Tearing stability in toroidal plasmas with shaped cross section

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Tearing stability in toroidal plasmas with shaped cross section

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Abstract
Two methods for calculating tearing mode stability are described in this paper. A fast method using the recently improved T7 code (Ham *et al* 2012 *Plasma Phys. Control. Fusion* **54** 025009) and a new method based on the MARS-F MHD stability code (Liu *et al* 2000 *Phys. Plasmas* **7** 3681) which constructs the tearing mode solution from calculated basis functions in the full geometry of the problem. The effects of plasma toroidicity and cross-sectional shaping on tearing mode stability are investigated using both of the methods; the resultant stabilizing effects are in reasonable agreement over the range of parameters investigated. The parameter-space explored includes JET-like and ITER-like plasma shaping. While T7 can be used for rapid calculations and parameter scans, the MARS-F construction technique produces the more accurate value of the tearing mode stability index.

(Some figures may appear in colour only in the online journal)

1. Introduction

Tearing instabilities change the magnetic topology of a tokamak plasma, introducing magnetic islands where there were nested flux surfaces. In most cases when a tearing mode is formed the confinement is significantly degraded because transport across such islands is significantly enhanced [1]. Furthermore, tearing mode growth can lead to plasma disruptions. Thus, optimizing tokamak operation requires the identification of tearing mode stable regimes.

In the presence of plasma flow, tearing mode stability in a torus can be characterized by a single quantity, $\Delta'_{m,n}$, for each rational surface, where $m = nq(r)$. In this paper, we investigate two different methods for calculating Δ'. (A brief review of currently available codes for calculating Δ' was recently given in [2].) We then use these two methods to develop an understanding of the sensitivity of tearing mode stability to aspect ratio, elliptic and triangular plasma cross-sectional shaping and safety factor profile and to appreciate the respective merits and limitations of the two approaches. The role of plasma pressure is the subject of continuing research.

The tearing mode formalism, where the plasma is separated into interior regions, near to the rational surfaces, and exterior regions, away from the rational surfaces, can be used to build sophisticated models of the plasma at low computational cost. The exterior region is modelled using ideal magnetohydrodynamics (MHD) which is not computationally heavy. The narrow interior region can include extra physical effects such as resistivity, viscosity, inertial and kinetic effects. The interior and exterior solutions are matched using Δ' to determine stability. The numerical effort is thus focused on thin layers rather than the whole plasma.

The first method for calculating Δ' uses the recently improved T7 code [2, 3]. This code is very fast and so can be run a large number of times to explore the effect of changing quantities such as plasma shape. The T7 plasma equilibrium is based on a large aspect ratio and small shaping expansion so its application to finite values of these quantities may be limited. However, trends in increasing or decreasing stability should be robust.

We use the MARS-F code [4], which solves the resistive MHD equations in full toroidal geometry with no ordering assumptions, for the other method of calculating the pressure-free Δ'. MARS-F has been extensively used in the resistive wall mode stability study [5] and recently in plasma response calculations [6]. There are two solutions for the tearing mode eigenfunction when expanded around the rational surface in the pressure-free case. One behaves linearly with distance from the rational surface and is known as the 'small' solution, the other behaves as a constant and is called the 'large' solution. The first MARS-F method directly calculates the small solution with an innovative choice of boundary conditions. A 'response' solution is also calculated which is combined with the small solution to produce a solution satisfying the correct boundary conditions and hence the Δ'. This method uses no large aspect ratio assumption and so should provide a more accurate calculation of Δ'.

Although not a problem for T7, including pressure has a profound effect in resistive MHD codes like MARS-F because of the screening at the rational surface due to a negative pressure gradient with favourable average curvature, the 'Glasser effect' [7]. This screening effect means that the resonant harmonic tends to zero at the rational surface for all boundary perturbations so that the basis functions described above cannot be calculated. Overcoming this problem is the subject of ongoing research.

In a previous paper we investigated the effects of strong toroidal coupling on the tearing mode stability, both analytically and numerically using the T7 code [2]. In particular, we examined the stability of both the conventional and hybrid scenarios. In this paper we seek to explore the effects of finite shaping on the tearing mode stability of these plasmas at zero pressure. The effects of infinitessimal shaping yield a result of the form

$$\Delta' = \Delta'_{\mathrm{cyl}} + c_1\epsilon^2 + c_2E^2 + c_3T^2 + \cdots, \tag{1}$$

where ϵ, E and T represent the inverse aspect ratio, a/R, elliptic and triangular shaping defined in the appendix. However, this infinitessimal approach neglects higher order terms in these parameters. Such results were presented in [3] using the T7 code for particular q profiles using infinitessimal values of these parameters so that the expansion is fully justified. However, in this paper we are able to scan finite values of them and produce plots that allow the sensitivity of the tearing mode to these variables, including higher order terms, to be understood better.

Here, we only study cases where $q > 1$ throughout the plasma. When there is a $q = 1$ surface in the plasma, the particular nature of the $m = n = 1$ internal mode can produce an order one effect on Δ' [2, 8, 9], in some cases even producing a pole in Δ'.

