CCFE-PR(17)54

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Two Species Drag/Diffusion Model for Energetic Particle Driven Modes

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(Dated: 13 July 2017)

A nonlinear bump-on-tail model for the growth and saturation of energetic particle driven plasma waves has been extended to include two populations of fast particles - one dominated by dynamical friction at the resonance and the other by velocity space diffusion. The resulting temporal evolution of the wave amplitude and frequency depends on the relative weight of the two populations. This suggests an explanation for recent observations on the TJ-II stellarator, where Alfvén Eigenmodes transition between steady state and bursting as the magnetic configuration varied. The two species model is then applied to burning plasma with drag-dominated alpha particles and diffusion-dominated ICRH accelerated minority ions.

PACS numbers: 52.25.Dg, 52.55.Tn

I. INTRODUCTION

Instabilities driven by energetic particles in fusion plasmas are of great concern for the next-step burning plasma experiment as these instabilities may affect the alpha-particle heating profile, He ash accumulation, and cause damage to the first wall[1]. The temporal evolution of the instabilities observed in present-day machines varies from a steady-state saturated mode amplitude at nearly fixed frequency to a bursting amplitude and sweeping frequency scenarios (see *e.g.* Ref. [2] and references therein). Depending on the type of the nonlinear evolution, transport and peak loads of lost energetic particles to the wall vary significantly, so it is important to understand the key physics effects determining the type of the nonlinear evolution of waves excited by energetic particles.

Since energetic particle instabilities usually involve wave-particle resonant interaction, theory developments focus on the resonant particles. This approach simplifies the description of the multi-dimensional problem as the particle motion is effectively one-dimensional in the vicinity of a resonance if proper action-angle variables are employed[3]. A simple one-dimensional bump-on-tail model was proven to be one of the most effective in describing characteristic nonlinear scenarios in the past[4,5], and this model was successfully applied to the problem of the mode nonlinear evolution too[6,7]. It was found[7] that the different types of nonlinear evolution of modes driven via wave-particle resonances can be attributed to the nature and the rate of the relaxation effects restoring the unstable distribution function of the energetic particles at the position of the resonance. In particular, it was shown that the relaxation of a dynamical friction type (e.g. electron drag for energetic ions) causes only bursting evolution of the mode amplitude at a concomitant strong frequency sweeping, while a diffusive type relaxation may produce four types of nonlinear evolution: steady-state, periodic modulation of the mode amplitude (pitchfork splitting), chaotic evolution, and bursting evolution. This theory is in a robust agreement with observed Alfvén instabilities driven by auxiliary heating ions on many tokamaks.

A surprising recent result from the TJ-II stellarator[8] was the observation of a change in the temporal evolution of a beam-driven Alfvén Eigenmode (AE) during iota profile variation. In this experiment, AEs were excited by NBI-produced beam ions. As iota varied throughout a single discharge, the AE nonlinear evolution was transformed from bursting to steady-state, and then to bursting again. During the iota variation, neither the density nor the temperature of the plasma changed significantly enough to affect the ratio between the drag and diffusion relaxation mechanisms restoring the beam distribution. The observed correlation between the type of

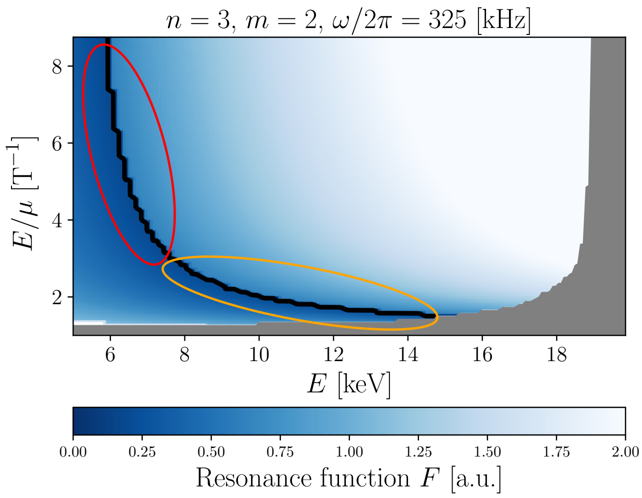

FIG. 1. (Color online) Map of the resonance function $F \equiv |n\omega_\phi - m\omega_\theta - \omega|$ for energetic particles launched from $\psi_N = 0.25$ in TJ-II. The resonance condition $F = 0$ is highlighted by the black line. The grey region corresponds to particles on loss orbits. Two regions of interest along the resonance are indicated.

nonlinear AE evolution and the magnetic configuration may be caused by a number of effects going beyond the assumptions used in the 1D Bump-On-Tail model. The most essential assumptions are: (1) the damping of the mode does not change throughout the mode evolution; (2) no other free energy source exists apart from the energetic particle drive (e.g., the MHD part playing a role in fishbones is excluded); (3) the width of the resonance is much smaller than the width of the mode (as the width of the mode is not present in the 1D bump-on-tail model).

