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Possible Electron Bernstein Wave Heating Scenarios For MAST

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Abstract The conversion of the O-mode to the X-mode is calculated for a sheared magnetic field. The damping of electron Bernstein waves by Doppler shifted cyclotron resonance is shown to be very strong. The energy of the Bernstein wave is totally absorbed well before the cold resonance.

1. Introduction

The principal challenge to the development of an electron Bernstein wave heating scheme for spherical tokamaks is that of coupling. Batchelor and Bigelow[1] have proposed the O-X conversion as one possibility and Bers, Ram and Schultz[2] have suggested a cut-off-resonance-cut-off triplet configuration as another. In Section 2 of this paper we analyze the effect of magnetic shear on the O-X conversion. The damping of electron Bernstein waves is described in Section 3 and a brief summary and conclusions are given in Section 4.

2. O-X Conversion

We suppose that radiation reaches the surface $\omega = \omega_p$ at an angle near the critical condition for total conversion from the O- to the X-mode. Consider slab geometry with a non-zero (but small) y-component of the wave vector. The magnetic field and density vary in the x−direction. The axes may be chosen so that, at the value of x where $\omega = \omega_p$, the magnetic field is in the z-direction. Magnetic shear is introduced as follows. The coordinates (x, y, z) will be assumed to be fixed and oriented as above at the critical surface. An x−dependent system (x', y', z') will be oriented with the z' axis along the magnetic field. If $\theta(x)$ is the rotation angle, then

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

and all other vector quantities are related in the same way. By definition $\theta(0) = 0$, and we shall assume that in the region of interest θ is small. In this case we can take $\cos\theta \approx 1$, $\sin\theta \approx \theta$ in (1) so that we have

$$\begin{aligned} n'_y &\approx n_y + \theta n_z \\ n_{z'} &\approx -\theta n_y + n_z \approx n_z \end{aligned} \tag{2}$$

where n_y is also assumed to be small and we ignore products of small quantities. In the rotated frame the local dielectric tensor takes the usual form and if we then express primed quantities in terms of unprimed quantities we get, to lowest

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order,

$$\begin{pmatrix} \varepsilon_\perp - n_z^2 - n_y^2 & n_x n_y + i\varepsilon_{xy} & n_x n_z \\ n_x n_y - i\varepsilon_{xy} & \varepsilon_\perp - n_x^2 - n_z^2 & n_y n_z + \theta n_z^2 \\ n_x n_z & n_y n_z + \theta n_z^2 & \varepsilon_\parallel - n_x^2 - n_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y + \theta E_z \\ E_z - \theta E_y \end{pmatrix} = 0. \quad (3)$$

In the vicinity of the critical surface, n_x is small and neglecting terms quadratic in small quantities n_y and n_x, the middle line of (3) gives

$$E_y = \frac{i\varepsilon_{xy}}{(\varepsilon_\perp - n_z^2)} E_x - \frac{(n_y n_z + \varepsilon_\perp \theta)}{(\varepsilon_\perp - n_z^2)} E_z \quad (4)$$

Using (4) in the other two lines of (3), making the substitution, $n_x \to -id/dx$ where distance is scaled in units of c/ω and again neglecting products of n_y and n_x, we obtain a pair of differential equations for the inhomogeneous system

$$\frac{dE_x}{dx} = \frac{\varepsilon_{xy}(n_y + \theta n_z)}{(\varepsilon_\perp - n_z^2)} E_x - i\frac{\varepsilon_\parallel}{n_z} E_z \quad (5)$$

$$\frac{dE_z}{dx} = -i\frac{\left\{(\varepsilon_\perp - n_z^2)^2 - \varepsilon_{xy}^2\right\}}{n_z(\varepsilon_\perp - n_z^2)} E_x - \frac{\varepsilon_{xy}(n_y + \theta n_z)}{(\varepsilon_\perp - n_z^2)} E_z. \quad (6)$$

