Integrated tokamak modelling taskforce: Validation of the equilibrium reconstruction from experimental data

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Abstract.
The Integrated tokamak modelling taskforce was set up to provide the European scientific community with simulation tools for preparing and analysing discharges of fusion experiments. We will report on recent progress made on the taskforce project on equilibrium and linear stability. A generic data structure has been devised to describe the geometry of a machine and physical processes in the discharge. This data structure is used to interface all individual analysis program within the taskforce. One of the analysis tools, the equilibrium code EFIT_ITM, based on the EFIT code written by L.L.Lao, has been completely rewritten in order to make it suitable for the ITM. It has algorithm enhancements to increase execution speed, and the ability to treat anisotropic pressure and deviation from axisymmetry. The reconstruction code is now completely independent of the machine description. First results on verification and validation of the new tool are presented.

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INTRODUCTION

The aim of the Integrated Tokamak Modelling taskforce (`http://www.efda-taskforce-itm.org`) is to provide the European scientific community with a set of simulation tools for preparing and analysing discharges of fusion experiments. The task force consists of the infrastructure and software integration project (ISIP) and five integrated modelling projects IMP-1 to -5. ISIP is in charge of the software and hardware for the project, in particular the definition of the data structures, the code platform and the database. The five integrated modelling projects address physics issues (IMP1: Equilibrium and Linear MHD Stability, IMP2: Nonlinear MHD and Disruptions, IMP3: Transport Code and Discharge Evolution, IMP4: Transport Processes and Micro-Stability and IMP5: Heating, Current Drive and Fast Particles). We report on recent progress made in the projects IMP-1/ISIP on the integration of the codes for the reconstruction of the equilibrium state of tokamak discharges.

The task of calculating the magnetic field from measured data leads to an optimisation problem with constraints from magnetohydrostatic theory (e.g. [1]). Taking into account only Maxwell's equation as constraints leads to a linear least squares problem. This is the basis for all codes to obtain the plasma boundary, global parameters of the discharge, and an approximation of the current distribution inside the plasma. Suitable algorithms are based on a parameterised current distribution or expansion into suitable functions. These solutions however, do not provide the equilibrium state accurate enough to serve for subsequent analysis, e.g. for the MHD stability. One therefore has to assume force balance of the plasma, introducing a nonlinear constraint. An approach to avoid the solution of the constrained nonlinear optimisation problem is the function parameterisation or neural network method [2]. These methods establish a large database of equilibria with an accurate predictive solver. The actual equilibrium is found by interpolating using the database. The quality of the reconstruction is directly linked to the precision of the solver and the completeness of the database. An obvious problem poses the accurate predictive modelling of the nonlinear behaviour of the ferromagnetic material in tokamaks with a transformer.

In contrast, the direct method calculates the equilibrium for each set of measurements individually by solving the static MHD equation. One parameterises the unknown current profile with a suitable set of test functions and determines the free parameters by fitting to measurements. Various codes have been developed for axisymmetric

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plasmas [3, 4, 5, 6, 7, 8, 9, 10]. One of the most widely used codes is EFIT [3], which is in routine use for many tokamaks, for example DIII-D, JET, and Tore Supra. We derive the algorithm of direct equilibrium reconstruction for a tokamak in the general case of anisotropic pressure, and in the presence of a ferromagnetic transformer. The equilibrium code EFIT_ITM is based on EFIT [3], but has been completely rewritten in order to make it suitable for the ITM. It was optimised to achieve greater execution speed. EFIT_ITM includes an iron model for tokamaks with a ferromagnetic transformer.

ITM DATA STRUCTURES FOR EQUILIBRIUM ANALYSIS

One of the aims of the taskforce is to integrate existing analysis codes to obtain a complete simulation environment for fusion machines. An important objective for achieving this goal is to define a generic data structure suitable for all existing and future tokamaks. The individual modules communicate with each other only via these data structures. Existing software, in particular equilibrium reconstruction codes are traditionally closely linked to a particular experiment, often hard-wired in the code. These dependence has to be removed completely in order to be ITM compliant.

