

C. A. Bowie, R. O. Dendy and M. J. Hole

# Delay time embedding of mass loss avalanches in a fusion plasma-oriented sandpile model

Enquiries about copyright and reproduction should in the first instance be addressed to the Culham Publications Officer, Culham Centre for Fusion Energy (CCFE), K1/083, Culham Science Centre, Abingdon, Oxfordshire, OX14 3DB, UK. The United Kingdom Atomic Energy Authority is the copyright holder.

# Delay time embedding of mass loss avalanches in a fusion plasma-oriented sandpile model

C. A. Bowie<sup>1</sup>, R. O. Dendy<sup>2,3</sup> and M. J. Hole<sup>1</sup>

<sup>1</sup>*Res. School of Physics and Eng., Australian National University, Canberra, ACT 0200, Australia*

<sup>2</sup>*CCFE, Culham Science Centre, Abingdon, Oxon. OX14 3DB, UK*

<sup>3</sup>*Centre for Fusion, Space and Astrophysics, Warwick University, Coventry CV4 7AL, UK*



# Delay time embedding of mass loss avalanches in a fusion plasma-oriented sandpile model

C. A. Bowie\*

*Res. School of Physics and Eng., Australian National University, Canberra, ACT 0200, Australia.*

R. O. Dendy

*CCFE, Culham Science Centre, Abingdon, Oxon. OX14 3DB, UK;*

*Centre for Fusion, Space and Astrophysics, Warwick University, Coventry CV4 7AL, UK*

M. J. Hole

*Res. School of Physics and Eng., Australian National University, Canberra, ACT 0200, Australia*

The sandpile paradigm is widely used to model aspects of the phenomenology of magnetically confined fusion (MCF) plasmas, including enhanced confinement, edge pedestals and, potentially, the impulsive energy and particle release process known as ELMing. Here we identify new points of contact between ELMing and the systemwide avalanches in a sandpile. We compare the quantified response (F A Calderon *et al.*, Phys. Plasmas **20** 042306 (2014)) to increased fuelling of the time sequence of ELM events in a series of similar JET plasmas to the response to increased fuelling of the time sequence of systemwide avalanches in a sandpile model (S C Chapman *et al.*, Phys. Rev. Lett. **86** 2814 (2001)) that has well established links to MCF plasma phenomenology. Both the probability density functions of inter-event time intervals, and delay time embeddings of event time sequences, at different fuelling rates, show common features and point to shared underlying physics.

## INTRODUCTION

Sandpile models[1, 2] have been extensively used to simulate aspects of the global phenomenology of macroscopic plasma systems, in both magnetic confinement fusion[3–10] and space and astrophysics[11–20]. In the sandpile paradigm, the system is typically fuelled by the addition of grains at its centre. If, as a consequence of fuelling, the local gradient of the sandpile exceeds a specified critical value  $Z_c$ , then local redistribution of sand takes place, following a simple algorithm. The resulting local movement of sand may cause the local gradient to exceed the critical gradient at neighbouring points, triggering further redistribution of sand there. This may continue progressively, giving rise to an avalanche, which may halt before it leaves the system, or propagate to the system boundary. The latter case is a systemwide avalanche, a category which provides a focus for the present paper. After each avalanche, the local gradient is everywhere at a value below critical. In the classical sandpile paradigm (as distinct from the running variant [18, 19], discussed further below), only then does fuelling recommence. The distribution of avalanche events, with respect to magnitude and their sequence in time, assists statistical characterisation of the system dynamics. The key components of the sandpile paradigm are thus energy fuelling, storage, and release through nonlocal nondiffusive transport events conditioned by a critical gradient. These map across to key aspects of the observed phenomenology of macroscopic plasma systems, hence the continuing topicality of sandpiles in plasma physics research.

Carreras *et al.*[3] and Newman *et al.*[4] first proposed sandpile models to interpret non-diffusive transport events, together with the associated radial temperature and density profiles, in magnetically confined fusion plasmas. They conjectured that these profiles might lie close to the critical values for the onset of instabilities, driven to this state by externally

