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A possible excitation mechanism for observed superthermal ion cyclotron emission from tokamak plasmas

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Intense superthermal ion cyclotron emission (ICE) has been observed from tokamak plasmas. The power spectrum displays narrow peaks at multiple harmonics of the background ion cyclotron frequency [Cottrell and Dendy, Phys. Rev. Lett. **60**, 33 (1988)] in Ohmic deuterium plasmas, and the radiation appears to be driven by the fusion ion population in the edge plasma. Theoretical investigation of this phenomenon may be rewarding, in terms of the information about the behavior of energetic ions in tokamaks that can be extracted from ICE observations. The interpretation presented here is based on the resonant excitation of fast Alfvén waves with ion Bernstein waves supported by an energetic ion species (α), in the presence of a more numerous thermal ion species (i). Because the ion cyclotron frequencies may be commensurate ($l\Omega_\alpha = s\Omega_i$ for some low integers l,s), and observations indicate that ω is comparable in magnitude to Ω_i, the standard theory [Belikov and Kolesnichenko, Sov. Phys. Tech. Phys. **20**, 1146 (1976)] which assumes $\omega \gg \Omega_i$ is not immediately applicable, and is accordingly extended here to the low-frequency regime. The results show that excitation of the fast Alfvén wave at proton cyclotron harmonics can occur for fusion proton concentrations n_α/n_i as low as 10^{-7}, and that multiple cyclotron harmonics can be simultaneously unstable. Furthermore, while fusion protons born at 3.0 MeV are above the energy threshold required to drive the instability, the other primary fusion products in deuterium—1.0 MeV tritons and 0.82 MeV helium-3 nuclei—fall below it, consistent with the observation that radiation at cyclotron harmonics of the latter is not detected. However, the mechanism considered here cannot easily explain the generation of radiation which is observed at deuterium cyclotron harmonics that are degenerate with proton half-harmonics.

I. INTRODUCTION

The observation of superthermal radiation in the ion cyclotron range of frequencies provides, in principle, a method for detecting and studying energetic charged particles in magnetic fusion experiments. The main features of ion cyclotron emission (ICE) observed on Joint European Torus (JET)[1–4] include the following: in Ohmic deuterium discharges, the observed ICE intensity is proportional to the measured fusion reactivity R_{DD} over three orders of magnitude in signal intensity;[2] in both neutral-beam injected (NBI) and Ohmic discharges, the ICE frequency spectrum contains intense (10^4 times background), regularly spaced peaks whose narrowness ($\Delta\omega/\omega \simeq 0.1 \ll a/R_0 \simeq 0.4$) implies a spatially localized origin;[1,3] frequency-matching calculations suggest the edge plasma as the emitting region;[1] and recent observations of inverted ICE sawtooth oscillations,[4] which coincide with the arrival of the sawtooth heat pulse in the edge plasma, give further support to this interpretation. In Fig. 3 of Ref. 3, it is shown that the ICE spectral peaks from an Ohmic deuterium discharge (with protons as the most energetic fusion product) uniquely match the first nine cyclotron harmonics of deuterium evaluated at $R = 4.0 \pm 0.1$ m, in the near-field region of the detecting antenna. Thus, there is strong experimental evidence that a minority energetic proton population in a deuterium plasma can excite superthermal ICE with spectral peaks at multiple harmonics of the deuteron cyclotron frequency (harmonics and half-harmonics of the proton cyclotron frequency), evaluated in the near-field region of the antenna at the plasma edge. It also appears reasonable to infer from the experiments that the velocity distribution of the fusion protons in the emitting region possesses, at least transiently, a positive gradient in some region of velocity space. In addition to the highly superthermal intensity of ICE already discussed, it was noted in Ref. 4 that mechanisms such as the change in fusion reactivity associated with the sawtooth heat pulse, coupled with the edge loss term, could plausibly create a hollow shell distribution in velocity space. A similar argument presumably applies to the velocity space distributions of the fusion tritons and helium-3 nuclei, which are born at 1.0 and 0.82 MeV, respectively, at a rate roughly equal to that of the 3.0 MeV fusion protons. However, in the ICE experiments, while the intensity is proportional to measured fusion reactivity R_{DD}, no spectral peaks have been identified at frequencies corresponding to cyclotron harmonics of tritium or helium-3 that are not degenerate with deuterium harmonics—for example, $4\Omega_d/3$ or $8\Omega_d/3$. This appears to be an important null result, which any candidate theory of the driving instability should replicate.