The aim of this Letter is to demonstrate that the specific TJ-II stellarator configuration gives rise to wave-particle resonances in two very different regions of the phase space. Guiding center particle orbits, starting on a core flux surface (normalized poloidal flux $\psi_N = 0.25$) have been simulated for experimental parameters of interest. A resonance map of a typical mode with toroidal and poloidal mode numbers of $n = 3$, $m = 2$ respectively for a wave with $f = 325$ kHz, is shown in Fig. 1. The resonance function $F \equiv |n\omega_\phi - m\omega_\theta - \omega|$ is plotted as a function of the particles' energy and pitch angle; a black line indicates the resonance condition $F = 0$. This very special shape of the resonance map suggests a natural division of the whole population of the fast ion resonant phase space into two regions. Region 1 (red oval) is narrow in energy ($6 < E < 8$ keV), but insensitive to pitch angle, while region 2 (orange oval) covers only a narrow range of pitch angles and extends in energy ($8 < E < 14$ keV). In region 1 drag alone determines the replenishment of the distribution function at the resonance, while region 2 is determined by the pitch-angle scattering effect (diffusive collisional operator). Therefore one arrives at the possibility of describing the nonlinear evolution of the AEs by a sum of two ion species with different weighting factors, one of which is dominated by drag and the other by diffusion.

II. MODEL

We consider marginally unstable modes, with the linear growth rate and damping rate respectively satisfying $\gamma_L > \gamma_d \gg |\gamma_L - \gamma_d|$. For an electrostatic wave with wavenumber $k = 2\pi/\lambda$, the kinetic equation in the frame of the wave becomes

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial \xi} - \frac{1}{2}\left(\omega_B^2 e^{i\xi} + \omega_B^{*2} e^{-i\xi}\right)\frac{\partial F}{\partial u} = \left.\frac{dF}{dt}\right|_{\text{coll}} \quad (1)$$

where $\xi = kx - \omega t$, $u = kv - \omega$, the electron bounce frequency $\omega_B^2 = ek\hat{E}/m_e$ and other symbols have their usual meanings. The distribution function is decomposed as a Fourier series $F = F_0 + f_0 + \sum_{n=1}^{\infty}[f_n \exp(in\xi) + f_n^* \exp(-in\xi)]$; in a similar electric field decomposition, $\hat{E}$ is the $n = 1$ component. The collision term is taken to be

$$\left.\frac{dF}{dt}\right|_{\text{coll}} = \alpha^2\frac{\partial F}{\partial u} + \nu^3\frac{\partial^2 F}{\partial u^2} \quad (2)$$

where the α term corresponds to velocity-space friction and ν to diffusion. A complete set of equations is formed with the addition of Ampère's law,

$$\frac{\partial \omega_B^2}{\partial t} = -\gamma_d\omega_B^2 + \frac{4\pi e^2\omega}{m_e k}\int f_1 du. \quad (3)$$

By comparing coefficients and integrating iteratively[4,6] the electric field satisfies the following relation, to cubic order,

$$\frac{dA}{d\tau} = A - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \times \int_0^{\tau-2z} dx\, K(\hat{\alpha},\hat{\nu})A(\tau - z - x)A^*(\tau - 2z - x) \quad (4)$$

where $A = \omega_B^2[\gamma_L/(\gamma_L - \gamma_d)^5]^{1/2}$ and $\tau = (\gamma_L - \gamma_d)t$. The integral's kernel, arising from the collision operator, is given by

$$K(\hat{\alpha},\hat{\nu}) = \exp\left[i\hat{\alpha}^2 z(z+x) - \hat{\nu}^3 z^2(2z/3 + x)\right], \quad (5)$$

where the normalized coefficients $\hat{\alpha} = \alpha/(\gamma_L - \gamma_d)$, $\hat{\nu} = \nu/(\gamma_L - \gamma_d)$. It has previously been shown analytically[6] that for a single fast particle species to cubic order in electric field, a stable steady state exists only for $\hat{\nu} \geq 2$ and $\hat{\nu} > \hat{\alpha}$.