applied heating. In this near-marginal state, a locally excited fluctuation could propagate radially: the resulting enhanced local transport would suppress the fluctuation by flattening the local profile, but this would cause steepening of the profile at neighbouring locations. This in turn would trigger local instability, thus propagating an avalanche. Evidence supporting the sandpile approach is also obtained from direct observation of avalanche events in experiments and numerical simulations, particularly where event statistics are power law or strongly non-Gaussian. Such observations are widespread in space and laboratory plasma systems[16, 17, 20]. The sandpile paradigm is also attractive in that it enables global modelling to be carried out, albeit at a relatively coarse-grained level, for plasma systems that are too computationally expensive to simulate using the first-principles equations of plasma physics. This reflects the fact that macroscopic plasmas are complex systems, whose global phenomenology emerges from the interaction of multiple physical processes, coupled across a very broad range of lengthscales and timescales. The sandpile paradigm has assisted understanding, in fusion plasmas, of the existence of different classes[21] of energy confinement regimes [7–10]. In solar-terrestrial and astrophysical plasma physics, the sandpile paradigm has been used successfully to help interpret impulsive energy release events originating in the magnetosphere[11, 13, 14], the solar corona[15], and accretion discs[12].

Edge localised modes (ELMs) are pulsed relaxation events that typically accompany enhanced confinement regimes in magnetically confined toroidal plasmas. Understanding the ELMing process is central to research into the physics of large tokamak experiments such as the Joint European Torus (JET), and to the success of future burning plasma experiments in ITER [22–24]. The ELMing process encompasses both the birth-to-death life cycle of each individual ELM, and the way in which the sequence of many successive ELMs arises under

specific plasma conditions. The characterisation of measured sequences of inter-ELM time intervals from the perspective of statistical physics and complex systems science was pioneered in Refs [25, 26]. For JET plasmas in particular, this approach has recently produced results which are novel and unexpected. These include: period doubling of Type-I ELMing in a sequence of similar JET plasmas in response to different gas puffing rates [27]; first principles differentiation between Type-I and Type-III ELMs for a large sample of JET plasmas, in terms of extreme value distributions of inter-ELM time intervals [28]; and the identification of structure in the distribution of inter-ELM time intervals from a uniquely long and consistent sequence of 151 quasi-identical JET plasmas [29]. In the present paper, we build on a recent study by Calderon *et al.* [27] of the sequence of inter-ELM time intervals in a group of similar JET plasmas, which differ only in the rate at which they are fuelled through gas puffing at the outer plasma edge. This set of plasmas yields clear evidence [27] of low dimensional system dynamics in the ELMing process, including period doubling. Figure 1, reproduced from Calderon *et al.*, shows the probability distribution function (pdf) of inter-ELM intervals  $\delta t_n$  together with delay time embedding plots of  $\delta t_{n+1}$  versus  $\delta t_n$  for successive pairs of inter-ELM time intervals. The ELM occurrence times are defined, for present purposes, from the measured peaks of the recombination radiation signal  $D_\alpha$ . The time interval  $\delta t_n \equiv t_{n+1} - t_n$ , where  $t_n$  is the measured occurrence time of the  $n$ th ELM, and  $t_{n+1}$  that of its immediate successor. Period doubling can be identified from the two peaks in the pdfs in Fig. 1, while the four discrete clusters of data points in the time delay plots reveal dynamical switching between the two periods. The question arises: what light does this newly identified phenomenology shed on the underlying physics of the ELMing process in these plasmas?

### SANDPILE MODEL

In the present paper, we address this question by identifying fresh parallels between ELMing and systemwide avalanches in a sandpile. The possibility that, in some circumstances, ELMing may resemble avalanching was raised [7] in studies of the specific sandpile model of Ref. [30]. This simple one-dimensional  $N$ -cell sandpile model [7, 30] incorporates other established models [2, 31] as limiting cases. It is centrally fuelled at cell  $n = 1$ , and its distinctive feature is the rule for local redistribution of sand near a cell (say at  $n = k$ ) at which the critical gradient  $Z_c$  is exceeded. The sandpile is conservatively flattened around the unstable cell over a fast redistribution lengthscale  $L_f$ , which spans the cells  $n = k - (L_f - 1), k - (L_f - 2), \dots, k + 1$ , so that the total amount of sand in the fluidization region before and after the flattening is unchanged. Because the value at cell  $n = k + 1$  prior to the redistribution is lower than the value at the cells behind it, the redistribution results in the relocation of sand from the fluidization region, to the cell at  $n = k + 1$ . The system is then iterated to stability, resulting in an avalanche as redistribution is