While the leading hypothesis is that the fusion proton population provides the free energy that drives the ICE in Ohmic deuterium plasmas, the precise nature of the underlying instability and of the wave modes excited remains unknown. In particular, there exists no well-defined analytical model for the proton distribution function in the edge plasma; and, because the ICE signal apparently originates in the near-field region of the antenna, it may be composed of fast Alfvén waves, Bernstein waves, or both. Furthermore, the energy coupling mechanism that results in spectral peaks at odd harmonics of the deuteron cyclotron frequency, which are not resonant with the cyclotron harmonics of the driving protons, is also unknown. In Ref. 3, generic instabilities (see, for example, Refs. 5–10) and a simple analytical model for the proton distribution function were used to illustrate the physical principles. Further constraints emerged from kinetic modeling associated with the sawtooth oscillations in Ref. 4. These studies indicate that further theoretical investigations may be rewarding in terms of the information about the behavior of energetic charged particles in tokamak plasmas that can be extracted from the ICE observations. In particular, a better understanding of the physics underlying ICE would help in assessing the potential of ICE as a diagnostic of fusion alpha particles in deuterium–tritium plasmas. While the production rate of alpha particles can be deduced from the measured neutron flux, and thermal alpha particles can be detected by charge-exchange scattering, the behavior of alpha particles in their slowing-down phase is difficult to monitor. If some fraction of the alpha-particle energy is released by collective instability, it might be possible to detect and measure the process using ICE.

In the present paper, we concentrate on the essential physics of a candidate excitation mechanism for the ICE, which is independently of interest as a possible collisionless relaxation mechanism for energetic ions. This is the so-called magnetoacoustic cyclotron instability,[11] which occurs for propagation perpendicular to the magnetic field, at cyclotron harmonics of an energetic ion species in the presence of a more numerous thermal ion species. It is a reactive instability, involving resonance of the fast Alfvén wave with ion Bernstein waves supported by the energetic ion species and, if their cyclotron frequencies are commensurate, ion Bernstein waves supported by the thermal majority species. We concentrate on perpendicular propagation; in this case, cyclotron damping of the wave through resonance with background ions cannot, strictly speaking, occur. Instead, an equivalent effect, acting to stabilize the system, is produced by the positive-energy loading due to the background ions, which must be induced to oscillate coherently in the field of the excited wave. The existing theory[11] of the magnetoacoustic cyclotron instability is restricted to the frequency regime $\omega \gg \Omega_i$, where Ω_i is the cyclotron frequency of the thermal ion population. In view of the experimental results that we have discussed, after reviewing the existing theory in Sec. II, we carry out an extension of the theory to accommodate lower frequencies $\omega \gtrsim \Omega_i$ in Sec. III. In Sec. IV, we carry out a computational study, which relates the rather complex dispersion relation that emerges from the new analysis to the observed experimental parameters, particularly thresholds. Finally, in Sec. V, we present our conclusions on the role of the magnetoacoustic cyclotron instability as a potential excitation mechanism for the observed ion cyclotron emission.

II. OUTLINE OF THE MAGNETOACOUSTIC CYCLOTRON INSTABILITY

An excellent early treatment of this instability in the regime $\omega \gg \Omega_i$ was given by Belikov and Kolesnichenko,[11] who outlined many of the essential physical principles; see also Refs. 12 and 13. In this paper, we give a more general treatment, which extends the frequency range downward and is valid when $\omega \lesssim \Omega_i$. In the present section, after deriving the full dispersion relation in terms of dielectric tensor elements, we review the more restricted case considered in Ref. 11. This is a useful basis for comparison with our treatment of the full dispersion relation in the next section, and also allows us to correct a number of misprints that occur in Ref. 11.

