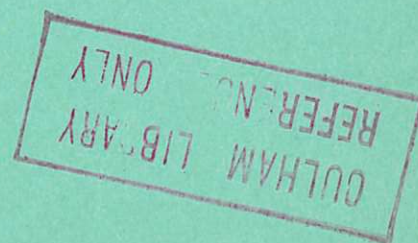United Kingdom Atomic Energy Authority

RESEARCH GROUP

Report

THERMONUCLEAR REACTORS BASED ON MIRROR MACHINE CONFINEMENT

A report from the Culham Mirror Study Group, 1967-1968

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A report from the Culham Mirror Study Group, 1967-1968

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C O N T E N T S

A P P E N D I C E S

1. INTRODUCTION

Research into the confinement of plasma by adiabatic magnetic mirror machines has been actively pursued for some seventeen years, and from this work much has been learned of the principles which must underlie successful confinement of plasma by magnetic fields. Present experimental work is largely concerned with understanding and overcoming the micro-instabilities which at present limit confinement in many mirror machine experiments. However, even if there were no impairment of confinement by micro-instabilities, it is by no means clear that the loss of plasma due to scattering into the loss cone would be sufficiently small to permit a thermonuclear reactor based on mirror confinement to operate. The Study Group was set up to examine this and other questions related to the long-term mirror machine programme at Culham. The present report deals essentially with two questions:

  (a)   the magnitude of the scattering loss and its relation to reactor economics;

  (b)   the possible reduction of this loss by auxiliary mechanisms.

2. REACTOR FEASIBILITY

In assessing the feasibility of a thermonuclear reactor, it is necessary to make considerable extrapolations beyond existing knowledge. In part, these extrapolations are of a technological nature - it is, for example, necessary to estimate the efficiency with which it will be possible to carry out various processes in the reactor power cycle (e.g. injection of plasma, conversion of thermal neutron energy to electricity, perhaps the direct conversion of charged particle energy into electrical power), and to estimate the maximum magnetic fields that superconductors will produce. To allow for technological progress we have made relatively optimistic assumptions about such factors - for example, the efficiency of a steam cycle has been taken to be 50%, and a peak magnetic mirror field of 150 kG has been assumed feasible.

It is also necessary to make physical extrapolations in view of the approximate nature of all existing energy balance calculations. In discussing this balance, it is conventional to define a quantity Q given by

$$Q = \frac{\text{Rate of production of fusion energy}}{\text{Rate of energy input to the plasma}} \ .$$

In Appendix I we review the calculations of this quantity which have been published to date. The results vary considerably: for example Post[1] and Fowler and Rankin[23,24] give estimates of the maximum value of Q (i.e. optimised with respect to the injection energy) of

$$Q_{Post} = 11 \ \log_{10}R$$

$$Q_{FR} \ = 3 \cdot 1 \ \log_{10}R$$

where R is the mirror ratio. On examining the physical assumptions and mathematical approximations underlying these calculations, we have concluded that neither of these expressions can claim to be the definitive result. The physical assumptions of Post (in particular his neglect of the role played by the electrostatic potential of the plasma, and of the cooling of ions by electrons) led him to underestimate the particle loss rate. On the other hand, the mathematical approximations used by Fowler and Rankin

may possibly exaggerate the importance of these effects. A further source of uncertainty is the role played by the alpha-particles produced by the thermonuclear reactions: a recent paper of Rose[17] has shown that alpha-particle heating of the plasma can lead to a substantial improvement in Q provided that the basic particle confinement times are sufficiently long. It follows from Rose's work that if the true particle loss rate were close to Post's value, alpha-particle heating would be an important favourable effect, whereas if the true loss rate were close to that of Fowler and Rankin, alpha-particle heating would be negligible. It is therefore necessary to know the particle loss rate to within a factor of about 2 in order to calculate Q with reasonable precision, and it is doubtful whether any calculation to date can legitimately claim such accuracy.

Faced with this situation, we have concluded that it is premature to make a final assessment of the feasibility of a mirror reactor. Our procedure has therefore been to take the Q-values of Post and Fowler and Rankin as two points of reference in the spectrum of possible values, and to investigate the economic and technological problems of mirror reactor design which would arise in each case.

In what follows, we begin by making a set of basic assumptions about the main physical and technological features of a mirror reactor: although these are not the only possible ones, they provide a basis for discussion. We then set up a model of the reactor which reproduces the main features of the interaction between the physical, technological and economic aspects of its design.

THE BASIC ASSUMPTIONS

We assume that :

(a)   A static magnetic field in some form of magnetic mirror is used to confine the plasma.

(b)   There are no macroscopic instabilities (in this connection we note that high values of beta are theoretically permissible in well-geometry) and that any micro-instabilities which remain have a negligible effect on the particle loss rate.

(c)   There is no loss of energy from the plasma due to instabilities or electro-magnetic radiation.

(d)   A D-T reactor (50-50 mixture) is envisaged in which tritium is bred in a lithium blanket, yielding a total energy release of 22.4 MeV per nuclear reaction in the plasma.

(e)   The conversion efficiency from heat to electricity is 50%.

(f)   The energy required to maintain the confining magnetic field and auxiliary equipment is negligible (less than a few percent of the gross output).

(g)   The maximum magnetic field (in the mirror region) is restricted to 150 kG so as to be within possible range of superconducting magnets.

THE MODEL

In our basic model of a reactor (described in detail in <u>Appendix II</u>), we have assumed that the energy released in neutrons is converted to electricity by a conventional thermal cycle (efficiency η_T taken equal to 0.5), whilst the energy of the charged particles (reaction products plus escaping plasma) is converted to useful electrical power at a (possibly higher) efficiency η_D. Some of this latter output is used to provide the power for the injection source, which is assumed to operate at an efficiency η_S. (If a large fraction of the total output power needs to be circulated, then clearly, in addition, some of the power from the neutrons must be used to drive the injector.) The model has been extended to try to take into account, in an idealised way, the effects of 'end-stoppers', devised to reduce the normal binary scattering losses through the mirrors.

On the basis of this model we have examined the implications of various estimates of the attainable Q of a mirror reactor. Clearly there is a minimum value of Q below which it is not possible to make a self-sustaining reactor at all; when Q exceeds this value, the economic cost of the power produced depends critically upon two factors:

(i)   The fraction of the gross power output which must be fed back in order to sustain the reactor - the circulating power. (This obviously falls as Q increases.)

(ii) The maximum mirror ratio (R) achievable, consistent with a reasonable power production per unit volume in the plasma.

In present power stations it is generally regarded as uneconomic to circulate more than 10% of the output power; but this arises chiefly from the capital cost of the plant required. We have not restricted ourselves to this 10%, but have also considered larger circulating powers, and have derived a relationship between the efficiency of the circulating loop and its permissible capital cost. To do this, we have supposed (following reference 27) that, in the absence of a circulating loop, the capital cost of a fusion reactor would be 16% cheaper than that of a fast fission reactor of equal net output; this margin has been assumed to be available for the loop and any other devices considered – such as end-stoppers. This assumption must be regarded only as an illustration of the cost restrictions which might be encountered.

CONCLUSIONS

(a)   Taking the Fowler and Rankin calculations of Q, and a mirror ratio R = 10, a self-sustaining reactor can be devised provided that the product $\eta_S \eta_D$ exceeds about 0.6, where η_S is the efficiency with which input electrical energy is converted into particle energy of the injected fuel; and η_D is the efficiency with which fusion produced charged particle energy is converted into output electrical energy. However, even if these conditions are well satisfied a large fraction G of the gross output power must be circulated: typically, with the Fowler-Rankin Q, if η_S = 90% and η_D = 80% then G = 45%, (and even if $\eta_S = \eta_D$ = 90% then G = 40%). If Post's Q value is used, then G = 16% when $\eta_S \eta_D$ = 0.7.

(b)   The economically permissible capital costs of the injector and direct output converter are governed by η_S, η_D and Q_{max}. Assuming particles with energies about 200 keV

(around which there is a broad maximum in Q) and a mirror ratio $R = 10$, the values of Q range from 3·1 to 11 according to whether the Fowler-Rankin or the Post expression is taken. If $\eta_S = \eta_D = 90\%$ and $Q = 3·1$, then the capital costs of the injector and output converter must each be kept below £2 per kW(e) output. On the other hand: if $\eta_S = \eta_D = 90\%$ and $Q = 11$, then the capital costs of the injector and output converter may each be permitted to rise to £20 per kW(e) output.

It should be noted that this choice of mirror ratio, combined with the maximum field available from superconductors and the need to maintain an economic power density, implies a high plasma pressure. For example with $B_{max} = 150$ kG and the above parameters $\beta = nkT/(nkT + \frac{B^2}{8\pi}) = 0.75$ at the centre of the well.

In assessing the significance of these capital cost figures it is relevant to consider the present day cost of conventional boiler-turbine equipment, which is of broadly comparable technical complexity. This approaches £30 per kW(e) of output. The upper limit calculated on the basis of Fowler and Rankin's Q is, to say the least, uncomfortably small. On the other hand the upper limit calculated on the basis of Post's Q is perhaps acceptable. Thus the uncertainties which surround the calculation of Q, discussed in Appendix I, are sufficient to make it impossible at the present time to evaluate definitively the economic prospects of a simple mirror reactor.

(c) Strictly speaking, the model described above is not generally applicable to a mirror reactor which incorporates 'end-stoppers'. However, an indication of the role which these might play is given by the following considerations. In evaluating the potential importance of end-stoppers it is necessary to take into account both the improvement in Q which they might effect, and the capital cost and energetic efficiency of the stopper. The relationship between the costs and efficiencies of the end-stoppers and the injection and output conversion equipment are complicated. However, assuming that Q is improved to $30 \log_{10}R$, then if η_S is 90% and η_D and end-stopper efficiencies are each 80%, the capital cost of each must not exceed about £10 per kW output.

(d) Finally, although it lies somewhat outside the general scope of this report, one paper on the feasibility of a fusion-fission reactor should be mentioned. Lontai[28] shows that a U^{238} blanket arranged to ensure adequate tritium production gives an optimum heat output about twice that of a non-fissile blanket, plutonium being produced at the rate of one atom per 5 D-T neutrons, and the fission of this plutonium, in some other device, could produce an overall heat output four times greater than a D-T-Li system.

3. REDUCTION OF END LOSSES BY AUXILIARY MEANS

The conclusion reached in the first part of this report is that although it may be possible to design a thermonuclear reactor based upon simple magnetic mirror plasma confinement, any auxiliary device which could improve the plasma confinement time without causing a substantial increase in the capital cost would be welcome. In this context we have examined a number of possible new developments – in particular the use of multiple mirrors or alternating fields in the mirror regions. Our conclusions are described in detail in Appendices III to V.

Appendix III - deals with the effect of supplementing magnetic mirrors by RF fields resonant with the ion cyclotron frequency.   It shows that RF 'end-stoppers' driven in exact resonance with the ion cyclotron frequency would not reduce scattering losses sufficiently because the energy of each particle is increased each time it is reflected by such a system.

Appendix IV - describes the properties of a system of multiple mirrors, in which large fractional improvements are possible only when the initial containment is very poor.   The absolute increase in containment time obtained by this method is only about 2R times that corresponding to the free flow escape time.

The application of RF fields at frequencies other than the ion cyclotron frequency has been examined in Appendix V, and by Watson [29].   It is shown there that, in general, non-resonant RF fields would have to produce a radiation pressure comparable with the central plasma pressure and the power needed for such 'end-stoppers' would be excessive. However, a particular method of overcoming this drawback is suggested, in which the RF fields are tuned to be nearly resonant with the ion cyclotron frequency.   By this method the magnetic moment of the ions is reversibly changed so that, in the absence of collisional scattering, no net power is absorbed by the plasma.   Because the magnetic moment of the ions increases within the mirror throats, most of the plasma pressure is sustained by the mirror magnetic field.   It is estimated that by this means a nearly resonant RF field could make a large contribution to containment whilst having to support rather less than 1/10 of the plasma pressure.   The irreversible non-adiabatic effects have not been included in this assessment.

The conclusion is that 'nearly resonant' RF fields appear, at this stage, to offer the best means of reducing end losses from mirror systems and thus of improving the prospect of using mirror confinement as the basis of a reactor.

THE RELIABILITY OF MIRROR MACHINE ENERGY BALANCE CALCULATIONS

(B. McNamara and C.J.H. Watson)

1. INTRODUCTION

The energy balance in a mirror machine results from the competition of two processes -
the scattering of particles into the loss cone and the thermonuclear reactions.   In the
regime of interest, both of these processes occur at rates which are sensitive to small
changes in the physical assumptions made in calculating them.   It is therefore not sur-
prising that the literature should contain a number of conflicting conclusions (e.g.
Post[1] , Sivukhin[2]) about the maximum value of Q attainable in a mirror reactor operating
under optimal conditions.   It is therefore necessary to consider the assumptions underly-
each calculation in some detail.