The plasma equilibrium will be discussed in section 2. An explanation of the methods used by the MARS-F code to calculate pressure-free Δ' will be given in section 3 along with a comparison with T7 at large aspect ratio and finite shaping cases. The response of $\Delta'_{2,1}$ to safety factor and aspect ratio will be explored using T7 in the following section. Finally, we discuss our results in the conclusions.

2. Plasma equilibrium

The safety factor profile is assumed to have the form

$$q(r) = q_0(1 + \lambda r^{2\nu})^{1/\nu}, \tag{2}$$

where r is the radial coordinate and q_0 is the minimum value of the safety factor, λ and ν are parameters which change the shape and edge value of the safety factor, q_a. We use three values of ν which correspond to the three values used by Furth *et al* [10] in their study of tearing stability in a cylinder. They describe a peaked current profile, $\nu = 1$, a rounded current profile, $\nu = 2$, and a flattened profile, $\nu = 4$. The minimum safety factor is generally assumed to be $q_0 = 1.2$ and the edge safety factor is $q_a = 3.5$, although we consider the effect of varying these for specific ϵ, κ and δ. The relationship of κ and δ to the expansion parameters used in the T7 equilibrium, E and T is given in the appendix. We will only consider pressure-free equilibria in this paper.

The T7 code currently uses a large aspect ratio approximation for its equilibrium. Although in most cases the code can run with a higher inverse aspect ratio, the expansion underlying T7 is further from its region of validity. In principle, the region of validity of T7 could be extended by introducing the expansion in poloidal harmonics of a numerically generated equilibrium, rather than the existing analytic expressions. The equilibria for MARS-F is calculated in a fully consistent manner with the safety factor profile as an input.

The plasma elongation is varied from $\kappa = 1$ to $\kappa = 1.9$ which includes JET-like values ($\kappa \sim 1.8$) and ITER-like values ($\kappa \sim 1.9$). Plasma triangularity is varied from $\delta = 0$ to $\delta = 0.5$ which again include the JET-like value ($\delta \sim 0.25$) and the ITER-like value ($\delta \sim 0.5$) [11]. The JET-like shaping values is used as the default. Finally, we assume that there is a perfect wall at the plasma edge.

3. Pressure-free Δ' using MARS-F

There are several different approaches to calculating Δ' for a given equilibrium; for example, a resistive MHD code can be used to calculate the tearing mode growth rate from which Δ' can be deduced using an appropriate dispersion relation [12]. Another method of calculating Δ' involves producing basis functions which are then themselves combined to satisfy the appropriate boundary conditions and evaluated at certain points to give Δ' [13]. For example, two basis functions are required for an equilibrium with one rational surface. The first basis function, the 'small' solution $\boldsymbol{\psi}^{\mathrm{s}}$ (where $\boldsymbol{\psi}$ denotes a vector of M poloidal harmonics and M is the total number of harmonics considered), is defined to have zero amplitude at the rational surface, $\boldsymbol{\psi}^{\mathrm{s}}(r_{\mathrm{s}}) = \mathbf{0}$, and for the resonant harmonic to have unit gradient, $\psi_m'^{\mathrm{s}}(r_{\mathrm{s}}) = 1$, where m is the resonant harmonic. The first derivatives of all the non-resonant harmonics are required to be zero at the rational surface, $\psi_{j\neq m}'^{\mathrm{s}}(r_{\mathrm{s}}) = 0$, where $j = 1\ldots M$. These boundary conditions then allow the ideal MHD equations to be solved numerically from the rational surface to the wall. Each harmonic of the small solution will then have a resulting amplitude at the wall, $\psi_j^{\mathrm{s}}(a)$. The second basis function, the 'response' solution $\boldsymbol{\psi}^{\mathrm{R}}$, is calculated by imposing a zero amplitude boundary condition at the magnetic axis on all the harmonics, $\boldsymbol{\psi}^{\mathrm{R}}(0) = \mathbf{0}$, and the values of the small solution at the wall, $\boldsymbol{\psi}^{\mathrm{R}}(a) = \boldsymbol{\psi}^{\mathrm{s}}(a)$, so that subtracting it from the small solution yields a solution satisfying wall boundary conditions. The harmonics are allowed to reconnect across their resonant surfaces. The small and response solutions are then combined to produce a solution, $\boldsymbol{\psi}(r) = \boldsymbol{\psi}^{\mathrm{R}}(r) - \boldsymbol{\psi}^{\mathrm{s}}(r)$, that respects all the required boundary conditions. The Δ' for a pressure-free equilibrium is then defined as

$$\Delta' \equiv \left[\frac{r\psi_m'}{\psi_m}\right]_{r_s} \tag{3}$$

where $[f(x)]_y \equiv f(y+\delta) - f(y-\delta)$ for δ small, and ψ_m is the perturbed poloidal flux of the resonant harmonic and this can be straightforwardly calculated from the final solution vector $\boldsymbol{\psi}$. This approach can be readily generalized to calculate Δ' in the presence of multiple rational surfaces.

