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Translation

METHODS FOR THE SYSTEMATIC STUDY OF MAGNETIC WELLS

J. ANDREOLETTI

Culham Laboratory,
Culham, Abingdon, Berkshire
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METHODS FOR THE SYSTEMATIC STUDY OF MAGNETIC WELLS

by

J. ANDREOLETTI

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ABSTRACT

Investigation of the stability of a low pressure plasma in a strong magnetic field(1,2) draws attention to the importance of 'Magnetic Wells', i.e. fields such that a finite volume exists within which the energy density $B^2/2\mu_0$ has a minimum. Two methods are developed permitting systematic investigation of these fields.

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U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
Berks.

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CONTENTS

THE METHOD OF CENTRAL TUBES

This is a local method based on the infinitesimal geometrical properties of a vacuum magnetic field in the neighbourhood of a non-zero minimum.

Centre, Central Tube. To start with the following definitions are introduced:

The 'Centre of a Magnetic Well' is the set of points in the internal region ($B \neq 0$ and $j = 0$) where the magnetic energy density is minimum.

A 'Central Tube' is a tube of infinitesimal magnetic flux ($d\Phi$) containing a magnetic line of force passing through a point of the centre.

Structure of the Centre: Structure of a Central Tube. The following geometrical properties may be shown to hold:

The centre of a magnetic well is a space of at most one dimension; that is to say, it is a curve which possibly may reduce to a point, or to a set of isolated points.

The root (neighbourhood of tube centre) of a central tube has the same form as that resulting from the application of a deformation perpendicular to the axis of a cylindrical tube, the amplitude of the deformation being proportional to the algebraic distance along the axis of the tube.

We differentiate between the 'divergence faces' of a central tube, which show a pure divergence of the magnetic lines of force without shear, and the 'shear faces', presenting a pure shearing of the magnetic lines of force without divergence. These two types of surface correspond to directions forming an angle of $\pi/4$ with one another.

Topological Restriction. The field under investigation is limited by eliminating from the problem all topologies of greater complexity than that of a torus : this amounts in effect to assuming that the centre is a simple continuous curve homotopic to a point. Thus, the centre will either be a point, or a curve obtained by transformation of a circle by continuous deformation.

Systematic Construction of the Magnetic Wells. The various types of magnetic well will be investigated by choosing systematically, starting from the simplest and the most highly symmetrical possibilities, the centre and the mode of ordering of the central tubes along it. The geometrical properties which can be deduced permit the characterisation of the class of possible wells amongst the set of fields $\bar{B}$, such that $\bar{B} = \nabla\psi$ with $\nabla^2\psi = 0$. To verify that such a class of fields actually exists it is then sufficient to find an analytical example having the geometrical properties of this class, and to provide proof by calculation.

A. THE CENTRE REDUCES TO A POINT

The problem of ordering the central tubes vanishes since there is in this case only a single central tube. The geometrical structure of the field is analogous to that of a central tube.

Analytical example:

$$\psi = z + \beta^2\left(\frac{z^3}{3} - z\,\frac{x^2+y^2}{2}\right) \quad \alpha\left(\frac{x^2}{2} - \frac{y^2}{2}\right).$$

Note: It may be shown that a system of currents having the same symmetry properties as the root of a central tube [the product of a rotation through $\pi/2$ about the axis, and a reflection in the median plane, is equivalent to a change of direction of $\bar{B}$ (and of the source currents)] produces a magnetic field whose components, to second order in the coordinates x , y , z about the central point (intersection of the two planes of symmetry of the coils and of the median plane), are of the same type as those obtained for a central tube.

B. THE CENTRE DOES NOT REDUCE TO A POINT

The possibilities of ordering which present the maximum symmetry are those where the 'coupling' faces (i.e. the faces common to two adjacent central tubes) are independent of the coordinate s along the central curve, and those where they are dependent of s periodically.

a. Central Tubes Coupled (independently of s) by Divergence Faces. The geometrical properties which may be deduced show two possible classes with axial symmetry, the one with radial central tubes, the other with non-radial central tubes (the azimuthal component, B_θ , non-zero).

Analytical examples:

$$\psi = \left(\frac{r^2}{2} - z^2\right) - \lambda^2 \operatorname{ch} z\, J_0(r) \qquad \text{(radial type)} ,$$

$$\psi = \left(\frac{r^2}{2} - z^2\right) + a^2 O \qquad \text{(spiral type)} .$$

b. Central Tubes Coupled (independently of s) by Shear Faces. The geometrical properties which may be deduced show one possible class of fields, with axial symmetry, composed of the azimuthal field, $B_\theta \sim 1/r$, of a central conductor, and of a meridian field which is convex toward the axis of symmetry.

Analytical example:

$$\psi = -z\left[1 + \frac{4}{3}\left(\frac{z}{a}\right)^2 - 2\left(\frac{r}{a}\right)^2\right] + 2a\theta .$$

This example has been adapted to magnetic compression with the flux through the central circle of the well being zero.

c. Central Tubes Coupled by Faces Rotating Periodically as a Function of the Coordinate s along the Centre. The geometrical properties deduced indicate one possible class of fields composed of a meridian mirror field of conventional shape and of a multipolar field with three or more pairs of poles. This class already falls beyond the region defined by the topological restriction 'simple continuous', for the centre is broken up into 2n isolated points.

Analytical example

$$\psi = z - \alpha^2 \sin z\, I_0(r) + \beta r^n \cos n\theta \ .$$

The field used in Ioffe's experiment(3) is of this type.

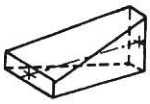
METHOD OF COILS WITH RULED SURFACES

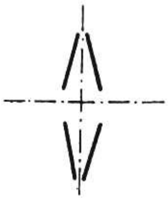
This is a macroscopic method using the average curvature of the magnetic field lines (taking into account their divergence at the ends of a single-turn coil with skin current) and the variations in cross-sectional area of the winding, as measures of the transverse gradient ($\nabla_\perp B$) and the longitudinal gradient ($\nabla_\parallel B$) respectively.

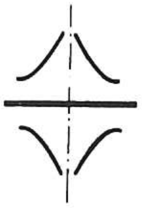
We limit the field under investigation by considering only the simplest ruled surfaces (first and second degree), and the most symmetrical ones: plane, circular cone, hyperboloid of one sheet (this latter is always associated with a central conductor, since its rectilinear generators form a non-zero angle with the symmetry axis).

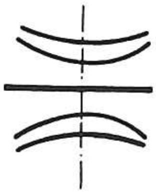
This method leads to the same types of well as previously, the coils having the following shapes:

tetrahedral winding (a tetrahedron truncated parallel to two edges) ;
a couple of conical windings ;
a couple of hyperbolical windings in line ;
a couple of concentric hyperbolic windings (see figures below) .









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