2. DISCUSSION

All calculations to date start from some form of Fokker-Planck equation i.e. from an
equation of the form

$$\frac{\partial}{\partial t} f_i + \vec{v} \cdot \frac{\partial f_i}{d\vec{x}} + \frac{e_i}{m_i} \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_i}{\partial \vec{v}} = -\frac{\partial}{\partial \vec{v}} (f_i \vec{V}_i) + \frac{1}{2} \frac{\partial}{\partial \vec{v}} \frac{\partial}{\partial \vec{v}} : (\vec{\vec{D}}_i f_i) \qquad \dots (AI.1)$$

where f_i is the distribution function of the ith charged species, and the vector $\vec{V}_i$
and the tensor $\vec{\vec{D}}_i$ are the friction and diffusion coefficients for that species due to
particle interactions.   The precise functional form of the friction and diffusion coeffi-
cients depends upon the starting point from which equation (AI.1) is derived. Rosenbluth,
MacDonald and Judd[3] , following Landau[4] , start from the Boltzmann equation. This approach
rests upon the physical assumption that the dominant scattering process is due to binary
collision.   In the case of a plasma with many particles in the Debye sphere, this assump-
tion is incorrect, and it leads to an expression for the Fokker-Planck coefficient, in the
form of a logarithmically divergent integral, which RMJ 'cut off' by appealing to the idea
of Debye screening.   An alternative approach starts from the relatively secure foundation
of Liouville's equation, and derives a Fokker-Planck equation commonly known as the Balescu-
Lenard (B-L) equation, by means of the cluster expansion (Mayer[5] , Bogolyubov[6] , Rostoker
and Rosenbluth[7] , Balescu[8] , Lenard[9]).   The coefficients obtained in this way differ from
the RMJ coefficients in that they depend upon integrals involving the dielectric coefficient
of the plasma i.e. they include collective effects which are ignored by the RMJ approach.
The importance of these effects has not yet been fully explored.   For an unmagnetised
plasma in thermal equilibrium, it can be shown (e.g. Montgomery and Tidman[10]) that the B-L
equation can be reduced to the RMJ equation apart from terms of order $1/\ell n \Lambda$, where Λ is
the number of particles in a Debye sphere.   In the case of a magnetised plasma, Rostoker
and Rosenbluth[11] have examined the coefficients for a test particle in an equilibrium
plasma and shown that the RMJ coefficients are recovered if the ion and electron Larmor

radii, (a_i, a_e) are large compared with the Debye length L_D. This assumption is unfortunately not necessarily valid for a thermonuclear plasma (e.g. if $n = 10^{14}$ cm^{-3} and $B = 40$ kG, $L_D = a_e \ll a_i$), so the RMJ coefficients may over-estimate the Fokker-Planck coefficients for electrons, although the asymptotic expressions in reference 11 suggest that the error should be small unless $\ell n(L_D/a_e) \gg 1$. Rostoker[12] has also examined the effect of non-thermal distribution functions, and shown that as the threshold of an instability is approached, deviations from the RMJ coefficients become important. In view of the prevalence of micro-instabilities in existing mirror machines, this possibility needs to be borne in mind.

It may be remarked at this point that even the Liouville equation (in its classical form) omits certain physical effects which may play a significant role under thermonuclear conditions. Its neglect of quantum effects, though certainly justifiable for the ions, is probably incorrect for the electrons. Both theoretically and experimentally (Williams[13]) quantum effects reduce the scattering of electrons by the screened Coulomb potential of an individual atom, and it can be inferred from the quantum kinetic equation of Silin[14] that the same effect should occur in a plasma. This leads to the replacement of Λ by $\frac{c\Lambda}{137V}$, where V is a typical electron velocity, and hence decreases $\ell n\Lambda$ by about 4; i.e. the dynamic friction of the electrons would be reduced by about 20%

The classical Liouville equation also neglects relativistic effects, which may again be significant. The two main relativistic effects are the contribution of transverse interactions between particles to their scattering, and the synchrotron radiation of electrons in the magnetic field. The former effect has been examined by Silin[14] and Harris[15] and is probably unimportant. The latter effect prevents the electron temperature from rising above about 50 keV and consequently indirectly affects the plasma potential and the cooling of ions by the electrons.

We conclude that in order to use the RMJ Fokker-Planck equation, it is necessary to ignore:

(i)   the influence of the magnetic mirror field on electron-electron and electron-ion interactions;

(ii)   all collective processes in the plasma except Debye screening;

(iii)   relativistic and quantum effects.

We shall now consider the literature to date on the scattering losses from a mirror machine, assuming that the RMJ Fokker-Planck equation adequately describes these losses. Since this equation is a set of coupled non-linear partial integro-differential equations (one for each species of particle) also coupled to Poisson's equation for the electrostatic field, it is clear that many further approximations are required and we shall see in what follows that four further assumptions have generally been made. The first step is to eliminate all three spatial variables from the problem. This is possible because diffusion across the field lines is negligible compared with the loss along field lines so (apart from a form factor which could probably be made close to unity by careful control of the plasma and magnetic profiles in the perpendicular direction), it is sufficient to compute the loss rate along the central flux tube. Since the plasma density and the loss-cone angle both vary with distance S along this flux tube, it is still necessary to consider the profiles of the magnetic and electrostatic fields as a function of S. This question

has been partially examined by BenDaniel and Allis[18] who show that for a single species plasma in a magnetic mirror field of arbitrary shape, the reduction in scattering rate due to the decrease in plasma density in the mirror region is almost exactly cancelled by the increase due to the wider loss cone angle, so in good approximation the actual profiles can be replaced by a 'square-well' profile in which $B = B_0$ (the central value) everywhere up to the mirrors, at which it discontinuously jumps to $B_m = RB_0$ (where R is the mirror ratio). In all calculations to date it has been supposed that if there is an electrostatic potential it is permissible to assume that it likewise jumps by an amount φ at the same point in space. Even if the peak of the electrostatic potential does in fact coincide with that of the mirror field this assumption may over-estimate the importance of φ. However, it should also be noted that various suggestions have been made of ways to elimi- nate the harmful effect of the electrostatic potential on plasma confinement by displacing the peak of φ away from the mirror region (for example Kelley[19]), so the results des- cribed below depend upon the assumption:

(iv) that effects which depend upon the spatial profile of the electrostatic field can be neglected, and that no attempt has been made to displace this profile.

The next step is to write the Fokker-Planck equation in spherical polar co-ordinates (v, θ, φ) in velocity space and eliminate the angular variables. The φ dependence dis- appears by symmetry in an axisymmetric system (though it has been suggested by Sadowski[20] that there is an advantage in large departures from this); the θ-dependence requires more care, however. For a single-species plasma it is now well established that in good approxi- mation the distribution function $f(v,\theta)$ can be factorised $f(v,\theta) = F(v)\,G(\theta)$ provided that the injection current is likewise factorisable. This leads to independent equations for F and G. The validity of this approximation has been examined numerically by Bing and Roberts[21] who solved the Fokker-Planck equation for a mirror machine with $R = 3\cdot3$ both directly and in the above approximation. Although they reported a discrepancy amounting to a factor of 2 in the loss rates, this was subsequently explained by BenDaniel and Allis[18] who showed that it was due to an over-simplification which Bing and Roberts made in applying the separability approximation, and that the actual error was only about 7%. This approach leads immediately to an expression for the loss rate which (for $R \gtrsim 3$) is proportional to $1/\log R$. It should be remarked that some rather unprecise calculations of Budker[22] indi- cate that a small reduction in the loss rate is obtained if the injection current is exactly perpendicular to the magnetic field, the mirror ratio R being effectively increased by a factor of $1\cdot8$.

When one takes into account the existence of an ambipolar potential φ in a plasma of two or more species, the use of the factorisation approximation is more questionable, and it is at this point that the methods of Post[1] and Fowler and Rankin[23,24] diverge. Post, following Bing and Roberts[21], argues that for a plasma in which the electron temperature is significantly less than the ion temperature, the effect of the ambipolar potential can be neglected, except insofar as it requires the electron loss rate to equal the ion loss rate, and that it is sufficient to calculate the ion loss rate using a one-species Fokker- Planck equation. Fowler and Rankin however, show that the ambipolar potential should be retained in the calculation because of the enhanced loss of slow ions which it should cause. They assume:

(v) that in calculating the loss rate for a given species of energy ε, one should use the analytic expression obtained by means of the separability assumption, but replace the actual mirror ratio R by an effective mirror ratio $R_{eff} = R/(1 - e\varphi/\varepsilon)$.

Since R_{eff} is energy dependent, it is clear that this assumption is not internally consistent, and it is not easy to estimate the error which it may introduce. Fowler and Rankin claim no more for it than that it gives a more realistic representation of the effect of the potential than that of Kaufman[25], upon which Post's assessment was based. It is difficult at the present time to justify this assumption as accurate to within a factor of less than 2.

The problem has now been reduced to a set of coupled non-linear ordinary integro-differential equations (one for each species), and is therefore within the scope of existing computers. However it has not yet been tackled in its full form. As we have seen, Bing and Roberts considered only a single species. Fowler and Rankin used a computer code which allowed them to consider two species at a time. In relating their results to the actual problem it is necessary to assume:

(vi) that the two fuel species can be replaced by a single equivalent ionic species (taken by Fowler and Rankin to have the mass of the deuteron).

(vii) that it is adequate to use a two-species code to examine successively electron cooling and alpha-particle heating, but not both together.

On the basis of assumptions (vi) and (vii), Fowler and Rankin conclude that when the ambipolar diffusion and electron cooling effects are taken into account the mean particle lifetime in the reactor is a factor of 8 smaller than Post's value. They also conclude that for an $R = 3$ mirror the alpha-particle heating effect is small, and not capable of sustaining the reaction.

Assumption (vi) introduces two kinds of error - a direct error in the loss rate, since the loss-rate coefficient is proportional to $1/\sqrt{m}$, and an indirect error, since it leads to incorrect distribution functions and in consequence affects the loss rate and the thermonuclear reaction rate. The direct error is small; the indirect error is difficult to assess.

Assumption (vii) has been criticised by Rose[17] on the grounds that the heating of the electrons by alpha-particles (and the consequent reduction in the cooling of ions) may be a significant, favourable process in a mirror reactor. Although Rose's calculations are in some respect less accurate than those of Fowler and Rankin (e.g. he assumes Maxwellian distributions in calculating reaction rates), his calculations show (see for example his Fig.3-25) that the importance of alpha-particle heating depends sensitively upon the plasma confinement time. He defines a parameter $\psi = \tau_{90}/\tau_{FR}$ where τ_{90} is the lifetime of an injected ion, computed on the assumption that it is lost after one classical 90^0 scattering time, and τ_{FR} is the actual mean diffusional lifetime. For $\psi \gtrsim 0.3$, alpha-particle heating is unimportant: for $\psi \lesssim 0.1$ there are regions of parameter space where it is important, and for $\psi < 0.05$ the reaction can become self-sustaining. If we compute ψ on the basis of Fowler and Rankin and Post's values of τ we obtain (estimating that the average energy equals the injected energy in both cases) for $R = 3$

$$\psi_{FR} = 1.4$$
$$\psi_{Post} = 0.19 \ .$$

For $R = 10$ these figures would both be reduced by a factor of 2. Thus although the calculations of Rose are not sufficiently accurate to permit firm conclusions to be drawn, they suggest:

(a)   that if Post's value of $n\tau$ were achievable, the maximum value of Q might be substantially better than his optimum value;

(b)   that if the true value of $n\tau$ lies between those of Fowler and Rankin and Post, the alpha-particle heating may take Q up to a value around that of Post;

(c)   that if Fowler and Rankin's value of $n\tau$ is correct, then alpha-particle heating is unlikely to play an important role.

In short, the discrepancy between the $n\tau$ values of Fowler and Rankin and Post shows that this quantity depends sensitively upon the physical assumptions made in calculating it, and Rose's calculations show that the importance of alpha-particle heating depends sensitively upon $n\tau$. In these circumstances, the uncertainties which surround the assumptions (i) to (vii) underlying the Fowler-Rankin calculation seem sufficient to make it impossible to draw any final conclusions from its outcome. Its value lies more in the emphasis which it puts on the potential importance of certain unfavourable effects which were ignored in Post's calculation.

In view of the uncomfortably delicate balance between the energy production and loss rates in a mirror reactor, the unfavourable factor of 4 in Q which Fowler and Rankin finally obtain is clearly significant, and it is therefore a matter of some importance whether their numerical procedures (as opposed to their physical assumptions) were correct. This question has been examined by Fisher, McNamara and Mason[26] who used their independent Fokker-Planck programme (which incorporates the same physical assumptions) to approach the equilibrium described by Fowler and Rankin, and obtained good agreement with their $n\tau$ values. It is not clear from Fowler and Rankin's paper how the thermonuclear reaction rates were calculated (e.g. what allowance, if any, was made for centre of mass and anisotropy effects) and this aspect of their calculations has not yet been checked.

3. CONCLUSIONS

The energy balance calculations of Post were a reasonable first estimate. Fowler and Rankin introduced two important additional physical effects - electron cooling and the plasma potential - and found a much lower net gain from a thermonuclear mirror machine. For the short confinement times found by Fowler and Rankin the effects of alpha-particle heating are small; however, simple calculations by Rose show that the beneficial effects of alpha-particle heating increase very rapidly with containment time. An examination of the basic RMJ Fokker-Planck equations reveals many small beneficial effects which in concert could readily increase Fowler and Rankin's containment times by a factor of 2.

The various effects considered here are listed in Table I with an estimate of the size of the effect and some remarks on the results of each effect. The experimental evidence in favour of the RMJ equations is not sufficient to determine the importance of these corrections.

More detailed calculations are being performed by K. Marx of Livermore to investigate the effects of spatial variation of plasma potential, the separability assumption, and the use of an effective mirror ratio. Other calculations by G. Kuo-Petravic, M. Petravic and C.J.H. Watson are in progress on the role of alpha-particle heating and more exact

calculations of the thermonuclear reaction rates.

Until these results are available it is only possible to conclude that the Q-values of Post ($Q = 11 \log_{10}R$) and Fowler and Rankin ($Q = 3.1 \log_{10}R$) should be regarded as points of reference. To be specific, if Fowler and Rankin's Q without alpha-particle heating were raised to about $7 \log_{10}R$ by a better theoretical treatment than the RMJ equation, then Rose's calculations indicate that alpha-particle heating could then yield a $Q \approx 15 \log_{10}R$.

ACKNOWLEDGEMENTS

TABLE I

Effect	Estimate of size	Remarks
Collective effects	$\pm O(1/\ell n \Lambda)$	Depends on distribution function[12]: small for Maxwellian. Ramazashvili et al[16].
Electron gyro motion	$+ O(\ell n \lambda_D / a_e)$	Decreases electron scattering rate. Rostoker and Rosenbluth[11].
Quantum effects for hot electrons ($T_e > 100$ eV)	$+ O(20\%)$	Decreases electron scattering rate. Silin[14].
Transverse field interactions	Negligible	Harris[15], Silin[14].
Synchrotron radiation	$O(20\%)$ increase in electron energy loss	Impossible to decide whether decreased plasma potential or increased ion cooling more important. Electron temperature limited to 50 keV. Rose[17].
Spatial variation of plasma potential	$\pm O(10\%)$	Kelley's proposal could make a much larger change.
Separability of $f(v,\theta)$	$\pm O(7\%)$	This estimate based on the single species calculations of BenDaniel and Allis; the assumption may make a larger error in a multi-species system.
Effective mirror ratio	$\pm (200\%)$	Fowler Rankin model is physically reasonable and gives correct sign and order of magnitude.
Single ion species	$+ O(10\%)$	The Fokker-Planck coefficients are proportional to $1/\sqrt{m}$.
Neglect of trapped alpha-particles	$+ O(1\% - 200\%)$	Could have dramatic effect if containment already good. Rose[17].
Thermonuclear reaction rates	$\pm 20\%$	Substantial discrepancies in existing literature; Roberts and Carr[35], Rose[17], Fowler and Rankin[24].