The Bump On Tail (BOT) code[7] efficiently solves this system of Equations to arbitrary order in n by working in reciprocal space (Fourier transforming in velocity), where the collision operator in the resulting advection equations becomes purely algebraic.

III. TWO DISTRIBUTIONS

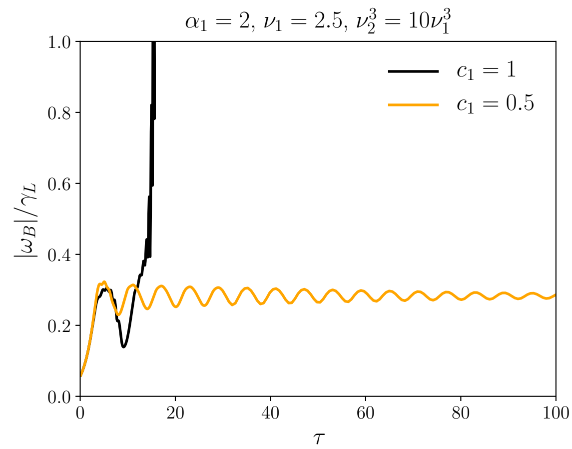

FIG. 2. (Color online) Bounce frequency solved to cubic nonlinearity, showing "explosive" evolution (black curve) for a single particle population with a given choice of drag and diffusion. This is stabilized by a second highly diffusive population (orange curve), with $c_1 + c_2 = 1$ in both cases.

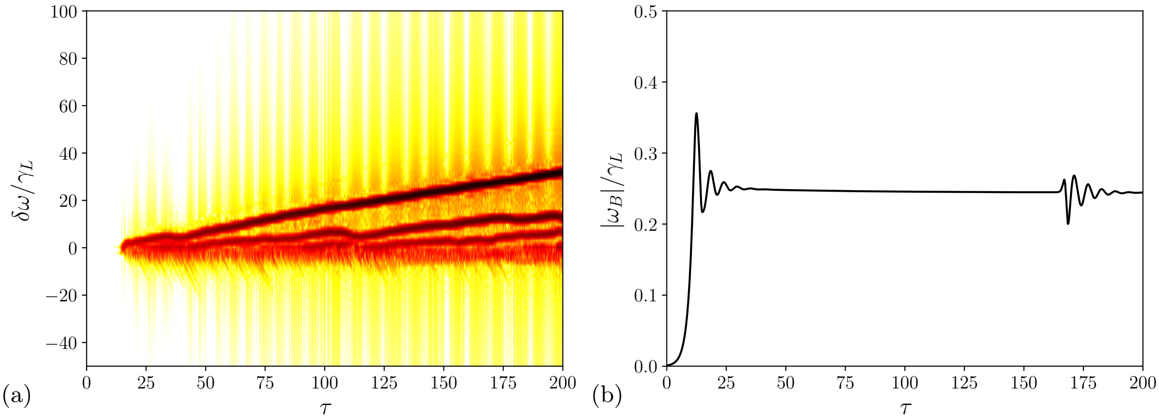

FIG. 3. (Color online) Cases with two distributions of fast particles, one with diffusion $\nu_1 = 2.55$ and one with drag $\alpha_2 = 4.5$. (a) With a relatively low proportion of the diffusion distribution, $c_1 = 0.36$ ($c_2 = 1 - c_1$ in both cases) a spectrogram of ω_B shows the formation of several frequency sweeping branches. (b) As the diffusive proportion is increased to $c_1 = 0.4$, the activity is stabilized, with the amplitude saturating and $\delta\omega \approx 0$.

We consider two perturbative interacting distributions $F = F_0 + F_1 + F_2$, with $F_1 = f_{1,0} + \sum_{n=1}^{\infty}[f_{1,n}\exp(in\xi) + f_{1,n}^{*}\exp(-in\xi)]$ and similarly for F_2. The collision operator acts on each separately as follows,

$$\left.\frac{dF}{dt}\right|_{\rm coll} = \sum_{j=1,2} \alpha_j^2 \frac{\partial F_j}{\partial u} + \nu_j^3 \frac{\partial^2 F_j}{\partial u^2}. \qquad (6)$$

This operator arises from the differences in the sensitivity of the two groups of fast particles to changes in energy and pitch angle.

