

United Kingdom Atomic Energy Authority

RESEARCH GROUP

Report

# A THERMIONIC LITHIUM PLASMA SOURCE FOR DRIFT WAVE EXPERIMENTS

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1969

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### A THERMIONIC LITHIUM PLASMA SOURCE FOR DRIFT WAVE EXPERIMENTS

by

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#### ABSTRACT

For experiments on drift waves in thermionic plasmas, it is important that the surface temperature of the ionizing plate should be carefully controlled in order to minimise the effects on the plasma of electric fields produced in the end plate sheaths. The ion Larmor radius may be made small by using a lithium plasma. Details are given of the design and construction of a source which has been used to produce a 5 cm diameter column of lithium plasma at densities in the range  $10^7 - 10^9$  cm<sup>-3</sup>.

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January, 1969 (C/18 MEJ)

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#### 1. INTRODUCTION

We are building an experiment to investigate drift waves in a cylindrical column of plasma produced by the thermal ionization of lithium atoms at a hot metal surface. Considerations of power consumption in the field windings limit the column radius R and we are therefore working with a lithium plasma in order to make the ion Larmor radius  $a_i$  small in order to avoid the ion-cyclotron instability, which Mikhailovskii  $\binom{1}{1}$  predicts will disappear if  $R/a_i > (m_i/m_e)^{\frac{1}{2}}$ ;  $m_i$  and  $m_e$  being the ion and electron masses respectively.

The efficiencies of various refractory metals for the thermal ionization of lithium (ionization potential  $V_i$  = 5.36 V) are compared in Table 1, using values for the effective work function for ionization  $\phi_i$  measured by Zandberg and Tontegode (2).

TABLE 1

Plate Material	Та	Mo	W	Re
Work Function φ <sub>i</sub> volt.	4.88 ± 0.05	5.02 ± 0.03	5.14 ± 0.03	5.43 ± 0.03
Ratio of ion and atom fluxes leaving the surface at 2000 <sup>0</sup> K	0.04	0.10	0.14	0.61
Ionization efficiency at 2000 <sup>0</sup> K	4%	10%	12.5%	38%

In collision-dominated plasmas, for which it is satisfactory to assume thermal equilibrium between plasma and hot plate, the plasma density is predicted  $^{(3)}$  to be independent of the value of  $\phi_i$ , suggesting that any refractory metal surface could be used to produce a lithium plasma. However low density plasmas may not be in thermal equilibrium and it is desirable to use rhenium hot plates in order to optimise the ionization of lithium.

We have produced lithium plasmas by ionization at tantalum, tungsten and rhenium surfaces and have found ionization rates consistent with those quoted in Table 1.

#### 2. EFFECTS DUE TO NON-UNIFORMITIES IN THE HOT PLATE TEMPERATURE

It is well known<sup>(4)</sup> that temperature gradients across the hot plate surface can have a serious influence on the behaviour of a thermionic plasma. We will consider a surface

at a temperature T eV with a uniform work function for electron emission  $\phi_e$ . The surface potential will be taken as zero and the plasma potential as U. For an electron sheath, i.e. an electron excess outside the plate, U < 0; for an ion sheath U > 0. We will define

$$\alpha = 1$$
,  $\beta = 0$ ; for  $U < 0$   
 $\alpha = 0$ ,  $\beta = 1$ ; for  $U > 0$ 

and

$$y = \exp - \frac{|U|}{T}$$

The electron flux emitted by the plate is given by the Richardson current

$$j_e = R(T)y^\alpha = AT^2 \exp (-\phi_e/T)y^\alpha$$

where  $A = 1.0 \times 10^{29} \text{ electrons cm}^{-2} \text{ sec}^{-1} \text{ V}^{-2}$ .