THE FEASIBILITY OF A THERMONUCLEAR REACTOR
USING THE MIRROR MACHINE PRINCIPLE

(D.W. Mason and G. Francis)

1. INTRODUCTION

In this Appendix the prospects of producing useful power from the fusion of light nuclei in a mirror machine are examined. We consider first in Section 2 the general requirements for the feasibility of any reactor, then in Section 3 their application to mirror confinement; the probable costs of a mirror reactor are estimated in Section 4, and the economic consequences of attempts to reduce the end losses are examined in Section 5. Conclusions are contained in Section 6.

2. BASIC REQUIREMENTS

Before discussing the mirror machine reactor some general results applicable to any fusion reactor/ are derived.

Definition of Q

It is assumed that the reactor operates in a steady state and that a fraction φ of the injected fuel undergoes fusion before it escapes from the containment region. The quantity Q, defined by

$$Q = \frac{\text{Rate of production of fusion energy}}{\text{Energy input rate to plasma}} = \frac{\varphi \bar{W}}{E} \qquad \ldots \text{(AII.1)}$$

where $\bar{W}$ is the mean energy released per fusion and E, the energy of the injected fuel, is a measure of the energy gain of the reactor.

Conversion of fusion energy to electrical energy

From the definition of Q the total energy output per second W_T is given by

$$W_T = I.E.(Q + 1) \qquad \ldots \text{(AII.2)}$$

where I is the rate at which fuel is injected into the reactor. If h represents the fraction of the nuclear energy output that resides in the neutrons, then the power output W_n from them is

$$W_n = I.E.\ hQ \qquad \ldots \text{(AII.3)}$$

and the charged particle power output, assuming no cooling of the fuel in the reactor*, is given by

$$W_D = I.E.\ (1 + (1-h)Q) \qquad \ldots \text{(AII.4)}$$

In order to change the neutron power W_n into electrical power, thermal conversion must be employed. For the charged particle power W_D however it may be assumed that some direct conversion process is used. Let the efficiencies of these two processes be η_T and

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/ Reactors in which the fuel is heated by fusion reaction products, rather than by the external application of power, are not, however, included in this discussion.

* i.e. ignoring cyclotron radiation losses, bremsstrahlung etc.

η_D respectively. Then for a net power output:

$$[\eta_T \, I \, E \, h \, Q + \eta_D \, I \, E(1 + (1-h)Q)](1-f) - \frac{IE}{\eta_s} > 0 \qquad \ldots (AII.5)$$

where f is the fraction of the gross electrical output that is required to operate the reactor and η_s is the efficiency with which electrical energy is converted into particle energy in the injected fuel.

Writing
$$\eta_T \, h \equiv \eta_T'$$
$$\eta_D(1-h) \equiv \eta_D'$$

and assuming $f \ll 1$ Eq. (AII.5) may be rearranged to give

$$\boxed{Q > \left(\frac{1}{\eta_T' + \eta_D'}\right)\left(\frac{1 - \eta_D \eta_s}{\eta_s}\right)} \qquad \ldots (AII.6)$$

This inequality must be satisfied if a net power output is to be obtained.

Circulating Power

From Eq. (AII.5) the net electrical output power P_{net} is

$$P_{net} = I \, E \left\{ \left[(\eta_T' + \eta_D') \, Q + \eta_D\right] (1-f) - \frac{1}{\eta_s}\right\}$$

and the circulating power P_c is

$$P_c = I \, E \left\{ \left[(\eta_T' + \eta_D') \, Q + \eta_D\right] f + \frac{1}{\eta_s}\right\}.$$

If $f \ll 1$ then the ratio G, defined by

$$G = \frac{circulating \ power}{gross \ output \ power}$$

is given by

$$\boxed{G = \frac{P_c}{P_{net} + P_c} = \frac{1}{\eta_s} \frac{1}{(\eta_T' + \eta_D')Q + \eta_D}} \qquad \ldots (AII.7)$$

If relation (AII.6) is used to define a critical value Q_c

$$Q_c = \frac{1}{\eta_T' + \eta_D'} \cdot \frac{1 - \eta_D \eta_s}{\eta_s} \qquad \ldots (AII.8)$$

Then Eq. (AII.7) may be written in the form

$$\boxed{G = \frac{1}{1 + (1 - \eta_D \eta_s)\left(\frac{Q}{Q_c} - 1\right)}} \qquad \ldots (AII.9)$$

Thus if $Q = Q_c$, $G = 1$ and all the gross output power is circulated in the reactor.

In curves I – IV (Figs. 1 and 2), Q is plotted against G for the following cases:

Curve I $\eta_T = \eta_D = \eta_s = 0.5$; $f = 0$

     II $\eta_T = \eta_D = 0.5$; $\eta_s = 0.8$; $f = 0$

     III $\eta_T = 0.5$; $\eta_D = 0.8$; $\eta_s = 0.5$; $f = 0$

     IV $\eta_T = 0.5$; $\eta_D = 0.8$; $\eta_s = 0.9$; $f = 0$.
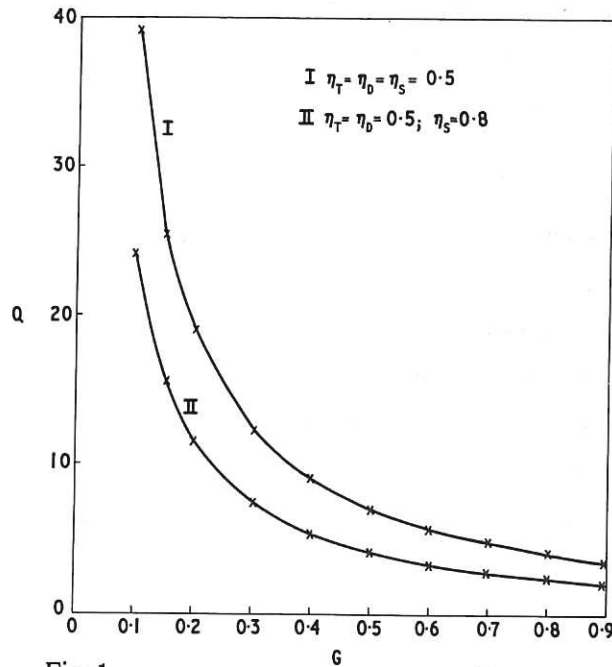
Fig. 1   (CLM-R94)

Minimum permissible Q value as a function of
the fraction G of gross output power which
must be circulated. (Curves I and II)
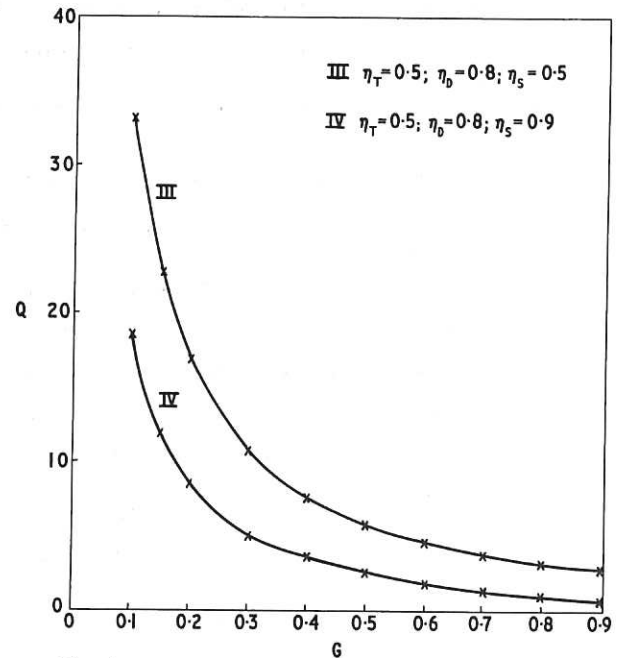

Fig. 2   (CLM-R94)

Minimum permissible Q value as a function of
the fraction G of gross output power which
must be circulated. (Curves III and IV)

In calculating values of η_T' and η_D' from the values of η_T and η_D given above it has been assumed that $h = 0.84$. This is the value applicable to a reactor operating with a 50 : 50 mixture of D and T with a lithium blanket. This system yields a value of $\bar{W}$ equal to 22.4 MeV and gives the highest value of $\bar{W}$ using light elements only.

It will be noted from the graphs that, for values of G less than about 0.5, direct conversion with an efficiency of 0.8 produces not more than a 33% reduction in the required value of Q from that obtained by total thermal conversion with an efficiency 0.5.

3. THE POSSIBILITY OF A MIRROR REACTOR

From the above discussion it may be concluded that, even if indirect losses are negligibly small, a thermonuclear reactor, operating with a source efficiency η_S of 0.5 and in which not more than 20% of the gross output power is circulated, requires a Q value of approximately 20. The analyses of Post and Fowler and Rankin gave Q values as follows:

$$Q_{Post} = 11 \quad \log_{10}R$$
$$Q_{FR} = 3.1 \quad \log_{10}R$$

Thus assuming a mirror ratio R equal to 10, Post's value falls short by a factor of about 2 and Fowler and Rankin's by a factor of about 7. An increase in source efficiency from 0.5 to 0.8 changes the situation considerably – a reactor with Post's value of Q becomes marginally possible (see curve II) but Fowler and Rankin's Q value is still a factor of about 4 too small.

Since Q increases only slowly with mirror ratio ($\sim \log_{10}R$) very large mirror ratios are required to make a reactor possible if Fowler and Rankin's Q values are considered. In order to maintain acceptable fusion power densities, mirror reactors, with very large mirror ratios, must operate at high beta values. In fact the achievable mirror ratio is determined by three factors:

(i)   the nuclear power output per cm^3 of plasma;

(ii) the value of beta at which stable operation is possible;

(iii) the maximum achievable magnetic field in the mirror region.

The significance of these parameters is discussed below.

The fusion power per cm^3, F, is given by (for a D-T reactor)

$$F = \frac{1}{4} n^2 \langle \sigma v \rangle_{DT} \cdot \bar{W} \qquad \dots \text{(AII.10)}$$

where n is the plasma electron number density and $\langle \sigma v \rangle_{DT}$ is the mean value of the product of fusion cross section and ion velocity averaged over the ion energy distribution. Since $\langle \sigma v \rangle_{DT}$ is not a very sensitive function of energy in the region $E \sim 200$ keV it is fairly insensitive to the ion energy distribution assumed. Thus, taking $\langle \sigma v \rangle_{DT}$ equal to 8×10^{-16} cm^3 sec^{-1} and $\bar{W} = 22 \cdot 4$ MeV, Eq. (AII.10) gives, after conversion of units

$$F = n^2 \times 7 \times 10^{-28} \text{ W } cm^{-3} \qquad \dots \text{(AII.11)}$$

Thus for $n = 3 \cdot 7 \times 10^{14}$, $F = 100$ W cm^{-3} .

If β is defined as the ratio

$$\beta = \frac{8\pi \, n \, kT}{B_0^2 + 8\pi \, n \, kT} \qquad \dots \text{(AII.12)}$$

where $kT = \frac{2}{3} E$, B_0 is the magnetic field strength <u>in</u> the plasma and E is the average particle energy, then

$$B_0^2 = 2 \cdot 7 \times 10^{-8} n \, E \, \frac{1-\beta}{\beta} \qquad \dots \text{(AII.13)}$$

where B is in gauss and E in keV .

From Eqs. (AII.11) and (AII.13) :

$$B_0 = F^{\frac{1}{4}} \left(\frac{1-\beta}{\beta} \right)^{\frac{1}{2}} E^{\frac{1}{2}} \times 10^3 \text{ G }, \qquad \dots \text{(AII.14)}$$

and the mirror ratio* R is given by :

$$R = \frac{B_M}{B_0} = B_M \left(\frac{\beta}{1-\beta} \right)^{\frac{1}{2}} F^{-\frac{1}{4}} E^{-\frac{1}{2}} \times 10^{-3} \qquad \dots \text{(AII.15)}$$

where B_M = magnetic field in mirror region.

In Fig.3, $\log_{10} R$ is plotted versus beta using Eq. (AII.15) and assuming that $F = 10$ W cm^{-3}, $B_M = 150$ kG , $E = 200$ keV. It will be noted from the graph that a mirror ratio of 10 implies a beta value of about 70%.

4. ECONOMICS OF A MIRROR REACTOR

The basic model is shown in Fig.4(a). The W's are the power outputs from each component, the η's the efficiencies, and the capital letters represent the capital cost (in £/kW) per unit output <u>of that</u> component. Thus there is a nuclear core - the reactor - producing a power output W_n in neutrons, and W_D in charged particles, having a total capital cost of $£(W_n + W_D) \cdot N$. I represents the injector, D a direct converter for the charged particle power, E conventional thermal-electrical turbine plant.

