE      (80Z)

C          H,LAMDA,NU    (80Z)

C          I,P(I)      (12,E13.4)
C          (THIS WILL BE A SEQUENCE OF CARDS, EACH HAVING ONE
C          NODE POWER P(I), AND THE CORRESPONDING
C          VALUE OF I. THE LAST CARD MUST BE BLANK)
C          I,A1,A2,A0,A4,A5,A6,A7,A8,A9,A10,      ( 14,2A8,60Z )
C          (A SEQUENCE OF CARDS, ONE TO EACH NODE, THE
C          LAST CARD HAVING I=0 AND
C          ZERO VALUES FOR VARIABLES A3 TO A10)

C          YGAMMA,OMEGA      (80Z)
C          (A SEQUENCE OF CARDS EACH CONTAINING
C          A PAIR OF VALUES CAN BE SUPPLIED. THE
C          LAST CARD SHOULD BE YGAMMA = -1.0.      OMEGA = 0.0)

```

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