We will assume a Maxwellian velocity distribution for the plasma electrons with a mean thermal velocity  $c_e = (2kT/m_e)^{\frac{1}{2}}$ . Neglecting electron losses other than at the end plate, the equation of electron conservation gives

$$R(T)y^{\alpha} = 1/4 \text{ nc}_{e} y^{\beta}$$
 ... (2.1)

It follows that

$$\frac{\partial U}{\partial T} = \ell n \ n - \frac{3}{2} \ell n \ T - 51.6 \qquad ... (2.2)$$

$$\frac{\partial U}{\partial n} = \frac{T}{n} \qquad (2.3)$$

and thus

$$\frac{\partial U}{\partial r} = T \left( \frac{\partial U}{\partial T} S_T^{-1} + S_n^{-1} \right) . \qquad (2.4)$$

The radial scale lengths of the temperature and density gradients have been introduced, defined by

$$S_{T}^{-1} = \frac{1}{T} \frac{dT}{dr}$$

$$S_n^{-1} = \frac{1}{n} \frac{dn}{dr} .$$

If the surface temperature is uniform, (2.4) indicates that a radial electric field  $E_{\Gamma}$  will exist wherever there is a radial density gradient. For the usual case where the density profile decreases smoothly towards the edge of the plasma, the centre of the column will be positive with respect to the edge. In a uniform axial magnetic field B this will produce an E/B rotation of the plasma which by equation (2.4) will be of opposite sign to the electron diamagnetic drift velocity  $C \frac{T}{B} S_{\Gamma}^{-1}$ . A drift wave would thus appear stationary

in the laboratory frame of reference if finite Larmor radius effects are negligible.

If the plate temperature is higher at the centre, the temperature gradient term in (2.4) will dominate and as (2.2) is negative, the centre of the plasma column will have a negative potential. This results in an E/B rotation in the same direction as the electron diamagnetic drift and drift waves would be observed at a Doppler shifted frequency (5).

The radial electric field can be made zero if the scale lengths  $\,{}^{\rm S}_{\rm T}\,$  and  $\,{}^{\rm S}_{\rm n}\,$  have the same sign and are adjusted to make

$$\frac{S_{T}}{S_{n}} = 51.6 + 1.5 \ln T - \ln n . \qquad (2.5)$$

For example, if  $n = 10^{10} \text{ cm}^{-3}$  and T = 0.2 eV;  $S_T \sim 25 \text{ S}_n$  to satisfy (2.5). Thus  $\Delta T/T \sim 4\%$  and the radial electric field of the uniform temperature case could be annulled by a radial variation in temperature of approximately  $80^0$  in  $2000^0$ K. A radial electric field variation will cause a rotation of the plasma column but will not lead to plasma loss.

Azimuthal temperature inhomogeneities are expected to be more damaging to the plasma equilibrium, since they will produce a radial component of the E/B drift, leading to enhanced plasma losses. To keep the temperature-induced loss rate less than that due to diffusion we will need

$$n \frac{c E_{\theta}}{B} < \frac{dn}{dr} D_{\perp} \qquad (2.6)$$

where  $D_{\perp}$  is the coefficient for diffusion across the magnetic field. Assuming azimuthal symmetry for the plasma density profile

$$E_{\theta} = \frac{1}{r} \frac{dU}{d\theta} = \frac{1}{r} \frac{\partial U}{\partial T} \frac{dT}{d\theta}$$

and taking Bohm's formula for the diffusion coefficient, (2.6) becomes

$$\frac{\text{nc}}{\text{B}} \frac{\partial \text{U}}{\partial \text{T}} \frac{1}{\text{r}} \frac{\text{dT}}{\text{d\theta}} < \frac{\text{dn}}{\text{dr}} \frac{\text{c}}{16\text{B}}$$

i.e.

$$\frac{1}{T}\frac{dT}{d\theta} < \frac{r}{n}\frac{dn}{dr}\left(16\frac{\partial U}{\partial T}\right)^{-1} \qquad ... (2.7)$$

If  $\Delta T$  is the temperature difference between opposite sides of the hotplate, we require

$$\frac{\Delta T}{T} < \frac{\pi}{16,25}$$

and thus the temperature must be uniform azimuthally to better than 1% to achieve containment times exceeding the Bohm time.

An additional requirement is that the time scale of the radial E/B drift should be long compared to the ion transit time between the ends of the plasma column,

i.e. 
$$\frac{R}{c E_{\theta}/B} > \frac{L}{C_{i}} \qquad ... (2.8)$$

where  $C_i$  is the ion thermal velocity and R and L are the radial and axial dimensions of the column, typically 1 cm and 1 m respectively. For a lithium ion in a magnetic field of 1 kG, the maximum permissible azimuthal asymmetry in temperature is about  $4^{\circ}$ K.

