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Wave Propagation through Cyclotron Resonance in the Presence of Large Larmor Radius Ions

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INTRODUCTION

In a fusion plasma ion cyclotron heating may be applied to a plasma in a regime where there is a population of ions whose Larmor radius is not small compared to the perpendicular wavelength. In this case the equations describing the propagation and absorption of the wave are integro-differential, describing the non-local response of the plasma to the wave field. In this paper we outline the derivation of such equations, using an extension of a technique already developed by us to obtain the differential equations appropriate to the small Larmor radius regime.[1,2] We then discuss the WKB approximation and the fast wave approximation[3,4] (the latter being similar to the Born approximation of scattering theory), both of which produce considerable simplification of the problem, while retaining information from the non-local response.

THE INTEGRAL EQUATIONS

We take an approximation in which the magnetic field (in the z-direction) varies as $1 - x/L$. The method we describe can be extended to include a general dependence on x of magnetic field, temperature and density, in which case it reproduces the equations given by others[5,6]. However, it is our view that the gradient in magnetic field, which determines the cyclotron resonance condition, is the most significant and that the simpler model includes the most important physics which can be included in a plane slab model. For purposes of illustration we consider the $z-z$ component of the dielectric tensor with the wave frequency close to the fundamental of the ion cyclotron frequency. The method is easily adapted to the other tensor elements and to any harmonic.

We begin with a standard integration along orbits for a uniform plasma, which gives

$$\sigma_{zz} = i\varepsilon_o \frac{\omega_p^2}{\omega} \int u\ v_\perp\ du\ dv_\perp d\theta \frac{\partial f_0}{\partial u} J_1(b) e^{i(b\sin\theta - \theta)}$$
$$\int_{-\infty}^{0} d\tau \exp\left\{-i\tau(\omega - k_{||}u - \omega_c)\right\} \tag{1}$$

where u is the parallel velocity and $b = k_\perp v_\perp/\omega_c$. The part of Eqn (1) where the spatial dependence of ω_c is important is the final resonant integral, and here we put

$$\omega - \omega_c \quad = \quad \omega_c \left(\frac{x}{L} + \frac{v_\perp}{L\omega_c}\ \sin\theta\right) \tag{2}$$

Note the crucial point that the field is evaluated not at the final position of the particle, but at the guiding centre. As a result σ_{zz} contains both the Fourier transform variable $k_\perp$ and an explicit dependence on x. This should be thought of as a two scale approach, with $k_\perp$ on the scale of the wavelength and x on the scale of the field gradient. We have

$$J(x) = \int_{-\infty}^{\infty} dk_\perp E(k_\perp)\sigma_{zz}(k_\perp, x)e^{ik_\perp x}. \tag{3}$$

where J is the component of J_z coming from E_z, which we denote simply by E. Also, if the plasma response is non-local, we might expect that

$$\begin{aligned} J(x) &= \varepsilon_o L \frac{\omega_p^2}{\omega^2} \int_{-\infty}^{\infty} E(x')G(x,x')dx' \\ &= \varepsilon_o L \frac{\omega_p^2}{\omega^2} \int_{-\infty}^{\infty} dk_\perp\ E(k_\perp) \int_{-\infty}^{\infty}\ dx'\ e^{ik'_\perp x'}\ G(x,x') \end{aligned} \tag{4}$$

Comparing (3) and (4) we see that

$$\int_{-\infty}^{\infty} G(x,x')e^{ik_\perp x'}dx' = \sigma_{zz}(k_\perp, x)e^{ik_\perp x} \tag{5}$$

The Fourier inversion of this gives $G(x,x')$. After detailed algebra, which is too long to give here, we obtain

$$G(x,x') = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk' e^{-ik'x'} \int_0^{\infty} dk e^{i(k+k')x - k^2\rho^2/4 - k^2 k_{||}^2 L^2\rho^2/4}$$
$$I_1\left(\frac{k'(k+k')\rho^2}{2}\right) e^{-k'(k+k')\rho^2/2}\left(1 - \frac{1}{2}k^2 k_{||}^2 L^2 \rho^2\right) \tag{6}$$

where ρ is the Larmor radius of a thermal particle, the distribution of particles having been assumed Maxwellian.

