LETTER TO THE EDITOR

Effect of plasma rotation on the beam-driven current

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Abstract. In a rotating plasma, with co-neutral beam injection (NBI), the Doppler shift of the NBI particles, as viewed in the frame of the plasma, can result in a significant reduction in the beam-driven (Ohkawa) current when the rotation is strong (*i.e.* with rotational Mach numbers, $M \geq 0.5$). The correction applies to the toroidal fast-ion current calculated for a non-rotating plasma and is independent of the normal $Z_{\rm eff}$ and electron trapping terms. A simple analytical model is presented to estimate the magnitude of the effect for plasmas with arbitrary toroidal rotation and the conditions where this is important have been identified. This model has been compared to the results from existing Monte Carlo neutral beam codes and found to reproduce their results. The important parameters in this problem are the ratio, $\rho^{\rm Lab} = \frac{v_{f0}^{\rm Lab}}{v_{\rm crit}}$, of the NBI injection particle velocity (in the laboratory frame) to the critical velocity of the plasma, and the ratio $\rho_\phi = \frac{v_\phi}{v_{\rm crit}}$ which is related to the rotational Mach number. A phase plot in dimensionless $(\rho^{\rm Lab}, \rho_\phi)$ space is presented which enables the fast ion current drive efficiencies to be compared for different tokamaks. For strongly rotating plasmas, the degradation in fast ion current efficiency is significant for $\rho^{\rm Lab} \leq 1$. However, when $\rho^{\rm Lab}$ is larger than this, the degradation in fast ion current drive is less severe. Approaches to improve the fast ion current drive efficiency are briefly discussed.

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1. Introduction

In simple derivations of the neutral beam (NB) driven, or Ohkawa, current, it is generally assumed that the NB particles are injected tangentially into a stationary background plasma and the current calculated in the laboratory frame of reference [1]. However, when the plasma is rotating (generally in the co-direction with respect to the NBI direction), there is a Doppler-shift of the beam particle energies in the frame of reference of the plasma which can affect the magnitude of the driven current.

The original motivation for making this calculation was to examine the efficiency of the NBI current drive in MAST spherical tokamak [2]. The relevant parameters for this device are given in Table 1.

Tokamak	**MAST**	**CTF**	**CTF(2)**
Normalised Radius, r/a	0	0	0.6
Plasma Ion Species	D	D + T	D + T
Plasma Density	2×10^{19} m^{-3}	2×10^{20} m^{-3}	1.6×10^{20} m^{-3}
Ion Temperature	1 keV	18.5 keV	6 keV
Electron Temperature	1 keV	15 keV	5 keV
Ion Sound Speed, c_S	3.1×10^5 m s^{-1}	9.8×10^5 m s^{-1}	5.6×10^5 m s^{-1}
NBI Energy (Lab frame)	60 keV	150 keV (nominal)	150 keV (nominal)
Neutral Beam Species	D	D + T	D + T
NBI Velocity (Lab frame, v_{f0}^{Lab})	2.4×10^6 m s^{-1}	3.39×10^6 m s^{-1}	3.39×10^6 m s^{-1}
Rotational Mach Number, M	≤ 0.7	1.0	0.54
Critical energy, E_{crit}	19 keV	301 keV	101 keV
Critical velocity, v_{crit}	1.34×10^6 m s^{-1}	4.8×10^6 m s^{-1}	2.8×10^6 m s^{-1}
Velocity ratio, $\rho^{\text{Lab}} = \frac{v_{f0}^{\text{Lab}}}{v_{\text{crit}}}$	1.79	0.71	1.21
ρ_ϕ $(= \frac{v_\phi}{v_{\text{crit}}})$, $M = 1$	0.23	0.2	0.2

Table 1. Typical parameters for the Mega Ampere Spherical Tokamak (MAST) and a conceptual spherical tokamak Component Test Facility (CTF) device [3, 4]. CTF(2) refers to the parameters at the normalised minor radius at which the NBI current drive is maximum, as determined by TRANSP simulations [5, 6].

In this paper, the rotational effect is considered using a simple analytical model. An expression for the Ohkawa [7] current with the Doppler-shift correction is derived for plasmas with arbitrary toroidal rotation and compared with results from a numerical Monte Carlo approach.

2. Rotating Plasma

Consider a monoenergetic beam of ions having charge $q_f = +Z_f e$ and mass $m_f = A_f m_p$. In the laboratory frame of reference, the beam has an energy E_{f0}^{lab}. The beam is injected tangentially into a toroidal plasma which is assumed to be co-rotating at velocity v_ϕ.

