LETTER

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The interaction between fishbone modes and shear Alfvén waves in tokamak plasmas

Hongda He\textsuperscript{1}, Yueqiang Liu\textsuperscript{2,1}, J.Q. Dong\textsuperscript{1}, G.Z. Hao\textsuperscript{1}, Tingting Wu\textsuperscript{3}, Zhixiong He\textsuperscript{1} and K. Zhao\textsuperscript{4}

\textsuperscript{1} Southwestern Institute of Physics, Chengdu, People’s Republic of China
\textsuperscript{2} CCFE, Culham Science Centre, Abingdon, OX14 3DB, UK
\textsuperscript{3} Dalian University of Technology, Dalian, People’s Republic of China
\textsuperscript{4} Southwest University for Nationalities, Chengdu, People’s Republic of China

E-mail: hehda@163.com

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Abstract
The resonant interaction between the energetic particle triggered fishbone mode and the shear Alfvén waves is computationally investigated and firmly demonstrated based on a tokamak plasma equilibrium, using the self-consistent MHD-kinetic hybrid code MARS-K (Liu et al 2008 Phys. Plasmas 15 112503). This type of continuum resonance, occurring critically due to the mode’s toroidal rotation in the plasma frame, significantly modifies the eigenmode structure of the fishbone instability, by introducing two large peaks of the perturbed parallel current density near but offside the $q = 1$ rational surface ($q$ is the safety factor). The self-consistently computed radial plasma displacement substantially differs from that being assumed in the conventional fishbone theory.

Keywords: interaction, fishbone mode, shear Alfvén wave

(Some figures may appear in colour only in the online journal)
modification of the fishbone eigenmode structure near the mode resonant surface. More specifically, two large peaks, for the perturbed parallel current density, appear near but offside the \( q = 1 \) rational surface. The gap between these two current peaks increases nearly linearly with the fishbone frequency. This is quantitatively confirmed by both toroidal computations and by the analytic estimate. The computed radial displacement of the mode switches sign near the \( q = 1 \) surface, being qualitatively different from the conventional theory assumptions, of being either a step-like function, or a hyperbolic tangent function describing the smooth transition within the inertial layer.

The aforementioned toroidal computations are performed with the MHD-kinetic hybrid code MARS-K [5]. The code is well benchmarked against other codes with similar drift kinetic effects [6, 7], and has been extensively applied to study MHD perturbations associated with external kink modes [8–11]. One key feature, which allows us to perform the study reported in this work, is the non-perturbative approach employed in the MARS-K formulation. The MHD equations and the drift kinetic equations are solved together in MARS-K, thus allowing (i) the self-consistent modification of the internal kink eigenfunction by the drift kinetic effects from trapped EPs, and (ii) the wave–wave resonance interactions between the fishbone and the plasma continua. Therefore, both types of resonance physics—the resonance between internal kink and EPs and the resonance between fishbone and Alfven waves, being of different physics origins, are included into the MARS-K model and employed in this study. We also mention that similar self-consistent MHD-kinetic hybrid formulations have been employed in other studies [12–15].

Assuming an ideal plasma with toroidal equilibrium flow, the core equations of the MARS-K model can be written as

\[
\begin{align}
\dot{\mathbf{v}} & = \mathbf{E} + \mathbf{J} \times \mathbf{B}, \\
p \dot{\mathbf{p}} & = \mathbf{F}, \\
\mathbf{J} & = \nabla \times \mathbf{B},
\end{align}
\]

where \( \mathbf{E} \) is the electric field, \( \mathbf{J} \) is the current density, \( \mathbf{B} \) is the magnetic field, \( \rho \) is the mass density, \( \mathbf{F} \) is the force density, and \( \mathbf{p} \) is the momentum. Here, \( \mathbf{v} \) is the velocity of the plasma particle, \( \mathbf{v}_i \) is the ion velocity, \( \mathbf{v}_e \) is the electron velocity, \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic field, \( \rho \) is the mass density, \( \mathbf{J} \) is the current density, \( \mathbf{F} \) is the force density, and \( \mathbf{p} \) is the momentum. The following equations can be used to describe the fishbone instability:

\[
\begin{align}
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f &= \nu \nabla^2 f, \\
\frac{\partial \mathbf{v}}{\partial t} &= -\nabla \mathbf{F} - \mathbf{E}, \\
\frac{\partial \mathbf{b}}{\partial t} &= \mathbf{J} / \mu_0,
\end{align}
\]

where \( f \) is the distribution function, \( \mathbf{v} \) is the velocity of the plasma particle, \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic field, \( \mu_0 \) is the magnetic permeability, \( \nu \) is the collision frequency, \( \nabla \) is the gradient operator, and \( \mathbf{J} \) is the current density.