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* In deriving Eq. (AII.15) the pressure produced by the reaction products has been ignored. According to Fowler and Rankin this pressure amounts to about 66% of the plasma pressure. If this factor is taken into account then the values of R should be reduced by 30% .
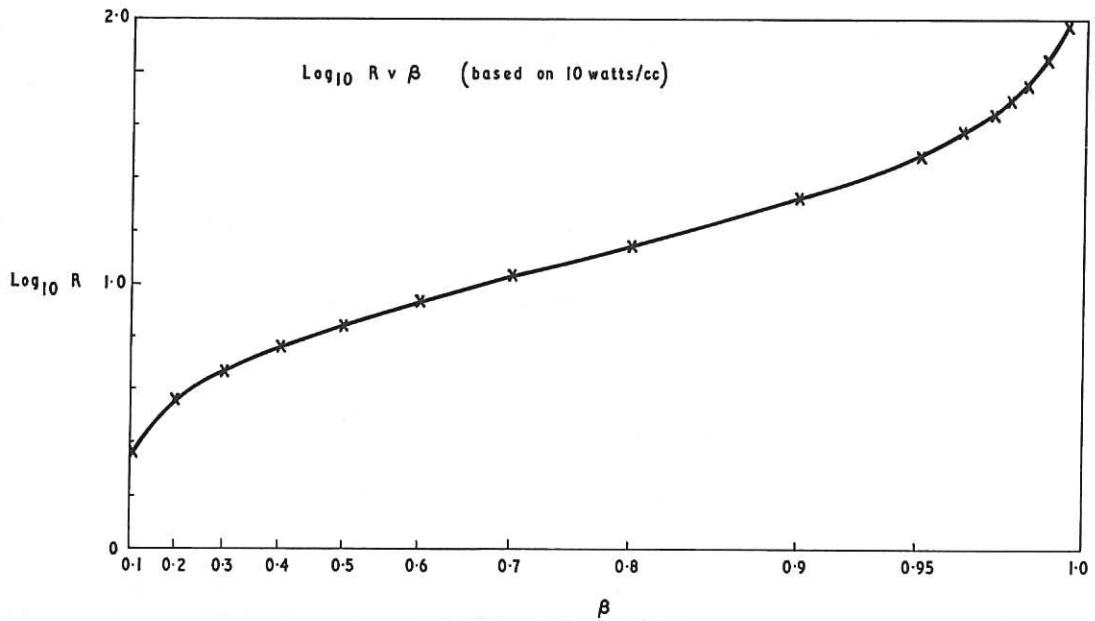
Fig. 3 (CLM-R94)

Mirror ratio (R) as a function of magnetic pressure (β). (Assuming a power density of $10 \, W \, cm^{-3}$ and a magnetic flux density in the mirror region of $150 \, kG$)
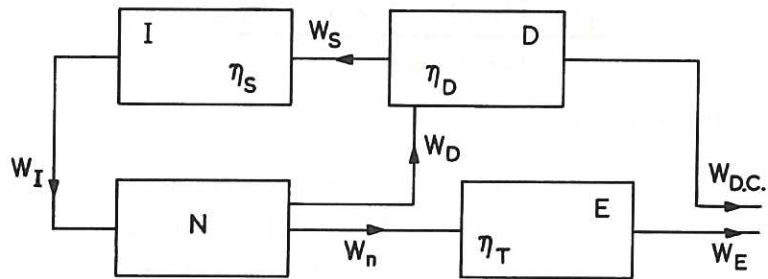

Fig. 4(a) (CLM-R94)

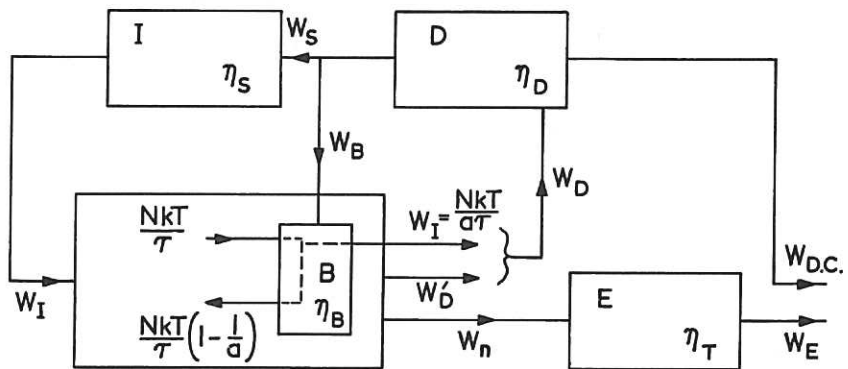Basic model of simple mirror reactor with direct converter D, injector I and conventional turbine plant E

Fig. 4(b) (CLM-R94)

Basic model of mirror reactor as for (a) plus an end-stopper B

Eqs. (AII.16) to (AII.26) then follow, leading to an expression for the cost per kilowatt of electricity produced.

$$\text{Cost} = W_E \cdot E + (W_n + W_D)N + (W_S + W_{DC})D + W_I \cdot I \qquad \dots \text{(AII.16)}$$

$$\text{Electrical output} = W_E + W_{DC} \qquad \dots \text{(AII.17)}$$

$$\text{Power Balance} = W_S = W_D \eta_D - W_{DC} \qquad \dots \text{(AII.18)}$$

$$\begin{array}{l}\text{Circulating power} = W_c = W_D - \dfrac{W_{DC}}{\eta_D} = \dfrac{W_S}{\eta_D} = \varepsilon (W_D + W_n) \\ \text{(before electrical} \\ \text{conversion)}\end{array} \qquad \dots \text{(AII.19)}$$

(Eq. (AII.19) <u>defines</u> ε)

$$W_D = (1 + 0.16Q) W_I \quad ; \quad W_n = 0.84 \cdot Q \cdot W_I = \gamma W_I \quad \dots \text{(AII.20)}$$

$$\frac{W_D}{W_n} = \frac{1 \times 0.16Q}{0.84Q} = \alpha \quad (\underline{\text{definition}} \text{ of } \alpha) \qquad \dots \text{(AII.21)}$$

$$W_{DC} = W_D \eta_D - W_S = W_n \left(\alpha \eta_D - \frac{1}{\gamma \eta_s} \right) \qquad \dots \text{(AII.22)}$$

So
$$W_{DC} + W_E = W_n \left[\eta_T + \alpha \eta_D - \frac{1}{\gamma \eta_s} \right] \qquad \dots \text{(AII.23)}$$

$$\text{Cost} = W_n \left[\eta_T \cdot E + (1 + \alpha)N + \eta_D \alpha D + \frac{I}{\gamma} \right] \qquad \dots \text{(AII.24)}$$

The electrical circulating power is $W_S = \eta_D \varepsilon \cdot W_n (1 + \alpha)$

So
$$G = \frac{\eta_D \varepsilon (1 + \alpha)}{\eta_T + \alpha \eta_D} = \frac{W_S}{W_n(\eta_T + \alpha \eta_D)} \quad ; \quad \text{also} \quad W_S = \frac{W_I}{\eta_s} = \frac{W_n}{\gamma \eta_s} \quad \dots \text{(AII.25)}$$

from (AII.25)
$$\frac{1}{\gamma \eta_s} = \frac{W_S}{W_n} = G(\eta_T + \alpha \eta_D). \qquad \dots \text{(AII.26)}$$

From Eqs. (AII.23), (AII.24) and (AII.25) the cost A in £ per kW output

$$= \frac{\eta_T E + (1 + \alpha)N + \eta_D \alpha D + I\gamma}{(\eta_T + \alpha \eta_D)(1 - G)} \qquad \dots \text{(AII.27)}$$

where $\gamma = 0.84Q$ for a D-T reactor.

Eq. (AII.27) is valid if $\alpha \eta_D \geqslant \dfrac{1}{\gamma \eta_s}$, i.e. provided that the direct converter provides all the power for the injector. In the absence of a circulating loop and a direct converter, Eq. (AII.27) reduces to a cost C in £ per kW, where

$$C = E + \frac{N}{\eta_T} .$$

Carruthers et al.[27] estimate $E \approx £30$ per kW , $N \approx £18$ per kW and show that on this basis the cost per kW output can be 16% below that of a fast fission reactor. We therefore assume that to be competitive with fast fission reactors

$$A \leqslant 1.16 \, C .$$

Taking the values of E and N above and $\eta_T = 0.5$ we get

$$A \leqslant £77 \text{ per kW} .$$

It should be noted that Carruthers' capital cost N of the reactor assumes a power density of 20 W cm^{-3} .

From Eq. (AII.27) it is then possible, assuming values of Q and the efficiencies η_s, η_D , to derive the permissible capital costs D , I of the direct converter and injector. Figs. 5 and 6 give some results for selected values, assuming Q = 10 and 3 respectively. These correspond roughly to the values given by Post and Fowler and Rankin, assuming a mirror ratio of nearly 10. (Note that, in order to achieve the power density of 20 W cm^{-3} , compatible with the assumed cost N of the reactor, the value of beta at the centre of the mirror has to be about 80% .
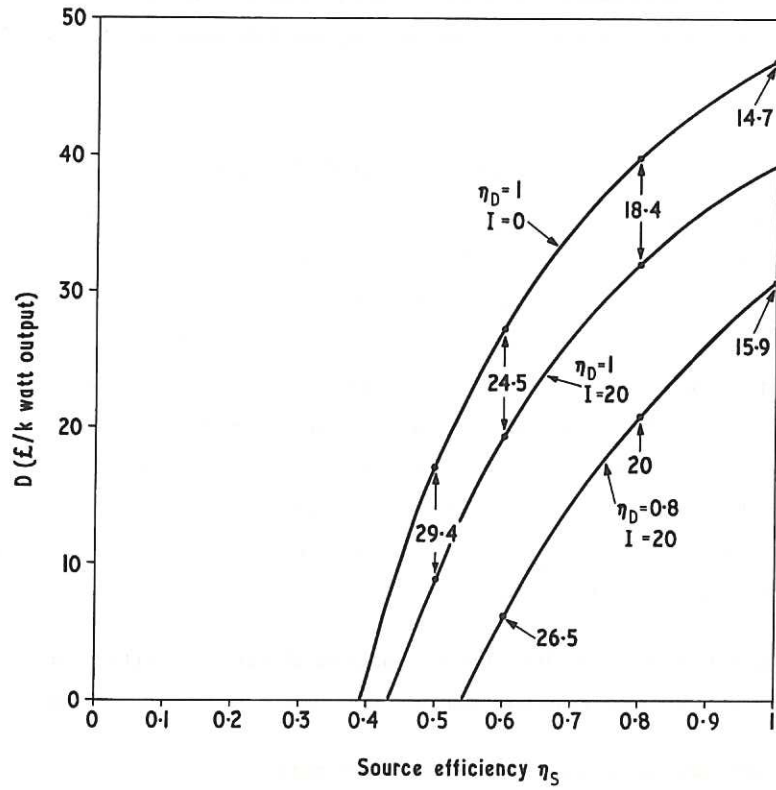
Fig. 5   (CLM-R 94)

Mirror without end-stoppers. Permissible capital cost (D) of direct converter as a function of source efficiency (η_S), for Q = 10. (The percentage of gross output power which must be circulated is indicated by arrows at selected points.)
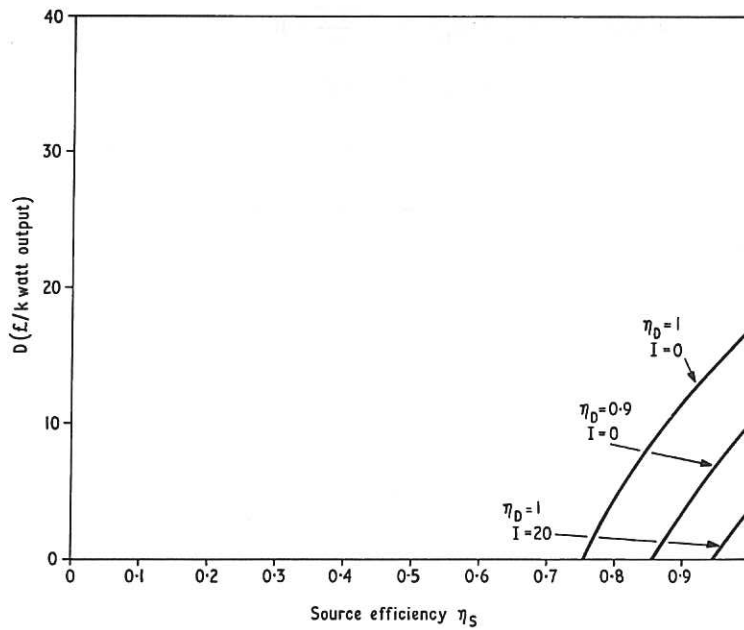

Fig. 6   (CLM-R 94.)

Mirror without end-stoppers. Permissible capital cost (D) of direct converter as a function of source efficiency (η_S), for Q = 3

We therefore conclude that if Fowler and Rankin's expression for Q is correct (Fig.6), the prospect of building an economic mirror reactor looks poor; and if we use Post's value (Fig.5) it is necessary to achieve efficiencies of 80 - 90% in both injector and direct converter <u>and</u> to build each of these devices for something like £20 per kilo-watt output.

5. THE POSSIBILITY OF END-STOPPERS

We postulate some sort of charge separating device acting upon the plasma streaming out of the throats of the mirrors. The plasma, having a total number of ions N , has a containment time τ in the absence of the end-stopper, and we suppose that this is increased by a factor a . In the normal scattering time τ , the ions have all their energy transferred into motion along the field lines, so that the power fed into the device is NkT/τ . A large fraction of ions are assumed to give up their energy into the device and then recover it in the opposite direction. The process is not 100% efficient, however, and part of the energy is lost as low-grade heat: an ion entering with energy W_i is assumed to leave with energy $\eta_B W_i$, so to maintain the energy of the plasma $(1 - \eta_B)W_i$ has to be supplied from an external source.

The essential new feature is that this blocking device, capital cost £B per kilowatt, efficiency η_B , lies inside the reactor core: the basic model is shown in Fig.4(b).