For waves propagating perpendicular to the magnetic field in a homogeneous plasma, the general dispersion relation is

$$\epsilon_{xx}\left(\epsilon_{yy}-\frac{c^2k^2}{\omega^2}\right)+\epsilon_{xy}^2=0; \tag{1}$$

see, for example, Eq. (2.43) of Mikhailovskii's review,[13] where we have used the fact that $\epsilon_{xy}=-\epsilon_{yx}$. Contributions to the dielectric tensor ϵ_{ij} arise from electrons (superscript e hereafter), a background thermal ion population (i), and a less numerous energetic ion population (α). The latter may, or may not, be of the same species as the thermal ions. We write

$$\epsilon_{xx}=1+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha, \tag{2}$$

$$\epsilon_{yy}=1+\epsilon_{yy}^e+\epsilon_{yy}^i+\epsilon_{yy}^\alpha, \tag{3}$$

$$\epsilon_{xy}=\epsilon_{xy}^e+\epsilon_{xy}^i+\epsilon_{xy}^\alpha. \tag{4}$$

Substituting Eqs. (2)–(4) into Eq. (1), multiplying by ω^2/c^2k^2, and rearranging, we obtain

$$\begin{aligned}1&+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha\\&=\frac{\omega^2}{c^2k^2}\,[(1+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha)(1+\epsilon_{yy}^e+\epsilon_{yy}^i+\epsilon_{yy}^\alpha)\\&\quad+(\epsilon_{xy}^e+\epsilon_{xy}^i+\epsilon_{xy}^\alpha)^2].\end{aligned} \tag{5}$$

It is convenient to group together terms that involve the same combination of species in Eq. (5), giving

$$\begin{aligned}1&+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha-\frac{\omega^2}{c^2k^2}\,[(1+\epsilon_{xx}^e+\epsilon_{yy}^e+\epsilon_{xx}^e\epsilon_{yy}^e+\epsilon_{xy}^{e^2})\\&+\epsilon_{xx}^i(1+\epsilon_{yy}^e)+\epsilon_{yy}^i(1+\epsilon_{xx}^e)+2\epsilon_{xy}^i\epsilon_{xy}^e\\&+\epsilon_{xx}^\alpha(1+\epsilon_{yy}^e)+\epsilon_{yy}^\alpha(1+\epsilon_{xx}^e)+2\epsilon_{xy}^\alpha\epsilon_{xy}^e+\epsilon_{xx}^i\epsilon_{yy}^i\\&+\epsilon_{xy}^{i^2}+\epsilon_{xx}^i\epsilon_{yy}^\alpha+\epsilon_{yy}^i\epsilon_{xx}^\alpha+2\epsilon_{xy}^i\epsilon_{xy}^\alpha+\epsilon_{xx}^\alpha\epsilon_{yy}^\alpha+\epsilon_{xy}^{\alpha^2}]=0.\end{aligned} \tag{6}$$

Thus far, the analysis has been completely general. We now restrict attention to frequencies ω much smaller than the electron cyclotron frequency Ω_e (we define Ω_e positive). In this limit,

$$\epsilon_{xx}^e=\epsilon_{yy}^e=\omega_{pe}^2/\Omega_e^2, \tag{7}$$

$$\epsilon_{xy}^e=i\omega_{pe}^2/\omega\Omega_e. \tag{8}$$

Assuming that plasma conditions are such that $\omega_{pe}^2/\Omega_e^2 \lesssim 1$, we may neglect certain terms involving unity, ϵ_{xx}^e and ϵ_{yy}^e in comparison to terms involving ϵ_{xy}^e in Eq. (6), leaving