The MARS-F code [4] solves the resistive MHD equations in full toroidal geometry using finite elements. The elements can be packed around the rational surfaces to ensure that the behaviour in the resistive layer is resolved for accurate calculation.

3.1. MARS-F construction method

This method constructs the small and response solutions as described above which are combined to calculate Δ' with no large aspect ratio approximation. So far, this method has only been implemented for the pressure-free case because of the Glasser effect, as discussed above.

Given that MARS-F is a finite element code the small solution cannot be directly calculated. In fact, the small solution must be calculated using a number of steps. A perfect conductor is assumed to occupy the region from the magnetic axis to the particular rational surface. The response of each harmonic to an external forcing, described below for the case of one rational surface, is then calculated and the result is called a 'primary' basis function, $\phi^j(r)$. This is carried out by setting the boundary condition of all harmonics to be zero at the rational surface, $\phi^j(r_s) = \mathbf{0}$, and all harmonics, except the jth, to be zero at the plasma edge, $\phi^j_{i\neq j}(a) = 0$, where $i = 1...M$. One harmonic in turn is given unit amplitude at the plasma edge, $\phi^j_j(a) = 1$ and the response calculated in each case. These primary basis functions, each with a different harmonic forced at the boundary, can then be combined to produce a single function which is the small solution

$$\sum_{j=1}^{M} \alpha_j \phi^j(r) = \psi^s(r), \tag{4}$$

where α_j are determined by satisfying the boundary conditions for the small solution, which are $\psi^s(r_s) = \mathbf{0}$, $\psi_m'^s(r_s) = 1$, where m is the resonant harmonic and $\psi'^s_{j\neq m}(r_s) = 0$, where $j = 1 \ldots M$.

Figure 1 plots the $m = 4$ primary basis function, i.e. the situation with a rigid perfect conductor from the magnetic axis to the rational surface and all but the $m = 4$ harmonic being zero at the outer boundary. Figure 2 shows a magnified version of figure 1 around the rational surface. The equilibrium for this case is pressure-free, has a circular cross section, inverse aspect ratio $\epsilon = 0.1$, minimum safety factor $q_0 = 3.6$ and toroidal mode number $n = 1$. Seven poloidal harmonics have been used in this calculation.

Figure 3 shows that the derivatives of the harmonics, as they approach the rational surface, are robustly constant. This

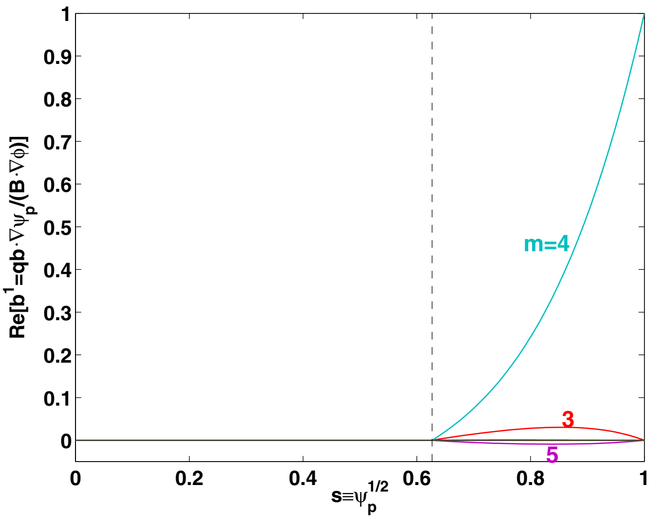

Figure 1. The harmonics of $b^1 = qb.\nabla\psi_p/(B\cdot\nabla\phi) \propto \tilde{\psi}_p$ across the minor radius, where ϕ is toroidal angle and ψ_p is poloidal flux. All boundary conditions at the rational surface and at the wall are set to zero except for the $m = 4$ harmonic at the wall which is set to unity.

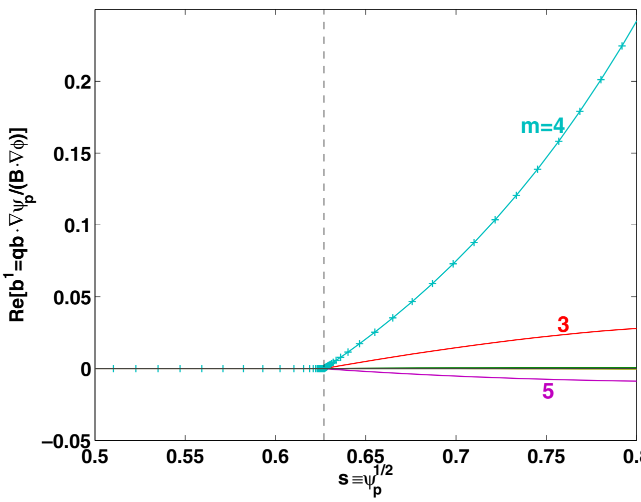

Figure 2. As figure 1 but zoomed in around the rational surface.

allows the set of solutions to be combined to produce a small solution. The amplitude of all the harmonics of the small solution can be calculated at the plasma edge.