It is obviously very important to ensure that the hot plate temperature isotherms are circular and closed within the diameter of the plate.

#### 3. CALCULATIONS OF THE HOTPLATE TEMPERATURE

In our experiment the hotplate is heated by electron bombardment of the rear surface from a hot filament maintained at a negative potential relative to the plate. Azimuthal temperature gradients on the front surface of the plate are minimised by ensuring accurate circular symmetry of the hot plate and filaments. The radial temperature profile across the front surface of the plate is determined by the diffusion of heat from the heated region on the rear surface.

Within a single solid plate the temperature T is satisfied by Laplace's equation

$$\nabla^2 T = 0 \qquad \dots (3.1)$$

The temperature gradient at the plate surface is given by the net radiative heat flux  $\dot{Q}$  to the surrounding surface which is at some temperature  $T_{\rm S}$ ,

$$\mathbf{K}\nabla\mathbf{T} = \mathbf{\dot{Q}} = \tau(\alpha\mathbf{T}^4 - \alpha_{\mathbf{S}}\mathbf{T_{\mathbf{S}}}^4) \qquad \dots \qquad (3.2)$$

where K is the thermal conductivity of the plate and  $\alpha$  is its emissivity.  $\tau$  is Stefan's constant =  $5.672 \times 10^{-5}$  erg sec cm<sup>-2</sup> deg<sup>-4</sup>. Conduction losses from the hotplate, which is supported at the edge in a thin walled tantalum cylinder, are negligible compared to the radiation losses.

The heat input W from a filament of effective area A can be represented conveniently by a radiating surface at a temperature  $T_{\rm F}$  given by

$$T_{F} = (W/AcJ)^{1/4}$$
 ... (3.3)

Assuming azimuthal symmetry for the hotplate and filaments, we have solved numerically in cylindrical geometry the 2-dimensional Laplace's equation by means of a successive over-relaxation method<sup>(6)</sup>. The calculated plate temperature is found to converge after several hundred iterative approximations.

A typical contour map of the temperature isotherms calculated for a hotplate of diameter 5.0 cm and thickness 2 mm is shown in Fig.1. The input power in this case was 2 kW from a 2 mm thick wire filament formed into a ring of diameter 3.0 cm. Experimentally measured values were taken for the surrounding wall temperatures.

Increasing the thickness of the plate improves the radial uniformity of the temperature across the front surface, Fig.2. A plate thicker than 1 cm would thus be needed to provide a reasonably uniform temperature. A second concentric ring filament improves the uniformity and the radial temperature profile may be controlled by varying the ratio of heating from the two filaments, Fig.3.

We found experimentally that the rear surface of a rhenium plate could be damaged easily by localised melting. To prevent this a tungsten disc is mounted behind the rhenium disc, separated from it by about 0.5 mm. This is also advantageous economically since it permits the use of thinner rhenium plates.

Temperature isotherms calculated for such a composite plate are shown in Fig.4.

Control of the radial profile can be effected by varying the ratio of the heating between two concentric ring filaments, Fig.5. A filament design using radial spokes to provide heating over a larger area than the ring filaments is described in the following section as part of a practical design, and calculations of the performance of this filament, Fig.6, show that temperatures uniform to within 5° can be expected.

#### 4. PLASMA SOURCE DESIGN

The construction of a reliable source of lithium plasma meeting the above requirements calls for careful attention to the design of the hotplate and filament structure in order to achieve the desired temperature uniformity. The materials chosen must be capable of withstanding high temperatures and should also be compatible with a clean high vacuum environment. The design of a 5 cm diameter source, shown schematically in Fig.7, is discussed below.