The data structures describe the geometry of a fusion machine and the physical processes to be treated. They are expressed in the object-oriented language XML (`http://www.w3.org/XML`). Once defined in XML, these schemas are translated into type definitions for various programming language, as Fortran, C, C++ and Java. Data contained in these structures is transferred to and from a database with a standardised interface, the universal access layer (UAL). One of the first codes that has been completely revised and adapted to the guidelines of the ITM taskforce is the code EFIT_ITM, based on the original version of L.Lao [3]. Suitable datastructures have been devised to describe all subsystems of a tokamak relevant for equilibrium reconstruction, described below:

PFSYSTEMS	Active poloidal field coils driven by the amplifiers, and passive structures with induced currents
TOROIDFIELD	Toroidal field coils
IRONMODEL	Model of the ferromagnetic transformer, if present
LIMITER	First wall surrounding the plasma
MAGDIAG	Magnetic diagnostics
MSEDIAG	Motional Stark effect diagnostics
INTERFDIAG	Interferometry
POLARDIAG	Polarimetry (Faraday)
COREPROF	Core plasma profiles as a function of a flux surface label, e.g. pressure
EQUILIBRIUM	Axisymmetric tokamak equilibrium

The figure 1 reveals the tree structure of these top nodes. The data structure equilibrium contains as subnodes the geometry of the plasma, profiles such as current density and safety factor, poloidal flux and magnetic field on a grid, and global scalar parameters. This structure is meant to be extensible, once further physical processes are formally defined. In particular, the data structure is also not limited to tokamak devices, but also for other types of fusion machines, e.g. stellarators.

DIRECT EQUILIBRIUM RECONSTRUCTION

The algorithm

We allow for the more general case of anisotropic pressure, but neglect plasma flow. Assuming toroidal symmetry of the tokamak and the discharge, the poloidal flux $\Psi(R,Z)$ is given by a Grad-Shafranov type equation [11, 12]. With the expressions for the magnetic field $\mathbf{B}$, kinetic pressure P and diamagnetic function F

$$\mathbf{B} = \frac{1}{R}(\nabla\Psi \times \mathbf{e}_\varphi) + \frac{F}{R}\mathbf{e}_\varphi, \quad P_\parallel' := \frac{\partial P_\parallel(\Psi,R)}{\partial\Psi}, FF' := F\frac{\partial F(\Psi)}{\partial\Psi},$$

one obtains

$$-\frac{\partial^2\Psi}{\partial R^2} + \frac{1}{R}\frac{\partial\Psi}{\partial R} - \frac{\partial^2\Psi}{\partial Z^2} = \mu_0 R J_{tor} = R^2\mu_0 P_\parallel' + FF' + \mu_0 R J_{ext}. \tag{1}$$

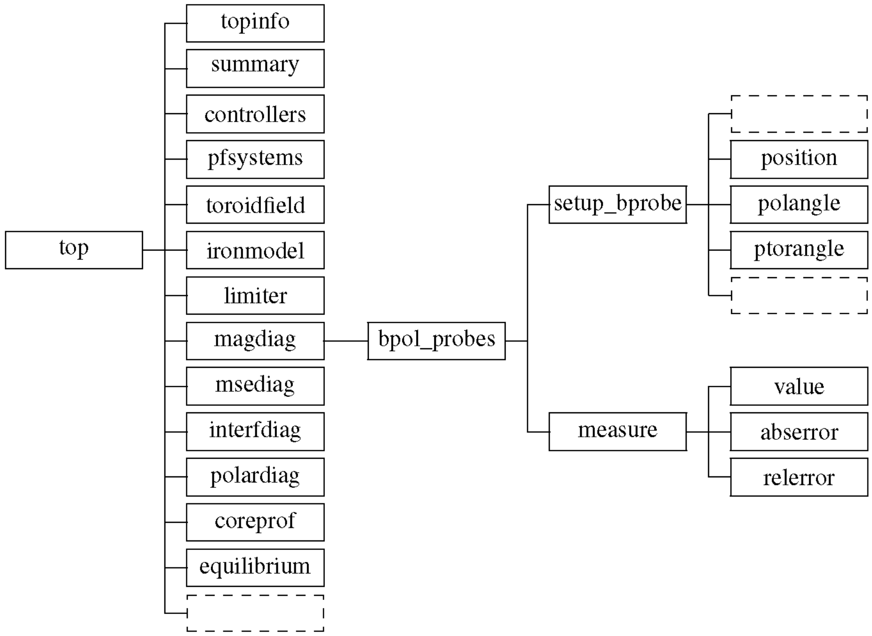

FIGURE 1. Excerpt of the ITM data structure. The figure shows nodes relevant for the equilibrium reconstruction. They contain further subnodes in a tree structure. The subnode "bpol_probes" contains the description of the geometry of the magnetic pick-up coils, and the measured value with its error.

