AIP | Conference Proceedings

A simple theory of relativistic damping of electromagnetic cyclotron and electron Bernstein waves in tokamaks

R. A. Cairns and C. N. Lashmore-Davies

Citation: AIP Conf. Proc. **595**, 430 (2001); doi: 10.1063/1.1424225
View online: http://dx.doi.org/10.1063/1.1424225
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=595&Issue=1
Published by the American Institute of Physics.

Related Articles

The dependence of extracted current on discharge gas pressure in neutral beam ion sources on HL-2A tokamak
Rev. Sci. Instrum. 83, 023302 (2012)

Diagnostics of the ITER neutral beam test facility
Rev. Sci. Instrum. 83, 02B103 (2012)

Analysis on the exclusiveness of turbulence suppression between static and time-varying shear flow
Phys. Plasmas 19, 020701 (2012)

Experimental investigation of geodesic acoustic mode spatial structure, intermittency, and interaction with turbulence in the DIII-D tokamak
Phys. Plasmas 19, 022301 (2012)

Energetics of runaway electrons during tokamak disruptions
Phys. Plasmas 19, 012507 (2012)

Additional information on AIP Conf. Proc.

Journal Homepage: http://proceedings.aip.org/
Journal Information: http://proceedings.aip.org/about/about_the_proceedings
Top downloads: http://proceedings.aip.org/dbt/most_downloaded.jsp?KEY=APCPCS
Information for Authors: http://proceedings.aip.org/authors/information_for_authors

ADVERTISEMENT

AIPAdvances

Submit Now

Explore AIP's new open-access journal

- Article-level metrics now available
- Join the conversation! Rate & comment on articles

Downloaded 07 Feb 2012 to 194.81.223.66. Redistribution subject to AIP license or copyright; see http://proceedings.aip.org/about/rights_permissions

A Simple Theory of Relativistic Damping of Electromagnetic Cyclotron and Electron Bernstein Waves in Tokamaks

R A Cairns[a] and C N Lashmore-Davies[b]

[a]*School of Mathematics and Statistics, University of St Andrews, KY16 9SS, UK*
[b]*EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX14 3DB, UK*

Abstract. An approximation to the relativistic broadening of an electron cyclotron resonance is obtained by including only the effect of the momentum parallel to the magnetic field. This approximation is then applied to the calculation of the absorption of the electromagnetic X-mode and compared to the full relativistic analysis with which it agrees very well. The propagation and damping of electron Bernstein waves is calculated using the approximate electrostatic dispersion relation and compared with the result obtained from the full electromagnetic dispersion relation.

INTRODUCTION

Electron cyclotron waves are important for heating and current drive in tokamaks. Spherical tokamaks offer a special challenge to wave heating methods and electron Bernstein waves appear to be a promising candidate [1]. The damping of electron Bernstein waves is very strong and for heating and current drive applications in tokamaks the location of the absorption region is very important. The width of the electron cyclotron resonance is due to the Doppler effect and to the relativistic dependence of mass on energy. The relativistic resonance condition is

$$\omega - k_{\|}v_{\|} - n\Omega_e\left(1-\frac{v_{\|}^2}{c^2}-\frac{v_{\perp}^2}{c^2}\right)^{1/2} = 0 \qquad (1)$$

where Ω_e is the electron cyclotron frequency calculated with the rest mass and $\Omega_e > 0$, ω, $k_{\|}$, $v_{\|}$, n, $v_{\perp}$ and c are the wave frequency, the wave number parallel to the magnetic field, the velocity parallel to the magnetic field, the cyclotron harmonic number, the velocity perpendicular to the magnetic field and the velocity of light respectively. For weakly relativistic conditions, $v_{\|}^2 << c^2$, $v_{\perp}^2 << c^2$, the resonant values of $v_{\|}/c$ given by Eq (1) are

CP595, *Radio Frequency Power in Plasmas: 14th Topical Conf.*, edited by T. K. Mau and J. deGrassie
2001 American Institute of Physics 0-7354-0038-5

$$\frac{v_{\|}}{c} = \frac{n_{\|}\omega}{n\Omega_e} \pm \frac{n_{\|}\omega}{n\Omega_e}\left[1 - \frac{2(\omega/n\Omega_e - 1)n^2\Omega_e^2}{n_{\|}^2\omega^2} - \frac{n^2\Omega_e^2 v_{\perp}^2}{n_{\|}^2\omega^2 c^2}\right]^{1/2} \tag{2}$$