The hotplate consists of a rhenium disc, of diameter 6 cm and thickness 1.5 mm, supported in one end of an 8 cm long tantalum cylinder. The rhenium is backed by a

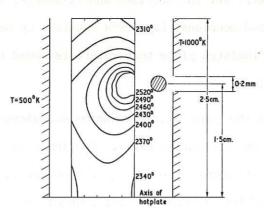


Fig. 1 (CLM-R98)
Contour map of temperature isotherms computed for a hotplate of diameter 5.0 cm and thickness 0.2 cm. Heat input was 2000 W from a 0.2 cm thick wire filament formed into a ring of diameter 3.0 cm

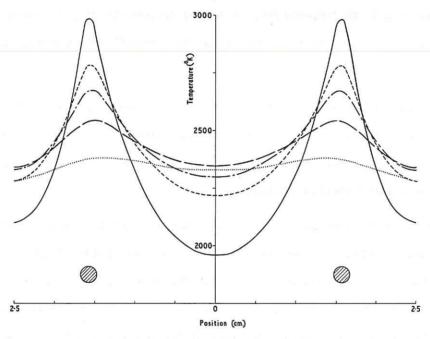


Fig. 2 (CLM-R98)

The temperature profile across the front surface of a single hotplate showing the dependence on plate thickness. — 0.1 cm; ----- 0.2 cm — 0.3 cm; — 0.5 cm; ·----- 1.0 cm

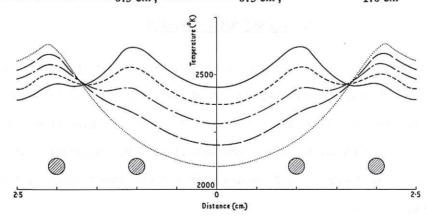


Fig. 3 (CLM-R98)
Radial temperature profile for a 0.3 cm thick plate heated by two concentric ring filaments:

dere. The recount is r	Inner filament	Outer filament
( <del></del>	— 1000 W	1000 W
	- 750	1250
	<b>—</b> 500	1500
	<b>—</b> 250	1750
	0	2000

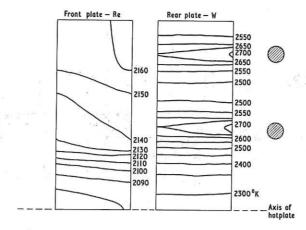


Fig. 4 (CLM-R98)
Contour map of temperature isotherms computed for a double hotplate of diameter 5.0 cm, and plates of thickness 0.15 cm. Heat input from two concentric ring filaments

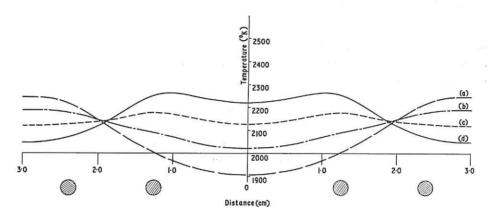


Fig. 5 (CLM-R98) Temperature profile computed for a double hotplate heated by two concentric ring filaments. Inner filament 750 W, Outer filament 1250 W

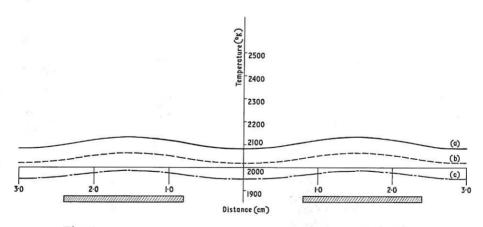


Fig. 6 (CLM-R98)
Temperature profile computed for a double hotplate heated by a spiral filament, input 1650 W