Figure 4 shows the 'response' which has been calculated with the same wall boundary conditions as the amplitudes of the harmonics of the small solution at the wall. The small and response solutions can then be subtracted to produce the full solution which respects all the boundary conditions. The exterior Δ' can now be calculated directly. This calculation has not resorted to a large aspect ratio approximation at any point.

If the equilibrium has several rational surfaces, for a given value of the toroidal mode number n, then there is no longer a single number which characterizes tearing stability, but there are relations between the tearing stability indices of each of the rational surfaces; however in the presence of flow they are distinct, a separate quantity $\Delta'_{m,n}$ holding at each rational

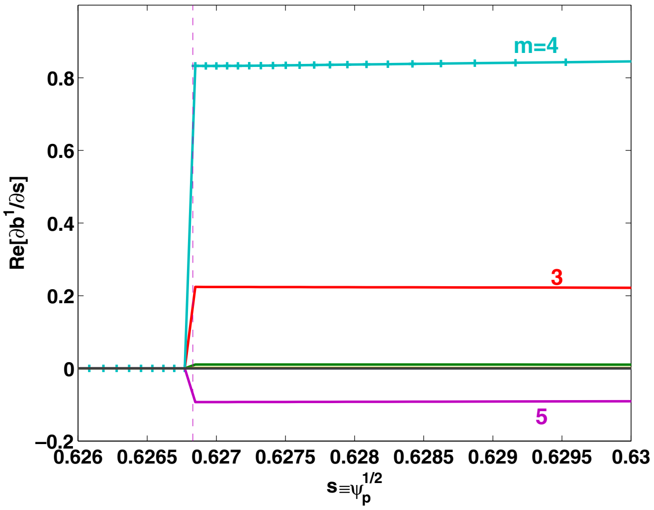

Figure 3. Derivatives of the poloidal harmonics as they approach the rational surface.

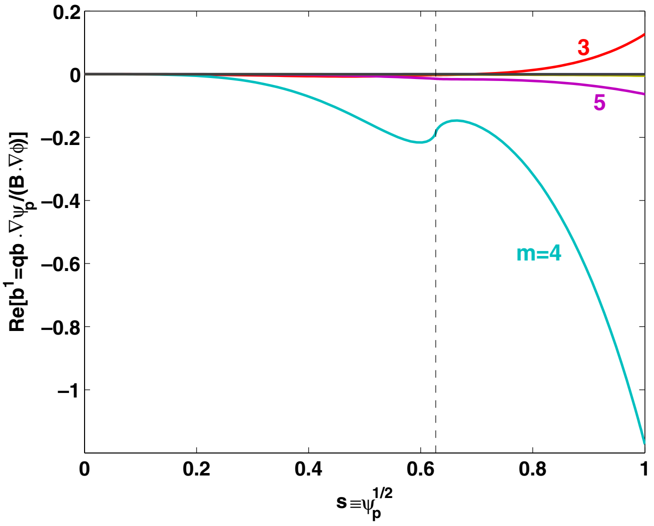

Figure 4. The 'response' solution for the problem. This combined with the 'small' solution will satisfy the boundary conditions at the magnetic axis and at the wall.

surface. A full solution basis function is calculated for each rational surface. The details of this are discussed in [3].

3.2. Comparison between MARS-F method and T7

The results from the MARS-F method have been compared to T7 results for a circular cross section plasma with inverse aspect ratio $\epsilon = 0.2$. The equilibrium safety factor has $q_0 = 1.2$, $q_a = 3.5$ and $\nu = 2$ and so it has two rational surfaces for $n = 1$. Figure 5 shows the safety factor profiles for $\nu = 1$, $\nu = 2$ and $\nu = 4$ with $q_0 = 1.2$ and $q_a = 3.5$. We seek to calculate the tearing stability at the $m = 2, n = 1$ surface i.e. $\Delta'_{2,1}$. The tearing mode at the $q = 3$ surface is usually stable and so we will not generally consider it. The MARS-F results are plotted as '×' in figure 6 and the T7 results are plotted as a solid line. The T7 and MARS-F results follow the same trend and agree to around 5%. The lack of agreement

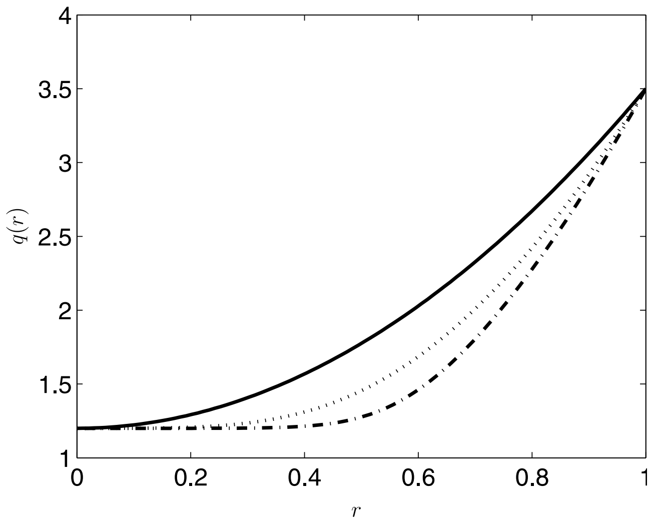

Figure 5. Safety factor $q(r)$ against minor radius, for $q_0 = 1.2$, $q_a = 3.5$ and $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line).