The complications of the relativistic theory arise because of the dependence of $v_{\|}$ on $v_{\perp}$. The theory is greatly simplified if $v_{\perp}/c$ is neglected in Eq (2) giving

$$\frac{v_{\|}}{c} \cong \frac{n_{\|}\omega}{n\Omega_e}\left[1 \pm \left(1 + \frac{2Xn^2\Omega_e^2}{n_{\|}^2\omega^2}\right)^{1/2}\right] \tag{3}$$

where $X = 1 - \omega/n\Omega_e$. The approximation given by Eq (3) is compared with the exact result of Eq (2) in Figures 1(a) and 1(b) where the resonant value of $v_{\|}/c$ is plotted against the parameter X which represents the spatial co-ordinate across the resonance. The full line is the approximate value from Eq (3) and the dotted and dashed dotted lines correspond to Eq (2) where $v_{\perp}^2$ corresponds to 5keV and 10keV respectively. The non-relativistic result, the straight broken line, is shown for comparison. The approximation given in Eq (3) agrees well and improves further from the cold resonance.

RELATIVISTIC DAMPING

The above approximation to the resonant values of $v_{\|}$ can now be used to obtain a much simpler dispersion relation. This is illustrated by obtaining an approximation to the weakly relativistic expression for the xx component of the dielectric tensor, ε_{xx}. For weakly relativistic conditions the main effect of the relativistic correction occurs in the resonance condition. By neglecting the correction due to $v_{\perp}/c$ the $v_{\perp}$-integration in the calculation of ε_{xx} is the same as in the non-relativistic case. Since there are now two values of the resonant $v_{\|}$, ε_{xx} is approximated by

$$\varepsilon_{xx} = 1 + \sum_{n=-\infty}^{\infty}\frac{\omega_{pe}^2}{\omega^2}\frac{n^2 e^{-\lambda^2}}{\lambda} I_n(\lambda)\varsigma_0\left[\mathrm{Re}\, Z(\varsigma_n) + i\pi^{1/2}\left(e^{-v_1^2/v_{T_e}^2} + e^{-v_2^2/v_{T_e}^2}\right)\right] \tag{4}$$

where $v_{1,2}$ are the two roots of Eq (3), $\lambda = k_{\perp}^2 v_T^2 / 2\Omega_e^2$, $\varsigma_0 = \omega / k_{\|} v_{T_e}$, $\varsigma_0 = (\omega - n\Omega_e)/k_{\|} v_{T_e}$ and Z(x) is the plasma dispersion function. The other elements of $\underline{\underline{\varepsilon}}$ can be approximated in a similar manner. This approximation to the weakly relativistic theory is now used to calculate Im $n_{\perp}$ for the X-mode at the second harmonic resonance and the results are compared with the corresponding calculation of Airoldi-Crescentini et al [2] who used the standard relativistic theory. The comparison is illustrated in Figures 2(a) and 2(b) in which the variation of Im $n_{\perp}$ across the resonance region is shown for different values of $n_{\|}$. It can be seen that the simple

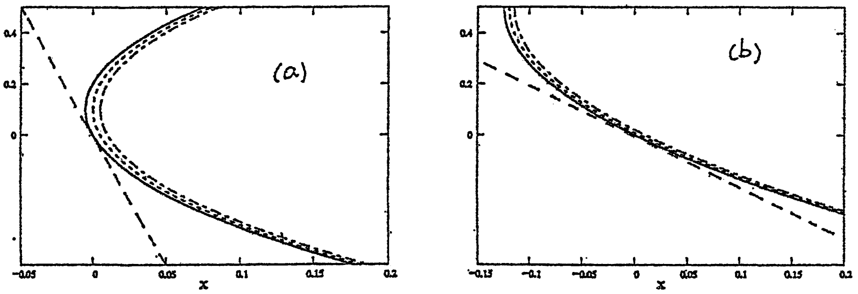

FIGURE 1 The resonant values of $v_{||}/c$ plotted as a function of $X = 1 - \omega/n\Omega_e$. The solid line corresponds to Eq (3) ($v_{\perp}^2 = 0$), the dotted and dashed dotted lines to Eq (2) for values of $v_{\perp}^2$ = 5keV and 10keV respectively. The straight dashed line is the non-relativistic value (a) $n_{||} = 0.1$, (b) $n_{||} = 0.5$.