The equations are now slightly modified. We have

$$W_I = \frac{1}{a} \frac{NkT}{\tau} \qquad \qquad \text{... (AII.28)}$$

$$W_B = \frac{NkT}{\tau}(1 - \eta_B) = aW_I(1 - \eta_B) \qquad \qquad \text{... (AII.29)}$$

and the capital cost of the reactor core is increased to

$$\text{Core} = (W_D + W_n) N + \frac{NkT}{\tau} \cdot B$$

$$= (W_D + W_n) N + W_I B . \qquad \qquad \text{... (AII.30)}$$

An analysis similar to that given in Section (4) now leads to a cost per unit output

$$A = \frac{\eta_T E + (1 + a)N + a\eta_D D + 1/\gamma(I + aB)}{(\eta_T + a\eta_D)(1 - G)} \qquad \qquad \text{... (AII.31)}$$

where G , the ratio of circulating power to gross electrical output, is now modified to

$$G = \frac{\frac{1}{\gamma}\{1/\eta_s + a(1 - \eta_B)\}}{\eta_T + a\eta_D} \qquad \qquad \text{... (AII.32)}$$

It can easily be seen that if the factor of improvement a is reasonably large (e.g. $a \geqslant 10$) the costs and efficiency of the injector do not play a large part in the economics. The most sensitive parameter is η_B the efficiency of the blocker. Fig.7 gives a typical result, based on the assumption that Fowler and Rankin's calculations give the correct basic value of Q $(= Q_0)$, and that the containment time is increased by a factor 10. The graph shows what capital cost is available to be distributed between direct converter, end-blocker and injector. It is clear that the blocking device must have an efficiency well in excess of 70%, and assuming that both it and the direct converter, being similar in principle, cost about the same, then (if $\eta_D = \eta_B = 80\%$) each must be built for £10 per kilowatt i.e. about one third the cost of conventional turbine plant.
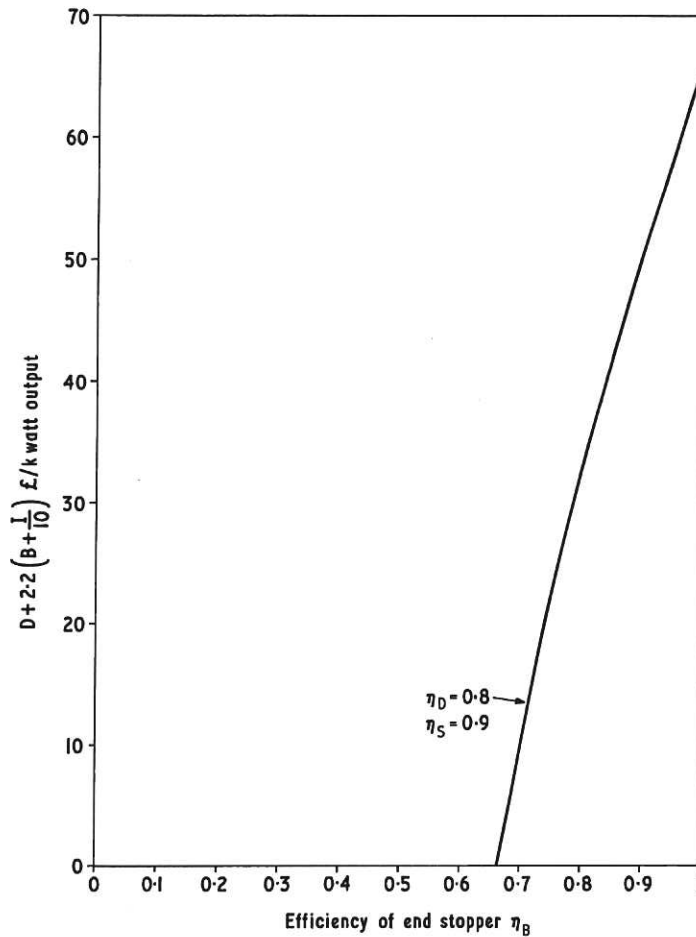
Fig.7   (CLM-R94)
Mirror with end stoppers. Permissible total capital cost,
comprising capital costs of direct converter (D), end-
stoppers (B), and injector (I), as a function of end-
stopper efficiency (η_B)

6. CONCLUSIONS

(1)   An economic reactor based on Post's Q value requires a large plasma pressure, corresponding to $\beta \sim 75\%$ at the centre.

(2)   An economic reactor based on Fowler and Rankin's calculation of Q is impossible unless stable operation at β values close to 1 are achievable and the particle motion remains adiabatic.

(3)   If Post's more optimistic value of Q can be achieved an economic reactor is possible provided certain quite stringent conditions are satisfied as shown in Fig.5. Typically we require that:

      (i)   efficiencies of about 80% are achieved for both injector and direct converter ; and

      (ii)   each of these devices be built for about £20 per kilowatt output.

(4)   If further increases in Q can be achieved by the use of an end-stopper then an economic reactor is possible, but the conditions on the efficiency and capital cost of the end-stopper are now the most stringent factor. Typically it must have an efficiency of about 80%, and be built for £20 per kilowatt output, or less.

PLASMA CONFINEMENT BY MAGNETIC FIELDS SUPPLEMENTED
BY RESONANT RF   FIELDS

(G. Francis)

1. INTRODUCTION

In this Appendix it is assumed that RF fields, resonating with the ion cyclotron
frequency, feed energy into the perpendicular motion of the ions at a rate sufficient to
counter diffusion into the loss cone.   The object is to calculate approximately how much
RF power is needed to replace all the scattering losses.   (The method used is due to
D.W. Mason and G. Francis.)   It is also assumed that the alternating fields, by some
means as yet unspecified, act only upon those ions which cross the edge of the loss cone,
and not upon the others.

2. DISCUSSION

Suppose that in the absence of RF fields plasma, of density $N \, cm^{-3}$, has a contain-
ment time τ .   In the crudest picture τ is the average time taken for a scattering
through 90°, into the loss cone: after this time the entire energy of the particle is
lost and thus the rate of energy loss which must be replaced by the RF field is simply
$NkT/\tau \, cm^{-3}$ of plasma.   This clearly conveys no economic benefit, being equivalent merely
to an alternative way of producing the initial plasma.

In more detail we examine the very small angular scatter $\delta\theta_1$, across the <u>edge</u> of
the loss cone during one transit between the mirrors, in a time t .   By random walk theory

$$\delta\theta_1 = \delta\theta \cdot \sqrt{\frac{t}{\tau}} \qquad \qquad \cdots \text{(AIII.1)}$$

where $\delta\theta$ is the normal scattering angle after a containment time τ: this we take
(approximately) to be $\pi/2$.

We also assume that the contained particles
have an isotropic velocity distribution described
by a function $n(\theta)$, which is constant between
the critical loss cone angles, θ_c , and zero out-
side (Fig.8).   The distribution here refers to
the velocity vectors at the mid-plane of the
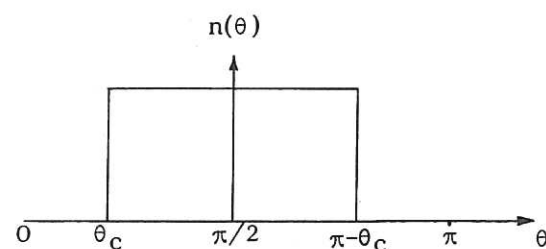mirror.   Then the number $N(\theta)$ lying between θ
and $\theta + d\theta$ is

Fig. 8   (CLM-R94)
Contained particles velocity distribution

$$N(\theta) = n(\theta) \cdot 2\pi \sin\theta \, d\theta \qquad \qquad \cdots \text{(AIII.2)}$$

and simple integration from θ_c to $\pi/2$ gives for the density N:

$$N = 4\pi \, n(\theta) \cdot \cos\theta_c \qquad \qquad \cdots \text{(AIII.3)}$$

The number (per cm^3 of plasma) of ions knocked into the loss cone in each transit of
the mirror is

$$N_{loss} = N(\theta_c) \cdot \delta\theta_1 . \qquad \qquad \cdots \text{(AIII.4)}$$
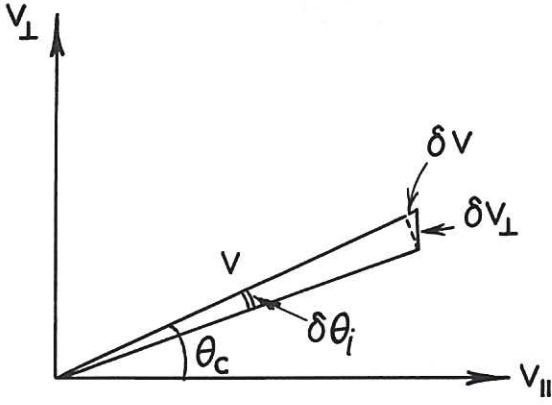
Fig. 9   (CLM-R94)
Vector diagram for ion moving with total velocity v

The vector diagram, Fig.9, for an ion moving with a total velocity v, shows that to push it out of the loss cone an increment $\delta v_\perp$ of velocity is required

$$\delta v_\perp = \frac{v \delta \theta_1}{\cos \theta_c} \qquad \ldots \text{(AIII.5)}$$

which is equivalent to an increment in total velocity

$$\delta v = v \delta \theta_1 \cdot \tan \theta_c \qquad \ldots \text{(AIII.6)}$$

or an increase in total energy

$$\Delta \varepsilon = mv^2 \tan \theta_c \cdot \delta \theta_1. \qquad \ldots \text{(AIII.7)}$$

From Eqs. (AIII.7) and (AIII.4) we calculate the total energy increment needed per cm^3 of plasma per transit, namely

$$
\begin{aligned}
\Delta \varepsilon \text{ per } cm^3 &= N(\theta_c) \, mv^2 \, \tan \theta_c \cdot (\delta \theta_1)^2 \\
&= N(\theta_c) \, mv^2 \, \tan \theta_c \cdot (\delta \theta_1)^2 \cdot \frac{t}{\tau} \\
&\approx n(\theta) \cdot 2\pi \sin \theta_c \tan \theta_c \cdot mv^2 \cdot \left(\frac{\pi}{2}\right)^2 \frac{t}{\tau} \qquad \ldots \text{(AIII.8)}
\end{aligned}
$$

Rearranging, this gives the RF power required.

$$
\begin{aligned}
P_{RF} \text{ per } cm^3 &= n(\theta) \cdot 4\pi \sin \theta_c \tan \theta_c \left(\frac{\pi}{2}\right)^2 \cdot \frac{\tfrac{1}{2} mv^2}{\tau} \\
&= \left(\frac{\pi}{2}\right)^2 \cdot \tan^2 \theta_c \frac{\tfrac{1}{2} Nmv^2}{\tau} \qquad \ldots \text{(AIII.9)}
\end{aligned}
$$

Using the expression that $\sin^2 \theta_c = 1/R$ where R is the mirror ratio this reduces to

$$P_{RF} \text{ per } cm^3 = \frac{\pi^2}{4} \cdot \frac{1}{R-1} \cdot \frac{\tfrac{1}{2} Nmv^2}{\tau} \qquad \ldots \text{(AIII.10)}$$

In this expression the last factor is simply NkT/τ as deduced from our crude initial argument. For $R \sim 10$ the multiplying factor is 0.28.

We conclude that if suitable RF fields are available, the scattering losses can be prevented by an expenditure of power of the order of $\frac{1}{4} \frac{NkT}{\tau}$. The questions remain: what are suitable RF fields and how can they be applied?

(a)   The Magnitude of the Field

A gyrating charge entering a resonant field ($\omega = \omega_{ci}$) can either gain or lose energy, depending upon the phase of the field, the initial velocity of the particle, and the time it stays in the field. If it stays in long enough it will always gain energy, which is what we require (a loss of perpendicular energy would cause an increase in diffusion into the loss cone).
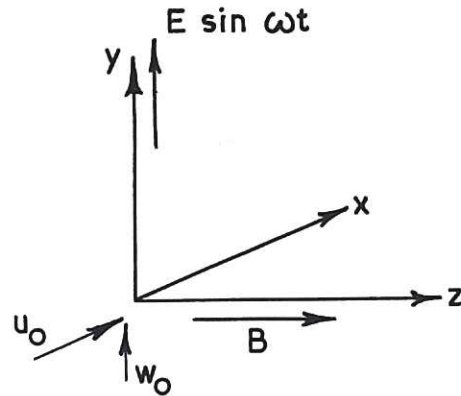

Fig. 10   (CLM-R94)
Case where particle enters a resonant field

Consider a particle charge e, mass M entering this region with components of perpendicular velocity initially u_0, w_0 in the x, y directions respectively at an instant when the phase of the field is φ (see Fig.10). The initial perpendicular energy

$$W_\perp = \tfrac{1}{2}M \,(u_0^2 + w_0^2) \qquad \ldots \text{(AIII.11)}$$

If we write the acceleration in the electric field $eE/M = f$, then straightforward equations of motion plus some tedious algebra gives the energy after a time t_1

$$
W_\perp + \delta W_\perp = \tfrac{1}{2}M\Bigg\{ u_0^2 + w_0^2 + \left(\tfrac{1}{2}\,f t_1\right)^2 + \left(\frac{f}{2\omega}\right)\; \sin^2\,\omega t_1
$$

$$
+ \, f t_1 \left[u_0 \sin\varphi - w_0 \cos\varphi - \frac{f}{2\omega}\sin\omega t_1 \, \cos(\omega t_1 + 2\varphi) \right]
$$

$$
+ \, \frac{f}{\omega}\sin\omega t_1 [u_0 \sin(\omega t_1 + \varphi) + w_0 \cos(\omega t_1 + \varphi)] \Bigg\} \qquad \ldots \text{(AIII.12)}
$$

Here the first two terms are obviously the initial $W_\perp$, the next term is always positive – and it is this term we want to dominate – the fourth term is never negative, but the subsequent brackets being complicated functions of u_0, w_0, φ and partly periodic in time can be negative. It is essential therefore that

$$
(\tfrac{1}{2}f t_1)^2 \gg \text{terms like } ft_1 \cdot u_0, \quad ft_1 \cdot w_0, \quad fu_0/\omega \text{ and } fw_0/\omega.
$$

Hence

$$
ft_1 \gg 4\,u_0 \text{ or } 4\,w_0. \qquad \ldots \text{(AIII.13)}
$$

But the basic idea of using resonant RF fields as a stopper is that the leaking ions shall be given increments of perpendicular energy $\delta W_\perp$ which are <u>small</u> compared with their total energy. This requires that

$$
(\tfrac{1}{2}f t_1)^2 \ll u_0^2 + w_0^2.
$$

Hence

$$
ft_1 \ll u_0 \text{ or } w_0 \qquad \ldots \text{(AIII.14)}
$$

– a condition clearly incompatible with the previous one. This result is so clear-cut that it is unnecessary to consider the motion of the ions in any further detail.