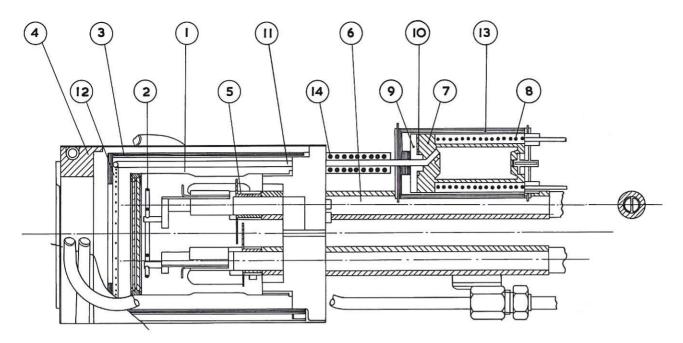


Fig. 7 (CLM-R98) Lithium plasma source. 1. Hot plate assembly. Tantalum, tungsten and rhenium. 2. Filaments. Tantalum-tungsten alloy. 3. Heat shields. Tantalum. 4. Water cooled shield. Copper and stainless steel. 5. Electron shields. Tantalum. 6. Tubes, double-dee cross section. Copper. 7. Lithium oven. Stainless steel. 9. Oven lid. Molybdenum. 10. Oven 8. Oven heater. Tantalum and alumina. seal. Stainless steel. 11. Lithium feed tube. Tantalum. 12. Lithium manifold. 13. Oven heat shields. Stainless steel. Tantalum. 14. Feed tube heater. Tantalum and alumina

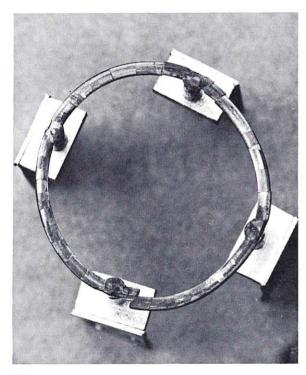


Fig. 8 (CLM-R98)
Ring filament of pure tantalum after crystallisation during high temperature operation

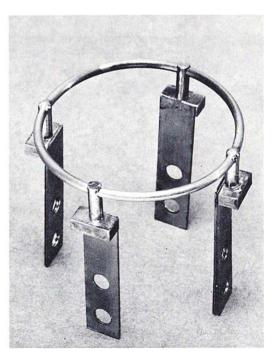


Fig. 9 (CLM-R98) Ring filament of tantalum-tungsten alloy showing no crystallisation

tungsten disc of similar dimensions, which serves the dual purpose of improving the temperature distribution and also protecting the rhenium plate against localised overheating and damage under fault conditions. The plates are separated by 0.5 mm and the facing surfaces have been roughened by spark erosion to improve the radiative heat transfer. In order that the plasma potential may be varied, the hotplate is insulated from earth at the cold end of the tantalum support tube by insulators of boron nitride. Having a vapour pressure of 10<sup>-6</sup> torr at 1500°C rising to 10<sup>-4</sup> torr at 1750°C this material is unfortunately unsuitable for insulation purposes within the hotplate itself.

A black body at 2000 K radiates approximately 90 W cm<sup>-2</sup> and, allowing for the lower emissivity of rhenium and tungsten surfaces, some 2 to 3 kW are lost by radiation from the hotplate. Conduction losses are reduced by holes cut in the tantalum support tube and are estimated as less than 100 W. The rear surface of the tungsten plate is heated by electron bombardment from a directly heated filament spaced 5 mm from the plate. Typical operating conditions are 1.5 A electron emission at 2 kV filament to plate potential. The emission current is stabilised by transistorised control of the filament heating current using the emission current as a control signal.

Two designs of filament have been developed; a pair of concentric ring filaments which permits variation of the radial temperature profile of the hotplate, and also a filament of a spiral design which provides a very uniform plate temperature. The concentric ring assembly consists of two circular filaments of diameter 2 and 5 cm made of 2 mm tantalum alloy wire containing 10% tungsten. This material, whilst being easier to fabricate than tungsten and possessing good welding properties, has a greater tensile strength at high temperatures than pure tantalum. The current through one half of each ring is typically 100 A and significant mechanical forces are exerted when the source is operated in magnetic fields up to 4 kG. After being used at high temperatures, filaments of pure tantalum developed a crystalline structure and became severely distorted, as in Fig.8. Fig.9 shows a similar filament made of tantalum-tungsten alloy which suffered no distortion after experiencing the same conditions.

The spiral filament is shown in Fig. 10. The construction is of tantalum with the exception of the filament wires which are of 0.5 mm diameter tantalum-tungsten alloy. The curvature of the filament wires is given by the curve shown in Fig. 11,

$$\theta = \sin (a/r) + (r^2/a^2-1)^{\frac{1}{2}} - \pi/2$$

which is calculated to produce a radially even heating of the hotplate. The curvature also takes up thermal expansion and electromagnetic stresses in the wires.