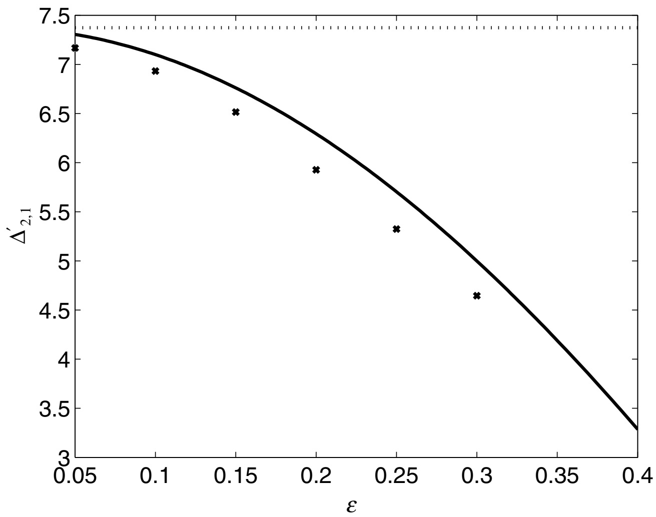

Figure 6. $\Delta'_{2,1}$ against increasing inverse aspect ratio ϵ calculated with T7 (solid line), and MARS-F Method-I ('×') for a plasma with a circular cross section ($\kappa = 1.0$, $\delta = 0.0$) and zero pressure with $q_0 = 1.2$, $q_a = 3.5$ and $\nu = 2$. The cylindrical value is shown by a dotted line.

is because T7 uses a large aspect ratio approximation for its equilibrium. The MARS-F results were also checked against the tearing mode growth rate method [12] and differed by less than 1% for calculations up to inverse aspect ratio, $\epsilon = 0.3$ and the case for $\epsilon = 0.15$ agrees to 0.1%. The slight difference is due to the aspect ratio expansion that is used to calculate the dispersion relation. The tearing mode growth rate results were checked for convergence in Lundquist number, mesh size and the number of harmonics used.

The MARS-F and T7 code results will differ more as inverse aspect ratio and shaping effects are increased, however the codes should exhibit the same trends with aspect ratio and shaping. Figure 7 shows the effect on tearing stability of increasing the plasma elongation, κ, calculated using T7. The elongation has been varied from $\kappa = 1$ to $\kappa = 1.9$ which

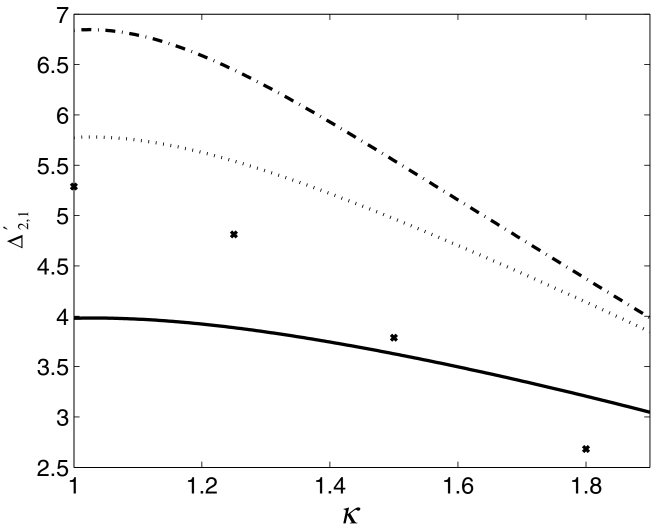

Figure 7. $\Delta'_{2,1}$ against increasing elongation κ calculated using T7 for a plasma with $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line) for a plasma with JET-like shaping ($\delta \approx 0.25$) and zero pressure with $q_0 = 1.2$ and $q_a = 3.5$. The '×' have been calculated with MARS-F for $\nu = 2$.

includes the ITER-like value ($\kappa \approx 1.9$). The three plots all show that increasing κ acts to stabilize the tearing mode when the q-profile is invariant. Four equilibria with $\nu = 2$ and increasing elongation have been calculated using the MARS-F code Method-I ('×' in figure 7). These results contain full aspect ratio effects and so do not agree closely with the T7 results; however, they do follow the same trend.

Figure 8 shows the effect on tearing stability of increasing the plasma triangularity, δ, calculated using T7. The triangularity has been varied from $\delta = 0$ to $\delta = 0.5$ which includes the ITER-like value ($\delta \approx 0.5$). The three plots all show that increasing δ without pressure acts to stabilize the tearing mode for the chosen q-profile. There is a very small destabilizing effect for small values of δ and $\nu = 2, 4$, however this is at an ignorable level within the approximations of T7. Calculation for four equilibria with $\nu = 2$ and increasing triangularity have been made using the MARS-F code ('×' in figure 8). Again, these results contain the full aspect ratio effects and do not agree closely with T7; however, they do confirm the magnitude and trend of the T7 results.