(b)   The Application of the Fields

The calculations given above are highly idealised, because we have assumed that the RF field acts upon the velocity vectors in the mid-plane, yet somehow selects only that group of ions diffusing over the edge of the loss cone. In practice the only method of selection is by taking advantage of the natural spatial separation, and applying the resonant field near (but not exactly at) the throat of the mirrors. The arrangement is shown in Fig.11. The RF field is applied in a region of uniform magnetic field,
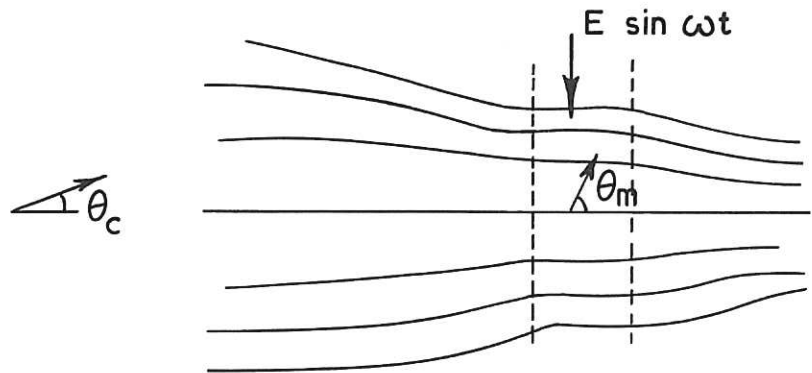

Fig. 11   (CLM-R 94)
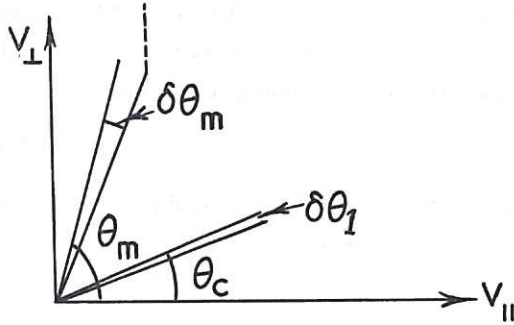Resonant field applied near (but not exactly at) the throat of the mirrors

- 25 -

Fig. 12   (CLM-R94)
Rotation of vector from θ_c to θ_m in
the resonant region

satisfying exact resonance; beyond this is a slightly larger static field to reflect those particles which have been operated upon. It is assumed that there are no scattering collisions between the resonant region and the reflection point.

A vector lying at the critical angle θ_c in the mid-plane is rotated to the angle θ_m in the resonant region (Fig.12). It can easily be shown that the angular scatter $\delta\theta_1$, referred to the velocity distribution in the centre (Eq. (AIII.1)) now becomes $\delta\theta_m$, when

$$\delta\theta_m = \delta\theta_1 \cdot \frac{\tan\theta_m}{\tan\theta_c} .$$

A calculation similar to the previous one performed around the angle θ_m now leads to an expression for the RF power

$$P_{RF} = \tan^2\theta_m \cdot \frac{\pi^2}{4} \cdot \frac{NkT}{\tau}$$

from which it is clear that when the RF field is applied close to the ends of the mirrors the amount of power required rises very rapidly, and gives no economic benefit.

3. CONCLUSION

Exactly resonant RF fields, acting to increase the perpendicular velocity of those ions which just leak into the loss cone, are not a practicable nor an economic method of reducing end losses.

It should however be noted that such fields applied to a non-uniform region of static magnetic fields exert an influence upon the parallel component of ion velocity. These are, however, not exactly resonant fields: their effect is considered by Watson in Appendix V.

MULTIPLE MIRRORS AND RF STOPPERS

(J.B. Taylor)

1. INTRODUCTION

It has been suggested that the best way to exploit RF stoppers might be in a multiple mirror arrangement. Briefly, the idea is that particles which just escape over the first mirror are scattered by a resonant RF field placed beyond the mirror; approximately half of them will then be reflected from the next mirror if it is of equal strength. On this naive picture a succession of such mirror/RF stoppers should reduce the loss by a factor which increases geometrically with the number of stages. However, multiple reflections will reduce the overall efficiency well below this.

Obviously this arrangement has much in common with the scheme discussed by Post[30], who carried out a series of numerical computations of the scattering loss from a multiple mirror arrangement and showed that in some circumstances considerable improvement could be obtained by the use of multiple mirrors.

The following simple model is useful for studying arrangements such as Post's or the present one. The model represents most of the important features and, for example, roughly reproduces the relevant results contained in Post's Monte Carlo calculations.

It shows that the improvement in containment over a simple mirror increases only linearly with the number of stages and that even this gain is only attainable when the basic single mirror containment is very poor.

2. MODEL

Consider a sequence of multiple mirrors between each of which there is a non-adiabatic scattering mechanism (e.g. RF, particle collisions or field perturbations). One such sequence is placed at either end of a simple mirror (see Fig.13).

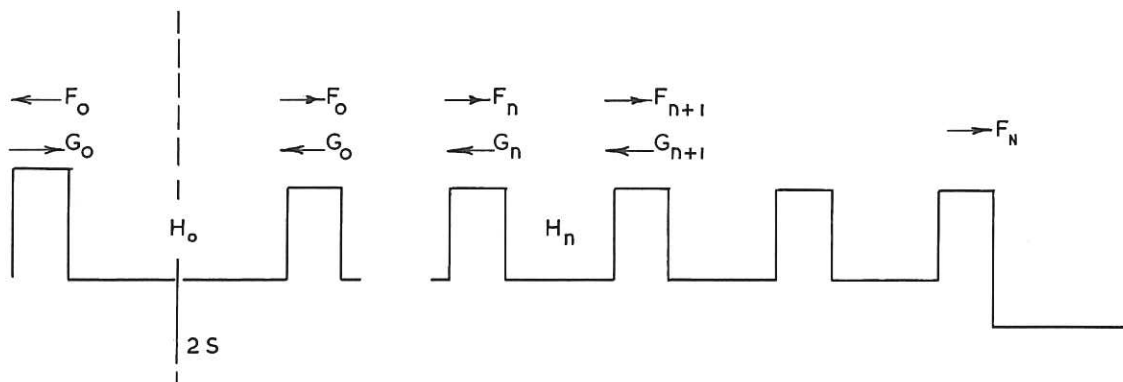Fig. 13   (CLM-R94)

Sequence of multiple mirrors between each of which there is a non-adiabatic scattering mechanism

A source of particles of strength $2S$ is injected into the central mirror so that in the steady state the leakage through each end is S. The flux in the outward direction through the nth mirror is F_n and the inward flux (due to reflection at mirrors beyond the nth) is G_n. The number of particles trapped between the nth and $(n+1)^{th}$ mirror is H_n and the number in the central mirror is H_0.

Let $(1-p)$ be the probability that a particle is 'caught', i.e. scattered into the containment cone while crossing from the nth to the $(n+1)^{th}$ mirror, and let τ be the mean time for which the particle then remains trapped. The following continuity equations can then be written down:

$$F_{n+1} = p\,F_n + H_n/2\tau \qquad \qquad \ldots \text{(AIV.1)}$$

$$G_n = p\,G_{n+1} + H_n/2\tau \qquad \qquad \ldots \text{(AIV.2)}$$

$$\frac{H_n}{\tau} = (1-p)\,F_n + (1-p)\,G_{n+1}\,, \qquad \qquad \ldots \text{(AIV.3)}$$

in which it is assumed that a trapped particle is equally likely eventually to escape over the left or the right mirror.

Eliminating H_n/τ,

$$F_{n+1} = \frac{(1+p)}{2}\,F_n + \frac{(1-p)}{2}\,G_{n+1} \qquad \qquad \ldots \text{(AIV.4)}$$

$$G_n = \frac{(1+p)}{2}\,G_{n+1} + \frac{(1-p)}{2}\,F_n\,. \qquad \qquad \ldots \text{(AIV.5)}$$

If there are N mirrors altogether then clearly

$$G_N = 0 \qquad \qquad \ldots \text{(AIV.6)}$$

and also

$$F_n - G_n = S\,. \qquad \qquad \ldots \text{(AIV.7)}$$

The solution of Eqs. (AIV.4) to (AIV.7) yields

$$F_n = \left[\frac{1-p}{1+p}\,(N-n) + 1\right] S \qquad \qquad \ldots \text{(AIV.8)}$$

$$G_n = \frac{1-p}{1+p}\,(N-n)\,S\,. \qquad \qquad \ldots \text{(AIV.9)}$$

In the central mirror, where the scattering may be different to that in the stoppers, the corresponding continuity equation is

$$\frac{H_0}{2\tau_0} = S + (1-p_0)\,G_0 \qquad \qquad \ldots \text{(AIV.10)}$$

so that using Eq. (AIV.9),

$$\frac{H_0}{2\tau_0} = S\left[1 + (1-p_0)\,\frac{(1-p)}{(1+p)}\,N\right]\,. \qquad \qquad \ldots \text{(AIV.11)}$$

Thus the effective containment time in the central mirror is increased by the 'stoppers' from τ_0 to

$$\tau_{eff} = \tau_0\left[1 + (1-p_0)\,\frac{(1-p)}{(1+p)}\,N\right]\,. \qquad \qquad \ldots \text{(AIV.12)}$$

3. INTERPRETATION

It will be seen immediately that, other quantities remaining fixed, the improvement in containment time increases only linearly with the number of stages. Furthermore, the maximum improvement which can be obtained is less than a factor $(N + 1)$. These features are roughly in accord with the numerical calculations of Post.

Perhaps the most important feature of the result Eq. (AIV.12) is that in order to get the full benefit of the stoppers we must have $p \ll 1$, i.e. there must be a significant probability that a particle is scattered during one transit between successive mirrors. Thus, the improvement by a factor $\sim N$ as found by Post, and predicted by Eq. (AIV.12), can only be achieved when the original mirror containment is very poor.

4. MULTIPLE RF / MIRROR STOPPERS

The use of RF (or field perturbations) as the scattering mechanism instead of the interparticle collisions envisaged by Post has the advantage that it can be applied in the stoppers without affecting conditions in the central mirror (i.e. $p_0 \neq p$ and $\tau_0 \neq \tau$).

In an attempt to get full benefit from the stoppers we consider a system in which the RF scattering is large, so that $p \ll 1$ and

$$\tau_{eff} = \tau_0 \left[1 + (1 - p_0) N \right] \qquad \dots \text{(AIV.13)}$$

but we also suppose that the initial mirror containment is good (i.e. particles are contained for many transits) – then $(1 - p_0) \ll 1$. In this situation there is a relationship between the probability $(1 - p_0)$ of a particle being 'caught' in one transit between main mirrors and the time τ_0 for which it remains so trapped. The relation is discussed at length by Robson and Taylor[31]. If the average time for a particle to pass between the central mirrors is taken as $L/V \cos\psi$, where ψ is the mirror loss cone angle, then detailed balancing[31] implies

$$(1 - p_0) \frac{V \cos\psi}{L} (1 - \cos\psi) \simeq \frac{\cos\psi}{\tau_0} \qquad \dots \text{(AIV.14)}$$

or for realistic mirror ratios

$$(1 - p_0) \frac{\tau_0 V}{L} \sim \frac{1}{1 - \cos\psi} \sim 2R. \qquad \dots \text{(AIV.15)}$$

Hence Eq. (AIV.13) can be written

$$\tau_{eff} \sim \tau_0 \left[1 + \frac{2RNL}{\tau_0 V} \right]. \qquad \dots \text{(AIV.16)}$$

This form for the effective containment time shows that large ratios of τ_{eff}/τ_0, of order N , can only be achieved when τ_0 is small compared to the 'free flow' loss time NL/V. The absolute increase in containment is limited to $\sim 2R$ times the free flow loss time.

5. CONCLUSIONS

The use of multiple mirrors with natural or imposed RF scattering between adjacent mirrors can give an improvement in containment time. However, while large fractional improvements can be obtained when the initial containment is very poor, the absolute increase in containment time is only $\sim 2R$ times the free flow escape time (where R is the mirror ratio).

MIRROR CONFINEMENT SUPPLEMENTED BY NEARLY-RESONANT   RF   FIELDS

(C.J.H. Watson)

1.   INTRODUCTION

Several papers in the literature propose the use of radio frequency fields to eliminate the loss of plasma along the field lines of a mirror machine.   The earliest proposal, by Glagolev et al.[32], was to use a non-resonant RF field transverse to the static magnetic field which (for simplicity) was taken to be uniform in the region concerned. They predicted theoretically that the plasma would be confined, provided that the RF radiation pressure equalled the central plasma pressure.   Experiments confirmed the validity of this pressure balance condition.   The advantage of this arrangement over one in which the plasma is confined by radiation pressure alone is that this pressure need only be applied over the small fraction of the total plasma surface which is intersected by magnetic field lines, and since the amount of RF power required to maintain a given radiation pressure is roughly proportional to the area over which it must act, there is a corresponding reduction in the required power.

Unfortunately, as shown in reference 33, a plasma of reasonable dimensions cannot generate enough thermonuclear power to confine itself by radiation pressure alone. However, the factor by which it fails to do so is only about 100.   Thus, if one is prepared to envisage a long thin mirror machine in which the surface area of the plasma intersected by field lines is only $1/100^{th}$ of the total surface, it appears[29] that it might just be possible to design a self-sustaining reactor in this way, but it is difficult to design one in which the circulating power is a small fraction of the total output.   A further difficulty is that the required electric field strengths – in excess of 10^6 V cm^{-1} – are somewhat beyond existing RF technology.

A somewhat different proposal has been made by Consoli[34], who pointed out that if a surface exists within the plasma at which the RF field is in exact resonance with the (ion or electron) cyclotron frequency in the mirror field, the RF field can be used to increase the transverse energy of the particles, and hence to ensure their confinement by the mirror field.   It is not easy to make a precise estimate of the RF power required by this approach.   In Appendix III, G. Francis has given a model of the mode of operation of the RF field, on the basis of which he shows that this proposal can likewise be rejected on energetic grounds;   if the RF field is to be effective, it has to transfer energy to the plasma at a significant fraction of the rate at which energy would otherwise be lost by the scattering of particles into the cone.   His argument depends essentially upon the fact that a particle passing through a region in which the RF field is in exact resonance with its cyclotron frequency experiences an irreversible change in the values of its energy and adiabatic invariants.   It therefore remains an open question whether the same conclusion would follow if the RF field was nowhere in exact resonance with the

cyclotron frequency, but closely approached this value in the mirror regions. In particular if it could be shown that such nearly-resonant RF fields could <u>reversibly</u> increase the magnetic moments of all the particles as they approach the mirrors, in such a way that they become trapped by the mirror field but do not on average absorb any RF power, then the use of supplementary RF fields might once more become a practical proposition. The purpose of this note is to show that under suitable circumstances nearly-resonant RF fields can act in precisely this way, and that the RF power required to maintain the necessary RF field strengths is substantially smaller than that required to effect non-resonant RF plasma confinement.