In the frame of reference of the plasma, the injected particles have apparent (Doppler-shifted) energies of

$$E_{f0} = \frac{1}{2} m_f (v_{f0}^{\text{Lab}} - v_\phi)^2 \tag{1}$$

where $v_{f0}^{\text{Lab}} = \left(\frac{2E_{f0}^{\text{Lab}}}{m_f}\right)^{1/2}$ is the velocity of the beam particles measured in the laboratory frame.

The Doppler shift of the beam ions can have a significant effect on the thermalisation time τ_{therm}, which is the time taken by a freshly injected beam ion to slow down to the thermal energy of the plasma. We calculate τ_{therm} for this situation following Wesson [8, p. 68] using standard Coulomb frictional drag forces and assuming for simplicity that the ion and electron temperatures and densities are equal ($T_i = T_e$, $n_i = n_e$). In this case the ion-electron slowing-down time is

$$\tau_{se} = \frac{3(2\pi)^{1/2} T_e^{3/2}}{m_e^{1/2} m_f A_D} \tag{2}$$

where

$$A_D = \frac{n e^4 \ln \Lambda}{2\pi \epsilon_0^2 m_f^2} \tag{3}$$

and the symbols have their usual meanings.

The solution to the Coulomb drag equations is

$$v_f(t) = v_{f0} \left[e^{-\frac{3t}{\tau_{se}}} - \left(\frac{E_{\text{crit}}}{E_{f0}}\right)^{3/2} (1 - e^{-\frac{3t}{\tau_{se}}}) \right]^{1/3} \tag{4}$$

where the critical energy (at which the fast ion is losing energy equally to the thermal ions and electrons) is

$$E_{\text{crit}} = \left(\frac{3\sqrt{\pi}}{4}\right)^{2/3} \left(\frac{m_i}{m_e}\right)^{1/3} \frac{m_f}{m_i} T_e. \tag{5}$$

The critical velocity, v_{crit}, is then the velocity of an NBI ion having energy E_{crit}. From (4) the thermalisation time

$$\tau_{\text{therm}} = \frac{\tau_{se}}{3} \log \left[1 + \left(\frac{E_{f0}}{E_{\text{crit}}}\right)^{3/2} \right] \tag{6}$$

can be found. Note that E_{f0} in Equation (6) refers to the Doppler-shifted energy of the beam ions as viewed in the plasma frame.

3. Fast Ion Current

To estimate the magnitude of the Doppler shift on the Ohkawa current, we use a model one-dimensional fast ion distribution function $f(v_f)dv_f$ giving the number of fast ions having velocities between v_f and $v_f + dv_f$. For a beam particle source of flux, S particles s^{-1}, the particle flux in velocity space is uniform

$$\frac{dv_f}{dt} f(v_f) = -S. \tag{7}$$

Using (4) we obtain

$$f(v_f) = \frac{S\tau_{se}}{v_f}\left(1 + \frac{v_{\rm crit}^3}{{v_f}^3}\right)^{-1} \tag{8}$$

which is normalised so that $\int_0^{v_{f0}} f(v_f)dv_f = S\tau_{\rm therm}$.

Fast ions in the velocity interval v_f and $v_f + dv_f$ make $\frac{v_f dv_f}{2\pi R}$ toroidally circulating fast ion loops whilst slowing down, where R is the major radius of the torus. In this velocity interval, the contribution to the fast ion current is

$$\delta I_f = f(v_f)dv_f\frac{v_f}{2\pi R}q_f. \tag{9}$$

The total circulating ion current, I_f, is obtained by integrating (9) over all actively slowing-down ions

$$I_f = \int_0^{v_{f0}} \delta I_f dv_f = \frac{q_f S\tau_{se}}{2\pi R}\int_0^{v_{f0}}\left(1 + \frac{v_{\rm crit}^3}{{v_f}^3}\right)^{-1} dv_f. \tag{10}$$

The integral in (10) can be evaluated exactly [9, p. 73]

$$\begin{aligned} I_f = \frac{q_f S\tau_{se}v_{\rm crit}}{2\pi R}\Bigg(\rho - \Bigg[\frac{1}{3}\ln(1+\rho) \\ + \frac{1}{\sqrt{3}}\left(\tan^{-1}\left(\frac{2\rho-1}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right) \\ -\frac{1}{6}\ln\left(\rho^2 - \rho + 1\right)\Bigg]\Bigg), \end{aligned} \tag{11}$$

where $\rho = \frac{v_{f0}}{v_{\rm crit}}$.