$$\begin{aligned}1&+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha\\&-\frac{\omega^2}{c^2k^2}\,[\epsilon_{xy}^{e^2}+(1+\epsilon_{xx}^e)(\epsilon_{xx}^i+\epsilon_{yy}^i+\epsilon_{xx}^\alpha+\epsilon_{yy}^\alpha)\\&+2\epsilon_{xy}^e(\epsilon_{xy}^i+\epsilon_{xy}^\alpha)+\epsilon_{xx}^i\epsilon_{yy}^i+\epsilon_{xy}^{i^2}+\epsilon_{xx}^i\epsilon_{yy}^\alpha\\&+\epsilon_{yy}^i\epsilon_{xx}^\alpha+2\epsilon_{xy}^i\epsilon_{xy}^\alpha+\epsilon_{xx}^\alpha\epsilon_{yy}^\alpha+\epsilon_{xy}^{\alpha^2}]=0.\end{aligned} \tag{9}$$

This is a generalization of the basic expression Eq. (2) of Ref. 11, which is (correcting a misprint)

$$1+\epsilon_{xx}^e+\epsilon_{xx}^i+\epsilon_{xx}^\alpha-\frac{\omega^2}{c^2k^2}\,(\epsilon_{xy}^{e^2}+2\epsilon_{xy}^e\epsilon_{xy}^\alpha+\epsilon_{xx}^i\epsilon_{yy}^\alpha)=0. \tag{10}$$

Let us consider the form of the dielectric tensor elements that occur in these equations. We assume that for the energetic ion population, only the contribution to ϵ_{ij}^α that is resonant when $\omega\simeq l\Omega_\alpha$, for some as yet unspecified value of l, is significant. Accordingly, we write

$$\epsilon_{ij}^\alpha=-\frac{l^2\omega_{p\alpha}^2}{\omega(\omega-l\Omega_\alpha)}\frac{1}{z_\alpha^2}\,\Pi_{ij}. \tag{11}$$

For example, for an isotropic shell distribution

$$f_\alpha(\mathbf{v})=(1/4\pi v_0^2)\delta(v-v_0), \tag{12}$$

we have

$$z_\alpha=kv_0/\Omega_\alpha, \tag{13}$$

and expressions for Π_{ij} are given in Ref. 11; unfortunately, as we discuss in the Appendix, it appears that there is a misprint in the expression given for Π_{yy}, which is the dominant contribution from the energetic ions in the analysis of Ref. 11. It seems clear from the observations of intense superthermal ICE[2–4] that the fusion product population in tokamaks can deviate significantly from an isotropic, monotonically decreasing slowing-down distribution. As discussed in the Introduction, it appears reasonable to adopt the somewhat simplistic model Eq. (12) in the absence of more detailed experimental information. This model enables us to concentrate on what appear to be the two key parameters for any candidate emission mechanism, namely the birth energy of the fusion products and their concentration in the plasma. A more detailed model for the velocity distribution would need to reflect a number of additional competing physical effects. These include: the effect of large Larmor radius on energetic ion orbits, which produces a significant excess of high $v_\perp/v_\parallel$ ions at the edge[14]—this phenomenon would act to counterbalance the effect of ripple loss; centrally born fusion products passing through the edge region on their way out of the plasma; the spread in birth energy in the laboratory frame introduced by the center-of-mass effect associated with thermal motion of the fusing nuclei, which is non-negligible; and edge losses. To combine all these effects so as to give a convincing model for the fusion product velocity distribution in the edge plasma, is a long-term objective beyond the scope of the present work, which is concerned with identifying a plausible candidate emission mechanism.

