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A SIMPLE DERIVATION OF RELATIVISTIC FULL-WAVE EQUATIONS AT ELECTRON CYCLOTRON RESONANCE

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ABSTRACT

When a wave passes through an electron gyroresonance, in a plasma in the presence of a magnetic field gradient, there is a small spread in the resonance due to the electron's Larmor radius. Mathematically this is represented by the inclusion of the so called gyrokinetic term in the resonance condition, Lashmore-Davies and Dendy[1]. The smallness of this term, compared with other effects such as relativistic broadening, suggests that it should be negligible. However, we shall show here, by extending the method of Cairns et al[2] into the relativistic regime, that its inclusion is vital for producing self consistent full-wave equations which describe electron gyroresonance. The method is considerably simpler than those used previously by Maroli et al[3], Petrillo et al[4] and Lampis et al[5] for obtaining similar equations. As an example we include a calculation for the O-Mode passing perpendicularly through the fundamental.

DERIVATION OF THE WAVE EQUATION

To illustrate the technique we consider the O-Mode at perpendicular incidence and simply note that it can readily be extended to oblique incidence and to the X-Mode. We begin by taking the usual uniform plasma expression for the current density,

$$J_z(x) = \int dk\, \sigma(k) E_z(k) \exp(ikx - i\omega t) \qquad (1)$$

Where,
$$\sigma(k) = \sum_{n=-\infty}^{\infty} \int d^3u\, \frac{S_n}{\omega\gamma - n\Omega}$$

And,
$$S_n = 2\pi^{-3/2}\, i\omega_p^2 \varepsilon_0\, J_n(k\rho u_\perp) \exp(ik\rho u_y - in\theta - u^2)$$

We have assumed here a Maxwellian distribution, with thermal velocity u_t.

We now consider an inhomogenous magnetic field. To simplify the algebra, let us consider wave absorption at the fundamental, and assume that in the vicinity of the resonance the gradient scale length is L, so that,

$$\Omega(x) = \omega(1 - x/L) \qquad (2)$$

The most straight forward way to proceed is to insert this expression for Ω into Eq. 1 and then solve to obtain a wave equation. However, the equations formed, by this method, do not conserve energy, because we have not considered the so called gyrokinetic effect. The gyrokinetic effect arises from the fact that Ω must be evaluated not at x, the point at which we are calculating the response of the plasma, but at $x + u_y/\Omega$, the position of the particle guiding centre.

Expanding γ to second order and inserting the corrected Larmor frequency into the resonant denominator, gives,

$$\sigma(k) = \frac{L}{\omega} \frac{S_1}{x + Lu^2/\mu + u_y/\omega} + \sum_{n \neq 1} \int d^3u \, \frac{S_n}{\omega\gamma - n\Omega} \tag{3}$$

The velocity integration can now be performed, after first applying the following identity to the resonant denominator,

$$\frac{1}{D} = -i \int_{\infty}^{0} dk' \exp(iDk') \qquad \text{where, Im } D>0 \tag{4}$$

Giving,

$$\sigma = \int_{\infty}^{0} dk' \frac{L}{\omega} \omega_p^2 \varepsilon_0 \frac{e^{ik'x}}{(1 - it)^{5/2}} I_l(\lambda) \exp(-k'^2\rho'^2/4-\lambda) \tag{5}$$

Where, $\lambda=k(k+k')\rho'^2/2$, $\rho'=\rho(1-it)^{-1/2}$ and $t=k'L/\mu$. To produce the second order wave equation we expand this to second order in $k\rho$ and $k'\rho$, to give the finished wave equation as,

$$\frac{d}{dx}\left[\left(1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2} F_{7/2}(\mu x/L)\right)\frac{dE_z}{dx}\right] + k_0^2 E_z = 0 \qquad (6)$$

This is the equation derived by Maroli et al[3].

DISCUSSION

By including the gyrokinetic effect into the standard method for calculating the homogenous dielectric tensor, we have produced a technique for deriving full-wave equations describing ECR, which is a considerable simplification on what has gone before. For the O-Mode we have shown it to be in full agreement with the previous work of Maroli et al[3], and the technique can easily be extended to produce wave equations for higher resonances and also to describe the X-Mode.

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