In the limit $\rho \ll 1$, (11) can be approximated by

$$I_f \simeq \frac{0.7 q_f S\tau_{se}}{8\pi R v_{\rm crit}^3}v_{f0}^4 = \frac{0.7 q_f S\tau_{se}}{8\pi R v_{\rm crit}^3}(v_{f0}^{\rm Lab} - v_\phi)^4. \tag{12}$$

The back electron current correction [1] has been omitted here as the primary interest is in assessing the overall effect of plasma rotation on current drive efficiency, and the correction term will cancel in (15).

The plasma-frame parameter ρ can be separated into in its laboratory-frame and Doppler-shift components:

$$\rho = \rho^{\rm Lab} - \rho_\phi = \frac{v_{f0}^{\rm Lab}}{v_{\rm crit}} - \frac{v_\phi}{v_{\rm crit}}. \tag{13}$$

When $T_i = T_e$, the Doppler-shift term in (13) is simply related to the rotational Mach number

$$\frac{v_\phi}{v_{\rm crit}} = \left(\frac{4}{3}\right)^{1/3}\left(\frac{m_e}{\pi m_i}\right)^{1/6}\frac{v_\phi}{c_{\rm s}} \tag{14}$$

where $c_{\rm s}$ is the ion sound speed.

To compare the driven currents in different rotating plasmas, an efficiency, η, can be defined:

$$\eta(\rho^{\rm Lab}, \rho_\phi) = \frac{I_f(\rho^{\rm Lab}, \rho_\phi)}{I_f(\rho^{\rm Lab}, 0)}. \tag{15}$$

It is also worth noting from (12) that as $v_{f0}^2 \propto E_{f0}$, and the beam power $P_{\rm beam} = SE_{f0}^{\rm Lab}$, then $I_f \propto P_{\rm beam}E_{f0}^{\rm Lab}$ for a non-rotating plasma: higher energy beams drive more current per unit power than lower energy beams. The efficiency η thus refers to the ratio of the actual beam-driven current in the rotating plasma compared to that expected in a non-rotating plasma for any particular neutral beam energy. Hence in a non-rotating plasma ($\rho_\phi = 0$), $\eta = 1$ regardless of particle energy, although the total current driven for different energies will differ.

4. Comparison with other models

By comparing the predictions of the analytic model presented in Section 3 with simulations on a "realistic" plasma, we can assess the effectiveness of the analytic model in capturing the direct effects of rotation on current drive.

A series of simulations of neutral beam current drive in MAST and a conceptual spherical tokamak Component Test Facility (CTF) [3, 4] were carried out using the TRANSP code [5, 6]. TRANSP models neutral beam behaviour using the Monte Carlo NUBEAM module [10, 11]. The predictions from this code are considered to be well validated against experimental data [12, 13].

Toroidal rotation in this model is dealt with by evaluating the Fokker-Plank collision operator in the frame of reference of the rotating plasma; this requires a Doppler shift of the particle energies as in Equation (1). The code calculates a distribution function for the slowing fast particles, taking into account the particles' trajectories in a plasma with density, temperature, and rotational velocity profiles, and from this a total beam-driven current can be calculated.

A series of TRANSP simulations was carried out with the central plasma parameters chosen to match those in MAST and those expected in CTF (Table 1). NUBEAM was configured to simulate tangential co-injection into the centre of the plasma for a range of different injection energies. Rotation profiles were imposed with a range of central velocities and the effects on the neutral beam current drive were calculated. The results are shown in Figure 1.

It is clear that the analytical predictions match the TRANSP predictions well and that therefore the overall current drive efficiencies given by Equation (15) provide a reasonable measure of the current drive in rotating plasmas without requiring a full Monte Carlo simulation. It is also apparent that in MAST, for rotation below Mach 1 ($\rho_\phi = 0.23$), NBI current drive is not substantially affected.

5. Examples

Figure 2 shows a plot of the variation of the NBI current drive efficiency η with Doppler-shift parameter ρ_ϕ for parameters typical of the conceptual Component Test Facility tokamak at the normalised minor radius ($r/a = 0.6$) where the maximum in the beam-driven current occurs (Table 1, case CTF(2)).