Turning to the thermal ion population, its contribution can be divided into a cold and a resonant part. The latter corresponds to some integer s, such that $s\Omega_i$ is the thermal ion cyclotron harmonic closest to the wave frequency; assuming $s\neq 1$, we may write

$$\epsilon_{xx}^i=-\frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}+\frac{\omega}{\omega-s\Omega_i}f_s(z_i)\right), \tag{14}$$

$$\epsilon_{xy}^i=i\,\frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\frac{\Omega_i}{\omega}+\frac{\omega}{\omega-s\Omega_i}sg_s(z_i)\right), \tag{15}$$

$$\epsilon_{yy}^i=-\frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}+\frac{\omega}{\omega-s\Omega_i}h_s(z_i)\right), \tag{16}$$

$$f_s(z_i)=s^2I_s(z_i^2)\exp(-z_i^2)/z_i^2, \tag{17}$$

$$g_s(z_i)=[I_s'(z_i^2)-I_s(z_i^2)]\exp(-z_i^2), \tag{18}$$

$$h_s(z_i)=f_s(z_i)+2z_i^2g_s(z_i), \tag{19}$$

$$z_i^2=k^2v_{Ti}^2/2\Omega_i^2, \tag{20}$$

where v_{Ti} is the thermal velocity of the background ion population and I_s is the modified Bessel function of order s. Note that in Eq. (7) of Ref. 11, a factor s^2/z_i^2 has been omitted erroneously from the expression equivalent to Eqs. (14) and (17). Finally, we note that if the thermal ions are much more numerous than the energetic ions, the Alfvén velocity in the plasma is

$$c_{\mathrm{A}}=B/(4\pi n_im_i)^{1/2}, \tag{21}$$

so that

$$\omega_{pi}^2/\Omega_i^2=c^2/c_{\mathrm{A}}^2. \tag{22}$$

Under these circumstances, the condition of charge neutrality is, to good approximation,

$$\omega_{pe}^2/\Omega_e=\omega_{pi}^2/\Omega_i. \tag{23}$$

Thus, using Eqs. (22) and (23), the electron contributions to the dielectric tensor, given by Eqs. (7) and (8), become

$$\epsilon_{xx}^e=\epsilon_{yy}^e=\frac{\Omega_i}{\Omega_e}\frac{c^2}{c_{\mathrm{A}}^2}, \tag{24}$$

$$\epsilon_{xy}^e=i\,\frac{\Omega_i}{\omega}\frac{c^2}{c_{\mathrm{A}}^2}. \tag{25}$$

Our primary objective in this paper is the solution of the full dispersion relation Eq. (9) for wave frequencies ω such that $\Omega_e\gg\omega\gtrsim\Omega_i$; this will be obtained in Sec. III. As a first step, for purposes of comparison with existing theory, in the remainder of this section we consider the simpler and more restrictive case $\Omega_e\gg\omega\gg\Omega_i$ discussed in Ref. 11. We therefore return to Eq. (10) which, as we have seen, is the limit considered in Ref. 11 of the more general expression Eq. (9). Since we are restricting attention to frequencies $\omega \ll \Omega_e$ in plasmas for which $c^2/c_A^2 \gg 1$, it follows from the results in the preceding paragraph that the term $1+\epsilon_{xx}^e$ in Eq. (9) may be neglected. Furthermore, for the relatively high-frequency case that we are now considering, we may replace $\omega^2/(\omega^2-\Omega_i^2)$ by unity in Eq. (14). Then, substituting from Eqs. (14) and (25) in Eq. (10), and using Eq. (22), we obtain

$$-\frac{c^2}{c_A^2}\frac{\Omega_i^2}{\omega^2}\left(1+\frac{\omega}{\omega-s\Omega_i}f_s(z_i)\right)+\epsilon_{xx}^\alpha+\frac{c^2}{c_A^2}\frac{\Omega_i^2}{c_A^2k^2}$$

$$-\frac{2i\omega\Omega_i}{c_A^2k^2}\epsilon_{xy}^\alpha+\frac{\Omega_i^2}{c_A^2k^2}\left(1+\frac{\omega}{\omega-s\Omega_i}f_s(z_i)\right)\epsilon_{yy}^\alpha=0. \tag{26}$$