Now consider the behaviour of (5) and (6) near the critical surface. If we take $x = 0$ at the point where $\omega_p = \omega$, then we have $\varepsilon_\parallel \approx -x/L_n$ with L_n a scale length associated with the density gradient. If δ is the mismatch in n_z compared to the critical value $n_{cr} = (\varepsilon_\perp + \varepsilon_{xy})^{1/2}$ at $x = 0$, then (5) and (6) take the following form in the critical region

$$\frac{dE_x}{dx} = -(n_y + n_{cr}\frac{x}{L_\theta})E_x + i\frac{x}{n_{cr}L_n}E_z \quad (7)$$

$$\frac{dE_z}{dx} = i(4\delta + \frac{2}{L_n n_{cr}}\frac{\omega}{\omega + \Omega}x)E_x + (n_y + n_{cr}\frac{x}{L_\theta})E_z \quad (8)$$

where the scale length of the magnetic shear has been introduced by taking $\theta \approx x/L_\theta$ near $x = 0$.

If we now follow previous work and calculate the transmission coefficient using a WKB method, we obtain, assuming a solution in the form $\exp(\pm i \int^x k(x')dx')$ in (7) and (8)

$$k^2(x) = (\frac{2\omega}{\omega + \Omega}\frac{1}{L_n^2 n_{cr}^2} - \frac{n_{cr}^2}{L_\theta^2})x^2 + (\frac{4\delta}{n_{cr}L_n} - 2\frac{n_y n_{cr}}{L_\theta})x - n_y^2 \quad (9)$$

If the roots of the quadratic in (9) are a and b, and assuming that the coefficient of the x^2 term is positive, the transmission coefficient is given by

$$T = \exp\left[-\frac{\pi}{4}(\frac{2\omega}{\omega + \Omega}\frac{1}{L_n^2 n_{cr}^2} - \frac{n_{cr}^2}{L_\theta^2})^{1/2}(a - b)^2\right] \quad (10)$$

Substituting the explicit values of the roots of (9) into (10), and introducing the ratio $r = L_n/L_\theta$, (10) becomes

$$T = \exp\left[-\pi\frac{L_n\omega}{c}\frac{(\frac{2\delta}{n_{cr}} - rn_y n_{cr})^2 + n_y^2(\frac{2\omega}{\omega + \Omega}\frac{1}{n_{cr}^2} - r^2 n_{cr}^2)}{(\frac{2\omega}{\omega + \Omega}\frac{1}{n_{cr}^2} - r^2 n_{cr}^2)^{3/2}}\right]. \quad (11)$$

A plot of the transmission coefficient for a 60GHz source and conditions relevant to MAST[3] is given in Fig 1. The transmission coefficient is plotted against the parameter r for four values of the mismatch parameter δ. For a mis-match in $n_{\|}$ as much as ten per cent it is still possible for more than fifty per cent of the incident O-mode to be converted to the X-mode and hence the electron Bernstein wave.

3. The Damping of Electron Bernstein Waves

We have solved the full, three by three, hot plasma dispersion relation for the electron Bernstein waves. The full Bessel function dependence is included since $k_{\perp}\rho_e \gg 1$ for electron Bernstein waves and the sum over harmonics was taken from $n = -5$ to 10. For a 60GHz source on MAST the upper hybrid resonance can be chosen to lie between the fourth anf fifth harmonics. The mode converted Bernstein wave will propagate from the upper hybrid to the fourth harmonic resonance. The imaginary part of the perpendicular refractive index $n_{\perp}$ has been calculated for slab geometry with a varying magnetic field and constant density and temperature for $n_{\|} = 0.3$ and is shown in Fig 2(a). Note that imaginary $n_{\perp}$ is negative corresponding to the backward wave nature of the electron Bernstein wave. Figure 2(b) shows a similar result for $n_{\|} = 0.5$. In both cases the damping is extremely strong so that the wave will be totally absorbed well before the cold resonance, a situation which is very favourable for current drive.