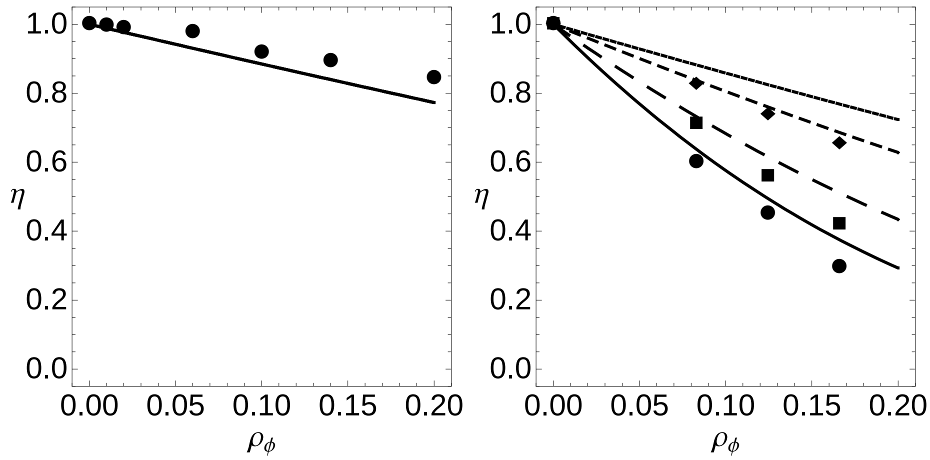

Figure 1. Comparison of NBI fast ion current drive efficiency η with Doppler-shift parameter, ρ_ϕ, for MAST (left) and CTF (right). The lines represent analytic model predictions and the points are TRANSP simulations. For MAST the beam energy is 60 keV, and for CTF the beam energies are 150 keV (solid line, circles); 250 keV (long dashes, squares); 500 keV (short dashes, diamonds); 750 keV (dotted line).

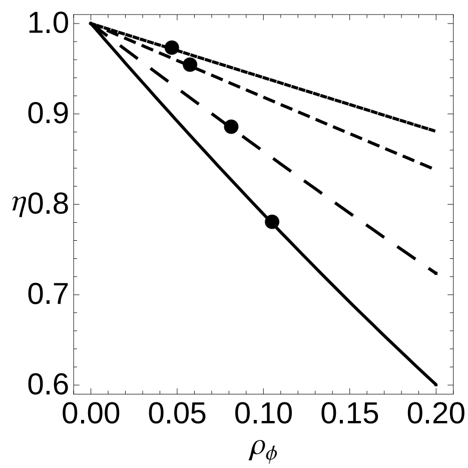

Figure 2. Variation of NBI fast ion current drive efficiency η with Doppler-shift parameter, ρ_ϕ, for CTF(2) parameters. Solid line: $\rho^{\rm Lab} = 1.21$ (equivalent to $E_{f0}^{\rm lab} = 150$keV); Coarse dashed line: $\rho^{\rm Lab} = 1.57$ ($E_{f0}^{\rm lab} = 250$keV); Medium dashed line: $\rho^{\rm Lab} = 2.23$ ($E_{f0}^{\rm lab} = 500$keV); Fine dashed line: $\rho^{\rm Lab} = 2.73$ ($E_{f0}^{\rm lab} = 750$keV). The points plotted are discussed in the text.

The results in Figure 2 show that, for the lowest assumed NBI energy (solid curve), the fast ion current drive efficiency is $\eta \sim 77\%$ (at $\rho_\phi = 0.105$, $M = 0.54$) of the beam-driven current expected in a non-rotating plasma, and increases as the NBI energy is raised (dashed curves). Noting that the ratio of NBI momentum to heating inputs to the plasma varies fundamentally as $\left(E_{f0}^{\rm lab}\right)^{1/2}$, it is expected that the plasma rotation will be reduced for higher NBI energies with constant net NBI power. In the design of a conceptual tokamak such as CTF, the total NBI power is constrained to a upper limit