$P'_{\|}$ and FF' are the two profiles describing the plasma current J_{plasma}, and J_{ext} denotes the current distribution of all external sources outside the plasma. The direct equilibrium reconstruction determines the unknown profiles $P'_{\|}$ and FF', and the source term J_{ext} for each time sample of measurement. For the isotropic case, $P_{\|}(\Psi,R)$ becomes $P(\Psi)$, such that the toroidal current depends only on the poloidal flux Ψ. In either case, the unknown current profile is parameterised as a linear superposition of suitable test functions with N_f unknown coefficients c_k

$$J_{plasma}(\Psi,R) = \sum_{k=1}^{N_f} c_k J_k(\Psi,R). \tag{2}$$

The external sources are constituted by the currents in the poloidal field coils, induced currents in the vacuum vessel, support structures, limiter and blanket. Finite domain codes as IDENTD [5] and equinox [9] avoid calculating the field of external sources and take them from interpolation of flux values at a suitably chosen boundary outside the plasma. Infinite domain codes calculate the field of outside sources J_{ext} which is straightforward for current-carrying structures as the poloidal coils. However, in tokamaks with an iron-core transformer, like JET or Tore Supra, calculating the field from induced magnetisation in ferromagnetic materials leads to a nonlinear problem. This case can be treated by introducing the amplitude of the magnetisation as additional free parameters to be determined by the fitting procedure [13]. The field from external sources is therefore split into a known part and a superposition of functions that reflect our knowledge of the geometry of the external sources with N_{ext} free parameters I_j

$$\Psi_{ext}(R,Z) = \Psi_{known}(R,Z) + \sum_{j=1}^{N_{ext}} I_j \Psi_j(R,Z). \tag{3}$$

The model is also called Green's functions model and allows for calculation of the external flux contribution at any point in space. The iron model for the EFIT code has been validated for the Tore Supra tokamak with a dry run, where the poloidal field coils are powered without plasma. It was shown that the iron model reproduced the measurement of the magnetic sensors with an error of less than 1 percent, well below the experimental error [13]. We define the vector

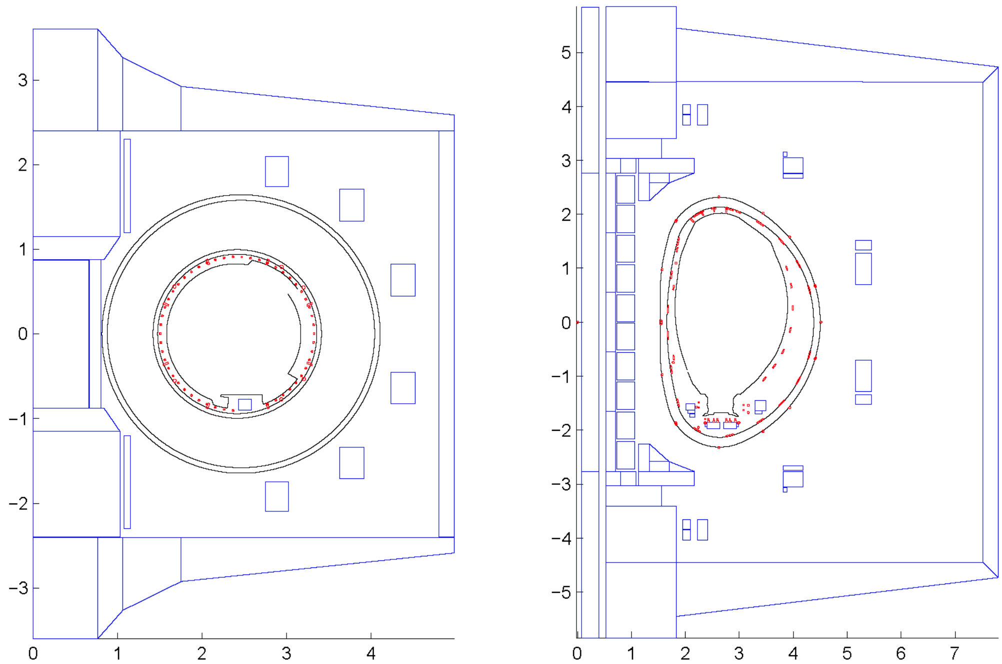

FIGURE 2. Machine description of two tokamaks with iron transformer, Tore Supra(left) and JET (right). The figures show the iron model, the poloidal field system and diagnostics relevant for the equilibrium reconstruction, using only information from the ITM machine description file. The ferromagnetic transformer is approximated by an axisymmetric model that preserves the radial cross section of the real transformer limbs.