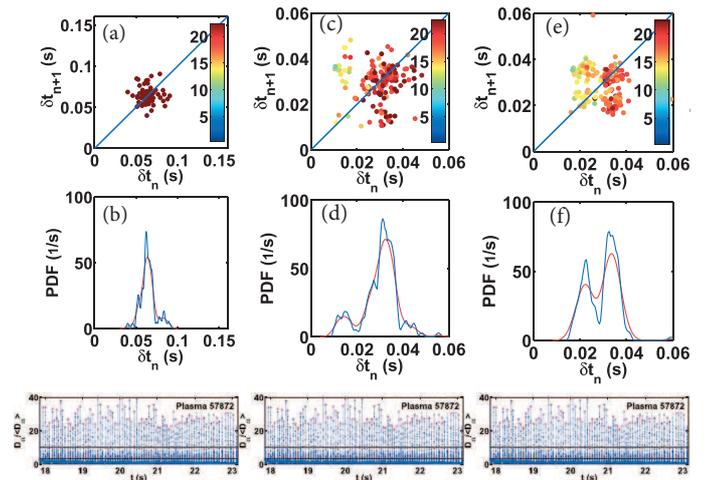


Figure 1. PDFs, delay plots, and  $D_\alpha$  plots for ELMs in JET plasmas, extracted from Figure 1 of Ref. [27]. Gas puffing rate increases from left to right.

sequentially triggered outwards across neighbouring cells. An avalanche may, or may not, propagate right across the sandpile and result in mass loss from the edge, which we refer to as a mass loss event (MLE). If a systemwide avalanche is triggered by the addition of a given grain of sand at the centre, the magnitude of the associated MLE is defined to equal the total amount of sand lost once the sandpile has iterated to stability, prior to addition of the next grain. The lengthscale  $L_f$ , normalized to the system scale  $N$ , provides the model's primary control parameter  $L_f/N$ , which governs different regimes of avalanche statistics and system dynamics. A further control parameter is the ratio between  $Z_c$  and the amount of sand,  $dx$ , added at each time step. Unlike some previous implementations of the model [6–8, 10, 32],  $Z_c$  here is single valued, rather than fluctuating randomly within a narrow range. The phenomenology generated by this model is known to include several features resembling tokamak plasmas, including edge pedestals, enhanced confinement [7] and self-generated internal transport barriers [10]. Potentially related to ELMs - we explore this conjecture in this paper - are the systemwide avalanches resulting in MLEs. The character of the MLE sequences varies with the confinement properties of the sandpile, exhibiting interesting quantitative correlations. For example, the mean time interval between MLEs scales with the energy stored in the sandpile, in a way resembling the scaling of the mean time intervals between ELMs and the stored energy in some JET plasmas [7].

Prompted by the recent discovery [27], see Fig. 1, of low dimensional behaviour in the dependence of the sequence of inter-ELM time intervals on the particle fuelling rate from gas puffing in some JET plasmas, we investigate here whether the sequence of inter-MLE time intervals from the sandpile of Refs. [7, 30] depends analogously on the sandpile fuelling rate. We simulate centrally fuelled sandpiles for different values of  $Z_c$ ,  $L_f$  and  $dx$ , and the system is iterated until stable

probability distribution functions of waiting times emerge.

## RESULTS

Figures 2 and 3 show examples of results produced by the model, dependant upon the fuelling rate,  $dx$ . To assist comparison with the ELM results of Figure 1, Figures 2 and 3 show a selection of pdfs of waiting times,  $\delta t_n$ , from the start of an MLE to the start of the following MLE, and delay plots of  $\delta t_n$  against  $\delta t_{n+1}$ , as well as the time series of MLEs. The delay plots, pdfs, and time series of MLEs are for common values of  $Z_c$  and  $L_f$ , and are obtained for multiple values of the fuelling rate  $dx$  in the low fuelling regime  $dx/Z_c \leq 0.01$ .

Figure 2 shows the results from the sandpile model of Refs. [4, 30], while Figure 3 shows results from a new running version of this model, which we now introduce. Our objective here is to test the robustness of the quantitative MLE phenomenology against minor variations of the sandpile model adopted. Whereas in models such as [4, 30] which follow the approach of Ref. [1], the sandpile is iterated to stability after addition of the next grain before the  $(n + 1)th$  grain is added, this is not the case in the running sandpile [18, 19] approach. Instead sand grains are added at intervals, regardless of whether the system has iterated to stability. Specifically, a grain is added after each sweep of the sandpile redistribution algorithm from the centre to the edge. This defines a de facto timestep. It follows that sand can be added while a systemwide avalanche is under way, hence ‘running’. A systemwide avalanche from our running sandpile is defined to continue for as long as sand is lost from the final cell at each successive time step. Systemwide avalanches in the running sandpile model thus have a time dependent profile of lost sand, whereas in the original model systemwide avalanches are impulsive in time. The magnitude of an MLE in the running version is determined by summing the sand lost between the first and last timestep of a systemwide avalanche. With this definition, we find that the MLE sizes for the original and running models are broadly similar.