The final, potentially double-resonant term proportional to $\epsilon_{yy}^\alpha/(\omega-s\Omega_i)$ in Eq. (26) is not included in the analysis in Ref. 11, which leads to the final expressions, Eqs. (9) and (10) of Ref. 11. It is not entirely clear why this term is omitted; it originates, of course, in the term proportional to $\epsilon_{xx}^i\epsilon_{yy}^\alpha$ in Eq. (9). Rearranging Eq. (26) with the aid of Eq. (22), we obtain

$$\frac{(\omega^2-c_A^2k^2)}{c_A^2k^2}\frac{(\omega-s\Omega_i)}{\omega}$$

$$=-\frac{(\omega-s\Omega_i)\omega}{\omega_{pi}^2}\left(\epsilon_{xx}^\alpha-\frac{2i\omega\Omega_i}{c_A^2k^2}\epsilon_{xy}^\alpha+\frac{\Omega_i^2}{c_A^2k^2}\epsilon_{yy}^\alpha\right)$$

$$+f_s(z_i)-\frac{\Omega_i^2}{c_A^2k^2}\frac{\omega^2}{\omega_{pi}^2}f_s(z_i)\epsilon_{yy}^\alpha. \tag{27}$$

Substituting for ϵ_{ij}^α from Eq. (11) in Eq. (27),

$$\frac{(\omega^2-c_A^2k^2)(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}{\omega^2c_A^2k^2}$$

$$=l^2\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{1}{z_\alpha^2}\frac{(\omega-s\Omega_i)}{\omega}\left(\Pi_{xx}-\frac{2i\omega\Omega_i}{c_A^2k^2}\Pi_{xy}+\frac{\Omega_i^2}{c_A^2k^2}\Pi_{yy}\right)$$

$$+\frac{(\omega-l\Omega_\alpha)}{\omega}f_s(z_i)+l^2\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{\Omega_i^2}{c_A^2k^2}\frac{f_s(z_i)}{z_\alpha^2}\Pi_{yy}. \tag{28}$$

Equation (28) is the basic dispersion relation, derived from Eq. (10) and applying when $\Omega_e \gg \omega \gg \Omega_i$. Generalizing the notation of Eq. (6) of Ref. 11, it is convenient to define

$$\chi_0=\left(\Pi_{xx}-\frac{2i\omega\Omega_i}{c_A^2k^2}\Pi_{xy}+\frac{\Omega_i^2}{c_A^2k^2}\Pi_{yy}\right)^{1/2}. \tag{29}$$

Recall from Eq. (11) that the functions Π_{ij} contain information about the distribution function of the energetic ions. Equation (28) can now be written

$$\frac{(\omega^2-c_A^2k^2)(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}{\omega^2c_A^2k^2}$$

$$=\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{l^2}{z_\alpha^2}\left(\frac{(\omega-s\Omega_i)}{\omega}\chi_0^2+\frac{\Omega_i^2}{c_A^2k^2}f_s(z_i)\Pi_{yy}\right.$$

$$\left.+\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\frac{z_\alpha^2}{l^2}\frac{(\omega-l\Omega_\alpha)}{\omega}f_s(z_i)\right). \tag{30}$$

We are particularly interested in the resonance of the fast Alfvén wave with the lth cyclotron harmonic of the energetic ions, and therefore write

$$\omega=\omega_0+\Delta\omega, \quad |\Delta\omega| \ll \omega_0, \tag{31}$$

$$\omega_0=c_Ak=l\Omega_\alpha. \tag{32}$$

For the time being, we do not specify how close ω lies to the nearest, sth cyclotron harmonic of the thermal ions. Substituting Eqs. (31) and (32) into Eqs. (29) and (30), we obtain

$$2\left(\frac{\Delta\omega}{\omega_0}\right)^2\frac{(\omega-s\Omega_i)}{\omega}$$

$$\simeq\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{c_A^2}{v_0^2}\left(\frac{(\omega-s\Omega_i)}{\omega}\chi_0^2+\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\frac{v_0^2}{c_A^2}\frac{(\omega-l\Omega_\alpha)}{\omega}f_s(z_i)\right.$$

$$\left.+\frac{\Omega_i^2}{\omega^2}f_s(z_i)\Pi_{yy}\right), \tag{33}$$

$$\chi_0=\left(\Pi_{xx}-\frac{2i\Omega_i}{l\Omega_\alpha}\Pi_{xy}+\frac{\Omega_i^2}{l^2\Omega_\alpha^2}\Pi_{yy}\right)^{1/2}. \tag{34}$$