To cubic nonlinearity, the kernel in Equation (4) is split into two components,

$$K_{\rm tot} = \sum_{j=1,2} c_j K(\hat{\alpha}_j, \hat{\nu}_j), \qquad (7)$$

where $K(\hat{\alpha}_j, \hat{\nu}_j)$ is an individual kernel from Equation (5) and c_j is the relative size of the respective distribution. In Figure 2, we show the electric field evolution for a single particle distribution with a choice of $\alpha = 2$ and $\nu = 2.5$ which evolves "explosively" - outside the parameter space of stable solutions. We add a second, purely diffusive population with $\nu_2^3 = 10\nu_1^3$ which is stable alone; we impose a normalization condition by keeping $c_1 + c_2 = 1$. The resulting solution is then stabilized to cubic order and tends to a constant value. This choice of ν_2 reflects the properties of ions accelerated by Ion Cyclotron Resonant Heating (ICRH), where strong quasilinear diffusion forms the energetic particle tail[5].

We have analyzed the velocity space coefficients of the two regions of interest of the TJ-II resonance map in Figure 1 with the Fokker-Planck approach[9]. We consider the experimental plasma parameters in the expression for the velocity space resonance. We estimate that the drag of the 6-8 keV beam (region 1 in the resonance map) and the 10-16 keV beam pitch-angle scattering corresponding to diffusion (region 2) satisfy

$$\frac{\nu}{1.7} \approx \frac{\alpha}{3}. \qquad (8)$$

By the same analysis, we confirm that drag in the first region dominates over diffusion and vice versa. We conjecture that the role of the magnetic configuration in the TJ-II experiment is in slightly shifting the proportion of the fast ions in region 1 with respect to that in region 2. Since the total beam density remains the same, the sum of the two densities during the variation of the iota parameter is assumed to satisfy $c_1 + c_2 = 1$.

We use a version of the BOT code, modified for two distributions, to explore the above situation in the fully nonlinear regime. The advective equations for each distribution function evolve separately as in the single case, but interact through a modified Fourier-space Ampère's law,

$$\frac{\partial \omega_B^2}{\partial t} = -\gamma_d \omega_B^2 + \frac{4\pi e^2 \omega}{m_e k} \sum_{j=1,2} c_j \mathcal{G}_{j,1}(0), \qquad (9)$$

with the Fourier transform of the distribution $\mathcal{G}_{j,1}(s) = \int f_{j,1}\exp(-isu/\gamma_L)du$.

Two regimes are seen in BOT simulations depending on the choice of c_1 and c_2 (which are taken to satisfy $c_1 + c_2 = 1$), as shown in Figure 3. A spectrogram of the amplitude $|\omega_B|$ is shown for the drag-dominated case, with distinct branches sweeping upwards in frequency, many comparatively small branches sweeping downwards and periodic broadband noise bursts. The frequency asymmetry arises due to the nature of the drag term. As the size of the diffusion term is increased, such rapid oscillations disappear and the amplitude saturates. Hence there is no frequency sweep; low-frequency bursts, such as that seen around $\tau = 175$, appear sporadically with long time separation.

IV. CONCLUSION

The resonance map obtained for the interaction of beam ions with AEs in stellarator TJ-II suggests two very different regions of the resonant beam phase space: the first with dominant drag relaxation and the second with dominant pitch-angle diffusive relaxation. A two-species model with two different relaxation effects was developed for describing the near-threshold nonlinear evolution of the beam-driven AEs. Within the lowest-order cubic nonlinear equation[4], a possibility was demonstrated of transforming explosive AE scenario driven by energetic particles with drag relaxation into a steady-state AE by adding a second species term with diffusion relaxation. For investigating the nonlinear AE evolution beyond the cubic nonlinearity, a two-species BOT code was developed. The results of the BOT modelling show a wide variety of nonlinear regimes including the steady-state ones and bursting, controlled by the proportion between the first and second species. This model may explain the experimentally observed TJ-II results and suggests that it may be possible to control the nonlinear evolution of alpha particle-driven AEs in ITER by adding ICRH-accelerated ions with dominant quasi-linear diffusive relaxation.

V. ACKNOWLEDGEMENTS

This work has been part-funded by the RCUK Energy Programme [grant number EP/P012450/1]. Support for the US group was provided by the US DOE under Grant Number DE-FG02-99ER54563.

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