For the running sandpile model, lost sand plots can resolve the amount of sand lost at each iteration, as well as the total amount of sand lost in an MLE, which typically continues for approximately 500 time steps. In order to see the time resolved behaviour of an MLE, an expanded view of a single vertical stripe in the lost sand plot is shown in Figure 4(c).

For the cases shown in Figure 3, two peaks appear, defining two characteristic time intervals between MLEs. Most, but not all, combinations of the two characteristic waiting times appear in the delay plots. Similar behaviour is observed in Figure 1(c), where two characteristic waiting times are present, and four groupings of points appear in the delay plot, representing the four possible combinations of short waiting times following short, long following long, and long following short and vice versa. The presence of such dynamical switching in both Figures 1 and 3 suggests that an underlying dynamical principle may be responsible for the observed response to in-

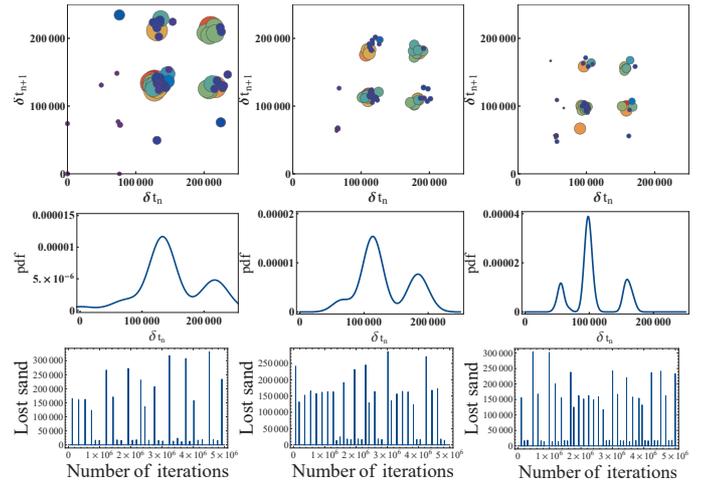


Figure 2. Delay plots (top), PDFs (middle), and lost sand plots (bottom) for  $dx$  ( $dx/Z_c$ )= (left) 0.72 (0.006), (centre) 0.84 (0.007), and (right) 0.96 (0.008) (original model). In each case,  $Z_c = 120$ ,  $L_f = 5$ , and results are shown for 5 million iterations in steady state, i.e., after the sandpile has filled. Horizontal and vertical axes represent  $\delta t_n, \delta t_{n+1}$  (delay plots),  $\delta t_n$ , proportion of occurrences (PDFs), and time, amount of sand lost per iteration (lost sand plots). Lost sand plots show the amount of sand lost during each avalanche, rather than at iteration. Fuelling is paused during avalanches until the sandpile ceases avalanching. Bubble size and colour represent MLE size in delay plots (small (blue)-large (red)). The smallest bubbles represent MLEs of about 600 units, while the largest bubbles represent MLEs of about 300000 units.