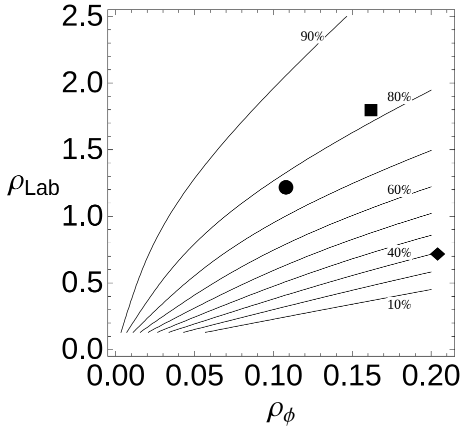

Figure 3. Contours of constant NBI fast ion current drive efficiency η in dimensionless $(\rho^{\rm Lab}, \rho_\phi)$ space. Contours from top of diagram: $\eta = 90\%, 80\%, \ldots\ 10\%$. Square point: MAST spherical tokamak parameters; disc point: CTF(2) parameters; diamond point: CTF (core) parameters.

by technological considerations such as the power handling capability of its divertor material surface and restrictions to the beam port size. The effect of changing the NBI energy at constant NBI power is shown by the series of points plotted in Figure 2. The first reference point has both the lowest NBI energy and efficiency in this plot and is based on a modelled rotational Mach number of 0.54. This value for CTF(2) data comes from TRANSP code [5] predictions in which the Prandtl number (ratio of angular momentum to ion thermal diffusivities) is assumed to be ~ 0.5. There is evidence from the MAST spherical tokamak that this assumption is justified [14]. The subsequent points in Figure 2 were obtained by assuming that the angular momentum diffusivity is independent of the NBI injection energy. As the NBI energy is increased, the value of ρ_ϕ decreases and η increases.

It is clear that the value of $\rho^{\rm Lab}$ needs to be as large as possible to avoid significant reduction of fast ion current drive efficiency for large rotational Mach numbers.

The behaviour of different tokamaks can be compared in the two-parameter dimensionless $(\rho^{\rm Lab}, \rho_\phi)$ phase diagram (Figure 3). Here, the contours are lines of constant NBI fast ion current drive efficiency η. MAST and CTF(2) data is shown in Figure 3. In the case of the MAST device, the value of $\rho^{\rm Lab}$ is large, leading to an efficiency of around 81%. For CTF(2) parameters, the efficiency is 77% at the normalised radius of 0.6 where TRANSP modelling indicates that most of the Ohkawa current is driven (see Table 1). However, for smaller radii the CTF fast ion current drive efficiency decreases rapidly and in the core it is $\sim 30\%$. Although the CTF is designed to have a hollow current profile, some current needs to be driven in the plasma core to maintain MHD stability. Higher-energy NBI beams are a possibility, but their use in CTF may increase beam shine-through and reduce the beam-plasma neutron yield, which for current designs makes up some 30% of the total CTF neutrons [3]. In this case, other methods such as Electron Bernstein Wave drive are under consideration [15].

6. Summary

A simple calculation of the NBI beam driven current, which includes toroidal plasma rotation, shows the strong effect it can have on reducing the current drive efficiency in certain co-rotating plasmas. The important parameters in this problem are the ratio, $\rho^{\mathrm{Lab}} = \frac{v_{f0}^{\mathrm{Lab}}}{v_{\mathrm{crit}}}$, of the NBI injection particle velocity (in the laboratory frame) to the critical velocity of the plasma, and the ratio $\frac{v_\phi}{v_{\mathrm{crit}}}$ which is related simply to the rotational Mach number. The use of these dimensionless parameters to plot current drive efficiency makes it convenient to compare different devices. The calculation of current drive efficiency compares well with simulations carried out using existing Monte Carlo neutral beam current drive codes.

In MAST plasmas, the typical critical energy is ~ 19 keV, the NBI particle energy is ~ 60 keV and the ratio of energies about three (with $\rho^{\mathrm{Lab}} = 1.79$). Thus, for MAST, the rotational effect is expected to be small, even though rotational Mach numbers of up to $M = 0.7$ have been observed.

However, when ρ^{Lab} is small, *i.e.* around unity or less, the plasma rotation plays a significant role, acting to reduce the thermalisation time and therefore the fast ion current. The effective beam velocity, as viewed in the rotating frame of reference of the plasma, is reduced by the Doppler shift; but this leads to an increased collisionality of the beam ions on plasma particles. Consequently the 'stacking factor' (or number of toroidal transits a beam ion makes before being thermalised) is reduced.

As Figures 2 and 3 show, when ρ^{Lab} is increased, the fast ion current drive efficiency also increases. To regain some of the full NBI current drive efficiency of the non-rotating $v_\phi = 0$ case, one option is to increase the energy of the beam ions.

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