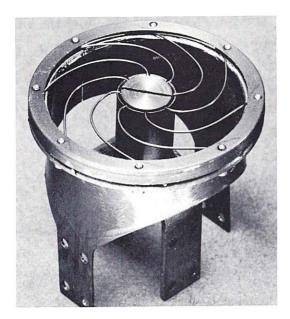


Fig. 10 (CLM-R98)
Spiral filament assembly. Tantalum is used throughout with the exception of the filament wires which are of tantalum-tungsten alloy

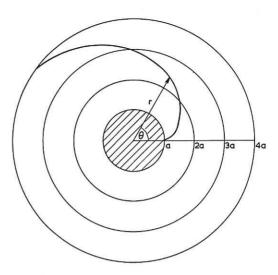


Fig. 11 (CLM-R98) Curvature of the spiral filament wires.  $\theta = \sin(a/r) + (r^2/a^2 - 1)^{1/2} - \pi/2$ 

The plasma source is always operated in a uniform magnetic field exceeding 100 G which prevents spreading of the electrons emitted from the filament. Electrons emitted in the backwards direction are captured on tantalum screens at the filament potential.

Current is fed to the filaments by four, 3/8 inch o.d. copper tubes, of double-dee cross-section enabling a separate flow and return of cooling water down each tube. Heat losses from the edge of the hot plate and the heat flux to the walls of the vacuum vessel are minimised by a series of radiation shields which surround the whole assembly. The outer shield is water cooled as it has been found to be important to define the temperature boundaries of the system if long term temperature drifts are to be avoided. A water-cooled ring of aperture 5.0 cm diameter in front of the hotplate restricts the diameter of the plasma column.

Lithium vapour is directed onto the front surface of the hotplate through a series of 0.2 mm diameter holes drilled in a circular manifold made from 3 mm o.d. tantalum tube. This produces a fairly uniform flux of lithium atoms over the whole plate. An alternative system giving a more localised flux, has been developed in which lithium vapour is fed through small holes in the centre of the hotplate itself. The corrosive nature of lithium vapour and liquid necessitates careful choice of materials for the feed system. The lithium is vaporised in a stainless steel oven (Fig.7) at a temperature of 700°C, when the vapour pressure is 1 torr. The oven is heated by a tantalum wire element wound on an alumina former and is surrounded by a series of heat shields. A tantalum feed tube is welded into the molybdenum oven lid which is sealed to the body by an inert gas-filled

stainless steel 'O' ring. The feed tube is heated by radiation from the hot plate and by a separate heater winding near to the oven.

A photograph of the partly assembled source with some heat shielding removed is shown in Fig.12. The performance of this design of source has been found to be very satisfactory and details of temperature measurements and plasma production experiments will be given in the following section.

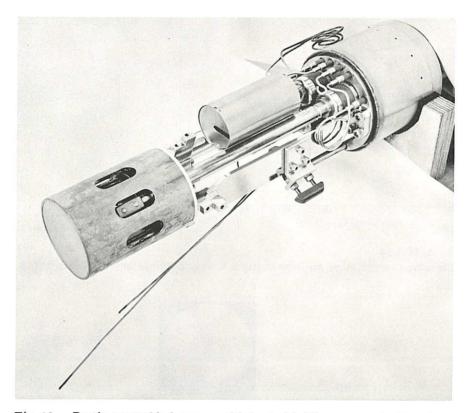
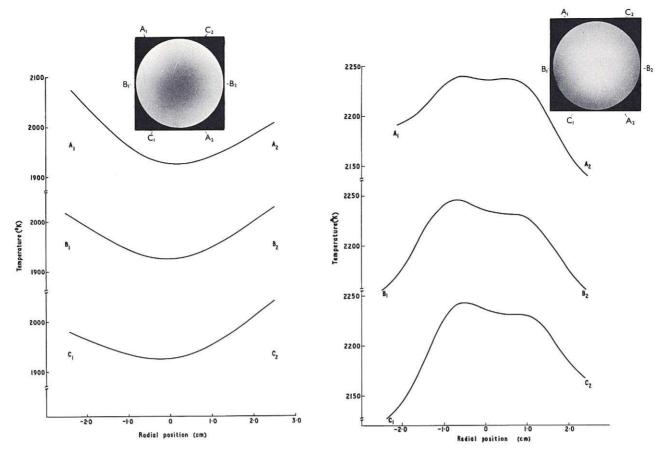