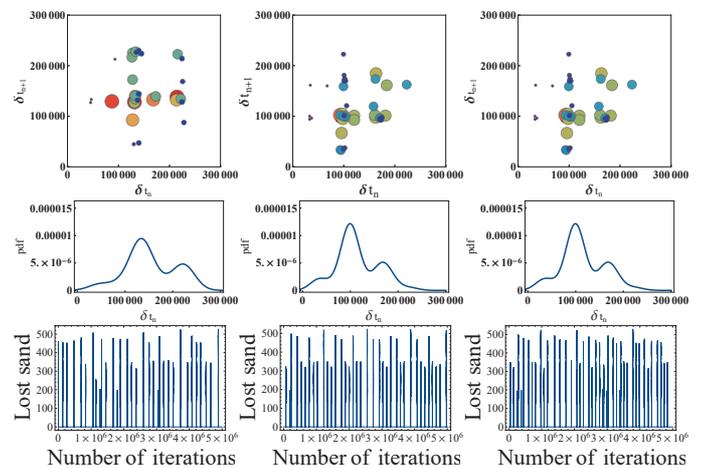


Figure 3. Delay plots (top), PDFs (middle), and lost sand plots (bottom) for  $dx$  ( $dx/Z_c$ )= (left) 0.72 (0.006), (centre) 0.84 (0.007), and (right) 0.96 (0.008) (running model). In each case,  $Z_c = 120$ ,  $L_f = 5$ , and results are shown for 5 million iterations in steady state, i.e., after the sandpile has initially filled. Horizontal and vertical axes represent  $\delta t_n, \delta t_{n+1}$  (top panels: delay plots),  $\delta t_n$ , proportion of occurrences (middle panels: PDFs), and time, amount of sand lost per iteration (bottom panels: lost sand plots). In the top panels, bubble size and colour represent MLE size (small (blue)-large (red)). The smallest bubbles represent MLEs of about 600 units, while the largest bubbles represent MLEs of about 300000 units.

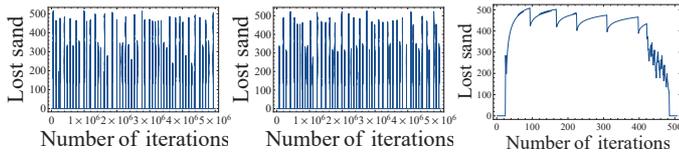


Figure 4. Lost sand plots for the running sandpile model. Left and centre panels show lost sand plots for  $dx$  ( $dx/Z_c$ ) = 1.08 (0.009) and 1.2 (0.01). The right panel shows an expanded view of the rightmost vertical stripe in the left sub-figure, which runs over approximately 400 iterations. At each iteration,  $dx$  units of sand are added, and a systemwide avalanche occurs. Following the first avalanche, the system becomes unstable, meaning that the avalanche would continue over many iterations, even if no further sand were added (as occurs in the classic model, in which addition of sand is suspended until avalanching ceases). The amount of sand lost at each iteration in the example shown is approximately 4-500 units. The overall profile comprises a series of smooth increases followed by drop-offs, before declining noisily after about 400 iterations, followed by complete cessation after about 480 iterations. This behaviour is typical for a large MLE. The full scale lost sand plots (left and centre) show that the amount of sand lost following a systemwide avalanche remains zero while the sandpile rebuilds through internal avalanches in response to continuing fuelling, until the next systemwide avalanche commences. The amount of sand lost in each systemwide avalanche varies from approximately 600 units to approximately  $3 \times 10^6$  units. The waiting time between systemwide avalanches is shown in the delay time PDFs (see Figure 3).

creased fuelling in both ELMing and sandpile cases.

## CONCLUSIONS

The sandpile models used here are deterministic, with all variables kept constant during each run. The sequences of mass loss events (MLEs) have interesting points of similarity with those of the ELMs studied in Ref. [27]. This suggests that ELMing phenomenology may include features that can be considered in terms of systemwide avalanching in the edge plasma, conditioned by the fuelling rate in relation to a critical gradient determined by the underlying plasma physics.

The similarities identified here in the differential response to increased fuelling rates in the time series of JET ELMs and of sandpile MLEs suggest that each ELM incorporates avalanche-type physics. Specifically, the avalanches to which this analogy applies are systemwide. These constitute the emptying of all the free energy stored in the sandpile, triggered by the addition of a "final grain"; as distinct from the far more frequent internal avalanches, which occur as the sandpile rises, without losing mass or energy to its boundary. A full nonlinear kinetic-transport-magnetohydrodynamic model for the birth-to-death ELMing cycle has not yet been constructed. The present study reinforces the physical basis for anticipating that critical gradient-conditioned physics, combined with a relatively minor trigger, will lead to full plasma pedestal emptying. It also supports the suggestion that intra-ELM pedestal building in MCF plasmas may incorporate dis-

continuous transport processes having some of the character of internal avalanches.

## ACKNOWLEDGMENTS

The authors thank S C Chapman for a helpful conversation. This work was jointly funded by the Australian Research Council through grant FT0991899 and the Australian National University. This work was part-funded by the RCUK Energy Programme [under grant EP/I501045] and the European Communities. The views and opinions expressed herein do not necessarily reflect those of the European Commission. One of the authors, C. Bowie, is supported through an ANU PhD scholarship and an AINSE Postgraduate Research Award.

