An Error Analysis of Dimensionless Confinement Scaling Experiments

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I. Introduction

Recently several tokamak experiments have been completed to try and elucidate the scaling of the confinement with the key dimensionless parameters ρ*, ν* and β. It has been shown theoretically that these 3 parameters are fundamental in determining the scaling of the energy confinement [1-3]. The experimental results, which consist of single scans in one of these variables, (for example β with ρ* and ν* fixed), are rather contradictory particularly with respect to the scaling with β and ν*. In some experiments the confinement time scaling is almost independent of β [4-7] whilst in others there is quite strong degradation with respect to β [8-9] $\left(B\tau_{\varepsilon} \sim \beta^{-0.9}\right)$.

There is also a discrepancy between many of the single scan results and that obtained from the multimachine databases, with respect to both β and ν*. For example the H-mode scaling IBP98(y, 2), given in the ITER Physics Basis [10] (P 2204), has the form in dimensionless parameters $B\tau_{IPB98} \sim \rho*^{-2.7}\beta^{-0.9}\nu*^{-0.01}$. Single scan experiments in the dimensionless parameters ρ*, β and ν* have been completed on both DIII-D [4,5] and JET [6,7]. These experiments have confirmed the ρ* dependence but disagree with the dependence on both β and ν*, having virtually no β dependence and a ν* dependence of the form $B\tau_E \sim \nu*^{-0.3}$. This discrepancy between the single scans and the database result was partly resolved by a re-examination of the database [11] in which the errors in the measurements were taken into account. The analysis showed that the β and ν* scaling of the single scan results, given in [4-7], could be obtained provided the error on the input power was sufficiently large. This analysis does not of course explain why there are discrepancies between the single scan results themselves. To address this problem, we examine in this note the effect of measurement errors on the single scan results. We show that with a basic gyro-Bohm confinement scaling, measurement errors in the stored energy and input power have a substantial effect on the determination of the β, ρ* and ν* scaling obtained from the single scans. For meaningful results either the range of each scan has to be very large indeed or the measurement errors have to be significantly reduced. In most of the beta scans [4-9] a rudimentary error analysis has been included in the publication, however apart from reference [9], no account is taken of the errors in the stored energy on the ρ* and ν* fits. This, as we will show, significantly increases the error on the determination of the beta scaling.

In this note as well as completing a complete error analysis for the β scans, similar analysis is completed for both ρ* and ν* scans. We also give a few examples at the end of each section, however the main point of the note is the presentation of the error analysis so that the results can be used in future scans.

The structure of the remainder of the paper is as follows:- in section II the general error analysis is determined for a gyro-Bohm scaling scenario, then in section III the errors in the determination of the β, ρ* and ν* scaling are determined in turn. The note is completed by a brief summary.

II. General Error Analysis

Our starting point is a general expression for the scaling of the dimensionless confinement time $B\tau_\varepsilon$ in terms of ρ^*, ν^* and β,

$$B\tau_\varepsilon \sim \rho^{*\alpha_1}\, \nu^{*\alpha_2}\, \beta^{\alpha_3} \qquad (1)$$

where τ_ε, ρ^*, ν^* and β are functions of the stored energy W, density n, volume V, major radius R, minor radius a, toroidal field B and input power P and are defined as follows:-

$$\tau_\varepsilon = W/P \qquad (2a)$$

$$\rho^* \sim W^{1/2}/(nV)^{1/2}\, Ba \qquad (2b)$$

$$\nu^* \sim n^3\, RV^2/W^2 \qquad (2c)$$

$$\beta \sim W/B^2 \qquad (2d)$$

In the analysis presented here we only consider errors in the stored energy W and input power P, since errors in the density and the dimensions R, a etc. are significantly lower than those in W and P. (In fact a full analysis which includes errors in the density and spatial parameters has been completed and we find that the errors in the determination of the scalings are only increased by about 1%).

Taking $W = W_o + \delta W$, $P = P_o + \delta P$ where W_o and P_o are the measured values. The indices α_n are written in the form $\alpha_n = \alpha_{no} + \delta\alpha_n$, with α_{no} the indices of the derived scaling and the $\delta\alpha_n$ are the errors in these indices. Completing a perturbation analysis of Eq. (1) and only retaining first order terms gives

$$B\tau_{\varepsilon o}\left[1+\left(1-\frac{1}{2}\alpha_{10}+2\alpha_{20}-\alpha_{30}\right)\frac{\delta W}{W_o}-\frac{\delta P}{P_o}\right] = \rho_o^{*\,\alpha_{10}+\delta\alpha_1}\; \nu_o^{*\,\alpha_{20}+\delta\alpha_2}\, \beta_o^{\,\alpha_{30}+\delta\alpha_3} \qquad (3)$$

After taking the log of Eq. (3) we arrive at

$$\delta\alpha_1 \log\rho_o^* + \delta\alpha_2 \log\nu_o^* + \delta\alpha_3 \log\beta_o = \left(1-\frac{1}{2}\alpha_{10}+2\alpha_{20}-\alpha_{30}\right)\delta W/W_o - \delta P/P_o \qquad (4)$$

Using Eq. (4), we now derive the error in the scaling for typical single scans in β, ρ^* and ν^*.