Here, we have used the fact that for the resonance Eq. (32), Eq. (13) gives

$$z_\alpha=lv_0/c_A. \tag{35}$$

The form of Eqs. (33) and (34) has been chosen so as to facilitate comparison with Ref. 11; clearly, Eq. (34) is equivalent to Eq. (6) of Ref. 11. In the limit where the possibility of cyclotron resonance with the thermal ions can be neglected, which is the first case considered in Ref. 11, we may set the second and third terms on the right of Eq. (33), which are both proportional to $f_s(z_i)$ defined at Eqs. (14) and (17), equal to zero. The factor $(\omega-s\Omega_i)/\omega$, which remains on both sides of the equation, may then be canceled, giving

$$\frac{\Delta\omega}{\omega_0}=2^{-1/2}\frac{\omega_{p\alpha}}{\omega_{pi}}\frac{c_A}{v_0}\chi_0; \tag{36}$$

this equation, which predicts instability when χ_0^2 is negative, agrees with Eq. (5) of Ref. 11.

Now suppose that the fast wave is simultaneously in cyclotron resonance with the thermal ions as well as the energetic ions. Then, instead of Eq. (32), we have

$$\omega_0=c_Ak=l\Omega_\alpha=s\Omega_i. \tag{37}$$

Substituting Eqs. (28) and (34) into Eq. (30), we obtain

$$2\left(\frac{\Delta\omega}{\omega_0}\right)^3-\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{c_A^2}{v_0^2}\left[\left(\chi_0^2+\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\frac{v_0^2}{c_A^2}f_s(z_i)\right)\frac{\Delta\omega}{\omega_0}\right.$$
$$\left.+\frac{f_s(z_i)}{s^2}\Pi_{yy}\right]=0, \tag{38}$$

where χ_0 remains defined by Eq. (34) with $\Omega_i/l\Omega_\alpha=1/s$. This is a cubic equation, in contrast to Eq. (36), which is the solution of a quadratic equation. The final term in Eq. (38) originates in the double-resonant term $\epsilon_{xx}^i\epsilon_{yy}^\alpha$ that we discussed after Eq. (26). In Ref. 11 this term is omitted so that, as can be seen from Eq. (38), a quadratic equation is recovered; using Eq. (17), its solution can be written

$$\frac{\Delta\omega}{\omega_0}=2^{-1/2}\frac{\omega_{p\alpha}}{\omega_{pi}}\frac{c_A}{v_0}\left(\chi_0^2+\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\frac{v_0^2}{c_A^2}\frac{s^2}{z_i^2}I_s(z_i^2)\exp(-z_i^2)\right)^{1/2}. \tag{39}$$

Equation (39) is equivalent to Eq. (10) of Ref. 11. This can be checked by noting that, by Eqs. (20), (35), and (37),

$$s^2v_0^2/c_A^2z_i^2=s^2z_\alpha^2/l^2z_i^2, \tag{40}$$

and by Eq. (37),

$$\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\equiv\frac{n_i}{n_\alpha}\frac{\Omega_i^2}{\Omega_\alpha^2}\frac{m_i}{m_\alpha}=\frac{n_i}{n_\alpha}\frac{m_i}{m_\alpha}\frac{l^2}{s^2}. \tag{41}$$

The solution Eq. (39) was obtained from Eq. (38) for the case where the final term in Eq. (38) is negligible in comparison with the second. However, it is not clear that the final term can always be safely neglected. This would require

$$|\Pi_{yy}|\ll s^2\frac{\omega_{pi}^2}{\omega_{p\alpha}^2}\frac{v_0^2}{c_A^2}\left|\frac{\Delta\omega}{\omega_0}\right|. \tag{42}$$