THE FAST WAVE APPROXIMATION

The method, similar to the Born approximation, has been shown[3,4] to work well in the small Larmor radius regime. The essential idea is that in the hot plasma resonant response we substitute a plane wave $E_o e^{ik_o x}$, with k_o given by the cold plasma dispersion relation. Thus, the integral response, defined by Eqs (4) and (6) can be obtained explicitly

$$J(x) = \varepsilon_o L \frac{\omega_p^2}{\omega^2} E_o e^{ik_o x} \int_0^\infty dk \exp\left\{ ikx - \frac{k^2\rho^2}{4} - \frac{k_\parallel^2 L^2 k^2 \rho^2}{4} \right\}$$
$$\times (1 - \frac{1}{2} k_\parallel^2 L^2 k^2 \rho^2) I_1 \left(\frac{k_o(k+k_o)}{2} \right) \exp\left\{ -\frac{k_o(k+k_o)\rho^2}{2} \right\} \qquad (7)$$

We write this as

$$J(x) = E_0 e^{ik_0 x} H(k_0, x) \simeq H(k_0, x) E(x) \qquad (8)$$

In the equations describing the plasma wave we replace the integral terms giving the resonant response with multiples of the field defined as in (8), while keeping the derivatives coming from $\nabla \times \nabla \times \boldsymbol{E}$.

We have applied this to minority heating, where, with the commonly used approximation that the parallel component of $\boldsymbol{E}$ is neglected, we obtain an equation of the form

$$\frac{d^2 E_y}{dx^2} + V(x) E_y = 0 \qquad (9)$$

with V a rather complicated expression containing integrals of the form shown in Eq (7). Such an approach yields reflection and transmission coefficients, but loses information on how the remaining energy is split between mode conversion and absorption around the resonance.

NUMERICAL RESULTS

The theory outlined above has been applied to minority heating in a D-He^3 plasma, with parameters appropriate to JET. We present results from Eq (9) in Fig 1 which also shows a comparison with results obtained from an equation similar to Eq (9) but with the warm plasma response obtained from locally uniform dielectric tensor elements. We have not, so far, implemented a numerical solution of the full integro-differential system. However, the reduction in absorption in the non-uniform theory as compared to a locally uniform theory appears to agree qualitatively with the results of Sauter and Vaclavik[5].

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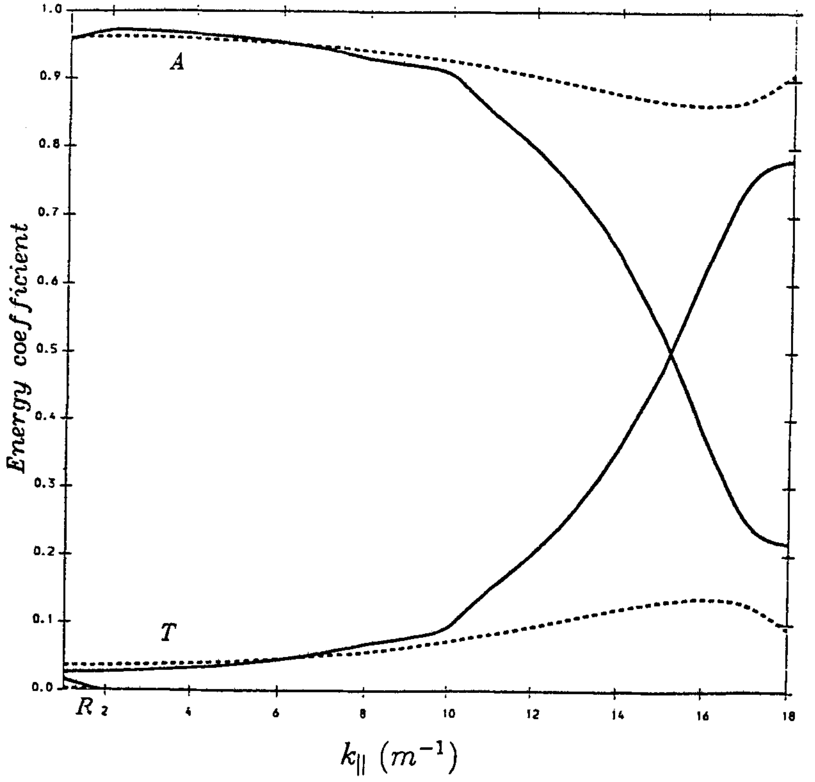

Figure 1. Energy coefficient variation with $k_{||}$ for non-uniform theory (full line) and for locally-uniform theory(broken line). T=Transmission, R=Reflection and A=Absorption. Plasma parameters are for a $D(He^3)$ plasma with the minority ion $b = He^3$ resonant at its fundamental cyclotron frequency with $B_T = 3.4T$, $L = 3.1m$, $n_e = 5.0 \times 10^{19} m^{-3}$, $n_b/n_e = 0.05$ and $T_b = 500keV$.