\* craig.bowie@anu.edu.au

- [1] P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. A* **38**, 364 (1988).
- [2] R. O. Dendy and P. Helander, *Phys. Rev. E* **57**, 3641 (1998).
- [3] B. A. Carreras, D. Newman, V. E. Lynch, and P. H. Diamond, *Phys. Plasmas* **3**, 2903 (1996).
- [4] D. E. Newman, B. A. Carreras, P. H. Diamond, and T. S. Hahm, *Phys. Plasmas* **3**, 1858 (1996).
- [5] R. O. Dendy and P. Helander, *Plasma Phys. Control. Fusion* **39**, 1947 (1997).
- [6] S. C. Chapman, R. O. Dendy, and G. Rowlands, *Phys. Plasmas* **6**, 4169 (1999).
- [7] S. C. Chapman, R. O. Dendy, and B. Hnat, *Phys. Rev. Lett.* **86**, 2814 (2001).
- [8] S. C. Chapman, R. O. Dendy, and B. Hnat, *Phys. Plasmas* **8**, 1969 (2001).
- [9] J. P. Graves, R. O. Dendy, K. I. Hopcraft, and E. Jakeman, *Phys. Plasmas* **9**, 1596 (2003).
- [10] S. C. Chapman, R. O. Dendy, and B. Hnat, *Plasma Phys. Control. Fusion* **45**, 301 (2003).
- [11] S. C. Chapman, N. W. Watkins, R. O. Dendy, P. Helander, and G. Rowlands, *Geophys. Res. Lett.* **25**, 2397 (1998).
- [12] R. O. Dendy, P. Helander, and M. Tagger, *Astron. Astrophys.* **337**, 962 (1998).
- [13] N. W. Watkins, S. C. Chapman, R. O. Dendy, and G. Rowlands, *Geophys. Res. Lett.* **26**, 2617 (1999).
- [14] N. W. Watkins, M. P. Freeman, S. C. Chapman, and R. O. Dendy, *J. Atmos. Sol.-Terr. Phys.* **63**, 1435 (2001).
- [15] D. Hughes, M. Paczuski, R. O. Dendy, P. Helander, and K. G. McClements, *Phys. Rev. Lett.* **90**, 4 (2003).
- [16] R. O. Dendy and S. C. Chapman, *Plasma Phys. Control. Fusion* **48**, B313 (2006).
- [17] R. O. Dendy, S. C. Chapman, and M. Paczuski, *Plasma Phys. Control. Fusion* **49**, A95 (2007).
- [18] T. Hwa and M. Kardar, *Phys. Rev. A* **45**, 7002 (1992).
- [19] D. E. Newman, R. Sanchez, B. A. Carreras, and W. Ferrelbaugh, *Phys. Rev. Lett.* **88** (2002).
- [20] R. Sanchez and D. Newman, *Plasma Phys. Control. Fusion* **57**, 123002 (2015).
- [21] F. Wagner, *Plasma Phys. Control. Fusion* **49**, B1 (2007).
- [22] H. Zohm, *Plasma Phys. Control. Fusion* **38**, 105 (1996).
- [23] A. Loarte, G. Saibene, R. Sartori, D. Campbell, M. Becoulet, L. Horton, T. Eich, A. Herrmann, G. Matthews, N. Asakura, *et al.*, *Plasma Phys. Control. Fusion* **45**, 1549 (2003).

- [24] R. J. Hawryluk, D. J. Campbell, G. Janeschitz, P. R. Thomas, R. Albanese, *et al.*, Nucl. Fusion **49**, 15 (2009).
- [25] A. W. Degeling, Y. R. Martin, P. E. Bak, J. B. Lister, and V. Llobet, Plasma Phys. Control. Fusion **43**, 1671 (2001).
- [26] J. Greenhough, S. C. Chapman, R. O. Dendy, and D. J. Ward, Plasma Phys. Control. Fusion **45**, 747 (2003).
- [27] F. A. Calderon, R. O. Dendy, S. C. Chapman, A. J. Webster, B. Alper, R. M. Nicol, and J. E. Contributors, Phys. Plasmas **20**, 042306 (2013).
- [28] A. J. Webster and R. O. Dendy, Phys. Rev. Lett. **110**, 5 (2013).
- [29] A. J. Webster, R. O. Dendy, F. A. Calderon, S. C. Chapman, E. Delabie, R. Felton, T. N. Todd, F. Maviglia, J. Morris, V. Riccardo, B. Alper, S. Brezinsek, P. Coad, J. Likonen, M. Rubel, and JET EFDA Contributors, Plasma Phys. Control. Fusion **56** (2014).
- [30] S. C. Chapman, Phys. Rev. E **62**, 1905 (2000).
- [31] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
- [32] S. C. Chapman, R. O. Dendy, and N. W. Watkins, Plasma Phys. Control. Fusion **46**, B157 (2004).