4. Scans in other equilibrium quantities

Now that the T7 and MARS-F results have been compared in circular and shaped cross sections we will investigate the effects of other equilibrium properties on the tearing mode stabiility using T7 because of its suitability for large parameter scans. The same basic equilibrium given in section 3.2 will be used in all the following cases, with just the quantity under investigation varied.

Figure 9 shows the effect on $\Delta'_{2,1}$ of changing the inverse aspect ratio from 0.01 to 0.4. It can clearly be seen that increasing inverse aspect ratio is stabilizing for tearing modes. This is in agreement with Fitzpatrick *et al* [3].

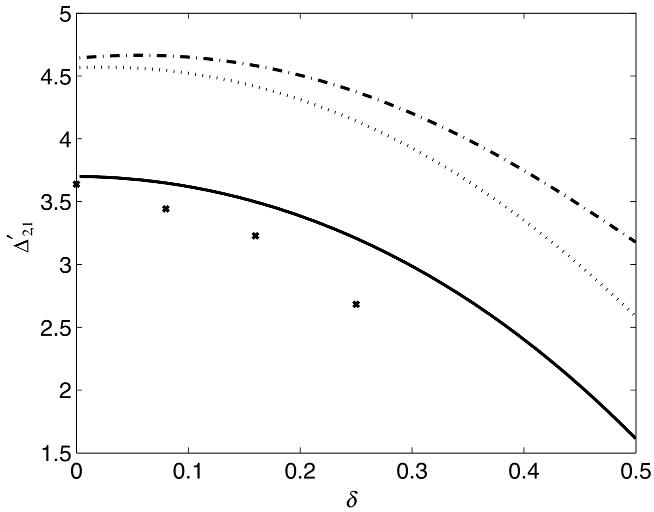

Figure 8. $\Delta'_{2,1}$ against increasing triangularity δ calculated using T7 for a plasma with $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line) for a plasma with JET-like shaping ($\kappa \approx 1.8$) and zero pressure with $q_0 = 1.2$ and $q_a = 3.5$. The '×' have been calculated using MARS-F for $\nu = 2$.

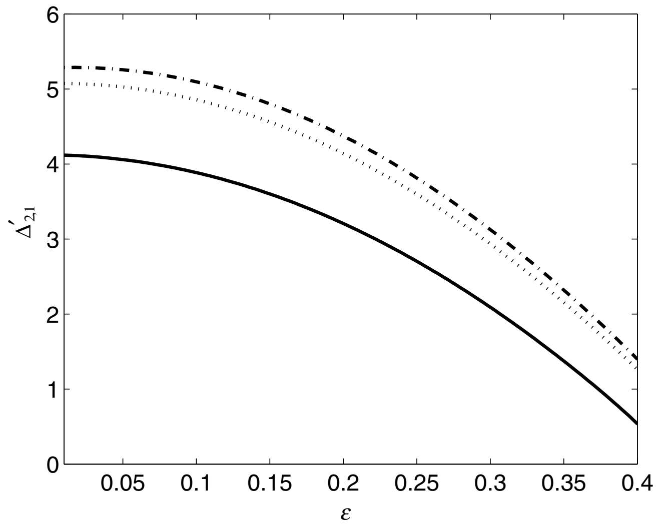

Figure 9. $\Delta'_{2,1}$, calculated using T7, against increasing inverse aspect ratio ϵ with $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line) for a plasma with JET-like shaping ($\kappa \approx 1.8$, $\delta \approx 0.25$) and zero pressure with $q_0 = 1.2$ and $q_a = 3.5$.

Figure 10 shows the effect on tearing stability of increasing the minimum safety factor calculated using T7. The minimum safety factor has been varied between $q_0 = 1.15$ and 1.5. The three plots all show that increasing q_0 acts to destabilize the tearing mode. This might be expected since as q_0 is increased, r_s is smaller which means the aspect ratio at the rational surface is larger and the cross-sectional shaping at the rational surface is smaller. These changes both have a destabilizing effect on the tearing mode.

Figure 11 shows the effect on tearing stability of increasing the edge safety factor calculated using T7. The edge safety factor has been varied between $q_a = 3.2$ and 4.0. The three plots all show that increasing q_a acts to destabilize the tearing mode. Again, this might be expected because r_s has decreased and the consequences for aspect ratio and