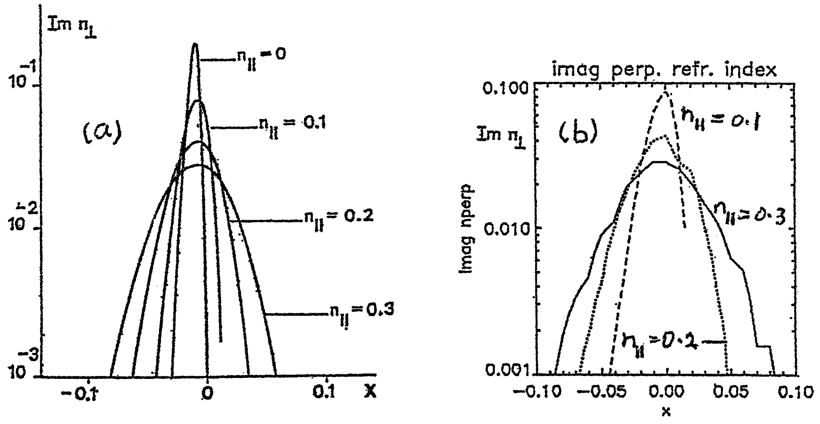

FIGURE 2 Im $n_{\perp}$ plotted as a function of distance across the resonance region for the X-mode at $\omega = 2\Omega_e$ with $\omega_{pe}^2/\omega^2 = 0.2$, $T_e = 0.42$keV and for various values of $n_{||}$, (a) relativistic results of Airoldi-Crescentini et al (1980), (b) relativistic approximation.

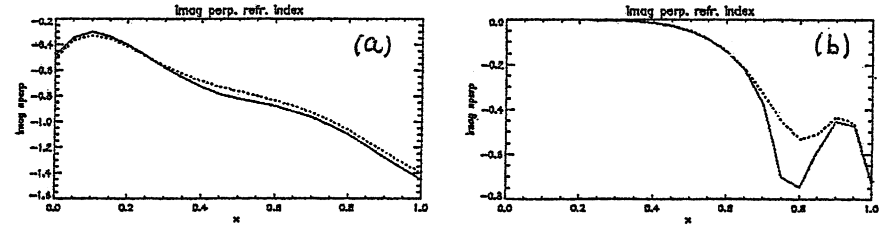

FIGURE 3 Im $n_{\perp}$ as a function of Ω_e/ω which varies linearly with x from 0.3 to 0.25 for electron Bernstein waves calculated from electromagnetic dispersion relation (solid line) and from $\varepsilon_{xx} - n_{||}^2 = 0$ (dotted line) both using the relativistic approximation (a) $n_{||} = 0.5$, (b) $n_{||} = 0.1$ and $\omega_{pe}^2/\omega^2 = 5$ and $T_e = 5$keV in both cases.

aproximation to the relativistic damping reproduces the results of the more exact theory very well, both qualitatively and quantitatively.

The approximate relativistic theory is now used to calculate the damping of electron Bernstein waves. For this case we compare the values of Re $n_\perp$ and Im $n_\perp$ obtained from the full electromagnetic theory with those obtained from the approximate dispersion relation

$$\varepsilon_{xx} - n_{\|} = 0 \qquad (5)$$

In both cases, the relativistic damping is represented by the approximation in which the $v_\perp^2/c^2$ correction is neglected. The real parts of $n_\perp$ agree very closely and the imaginary parts are shown in Figure 3(a) for $n_{\|} = 0.5$ and in Figure 3(b) for $n_{\|} = 0.1$. The solid line corresponds to the full electromagnetic dispersion relation and the broken line is obtained from the solution of Eq (5). The values for Im $n_\perp$ agree well for $n_{\|} = 0.5$ but not so well for $n_{\|} = 0.1$.

CONCLUSIONS

A simple approximate method of calculating relativistic electron cyclotron damping has been proposed. It is demonstrated that the approximation agrees well with the standard, weakly relativistic analysis for the second harmonic X-mode even for $n_{\|} = 0.1$. This relativistic approximation is then used to calculate the real and imaginary parts of $n_\perp$ for electron Bernstein waves using both the full electromagnetic and approximate electrostatic $(\varepsilon_{xx} - n_{\|}^2 = 0)$ dispersion relations. The simplified electron Bernstein dispersion relation agrees closely with the electromagnetic dispersion relation. The development of a simple approximation to the relativistic damping and propagation of electron Bernstein waves should be useful for current drive and ray tracing calculations. It could also be useful for similar calculations for the O- and X-modes for conventional ECRH.

ACKNOWLEDGEMENTS

This work was jointly funded by EURATOM and the UK Department of Trade and Industry.

REFERENCES

1. Ram A K and Schultz S D, *Phys Plasmas* 7, 4084 (2000).
2. Airoldi-Crescentini A, Lazzaro E, Lontano M, Maroli C, Oretice A, Petrillo V, Pozzoli R and Ramponi G, " " in *Prcoeedings 2nd Joint Grenoble-Varenna International Symposium on Heating in Toroidal Plasmas*, Como, Italy 1980 Vol I p127 Commission of the European Communities EUR7424 EN.