4. Summary and Conclusions

The most promising method of coupling power to electron Bernstein waves is by conversion at the upper hybrid resonance. The O-X conversion is one means of achieving this and we have shown that a significant fraction of the incident O-mode energy can be converted to the X-mode in a sheared magnetic field. The effect of a mismatch in the optimum $n_{\|}$ and a finite value of n_y has also been included.

The non-relativistic damping due to Doppler shifted cyclotron resonance has also been calculated. This is relevant to the O-X conversion which requires significant values of $n_{\|}$. For spherical tokamaks the hybrid resonance can be located between harmonic resonances as well as between the fundamental and second harmonic. The electron Bernstein wave is very strongly damped and would be totally absorbed on the low field side of the resonance well before the cold resonance. This property is advantageous for current drive.

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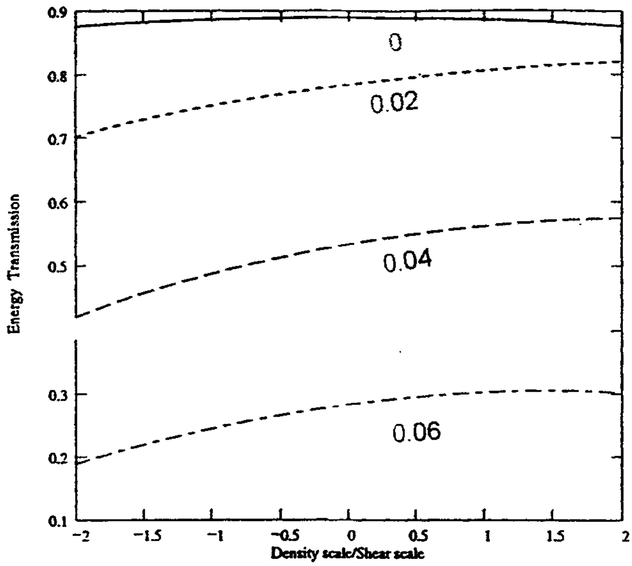

Figure 1 Energy transmission coefficient for the fraction of incident O-mode energy converted to X-mode as a function of density scale/shear scale. The curves are labelled by the mismatch parameter δ and the other parameters are $n_y = 0.03$, $f = 60$GHz, $L_n = 0.1$m, $\Omega/\omega = 0.22$.

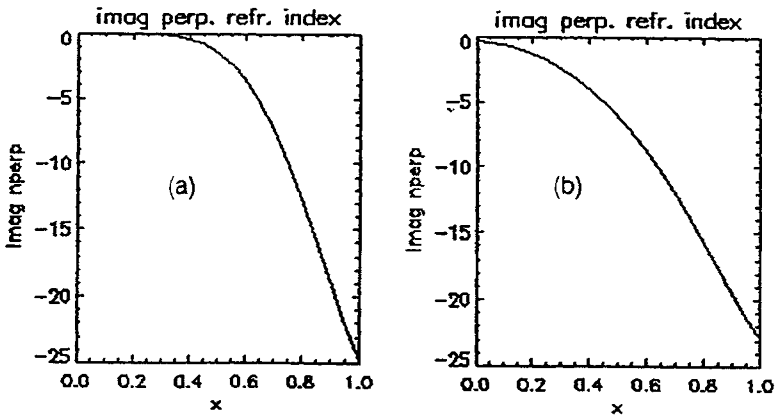

Figure 2 Imaginary $n_\perp$ in the vicinity of $\omega = 4\Omega$ as a function of x (normalized) for $T_e = 1.5$keV, density of 1.5 the O-mode cut-off density, (a) $n_\| = 0.3$ and (b) $n_\| = 0.5$. For MAST parameters the distance from $x = 0$ to $x = 1$ corresponds to 0.071m.