2. SINGLE PARTICLE MOTIONS

We may begin by outlining the derivation of the adiabatic equations which describe the motion of single particles in nearly-resonant RF fields. The basic equation of motion is

$$\dot{\vec{v}} = \vec{E}_\omega(\vec{r}) + \frac{\vec{v}}{c} \times \vec{B}_\omega(\vec{r}) + \vec{v} \times \vec{\Omega}(\vec{r}) \qquad \ldots \text{(AV.1)}$$

where we have for brevity written

$$\vec{E}_\omega = \frac{e}{m} \vec{E}_{RF} \, , \, \vec{B}_\omega = \frac{e\vec{B}_{RF}}{m} \, , \, \vec{\Omega} = \frac{e\vec{B}_o}{mc} \, .$$

If the length scales on which the static mirror field $\vec{B}_o$ and the RF fields $\vec{E}_{RF}$ and $\vec{B}_{RF}$ vary, L_Ω and L_ω respectively, are both long compared with the maximum displacement of the particle during a time $1/\Omega$ or $1/\omega$, we can calculate adiabatic invariants of the particle motion as follows. From the form of Eq. (AV.1) we expect that if $1/|\omega + \Omega|$ remains adequately bounded, a solution can be developed as a double series:

$$\vec{r}(t) = \sum_{m,n=o}^\infty \vec{a}_{mn}(t) \, e^{in\omega t + in\theta(t)} \qquad \ldots \text{(AV.2)}$$

where $\dot{\theta}(t) = \Omega$ and the coefficients $a_{mn}(t)$ are slowly varying functions of time. As we shall see, the lowest non-trivial approximation is that in which only the first five terms are retained, i.e.

$$\vec{r} = \vec{R} + \vec{\rho}_\omega + \vec{\rho}_\Omega \qquad \ldots \text{(AV.3)}$$

where

$$\vec{R} = a_{oo}(t) \, , \, \vec{\rho}_\omega = \vec{a}_{1o}e^{i\omega t} + \vec{a}_{-1o}e^{-i\omega t} \quad \text{and} \quad \vec{\rho}_\Omega = \vec{a}_{o1}e^{i\theta} + \vec{a}_{o-1}e^{-i\theta} \, .$$

Although the solution can be represented in this way, it is not possible to split the basic equation (AV.1) into three independent equations for $\vec{R}$, $\vec{\rho}_\omega$ and $\vec{\rho}_\Omega$, but a solution of the form (AV.3) can be obtained by iteration. We first obtain a lowest order equation for $\vec{\rho}_\omega$ by neglecting the term involving B_ω in Eq. (AV.1) and ignoring the spatial variation of $\vec{E}_\omega$ and $\vec{\Omega}$ (these approximations are justified by expansions in ρ_ω/L_ω and ρ_Ω/L_Ω):

$$\frac{d^2}{dt^2}\vec{\rho}_\omega = \frac{d}{dt}\vec{v}_\omega = \vec{E}(R)e^{i\omega t} + \vec{E}^*(R)e^{-i\omega t} + \vec{v}_\omega \times \vec{\Omega}(R) \qquad \ldots \text{(AV.4)}$$

Since the general solution of Eq. (AV 4) includes terms with time dependence other than $e^{\pm i\omega t}$, we take the particular integral only. This can be written as:

$$\vec{v} = \frac{e^{i\omega t}}{i\omega(\omega^2 - \Omega^2)} [\omega^2\vec{E} - (\vec{E}\cdot\vec{\Omega})\,\vec{\Omega} + i\omega\vec{\Omega} \times \vec{E}] + \text{c.c.} \qquad \ldots \text{(AV.5)}$$

or more elegantly as

$$\vec{v}_\omega = e^{i\omega t}\sum_{\nu=-1}^{1} \frac{E_\nu \vec{\tau}_{-\nu}}{i(\omega + \nu\omega)} + \text{c.c.} \qquad \ldots \text{(AV.6)}$$

where we have projected $\vec{E}$ onto a complex curvilinear co-ordinate system with unit vectors $\vec{\tau}_0, \vec{\tau}_{\pm 1}$ defined by $\vec{\tau}_0 = \vec{\Omega}/\Omega$, $\vec{\tau}_{\pm 1} = (\vec{\tau}_\alpha \pm i\vec{\tau}_\beta)/\sqrt{2}$, where $\vec{\tau}_\alpha$ and $\vec{\tau}_\beta$ are real unit vectors orthogonal to each other and to $\vec{\tau}_0$.

We now write $\vec{r} = \vec{r}_1 + \vec{\rho}_\omega : \vec{v} = \vec{v}_1 + \vec{v}_\omega$ and obtain from Eq. (AV.1) by Taylor expansion:

$$\frac{d\vec{v}_1}{dt} = \vec{v}_1 \times \vec{\Omega}(v_1) + \left\langle (\vec{\rho}_\omega \cdot \nabla) \ \vec{E}_\omega(r_1) + \vec{v}_\omega \times \frac{\vec{B}_\omega}{c}(r_1) + \vec{v}_\omega \times \rho_\omega \cdot \nabla \vec{\Omega}(r_1) \right\rangle \quad \ldots \text{(AV.7)}$$

In deriving Eq. (AV.7) we have ignored those terms obtained in the Taylor expansion of Eq. (AV.1) which have the time dependence $e^{\pm i\omega t}$ since these simply give a higher order correction to $\vec{\rho}_\omega$: we likewise ignore terms with the time dependence $e^{\pm 2i\omega t}$ and the brackets $\langle \rangle$ are used to signify their exclusion. We shall omit here the several pages of tedious tensor algebra required to show that the expression in the brackets $\langle \rangle$ can be simplified, using Eq. (AV.5) or Eq. (AV.6), to give

$$\frac{d\vec{v}_1}{dt} = \vec{v}_1 \times \vec{\Omega}(r_1) - \vec{\nabla}\Psi \qquad \ldots \text{(AV.8)}$$

where

$$\Psi = \frac{\vec{E}^* \cdot (\omega^2 \vec{E} - (\vec{E} \cdot \vec{\Omega})\vec{\Omega} + i\omega\vec{\Omega} \times \vec{E})}{\omega^2(\omega^2 - \Omega^2)} \equiv \frac{\vec{E}^* \cdot \vec{\vec{T}} \cdot \vec{E}}{\omega^2}$$

or

$$\Psi = \sum_{\nu = -1}^{1} \frac{|E_\nu|^2}{\omega(\omega + \nu\Omega)}$$

if we use the curvilinear co-ordinate system. The relevant algebra can be found in Motz and Watson[33]. Equation (AV.8) can immediately be integrated to give:

$$\tfrac{1}{2} v_1^2 + \Psi = \varepsilon$$

as an adiabatic invariant. Expressed in dimensional terms this is

$$\tfrac{1}{2} m \ (\vec{v} - \vec{v}_\omega)^2 + \sum_{\nu} \frac{e^2 |E_\nu|^2}{m\omega(\omega + \nu\Omega)} = \varepsilon \ . \qquad \ldots \text{(AV.9)}$$

The invariant ε is in fact the only invariant which we shall make use of in what follows: for the purposes of physical interpretation, however, it is useful to derive the other two invariants as well. Since

$$\Psi \sim v_\omega^2 , \quad \nabla\Psi \sim v_\omega^2/L_\omega \sim \omega v_\omega(\rho_\omega/L_\omega) \ ,$$

so for $\omega v_\omega \sim \Omega v_1$, the term $\nabla\Psi$ is a first order correction in Eq. (AV.8). Consequently the quantity $\mu \equiv \tfrac{1}{2} m v_{1\perp}^2 / B_0$ is an adiabatic invariant. Finally, if we write Eq. (AV.9), using the adiabatic invariance of μ , as

$$\tfrac{1}{2} m v_{1\parallel}^2 + \mu B_0 + \Psi = \varepsilon \qquad \ldots \text{(AV.10)}$$

and suppose that spatial variation of B_0 and Ψ are such that the particle is longitudinally confined, we can construct a third adiabatic invariant,

$$J = \oint (\varepsilon - \mu B_0 - \Psi)^{1/2} \ dr_{1\parallel} \qquad \ldots \text{(AV.11)}$$

which again differs from the usual expression for J in that the confining potential for B_0 is supplemented by the RF quasi-potential Ψ . If the magnetic field and the RF

field have, for example, axial symmetry then J is related to the drift invariant in the usual way.

For present purposes, the fact that we can derive an adiabatic invariant $\mu \equiv \frac{1}{2} mv_{1\perp}^2/B_0$ is less important than the fact that this is _not_ equal to the 'magnetic moment' of the particle, if we take the conventional definition of that quantity, $\mu_c \equiv \frac{1}{2} mv_\perp^2/B_0$. In fact $\mu_c = \frac{1}{2} m(\vec{v}_{1\perp} + \vec{v}_{\omega\perp})^2/B_0$ and we see that it can be written as

$$\mu_c = \mu + \frac{1}{2} mv_{\omega\perp}^2/B_0 + \delta\mu_c$$

where $\delta\mu_c$ is oscillatory and $\frac{1}{2} mv_{\omega\perp}^2/B_0$ monotonically increase as the particle moves into regions of increasing RF field strength. This is not, however, a useful piece of information, for we have no reason to believe that the force on the particle should have the form $\mu_c \vec{\nabla} B_0$: such an expression is only valid when μ_c results from magnetic gyrations alone. If on the other hand we define an effective RF magnetic moment μ_{RF} by the condition that the force acting on the guiding centre due to the gradient of the magnetic field can be written as

$$F = -(\mu + \mu_{RF})\vec{\nabla} B_0 \qquad \qquad \ldots \text{(AV.12)}$$

then we can obtain an expression for μ_{RF} from Eqs. (AV.9) and (AV.10) :

$$\mu_{RF} = \frac{e^3}{m^2 \omega c} \left[\frac{|E_{-1}|^2}{(\omega - \Omega)^2} - \frac{|E_{+1}|^2}{(\omega + \Omega)^2} \right] \qquad \qquad \ldots \text{(AV.13)}$$

Equation (AV.13) has a simple physical interpretation: the component of the RF field which is circularly polarised in the same sense as the magnetic gyrations of the particle tends to increase its magnetic moment; the component polarised in the opposite sense tends to decrease it or ven to give the particle an effective dipole moment orientated in such a way that it tends to move towards regions of higher magnetic field strength.

We are now in a position to select the parameters which are appropriate for our present purpose. Let us first note that it is unrealistic to contemplate exploiting the electron cyclotron resonance in a thermonuclear device which largely relies on magnetic confinement: the RF frequency required is in the millimetre waveband, and adequate power can neither be generated nor localised in the region required. We are therefore forced to use the ion cyclotron resonance. Let us assume for simplicity that we use a wave which is circularly polarised in the same sense as the ion gyrations, and that the frequency is slightly greater than Ω_i everywhere, so that there is no region where exact resonance conditions obtain. We then have

$$\Psi_i = \frac{e^2 |E_{-1}|^2}{\omega^2} \left(\frac{1}{m_i} \frac{\omega}{\omega - \Omega_i} \right) \quad \Psi_e = \frac{e^2 |E_{-1}|^2}{\omega^2} \left(\frac{1}{m_e} \frac{\omega}{\omega + |\Omega_e|} \right) = \frac{e^2 |E_{-1}|^2}{\omega^2} \left(\frac{\omega}{m_i \Omega_i} \right)$$

since $\omega \ll |\Omega_e|$ and $m_i \Omega_i = m_e |\Omega_e|$. Thus if we are _not_ close to resonance (so that $\omega - \Omega_i \sim \Omega_i$), the magnitudes of Ψ_i and Ψ_e are the same, but the directions of their gradients (for constant $|E_{-1}|^2$) are opposite. Under these circumstances, electrons would tend to get expelled from the machine, and space charge forces would develop which would destroy the ion confinement. If on the other hand we work close to resonance, $\Psi_i/\Psi_e = \frac{\Omega_i}{\omega - \Omega_i}$: i.e. the ions are more strongly confined than the electrons are expelled, and the space charge potential φ which is set up would be able to confine the electrons. It is clear that the presence of such an electrostatic field would modify Eq. (AV.10) simply by adding a term $+ e\varphi$ to the quasi-potential.

We may conclude this section by demonstrating that the RF field strengths required to confine single particles with thermonuclear energies in this way are well within the range of existing technology. We shall assume that we are working in the neighbourhood of the deuterium ion cyclotron frequency in a peak mirror field of 200 kG ($\Omega_i = 10^9$ sec^{-1}) and that we wish to confine ions with energy 100 keV. The electric field strength at which the quasi-potential Ψ reaches this value can be written as

$$E = 4 \cdot 5 \ 10^5 \left(\frac{\omega - \Omega}{\omega} \right)^{\frac{1}{2}} \text{V cm}^{-1}.$$

Even if we do not take advantage of the reduction obtained by working very near resonance, such field strengths are comparable with those used in conventional linear accelerators. It should be noted, however, that the radiation pressure associated with such a field is only of order $\frac{1}{10} \frac{\omega - \Omega}{\omega}$ atmospheres, whereas the pressure of a thermonuclear plasma of interesting density is some tens of atmospheres. We have therefore to examine whether such RF fields can be maintained in the presence of the plasma, and this requires a self-consistent solution of the plasma and field equations.