of unknowns or state vector as $\mathbf{x} := (I_j, c_k)^T$, which is obtained by minimising the least squares functional

$$\chi^2 = \sum_{m=1}^{N_{meas}} \frac{1}{\sigma_m^2} (F_m^{calc}\{\Psi;\mathbf{x}\} - F_m^{meas})^2 + \lambda^2 \mathfrak{R}, \tag{4}$$

with the Grad-Shafranov type equation (1) as a constraint. F_m^{meas} is the measured value, σ_m is the estimated uncertainty of the measurement, and F_m^{calc} a functional to recalculate it from the flux function and the coefficients. Since the reconstruction problem is inherently ill-posed, one has to add a Tikhonov regularising term $\mathfrak{R}$ [14] that controls unwanted oscillations of the current profile J_{plasma} on a numerical scale of the test functions.

Reconstruction of tokamak discharges

The reconstruction problem to determine the unknown current profile and external sources is nonlinear due to the Grad-Shafranov equation (1), and must therefore be solved iteratively. We apply the method of the Picard iteration, interleaving the solution of the Grad-Shafranov equation (1) for each of the test functions J_k in equation (2) with the minimisation of the least squares functional (4). The inversion of the Grad-Shafranov operator is performed using the very effective algorithm by Lackner [15]. Details of further optimisation are described in [13]. The program is written in ANSI Fortran 95 and uses the external libraries FFTW [16] and LAPACK [17]. The CPU time for one time slice was measured for a typical JET discharge and a grid size of 33×33 to be about 100msec on a HP Alpha workstation (1450 Linpack MFlops). Figure 3 shows an application to the randomly selected JET discharge 58094. The same code EFIT_ITM was applied to reconstruct an equilibrium of the ITER device, shown in figure 4. The data was produced from a previous simulation with the code DINA.

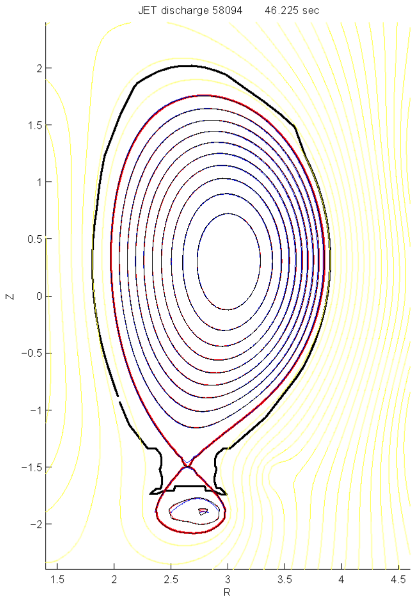

FIGURE 3: Equilibrium reconstruction of JET discharge 58094 at 46.2 sec. Comparison of the flux surfaces produced by the code EFITJ [18], installed at JET, with the results of EFIT_ITM. The results of the two codes show very good agreement.

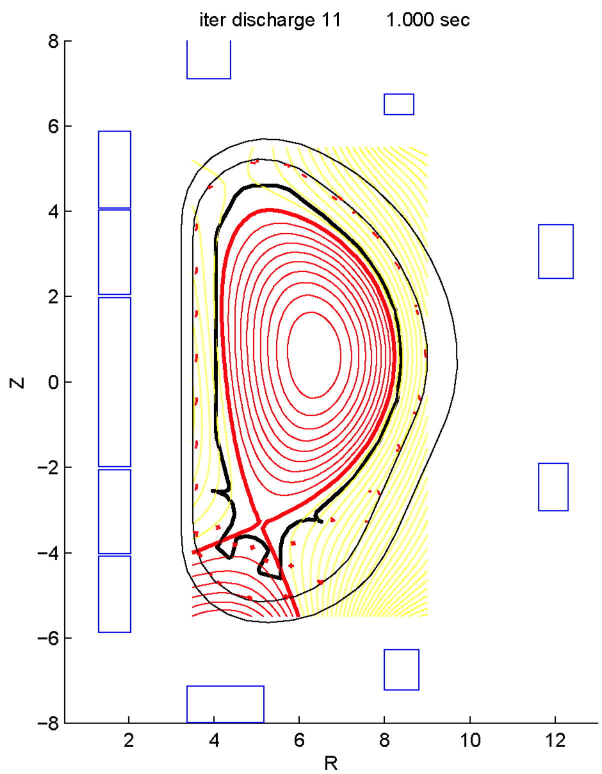

FIGURE 4: Equilibrium reconstruction of the ITER discharge internally numbered as 11. This is the inductive scenario 2 with a plasma current of 15 MA.