Here, \( \Gamma \) denotes the particle velocity space and \( M \) the particle mass, \( v_i \) and \( v_e \) denote the parallel and perpendicular velocities of the particle guiding center drift motion, respectively. The sign \( \sigma \) represents particle species (electrons, ions and energetic particles). The perturbed distribution function \( f^p \), defined in the Lagrangian frame, satisfies the following equation,

\[
\frac{df^p}{dt} = f^p_0 \frac{\partial H^p}{\partial \mathbf{v}} - f^p \frac{\partial H^p}{\partial \mathbf{v}} - \nu_{\text{eff}} f^p,
\]

where \( f^0(\varepsilon, \psi) \) is the equilibrium distribution function of particles, assumed to be Maxwellian for thermal ions and electrons, and slowing down distribution for energetic ions. \( f^0 \) and \( f^0_0 \) denote the derivatives with respect to the particle energy and canonical momentum, respectively. \( \psi \) is the poloidal magnetic flux, \( \epsilon = Mv^2 / 2 + Ze \Phi \) is the particle total energy with \( Z \) being the charge number and \( \Phi \) being the equilibrium electrostatic potential. The total canonical momentum is defined as \( P_\psi = MR^2 \phi + Ze \psi \), \( \nu_{\text{eff}} \) is the collision coefficient. The variable \( H^p = -\varepsilon \kappa H_0 \exp(-i\omega t + i\phi) \) represents particle Lagrangian perturbation with \( H_0 = [Mv^2_0 \kappa \cdot \mathbf{Q} + Mv^2_0 \kappa \cdot \mathbf{Q}] / \varepsilon \), \( \kappa = (b \times \nabla) \phi \) is the magnetic curvature and \( \mu = Mv^2_0 / 2 \) is the magnetic moment of particle, \( \epsilon \) is the poloidal magnetic flux, varying between 0 (magnetic axis) and 1 (plasma boundary). The second and the third variables, \( \chi, \phi \), correspond to a generic poloidal angle and the geometrical toroidal angle, respectively.

An experimental equilibrium configuration of HL-2A, with neutral beam injection (NBI) heated plasma, is considered. The equilibrium obtained from the EFIT code, as well as the relevant kinetic profiles, are shown in figure 1. The main parameters are: the toroidal magnetic field \( B = 1.3 \) T, the major/minor radii \( R = 1.65 \) m, \( a = 0.4 \) m, the central electron density \( n_e(0) = 1.6 \times 10^{19} \) (m\(^2\))\(^{-1}\), the electron temperature \( T_e(0) = 1.5 \) (keV), the ion temperature \( T_i(0) = 1.5 \) (keV), the temperature ratio of the hot to thermal ions \( T_h/T_i = 30 \), the density fraction of hot ions \( n_h/n_t = 0.04 \), the on-axis safety factor \( q(0) = 0.9 \) and the edge safety factor \( q(a) = 4.08 \). In order to avoid the stabilizing effect of the plasma flow on the fishbone instability, the plasma equilibrium is assumed to be static. The energy distribution of energetic ions is a slowing-down function, and the pitch angle distribution is specified by Gaussian functions [5]. In this work, the orbit width of EPs and the particle collision are neglected. The
versus the beta fraction of energetic ions with beam energy $E_b = 45$ keV. (b) The normalized toroidal precessional drift frequency of energetic ions $\tilde{\omega}_d = \omega_d/\omega_a(0)$, as a function of square root of the normalized poloidal magnetic flux.

Figure 2. (a) Real (solid line) and imaginary (broken line) parts of the fishbone eigenvalue, normalized to the on-axis Alfvén frequency, $\omega = \omega_a(0)$, versus the beta fraction of energetic ions with beam energy $E_b = 45$ keV. (b) The normalized toroidal precessional drift frequency of energetic ions $\tilde{\omega}_d = \omega_d/\omega_a(0)$, as a function of square root of the normalized poloidal magnetic flux.