Fig. 12 Partly assembled source with heat shielding removed (CLM-R98)

#### 5. PERFORMANCE OF THE SOURCES

The hotplate temperature was measured by an optical pyrometer sensitive to the wavelength range 0.5 to 1.2 microns (Land type NQO 9/500/20 AE). The instrument was mounted outside the vacuum vessel behind a glass window and could be scanned across the hotplate by means of a 45° mirror attached to a moving probe. When operated at its focal distance of 50 cm the pyrometer had a resolution of 0.15 cm. After amplification by 30 dB to millivolt level, the pyrometer output was displayed on a chart recorder. The readings were corrected for plate emissivity and transmission losses in the optical system and were converted to degrees using a black body source calibration chart. The sensitivity of the pyrometer at plate temperatures of 2000°K was of the order of 1°K and the absolute



Figs. 13&14 (CLM-R98) Measured temperature profiles using a double hotplate and concentric ring filaments

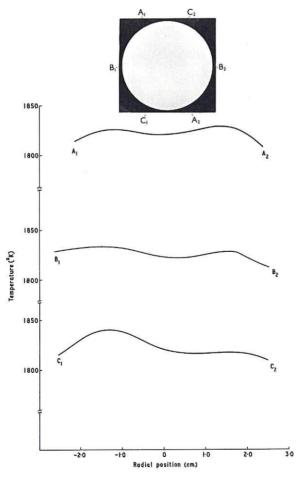


Fig. 15 Measured temperature profiles using a spiral filament (CLM-R98)

accuracy better than  $100^{0} \mathrm{K}$ , this being limited by the value taken for the emissivity of rhenium.

Photographs of the hotplate taken through a dark filter simultaneously with the pyrometer measurements were used to give a visual measure of the hotplate temperature variations.

Typical temperature profiles and photographs using two concentric ring filaments to heat the plate are shown in Figs.13 and 14. The shape of the radial temperature profile could be controlled by varying the electron emission from the two filaments. The azimuthal uniformity of temperature was found to be less than required with only about 50% of the plate area lying on closed isotherms. It is thought that these non-uniformities arise from the difficulty of accurately positioning the filaments whilst allowing freedom for thermal expansion and some modifications to the design are in hand which should produce improvements.

The spiral type of filament produces a more uniform hotplate, Fig.15, which it is thought would be improved further by a more accurate location of the ends of the filament wires assisted by an increased spacing between filament and hotplate.

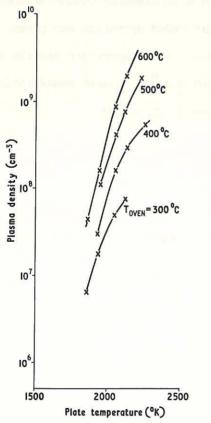


Fig. 16 (CLM-R98)
Variation of plasma density in single ended column, with plate and oven temperatures,
Tp and To respectively

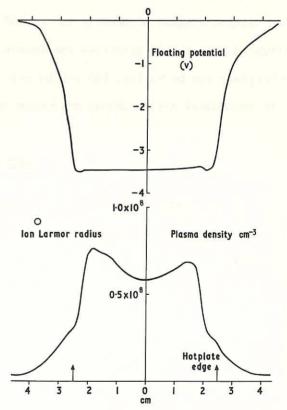


Fig. 17

(a) Radial density profile of single ended plasma column. (b) Radial variation of floating potential measured with a single probe connected to a high impedance voltmeter

Lithium plasmas have been produced from a single source operated in a uniform magnetic field of 10<sup>3</sup> G and densities of 10<sup>7</sup> to 10<sup>9</sup> cm<sup>-3</sup>, depending on the plate and oven temperatures, were measured by collecting ion current on the cold rhenium end plate used to terminate the plasma column (Fig.16). A typical radial density profile measured with a single Langmuir probe with a spherical tip of diameter 1 mm is shown in Fig.17. When the rhenium end plate was heated to the same temperature as the source hotplate the plasma density increased by a factor of 1.6 as would be expected if the probability of reionizing lithium ions at a rhenium plate is taken as 0.38.