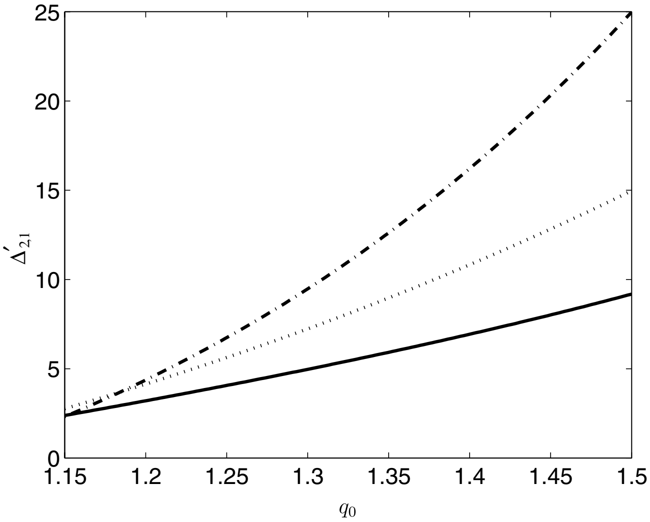

Figure 10. $\Delta'_{2,1}$, calculated using T7, against increasing q_0 for a plasma with $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line) for a plasma with JET-like shaping ($\kappa \approx 1.8$, $\delta \approx 0.25$) and zero pressure with $q_0 = 1.2$ and $q_a = 3.5$.

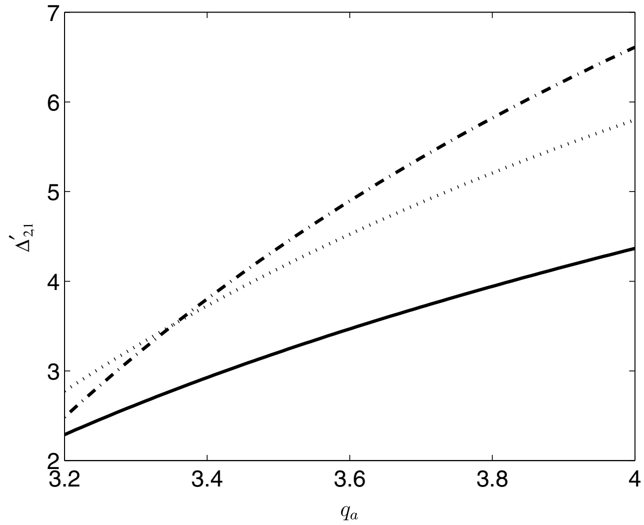

Figure 11. $\Delta'_{2,1}$, calculated using T7, against increasing q_a for a plasma with $\nu = 1$ (solid line), $\nu = 2$ (dotted line) and $\nu = 4$ (dashed–dotted line) for a plasma with JET-like shaping ($\kappa \approx 1.8$, $\delta \approx 0.25$) and zero pressure with $q_0 = 1.2$ and $q_a = 3.5$.

cross-sectional shaping are destabilizing. It is interesting to note that the magnetic shear is constant at the rational surface in this scan as q_a increases, but decreases as q_0 increases in figure 10.

The consequences of changing the shape of the safety factor profile on tearing mode stability are unclear because it is difficult to disentangle the competing effects. At a given value of q_0 each of the curves has a different location of the rational surface and magnetic shear and cross-sectional shaping at the rational surface. Figures 10 and 11 both have points where the $\nu = 2$ and $\nu = 4$ curves cross. This may be a physically accurate picture and due to the interaction of stabilizing and destabilizing effects as q_0 or q_a are varied. The effects of the approximations made within T7 may also vary with these parameters so some caution should be exercised when interpreting the fine details of these results.

5. Discussion and conclusions

Two methods for calculating the tearing mode stability index, Δ', have been described here. The first method uses the T7 code which allows fast calculations of Δ', and the more time consuming second method uses basis functions constructed by MARS-F to calculate Δ' without any aspect ratio approximation.

The effect of shaping on tearing mode stability for three current profiles; peaked, rounded and flattened, as used by Furth *et al* [10], has been investigated using these two methods for calculating Δ'. The values of elongation and triangularity considered include typical values for JET ($\kappa \approx 1.8$, $\delta \approx 0.25$) and ITER ($\kappa \approx 1.9$, $\delta \approx 0.5$). A number of effects are clear; increasing inverse aspect ratio, plasma elongation or triangularity are stabilizing for the tearing mode when the q-profile is held invariant, whereas increasing q_0 or q_a are both destabilizing. These results are similar to the fixed boundary results in Fitzpatrick *et al* [3] where an expansion for small values of plasma shaping was calculated using T7. Hender *et al* [14] also found that plasma shaping had a stabilizing influence on tearing modes in the zero pressure case when the q-profile was held constant. It is difficult to compare these results with experiment. A change to the boundary shape of the plasma produces a number of effects on the plasma which are difficult to disentangle. However, there is some experimental evidence from ASDEX-Upgrade [15] and TCV [16] that plasma shaping does act to stabilize tearing modes.

T7 is based upon a large aspect ratio, small shaping equilibrium expansion and so the absolute values of results at finite values of these parameters may be in doubt. The MARS-F method does not use any large aspect ratio or small shaping approximations. Nonetheless, the comparison of T7 with MARS-F results shows that the T7 trends are correct and the magnitude is close to the MARS-F values. However, experience with the T7 code has shown spurious results for Δ' can occur if the equilibrium parameters are pushed too far. These appear as large excursions in Δ' in narrow regions of parameter space and serve as warnings that the code is being applied beyond its regime of validity.