Accuracy of reconstruction

Gain of accuracy from measurements inside plasma

The accuracy of the direct reconstruction depends on measurements of variables effectively present in the equilibrium problem, which are magnetic field and total kinetic pressure for the static force balance. The most reliable and precise direct measurement of the magnetic field is only available outside the discharge. However, using this data alone is not sufficient to obtain the current profile accurately, because its extrapolation into the discharge is a mathematically ill-posed problem [6]. Errors of the input data are significantly amplified in the centre of the plasma, and especially quantities like the central safety factor can be quite inaccurate [19, 20, 21]. To achieve a high accuracy of the reconstruction, one has to obtain direct measurements of total pressure or safety factor, or the magnetic field inside the discharge. The Faraday rotation diagnostics [5] gives the polarisation angle α integrated over the line of sight $\alpha \sim \int n_e \mathbf{B} \cdot \mathbf{ds}$, with the electron density n_e. Another local measurement of the field is given by measurement of the motional Stark effect (MSE), which is present in neutral beam heated plasma due to the induced electric field $\mathbf{E} = \mathbf{v}_{NBI} \times \mathbf{B}$. Using internal plasma data constrains the confidence interval to be of the order of magnitude of the relative measurement error of internal data (see [21]).

Choice of profiles

Series of polynomials are often used for the parameterisation of the current profile (2). Due to their flexibility and local support, B-splines [22] seem to be a better choice for fine resolution of profiles, as shown with the reconstruction code CLISTE [21].

Regularisation

The equilibrium reconstruction requires regularisation, normally chosen as a constraint on higher derivatives of the current profile. It is important to determine the amount of regularisation for the current profile correctly, as it can strongly influence the results of the reconstruction [12]. A generally applicable algorithm does not seem to be available [23], therefore one often relies on trial and error. A more elaborate method is the a posteriori error analysis by calculating a set of equilibria with λ from equation (4) as parameter. The optimum parameter is then found by the method of generalised cross-validation [23] or the method of L-curve [24, 12, 25]. These techniques are computationally costly and are therefore used only for advanced analysis of individual discharges. Further work is necessary to develop a faster algorithm to select the regularisation automatically or even a priori.

Anisotropic pressure

Using data of internal plasma measurements sometimes does not lead to an increase of accuracy, but to inconsistent results. This may reveal the limit of the model used, for example the often used isotropic pressure model. An early example is the JET high performance discharge 40847, heated with 18 MW neutral beam and 6 MW ICRF power. When reconstructing this discharge using total pressure obtained from the TRANSP transport analysis code, it turns out to be impossible to obtain an equilibrium with a central safety factor above one. Given the absence of sawteeth, this seems to be unrealistic. Relaxing the isotropy condition decouples the position of the magnetic axis and the peak of the pressure distribution, and gives more freedom for the reconstructed equilibrium. With suitable test functions for the parallel pressure in equation (1), a reversed profile of the safety factor was achieved, with a central q of 1.8 [12]. Figure (5) shows field lines and isobars of the reconstructed equilibrium. The position of the magnetic axis and the q profile agree well with soft X-ray data for a similar reversed shear discharge. In spite of the better reconstruction, the fitting of the data proved to be difficult and required careful choice of weights on measurements and regularisation. This is traced back to the increased number of degrees of freedom, since one has now a two-dimensional function $P_{\|}$, and the absence of measurements (only data for pressure). It also seems that a more physical model of the anisotropy is favourable, that takes into account the power deposition in the plasma more accurately. A suitable approach was suggested by Cooper [26].