The floating potential measured with a probe connected to a high impedance voltmeter which is a measure of the plasma potential is also shown in Fig.17. The plasma potential has also been measured from the shape of the Langmuir probe characteristics. In the central region of the plasma radial electric fields of less than 0.1 V cm<sup>-1</sup> were observed. Plasma rotation due to such small electric fields would not effect the stability of the column<sup>(7)</sup>.

#### 6. CONCLUSIONS

The plasma source which has been described produces a 5 cm diameter column of thermionic lithium plasmas of density  $10^7$  to  $10^9$  cm<sup>-3</sup> for single ended operation and gives correspondingly higher densities for double ended operation. The temperature profile of the hotplate can be varied, and can be made uniform to within  $20^{\circ}$ K; slight modifications to the mechanical design should give even smaller temperature variations.

#### 7. ACKNOWLEDGEMENTS

It is a pleasure to acknowledge the assistance of R.F. Dyer and G. Gigg with the experimental work and of A. Sykes with the programming of the temperature distribution computation.

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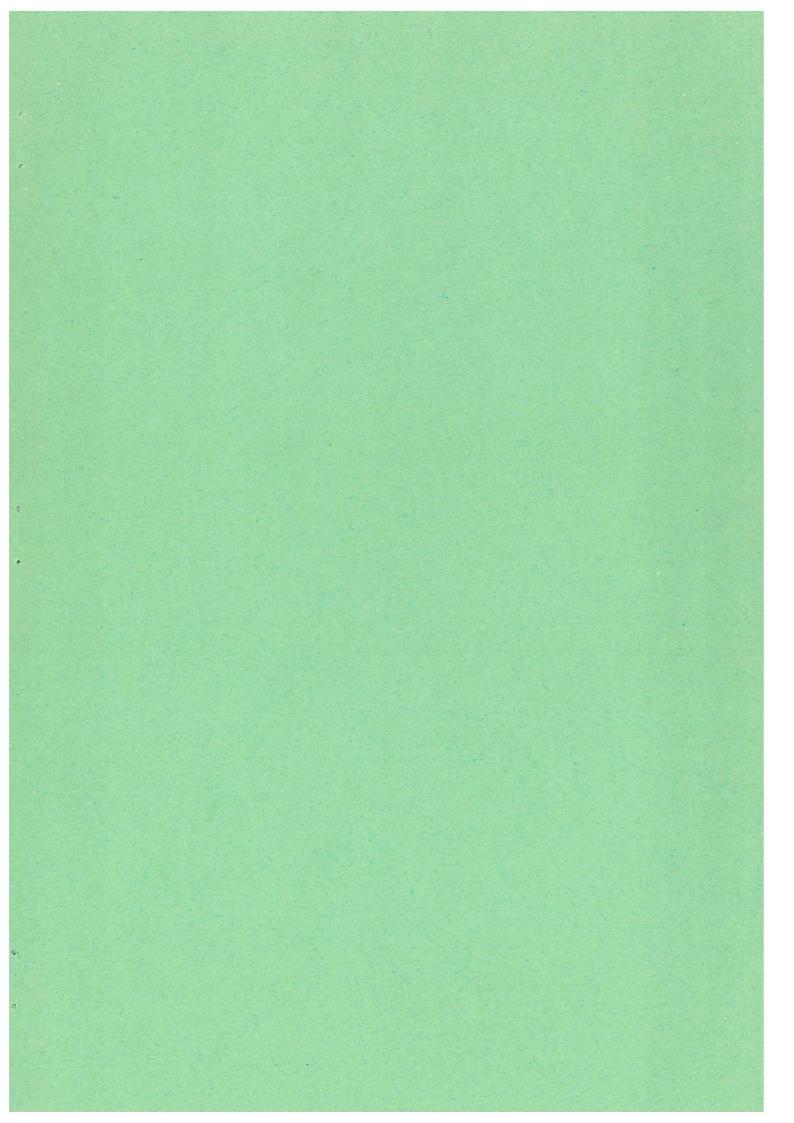
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