III. a) Error in the determination of the β scaling

Here we take the index $\alpha_{10} = -3.0$ (gyro-Bohm scaling) and $\alpha_{20} = -0.25$ which is a typical value from ν^* scans, then Eq. (4) reduces to

$$\delta\alpha_3 \log_e(\beta) = (2-\alpha_{30})\,\delta W/W_o - \delta P/P_o \qquad (5)$$

For a two point β scan, with an upper value of β_u and a lower value β_L we arrive at an expression for $\delta\alpha_3$,

$$\delta\alpha_3 \log_e\left(\beta_u/\beta_L\right) = \left(2-\alpha_{30}\right)\left(\frac{\delta W_u}{W_u} - \frac{\delta W_L}{W_L}\right) + \frac{\delta P_L}{P_L} - \frac{\delta P_u}{P_u} \qquad (6)$$

In table A1 of reference [11], the errors for $\delta P/P$ and $\delta W/W$ are given for the various devices. Typical values for AUG, DIII-D and JET are 12% for both $\delta W/W$ and $\delta P/P$.

With these errors and with $\alpha_{30} = -\frac{1}{2}$ and $\beta_u/\beta_L = 2$, $\delta\alpha_3$ would have a range of $\pm$ 1.2.

This is rather extreme and assumes that the errors are correlated with the values of the stored energy etc. A more realistic approach is to sum the errors quadratically and take the square root.

$$\delta\alpha_3 \log_e\left(\beta_u/\beta_L\right) = \left[\left(2-\alpha_{30}\right)^2\left\{\left(\frac{\delta W_u}{W_u}\right)^2 + \left(\frac{\delta W_L}{W_L}\right)^2\right\} + \left(\frac{\delta P_u}{P_u}\right)^2 + \left(\frac{\delta P_L}{P_L}\right)^2\right]^{1/2} \qquad (7)$$

Using the same errors for $\delta W/W$ and $\delta P/P$ of 0.12 we calculate the error $\delta\alpha_3$ for a few examples.

1) $\alpha_{30} = -0.5$, $\beta_u/\beta_L = 1.6$ gives $\delta\alpha_3 = \pm 0.98$.

2) $\alpha_{30} = -0.5$, $\beta_u/\beta_L = 2$ gives $\delta\alpha_3 = \pm 0.66$.

3) $\alpha_{30} = 0$, $\beta_u/\beta_L = 3$ gives $\delta\alpha_3 = \pm 0.35$

Hence we see from the above that even for case 3, in which there is a large range in β, the error is still quite significant. In fact to obtain a meaningful result, such that $\beta^{0.0}$ and $\beta^{-0.5}$ could be separated, one needs to reduce the measurement errors by a factor of 2 also. This may be possible in devices which have multiple measurements of the stored energy.

Note in cases where $\delta\alpha_3$ is large, such as in case 1) above, higher order terms in $\delta\alpha_3$ need to be retained in the perturbation analysis of Eq(1). Retaining these terms for case 1) reduces $\delta\alpha_3$ to 0.74 from 0.98, for the lower values of $\delta\alpha_3$ in cases 2) and 3) the reduction is much smaller.

b) Error in the determination of the ρ^* scaling

Here we take $\alpha_{10} = -3.0$, $\alpha_{20} = -0.25$, $\alpha_{30} = 0$, which is the typical scan result from DIII-D and JET. With these values Eq. (4) reduces to

$$\delta\alpha_1 \log_e\left(\rho_u^*/\rho_L^*\right) = 2\left(\frac{\delta W_u}{W_u} - \frac{\delta W_L}{W_L}\right) + \delta P_u/P_u - \delta P_L/P_L \qquad (8)$$

Using the same errors for $\delta W/W$ and $\delta P/P$ of 0.12 and assuming quadrature of errors as previously Eq. (8) reduces to

$$\delta\alpha_1 \log\left(\rho_u^*/\rho_L^*\right) = \pm 0.38 \qquad (9)$$

In a typical ρ* scan, in which the toroidal field B changes by a factor of 2, ρ* has a range of 1.6. Hence from Eq. (9) the error in the ρ* index $\delta\alpha_1 = \pm 0.81$. An error of this magnitude means that it is uncertain whether the ρ* scaling is gyro-Bohm ($\alpha_{10} = -3$) or Bohm ($\alpha_{10} = -2$). For an extended ρ* scan with a range of 2.6 say (equivalent to a range in B of a factor of 4), $\delta\alpha_1 = \pm 0.41$.

c) Error in the determination of the ν* scaling

Taking $\alpha_{10} = -3$, $\alpha_{20} = -0.25$ and $\alpha_{30} = 0$, then with the same errors and using quadrature of the errors gives,

$$\delta\alpha_2 \log\left(\nu_u^* / \nu_L^*\right) = \pm 0.38$$

For the typical range of ν* of 16, $\delta\alpha_2 = \pm 0.14$. This error is much smaller than those of the β and ρ* scans and is due to the large range in ν* that is possible in most experiments.

IV. Summary

In summary, it has shown that the errors in the measurements of the stored energy and input power can lead to large errors in the determination of the scaling of the confinement with respect to the dimensionless parameter β and to a lesser extent ρ*. To obtain a meaningful result for the beta scaling one needs to scan over a range of a factor of 3 and reduce the error in the stored energy and power to about 6%. Increasing the number of points in the scan will also improve the accuracy of the scaling.

Acknowledgements

The work was partly funded jointly by the United Kingdom Engineering and Physical Sciences Research Council and the European Communities under the contract of Association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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