The diamagnetic drift of energetic particles is the dominant factor driving the fishbone instabilities. We shall only consider the mode resonance with toroidal precessional drift motion (including both the $\nabla B$ drift and the curvature drift) of EPs. The other resonances, due to bounce/transit motions of EPs, are neglected. With non-uniform radial distributions of density or temperature, energetic particles usually interact with the (stable) internal kink mode, driving the fishbone instability, as long as the EPs pressure is sufficiently large. Moreover, the real frequency of the fishbone mode is usually about half of the precessional drift frequency of EPs at the particle birth energy (i.e. $\omega_r \sim \omega_d/2$). The growth rate of the mode is roughly a linear function of the EPs pressure [3]. Compared with previous studies based on the energy principle, the advantages of using the MARS-K code is the self-consistent treatment of not only the eigenvalue but also the mode eigenfunction for fishbone.

The MARS-K computed eigenvalues of the fishbone mode, for our equilibrium, are shown in figure 2(a), as function of $\beta_h/\beta_{th}$. The real and imaginary parts of the eigenvalues represent the real frequency (solid line) and the growth rate (broken line), respectively, of the fishbone instability. Shown in figure 2(b) is the toroidal precessional drift frequency $\tilde{\omega}_d$, averaged over the particle velocity space and normalized to the on-axis Alfvén frequency, versus the plasma.
The plasma displacement with a parabolic-like safety factor profile \(n = 1\), \(M = 1\), computed with MARS-K. The fishbone mode is computed at \(q_1 = 1\), \(E_b = 45\) keV, based on the equilibrium from figure 1. The minimum value of the toroidal precession frequency of EPs is about 0.1 in our case. Figure 2(a) shows that the fishbone is excited by energetic ions, when \(\beta_i/\beta_h < 1\) is larger than a critical value of 0.03. The real frequency of the excited mode stays nearly constant, being in the same order of magnitude as compared to the toroidal precessional drift frequency, agreeing with theoretical expectations [3, 4]. In MARS-K, the toroidal precession frequency is calculated as \(\omega_{th} = \partial J_\parallel/\partial s/\partial J_\parallel/\partial \varepsilon\), where \(J_\parallel = \int J_\parallel d\ell\) denotes the longitudinal adiabatic invariant of the particle motion. In the large aspect ratio approximation, the expression for the precession frequency is roughly reduced to \(\omega_{th} \propto q/r\). With a parabolic-like safety factor profile in the HL-2A plasma, the toroidal precession frequency is thus expected to decrease with \(s\) near the magnetic axis, and increase with \(s\) in the outer region. This is exactly what we numerically find, as shown in figure 2(b).

Figure 3 shows the MARS-K computed eigenmode structure, in terms of the (normalized) radial displacement of the plasma, for \(n = 1\), \(m = 1\) harmonic, versus the plasma minor radius. Compared are displacements between (a) the fishbone mode and (b) an unstable ideal internal kink mode within the single fluid approximation. It is important to note that these eigenfunctions are obtained from self-consistent MARS-K computations. The fishbone mode is computed at \(\beta_i/\beta_h = 0.84\), beam energy \(E_b = 45\) keV, and with an eigenvalue of \(\omega = (2.36 \times 10^{-2}, 1.1 \times 10^{-3})\). The plasma displacement associated with the ideal internal kink is, as expected, nearly constant inside the \(q = 1\) surface, and monotonically decays to zero outside the rational surface. What is interesting, however, is the more complicated mode structure for the fishbone instability. Sharp variation of the \(m = 1\) displacement across the \(q = 1\) surface is observed, with strong peaking occurring at the radial locations of \(s_1 = 0.354\) and \(s_2 = 0.389\).

This new feature of the fishbone mode structure results from the kinetic effects of energetic particles on the mode, as well as from the resonant interaction between the mode and the shear Alfvén waves, as will be elucidated next. We remark here that, in previous work, the eigenmode structure of fishbone is often assumed as a step-like function, making analytic utilization of the energy principal possible [3, 4]. What we find here shows that this may not always be a good approximation.

