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Describing Linear Second Harmonic ECA

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Abstract. Electron cyclotron absorption (ECA) of electromagnetic radiation at the second harmonic is of interest in Tokamak physics both for plasma heating and current drive, as well as for some diagnostics. Modelling of the linear wave propagation for such problems is usually done by WKB ray tracing, despite the fact that the WKB theory is not strictly valid in the neighbourhood of a resonance, where reflection and mode conversion can occur. Here, we present a full wave analysis of second harmonic ECA and compare it with various ray tracing models. The full wave equation used differs from previous ones in that it is valid even in regimes where significant absorption occurs on scale lengths comparable to the incident wavelength. The resulting analysis allows us to assess the accuracy of ray tracing in describing such problems.

FULL WAVE EQUATIONS

To derive the full wave equations to describe second harmonic ECA, we have used the gyrokinetic technique [1,2]; the resulting wave equations are,

$$\frac{d}{dx} \left[f(x, \omega) \frac{dE_x}{dx} \right] + i \frac{d}{dx} \left[f(x, \omega) \frac{dE_y}{dx} \right] + SE_x - iDE_y = 0 \quad (1)$$

$$-i \frac{d}{dx} \left[f(x, \omega) \frac{dE_x}{dx} \right] + \frac{d}{dx} \left[(1 + f(x, \omega)) \frac{dE_y}{dx} \right] + iDE_x + SE_y = 0 \quad (2)$$

where,

$$S = \frac{\omega^2}{c^2} \frac{\omega^2 - \omega_{UH}^2}{\omega^2 - \omega_c^2}, \quad D = \frac{\omega_p^2}{c^2} \frac{\omega \omega_c}{\omega^2 - \omega_c^2}, \quad f(x, \omega) = \frac{1}{2} \frac{\omega_p^2}{\omega_c^2} F_{7/2} \left(\mu \frac{\omega - 2\omega_c}{\omega} \right).$$

The interested reader is referred to the full derivation [3]. We see immediately that away from resonance, where $f(x, \omega) \rightarrow 0$, we have the standard cold

plasma result. The function denoted by F is the Dnestrovskii function [4,5], a relativistic plasma dispersion function.

The wave equations are valid even when the absorption scale length is comparable to the wavelength of incident radiation, and are consistent in that they can be shown to obey energy conservation through the energy equation,

$$\frac{d}{dx} \left[\text{Im} \left(E_y^* \frac{dE_y}{dx} + f(x, \omega) (E_x + iE_y)^* \frac{d}{dx} (E_x + iE_y) \right) \right] - \text{Im} (f(x, \omega)) \left(\left| \frac{dE_x}{dx} \right|^2 + \left| \frac{dE_y}{dx} \right|^2 \right) = 0. \quad (3)$$

We can identify the first two terms as the Poynting and kinetic power flux, and the final one as the power absorbed by the plasma, which is positive definite everywhere.

RAY TRACING EQUATIONS

The fact that we consider a model with inhomogeneity in one direction makes the ray tracing problem considerably simpler. The ray travels in one dimension, x , its path being defined exactly by its dispersion relation alone. The dispersion relation for our problem is,

$$\mathbf{D}(x, k, \omega) = \begin{pmatrix} S - fk^2 & -iD - ifk^2 \\ iD + ifk^2 & S - (1 + f)k^2 \end{pmatrix} \quad (4)$$

$$fk^4 - [2(S - D)f + S]k^2 + [S^2 - D^2] = 0. \quad (5)$$

From this expression we can see that away from resonance, in the limit $f \rightarrow 0$, we have two distinct types of solutions. The first, $k^2 \rightarrow (S^2 - D^2)/S$, corresponds to the usual extraordinary wave and the second, $k^2 \rightarrow S/f$, represents the Bernstein wave. The Bernstein wave will be evanescent below the resonant frequency and propagating above it.

Three variations on the basic WKB ray tracing are considered,

- (1) Direct solution of $\mathbf{D}=0$ in the complex plane.
- (2) Ray along $\text{Real}(\mathbf{D})=0$ with absorption calculated as a correction.
- (3) Cold plasma ray trajectory with absorption as correction.

NUMERICS

Our full wave equations were integrated using a Runge-Kutta scheme with boundary conditions corresponding to an incident wave from either the high

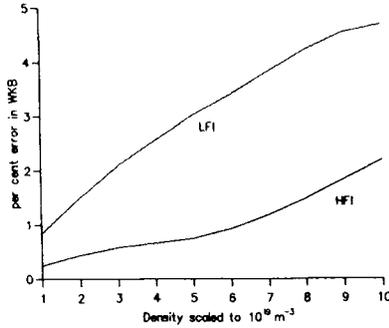


FIGURE 1. WKB error in transmission coefficient

field or low field side. The ray tracing was done by using another Runge-Kutta scheme to integrate along the rays, integrating the absorption, in turn, along these paths. Typical results of the full wave calculation were of an incident wave having energy absorbed from it, along with some reflection or mode conversion to Bernstein waves for the high field incident case.

Comparison with direct solution of the dispersion relation, method (1), gave good agreement with the full wave analysis, even outside of its regime of validity, for the transmission coefficient. This can be seen in figure (1) for the plasma parameters $B=2.4\text{T}$, $T=100\text{eV}$ and L , the scale length of the magnetic field, equal to 3m . Similar agreement was found with method (3), indicating that the ray trajectories differ little from their cold ones for the one dimensional problem. For method (2), however, sizeable differences were found, the theory having to be adjusted in some cases to avoid complete reflection or mode conversion to Bernstein waves. Information on mode conversion and reflection was not, of course, given at all by any of the ray tracing methods.

DISCUSSION

We have derived a full wave equation to describe the absorption of an extraordinary mode plasma wave passing through the 2nd harmonic electron cyclotron resonance for a slab model. The equation has been solved numerically, the results showing an incoming wave losing energy to the plasma as well as to a reflected wave and, through mode conversion, to a Bernstein wave. By comparing the results with those of a standard WKB analysis we have been able to study the accuracy of the WKB technique in regimes where it is not strictly valid. Although the WKB analysis represents a single mode analysis, and so cannot describe reflection or mode conversion, its description of wave absorption and, in particular, wave transmission turns out to be accurate, to within at least 10 per cent, for a wide range of parameters where its condi-

tions of validity are not met. This compares well with a similar analysis [6] for ordinary wave absorption near the fundamental resonance, which also found WKB theory to give good results well outside its range of validity.

The use of a slab model to describe the magnetic field in a tokamak is a large simplification. It can be shown [3] that for electron cyclotron absorption, where the absorption width is usually small compared to the dimensions of the tokamak, such an approximation can give useful results. However, the main aim of this paper has been to compare the full wave and ray tracing techniques to describe absorption, and it is not believed that toroidicity would introduce significant new physics into the differences between them. That is, the authors believe that reflection would still be the key feature separating the two methods. However, it would still be of interest to demonstrate this explicitly for a model which included toroidal effects.

A consequence of the fact that mode conversion does not seem to be as important effect on second harmonic electron cyclotron resonance heating, is that it may be possible to analyse such problems using a form of the fast wave theory [7,8]. The theory reduces such problems to second order ODEs at the expense of information concerning mode conversion. Although fast wave theory, as its name suggests, was designed to describe ion cyclotron resonance, it can equally be applied to our problem.

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