3. THE SELF-CONSISTENT FIELD EQUATIONS

The analysis given here is a slight extension of that given in Motz and Watson[33]. The general equilibrium solutions of the ion and electron Vlasov equations $f_{\pm}$ are $f_{\pm}(\varepsilon_{\pm}, \mu_{\pm}, J_{\pm})$, where we have put subscripts $\pm$ on all quantities to indicate the mass and charge parameters which feature in them. These enable us to calculate charge and current densities $\rho = \rho_+ + \rho_- = e \int (f_+ - f_-) d^3 v$; $\vec{j} = \vec{j}_+ + \vec{j}_- = e \int \vec{v} (f_+ - f_-) d^3 v$ and hence to write down Maxwell's equations for the electrostatic and transverse fields in the plasma. In general these constitute a set of four coupled non-linear partial differential self-consistent field equations; the following simplifying assumptions however lead to a single non-linear ordinary differential equation. We first suppose that the plasma remains approximately quasi-neutral, i.e. that $\int f_+ d^3 v = \int f_- d^3 v$ almost everywhere. If we take $f_{\pm}$ to be Maxwellian functions of $\varepsilon_{\pm}$ only (this assumption is appropriate if most of the collisions occur in the region of space between the two mirrors where the RF field strength is negligible), the quasi-neutrality condition gives us:

$$e^{-(\Psi_+ + e\varphi)/kT} = e^{-(\Psi_- - e\varphi)/kT} \qquad \text{... (AV.14)}$$

or $2e\varphi = \Psi_- - \Psi_+ \simeq -\Psi_+$ (for $\frac{\Omega_+}{\omega - \Omega_+} \gg 1$), and hence $\rho_{\pm} = \rho_0 e^{-\Psi_+/2kT}$ and $f_{\pm} = f(\frac{1}{2} m_{\pm} (\vec{v} - \vec{v}_{\omega\pm})^2 + \Psi_+/2)$. Inserting this into the wave equation, we obtain

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \frac{\omega^2}{c^2} \sum_{\nu} \left(1 - \frac{\omega_{p+}^2 \ e^{-\Psi_+/2kT}}{\omega(\omega + \nu \ \Omega_+)} - \frac{\omega_{p-}^2 \ e^{-\Psi_+/2kT}}{\omega(\omega + \nu \ \Omega_-)} \right) \vec{\tau}_{-\nu} \ \vec{\tau}_{\nu} \cdot \vec{E} \qquad \text{... (AV.15)}$$

when $\omega_{p\pm}$ are the ion and electron plasma frequencies at points when $\Psi_+ = 0$. We now assume that the RF field is a plane, circularly polarized standing wave $\vec{E} = E(z)(\cos \omega t, \pm \sin \omega t, 0)$ with its plane normal to the axis of an axially symmetric mirror machine whose field can be written in the neighbourhood of the axis as

$$\vec{B}_0 = (- \frac{x}{2} \Omega'_+, \quad - \frac{y}{2} \Omega'_+, \quad \Omega_+) m_+ \ c/e$$

where $\Omega(z)$ is equal to the ion cyclotron frequency on the axis of the machine and $\Omega'_+ \equiv \frac{d\Omega_+}{dz}$. If we expand Eq. (AV.15) in the neighbourhood of the axis and take the lowest

order terms only, we obtain (neglecting the non-resonant electron term)

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2} E = \frac{\omega^2_{p+}}{c^2} E \left(\frac{\omega}{\omega - \Omega_+}\right) e^{-\frac{e^2E^2}{4m_+\omega^2 kT}\left(\frac{\omega}{\omega - \Omega_+}\right)} \qquad \ldots (AV.16)$$

On multiplying by $\dfrac{c^2}{\omega^2}\dfrac{1}{4\pi}\dfrac{dE}{dz}$ and rearranging, this gives

$$\frac{d}{dz}\left(\frac{E^2 + B^2_{RF}}{8\pi} + 2n_0 kT\ e^{-\Psi_+/2kT}\right) = -2n_0 kT \left(\frac{\Psi_+}{2kT}\right) e^{-\Psi_+/2kT}\left(\frac{\Omega'_+}{\omega - \Omega_+}\right) \qquad \ldots (AV.17)$$

Equation (AV.17) shows that in the absence of a magnetic field gradient ($\Omega' \equiv 0$), the total pressure (i.e. the sum of the RF pressure and the ion and electron pressures) is constant, so that an equilibrium configuration in which a plasma is confined by the action of an RF field can only exist if the RF pressure outside the plasma equals the plasma pressure inside. If the magnetic field in non-uniform and of mirror type, however, Eq. (AV.17) shows that the static magnetic field can take up part of the pressure, since if we integrate Eq. (AV.17) from a point inside the plasma outwards along a field line, the right hand side is never positive, so the total pressure (in the above sense) can only decrease. It may appear surprising that the non-uniformity of the magnetic field does not simply add a term such as $B^2_0/8\pi$ to the expression for the total pressure balance: such a term is of course obtained if one writes down the z component of the divergence of the total stress tensor for the plasma plus electromagnetic fields, but it is cancelled by terms deriving from the off-diagonal terms in the magnetic stress tensor, and the term which appears on the right of Eq. (AV.17) arises from the off-diagonal terms of the plasma stress tensor. It follows that we do not expect to obtain an exact integral of Eq. (AV.17) and it is necessary to resort to numerical methods. Nonetheless the qualitative characteristics of the solutions can be inferred by inspection as follows.

For convenience we shall rewrite Eqs. (AV.16) and (AV.17) in dimensionless form, with

$$x = \left(\frac{e^2E^2}{4m_+\omega^2 kT}\right)^{1/2}, \quad t = \omega z/c, \quad A = \omega^2_{p+}/\omega^2 \quad \text{and} \quad b = \Omega_+/\omega = b(t).$$

We obtain

$$\ddot{x} + x = \frac{A}{1-b} \times e^{-x^2/(1-b)} \qquad \ldots (AV.18)$$

$$\frac{d}{dt}\left(\dot{x}^2 + x^2 + A\ e^{-x^2/(1-b)}\right) = -A\frac{x^2}{1-b} e^{-x^2/(1-b)} \cdot \frac{\dot{b}}{1-b} \qquad \ldots (AV.19)$$

The notation has been chosen to emphasize the formal analogy between Eqs. (AV.16) and (AV.17) for the field configuration as a function of the spatial variable z and Eqs. (AV.18) and (AV.19) which describe the motion in time of a particle in the time dependent force field

$$F = \left(\frac{A}{1-b}e^{-x^2(1-b)} - 1\right)x.$$

As Eq. (AV.19) indicates. the particle motion is non-conservative. and can be regarded as the motion of a particle in a potential well

$$V(x,t) = x^2 + A\ e^{-x^2/(1-b)}$$

which is dissipating its energy at a rate

$$A\frac{x^2}{1-b} e^{-x^2/(1-b)} \frac{\dot{b}}{1-b}.$$

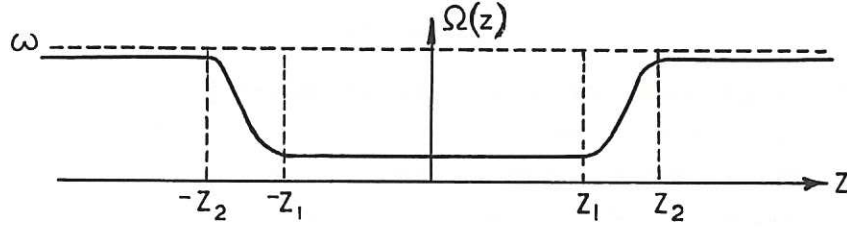Let us for definiteness consider a mirror field of the form indicated in Fig.14:

Fig. 14   (CLM-R94)
Mirror field with uniform sections joined by mirror regions

with uniform sections $-\infty \geqslant z \geqslant -z_2$, $-z_1 \geqslant z \geqslant z_1$, $z_2 \leqslant z \leqslant \infty$ joined by mirror regions.
Correspondingly, b is independent of t in the uniform regions, and increases with t
during the interval

$$\frac{z_1 \omega}{c} \equiv t_1 \leqslant t \leqslant t_2 = \frac{z_2 \omega}{c} .$$

Thus the particle energy remains constant in the intervals $-t_1 \leqslant t \leqslant t_1$ and $|t| \geqslant t_2$,
but declines in the interval $t_1 \leqslant t \leqslant t_2$. It is illuminating to represent the resulting
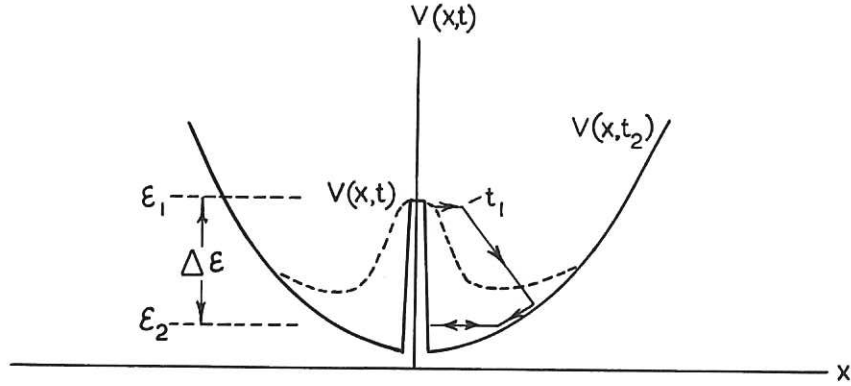trajectory in a diagram in which $V(x, t)$ is plotted as a function of x (see Fig. 15).

Fig. 15 $V(x, t)$ plotted as a function of x   (CLM-R94)

The dotted line indicates $V(x, t_1)$ and the continuous line $V(x, t_2)$: for intermediate
values of t the curve lies between the two limiting curves drawn. One possible trajec-
tory has been drawn, with the points t_1 and t_2 marked. It will be seen that before
t_1 and after t_2 the particle moves conservatively in the potential well : during the non-
conservative phase it drops in energy by an amount $\Delta\varepsilon$ which in general depends upon the
magnetic field profile, the values of x and $\dot{x}$ at the time t_1 , and the parameter A.
However, since $\varepsilon \equiv \dot{x}^2 + V$ is a positive definite function, the trajectory can never cross
the instantaneous curve $V(x, t)$, and hence ε cannot fall below the minimum value of
$V(x, t_2)$. Nevertheless ε_2 can be a very small fraction of ε_1 : if $\varepsilon_1 = A$ (a choice
which has the consequence that the RF field strength decays exponentially into the plasma),
the maximum value of

$$\varepsilon_1/\varepsilon_2 = \frac{A}{1-b_2} \left(1 + \ell n \frac{A}{1-b_2} \right) = \frac{\omega_{p+}^2}{\omega(\omega-\Omega)} \left(1 + \ell n \frac{\omega_{p+}^2}{\omega(\omega-\Omega)} \right) .$$

Recently obtained numerical results indicate that configurations exist in which this maximum
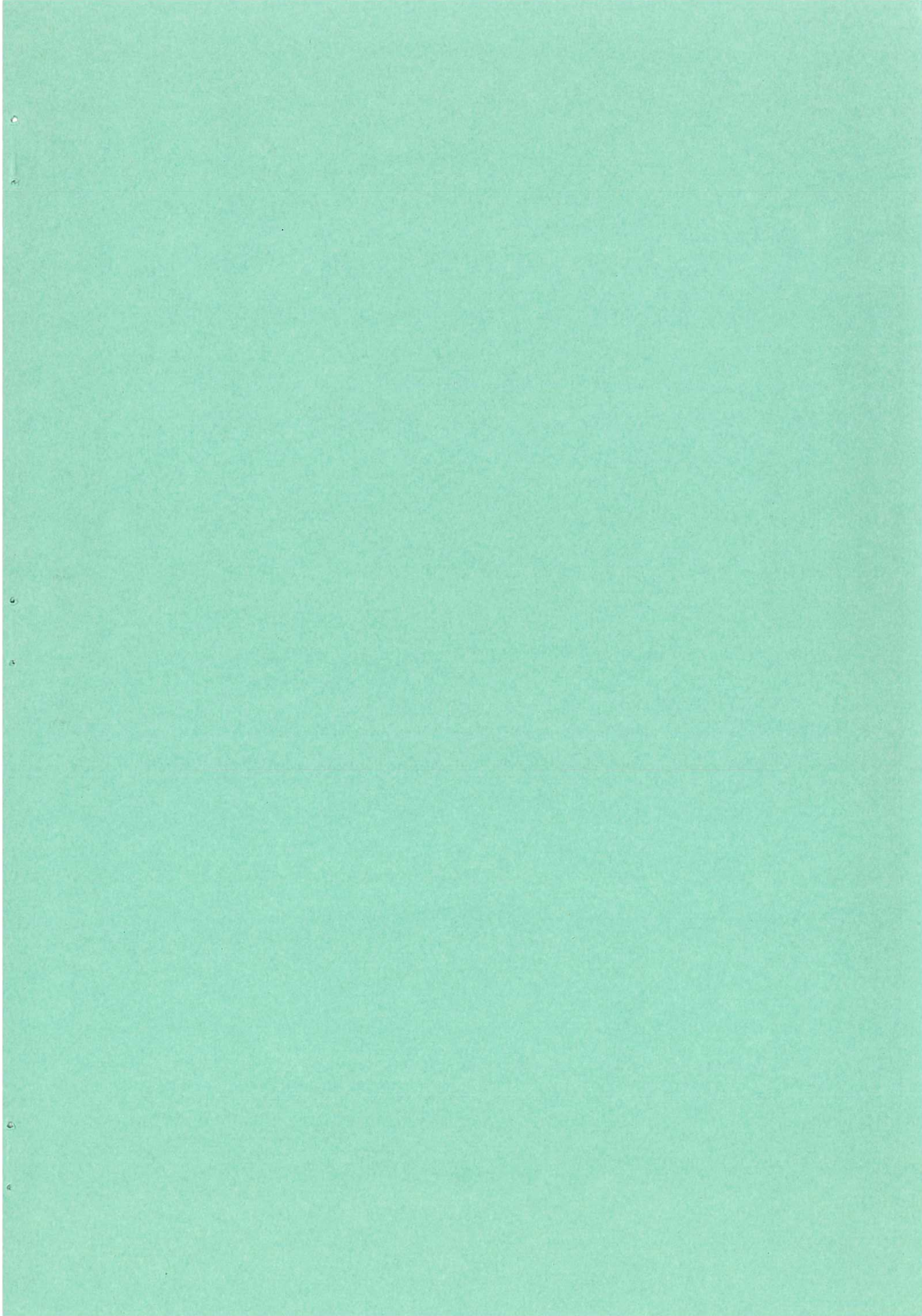value of the ratio $\varepsilon_1/\varepsilon_2$ is approached.

4. CONCLUSIONS

We have seen that particles moving in a magnetic mirror field upon which a nearly-resonant RF field has been superimposed can experience a reversible increase in their effective magnetic moments, such that they are reflected by the mirror field even though they would not have been reflected by the mirror field alone. It was shown that the RF field strength required to confine thermonuclear deuterons in this way is well within the range of existing technology but that the RF pressure created by such field strengths is much less than the pressure of a typical thermonuclear plasma. However, as the preceding section shows, with a suitable choice of magnetic field profile, self-consistent plasma field equilibria exist in which the external RF pressure is much less than the central plasma pressure. It is not possible at this stage to give a firm estimate of the pressure ratio which might be achieved in this way, but a ratio of 50 might well be obtained. Since the thermonuclear power output increases as p^2 if other quantities are held fixed, an improvement of three orders of magnitude might be achieved in the ratio of the thermo-nuclear power output to RF power required. Since the energy balance in a long mirror machine supplemented by radiation pressure is in any case no worse than marginal, this improvement, if realised, might provide the basis for a feasible reactor design.

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