The resonance between the fishbone mode and the shear Alfvén waves occurs, when the real frequency of the fishbone, in the plasma frame, matches that of the Alfvén waves

\[
\omega_i^2 = \omega_A^2(r) = (kqA)^2 = \left( \frac{ng(r) - m}{Rq(r)} \right)^2 v_A^2, \tag{9}
\]

where \(v_A\) is the toroidal Alfvén speed defined at the magnetic axis. Note that, with relatively slow mode frequency (~the toroidal precession frequency of fast ions) for the fishbone, the above resonance can only occur near the \(q = 1\) rational surface, where the parallel wave number \(|\ell|\) is small. Thus, for an ordinary \(q\)-profile, there are two resonant positions, located from both sides of the \(q = 1\) surface. The mode eigen-function is substantially modified near these two resonant surfaces, as shown in figure 3.

Even more interestingly, strong current sheets are formed at the radial locations of these two resonant surfaces, as shown in figure 4. These current sheets form part of the fishbone eigenfunction, as the self-consistent solution of the MHD-kinetic hybrid equations. For the case shown here, the perturbed parallel current density amplitude peaks at \(q_1 = 0.97796\) and \(q_2 = 1.02081\), corresponding to the radial positions of \(s_1 = 0.354\) and \(s_2 = 0.389\), respectively.

The radial gap between these two current sheets is numerically determined and shown in figure 5, as the distance between the upper and lower circles, at each computed mode frequency, which varies as a result of changing the temperature of EPs. The gap almost linearly increases with the mode frequency. On the other hand, the lines in figure 5 correspond to the analytic estimates following the exact solution of equation (9). Quantitative agreement between the numerical results and the analytic estimates confirm the shear Alfvén wave resonance nature for the fishbone mode. No sound wave resonance induced current sheets are observed in these computations, though theoretically such resonance, between the fishbone and the sound wave (or the slow magneto-acoustic wave) continuum, can also occur.

It is the conventional understanding that the EPs kinetic effects mainly act on the bulk part of the internal kink eigenmode (i.e. not inside the inertia layer which is normally very narrow). Toroidal computations allow us to investigate this aspect in more detail, also in the context of the mode coupling to the Alfvén waves as studied in this work. For this purpose, we apply an artificial window function along the plasma minor radius, of width \(2\delta\sigma\), as a multiplier to all the drift kinetic terms associated with the trapped EPs [16]. The function value is 0 inside the window and 1 outside the window. The window is symmetrically located from both sides of the \(q = 1\) rational surface. By varying this numerical
parameter $\delta_s$, we can study how the kinetic effects inside the layer affect the fishbone mode instability. The computational results, shown in figure 6, indicate that fishbone stability is not much affected, as long as the window width does not exceed the $q = 1$ inertia layer width, which is about 0.1 $a$ as shown in figure 3. On the other hand, a too large window, covering the whole inertia layer and beyond, quickly reduces the EPs drive, resulting in the loss of the fishbone excitation.

Figure 7 shows one example of the MARS-K computed fishbone eigenmode structure, but plotted in the 2D domain for the plasma core region. The same EPs parameters, as those in figure 3, are assumed. The $n = 1$, $m = 1$ internal kink structure is evident. Note, however, the sharp variations of the mode structure near the $q = 1$ rational surface, which is self-consistently generated due to the mode resonance with shear Alfvén waves. This is different from the conventional internal kink eigen-structure as predicted by the M3D code [12]. Finally, we remark that, as a linear eigenfunction, the toroidal phase of the computed plasma displacement is undetermined. Therefore, only the relative phase between the real and imaginary parts, shown in figure 7, has physics significance.

In summary, the non-perturbative toroidal modeling, using the well benchmarked MARS-K code, allows us to numerically investigate the resonant interaction physics between the trapped EPs triggered fishbone mode and the shear Alfvén waves in tokamak plasmas. Such interaction is possible, thanks to the finite mode frequency driven by EPs toroidal precession, even in a static equilibrium. The Alfvén resonance qualitatively modifies the eigenmode structure of the fishbone, by introducing a double-peak structure in the perturbed parallel current density near the $q = 1$ rational surface, and by causing substantial plasma radial displacement.
reversal near the same surface. These new features should be taken into account in further development of more accurate fishbone models, as well as in the future interpretation of experimental results, provided that fine measurements can be made within the narrow inertial layer in experiments. Finally, this modification of the mode structure may have consequences for the local (particle, thermal, as well as momentum) transport across the inertial layer, as has previously been studied [17].

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References