Threedimensional equilibria

Tokamaks are designed to provide a field structure that is axisymmetric around the torus axis. There are always small toroidal variations due to error fields, or applied on purpose to ergodise the field structure close to the plasma boundary for controlling heat and particle transport [27]. For tokamaks where the magnetic field is dominated by the toroidal field, the plasma response can be calculated with a perturbation method [28]. From the non-axisymmetric field $\tilde{\mathbf{B}}$ produced by external coils, the perturbed plasma current is given by $\tilde{\mathbf{J}}_{\mathbf{p}} = (RJ_T/B_0R_0)\tilde{\mathbf{B}}_{\mathbf{p}}$, where J_T is given by the Grad-Shafranov equation (1) and B_0 is the vacuum toroidal field at $R = R_0$, and the index p denotes poloidal component.

The only existing code to reconstruct fully threedimensional equilibria is V3FIT [29]. It is a combination of EFIT [3] with the fully three-dimensional code VMEC [30]. VMEC relies on a representation of the magnetic surfaces in

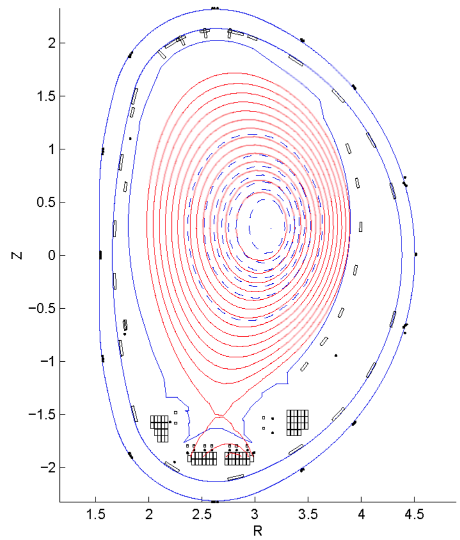

FIGURE 5: Equilibrium obtained earlier [12] for JET discharge 40847 with anisotropic pressure. The solid lines show the field lines and the dashed line the isobars of the parallel pressure. Note that the field lines do no longer coincide with the isobars. The magnetic axis is being shifted to 3.09 m. The standard reconstruction using magnetic data only would give a magnetic axis position of 3.02 m.

flux coordinates. V3FIT was designd for the reconstruction problem of stellarator discharges.

Verification and Validation

The verification proves correctness and numerical accuracy of code. This is code to code benchmarking, for example the comparison of two codes or two version of a code, as shown in figure (3). The well-known analytic Solov'ev equilibrium (e.g. [31]) has the disadvantage that it does not provide a plasma boundary, such that one has to use hybrid analytic-numeric solutions for testing numerical accuracy [32]. It is planned to formalise these comparisons to achieve a test suite for automatic consistency checking. Further development is needed to eliminate "user" parameters, in particular the resolution of the grid, number of test functions for the profiles or the amount of regularisation.

Validation, on the other hand, proves that a particular code describes the real world within the limits of the applied model. It shall explore the limits of the physical model applied, in particular when deviations from the usual Grad-Shafranov theory occur, e.g bulk flow, anisotropy or toroidal variation due to error fields. The equilibrium reconstruction code therefore has to be compared with diagnostic data that is completely independent . Measurements that rely on the knowledge of equilibrium have to be ruled out. Several diagnostics have been identified for comparison:

1. Scrape-off layer position from probes, camera
2. q from MHD measurements
3. Flux geometry from ECE (as function of field), X-ray tomography

The validation with the data of existing tokamaks ought to achieve a high standard of reliability and known confidence intervals of these tools designed for the preparation and analysis of ITER discharges.

CONCLUSIONS AND OUTLOOK

At present, the ITM data structure includes a formal description of all sources of the magnetic field, the first wall, and formal descriptions of diagnostics used for the equilibrium reconstruction. At present, there are data structures that receive data produced by equilibrium and stability codes. The processing chain provides by IMP-1 is therefore complete. It comprises data input from experimental data, equilibrium reconstruction made with EFIT_ITM, high-precision equilibrium with the fixed boundary code HELENA, and finally the universal code on linear stability MISHKA/ILSA. All these codes have standardised interfaces and are driven from the interactive simulation tool Kepler (`http://www.kepler-project.org`).

Several extension from earlier version of EFIT are planned to be implemented for EFIT_ITM, in particular the anisotropy model, toroidal flow, toroidal variations and possibly a scrape-off layer model [21]. There is more research needed to eliminate "user" parameters in order to run the code module in a simulation environment without user interaction. It is foreseen to extend the data structures also to other fusion machines.

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