These shortcomings maybe be remedied if an expansion in poloidal harmonics of a full numerical solution, rather than the existing analytic expansion, of the Grad-Shafranov equation is used for the equilibrium in T7. The number of poloidal harmonics used for each rational surface could also be increased from seven. The work here indicates that T7 may be sufficiently accurate to provide fast calculation of the tearing mode stability and so T7 could be run as part of a bigger plasma simulation package. The MARS-F construction method provides a new, very accurate, if more time consuming, approach to calculating the tearing mode stability. The time required to run the T7 code for a case is approximately an order of magnitude faster than MARS-F.

The work here has all been carried out for pressure-free equilibria. A pressure gradient around the rational surface produces the Glasser effect which is likely to be significant and complicates the use of MARS-F in this way. We hope to investigate this theoretically challenging effect in future work.

Acknowledgments

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Appendix A. T7 code

We include the following short appendix on the T7 code, most of which has been previously given in [2], for the convenience of readers.

A.1. Analytic equilibrium

The T7 code currently uses a large aspect ratio analytic equilibrium rather than the output plasma equilibrium from a Grad–Shafranov solver. The full details of this plasma equilibrium are presented in [3], however we give brief details here. A plasma equilibrium is constructed with coordinates (r, θ, ϕ) which are related to cylindrical coordinates (R, ϕ, Z), where Z is in the direction of the symmetry axis, by

$$\begin{aligned} R &= R_0 - r\cos\omega - \Delta_S(r) + E(r)\cos\omega + T(r)\cos 2\omega \\ &\quad + P(r)\cos\omega, \\ Z &= r\sin\omega + E(r)\sin\omega + T(r)\sin 2\omega - P(r)\sin\omega, \end{aligned} \tag{A.1}$$

where ω is the poloidal angle around the magnetic axis, Δ_S is the Shafranov shift, E is the flux surface ellipticity and T is the flux surface triangularity. The quantity

$$P(r) = \frac{1}{8}\frac{r^3}{R_0^3} + \frac{1}{2}\frac{r}{R_0}\Delta_S - \frac{1}{2}\frac{E^2}{r} - \frac{T^2}{r} + O(\epsilon^3 a) \tag{A.2}$$

is chosen so that the Jacobian of the transformation $(r, \theta, \phi) \rightarrow (R, \phi, Z)$ is given by

$$(\nabla r \wedge \nabla\theta \cdot \nabla\phi)^{-1} = \frac{rR^2}{R_0}. \tag{A.3}$$

The equilibrium coordinate θ is related to ω by

$$\theta = 2\pi \int_0^\omega \frac{J\,\mathrm{d}\omega}{R} \Bigg/ \oint \frac{J\,\mathrm{d}\omega}{R}, \tag{A.4}$$

where $J = (\partial R/\partial\omega\partial Z/\partial r - \partial R/\partial r\partial Z/\partial\omega)$ is the Jacobian for $(R, Z \rightarrow r, \omega)$. The field lines appear straight in the (r, θ, ϕ) system, i.e. $B.\nabla \propto ((\partial/\partial\theta) - q(r)(\partial/\partial\phi))$.

The ellipticity parameter is related to the elongation parameter by $\kappa \approx (a+E(a))/(a-E((a))$ and the triangularity parameters are related by $\delta \approx 4T(a)(a^2 - 3T(a)^2)/a^3$.

A.2. Definition of Δ'

In the presence of pressure and toroidal effects the Δ' needs to be more carefully defined than the expression given in (3). Full details of this are given in [3]. The resonant perturbed-flux harmonics near to their rational surface are given by

$$\begin{aligned} \psi_m &\approx A_L|x|^{\nu_L}[1 + \lambda_L x + \cdots] + A_S \mathrm{sgn}(x)|x|^{\nu_S}[1 + \cdots] \\ &\quad + A_C x[1 + \cdots] \end{aligned} \tag{A.5}$$

where $x = (r - r_m)/a$. The coefficients of the 'large' and 'small' solutions in the Newcomb sense [17] are A_L and A_S, respectively. The coefficients of the 'continuous' solution are A_C. The indices, ν_L and ν_S, are given by

$$\begin{aligned} \nu_L &= \tfrac{1}{2} - (-D_M)^{1/2}, \\ \nu_S &= \tfrac{1}{2} + (-D_M)^{1/2}, \end{aligned} \tag{A.6}$$

where D_M is given in [13], $D_M > 0$ corresponding to the Mercier instability criterion.

T7 is restricted to stability calculations of the tearing parity modes where the coefficient of the large solution is continuous across the rational surface. However, the coefficient of the small solution is not continuous and this provides the definition of Δ' as

$$\Delta' \equiv \left(\frac{A_{S+} - A_{S-}}{A_L}\right), \tag{A.7}$$

where A_{S+} is the amplitude of the small solution as the rational surface is approached from the right and A_{S-} from the left. A_L is the amplitude of the large solution at the rational surface.

In the special case of a pressure-free equilibrium the definition of Δ' is

$$\Delta' \equiv \left[\frac{r\psi'}{\psi}\right]_{r_s}, \tag{A.8}$$

where ψ is the perturbed poloidal flux of the resonant harmonic.

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