Assuming a solution whose magnitude approximately satisfies $|\Delta\omega/\omega_0|\simeq\omega_{p\alpha}c_A|\chi_0|/\omega_{pi}v_0$, following Eq. (39), the inequality Eq. (42) becomes

$$\frac{|\Pi_{yy}|}{\chi_0}\ll s^2\frac{\omega_{pi}}{\omega_{p\alpha}}\frac{v_0}{c_A}. \tag{43}$$

Referring to Eq. (34) and the expressions in the Appendix of Ref. 11, the left side of Eq. (43) may be of order unity. Thus, the inequality cannot be satisfied reliably unless

$$\left(\frac{n_i}{n_\alpha}\right)^{1/2}\frac{v_0}{c_A}\gg 1. \tag{44}$$

Under present experimental conditions, where the concentration of fusion products is very low, this inequality is well satisfied, so that the quadratic limit is a good approximation. However, in the interests of generality and future applications, in the next section we return to the full dispersion relation Eq. (9).

III. DISPERSION RELATION OF THE MAGNETOACOUSTIC CYCLOTRON INSTABILITY

The full dispersion relation for the fast Alfvén wave, propagating perpendicular to the magnetic field in the presence of an energetic ion species, will now be obtained from Eq. (9). As we noted in our discussion of Eqs. (24) and (25), the term proportional to $1+\epsilon_{xx}^e$ in Eq. (9) may be neglected in comparison with the term proportional to ϵ_{xy}^e that follows it, without loss of generality. It follows from Eqs. (14)–(16) and (19) that

$$\epsilon_{xx}^i\epsilon_{yy}^i+\epsilon_{xy}^{i2}=\frac{\omega_{pi}^4}{\omega^4}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\left\{1+\frac{2\omega}{\omega-s\Omega_i}\right.\right.$$
$$\left.\times\left[f_s-\left(\frac{s\Omega_i}{\omega}-z_i^2\right)g_s\right]\right\}+\frac{\omega^2}{(\omega-s\Omega_i)^2}$$
$$\left.\times(f_s^2-s^2g_s^2+2z_i^2f_sg_s)\right). \tag{45}$$

Using Eqs. (11), (14)–(16), and (19),

$$\epsilon_{xx}^i\epsilon_{yy}^\alpha+\epsilon_{yy}^i\epsilon_{xx}^\alpha+2\epsilon_{xy}^i\epsilon_{xy}^\alpha$$
$$=\frac{\omega_{pi}^2}{\omega^2}\frac{l^2}{z_\alpha^2}\frac{\omega_{p\alpha}^2}{\omega(\omega-l\Omega_\alpha)}\left[\frac{\omega^2}{\omega^2-\Omega_i^2}\left(\Pi_{xx}+\Pi_{yy}-2i\frac{\Omega_i}{\omega}\Pi_{xy}\right)\right.$$
$$\left.+\frac{\omega}{\omega-s\Omega_i}[f_s(\Pi_{xx}+\Pi_{yy})-2g_s(is\Pi_{xy}-z_i^2\Pi_{xx})]\right]. \tag{46}$$

A further combination of terms required in Eq. (9) follows from Eq. (11):

$$\epsilon_{xx}^\alpha\epsilon_{yy}^\alpha+\epsilon_{xy}^{\alpha2}=\frac{l^2}{z_\alpha^2}\frac{\omega_{p\alpha}^4}{\omega^2(\omega-l\Omega_\alpha)^2}(\Pi_{xx}\Pi_{yy}+\Pi_{xy}^2). \tag{47}$$

In Eqs. (45)–(47), the argument z_i of the functions f_s and g_s defined by Eqs. (17) and (18) has been suppressed for conciseness.

We now aim to collect coefficients in Eqs. (45)–(47), for use in the expression for the sum of these equations. It is convenient to define

$$\beta_1(s,z_i)=2\left[f_s(z_i)-\left(\frac{s\Omega_i}{\omega}-z_i^2\right)g_s(z_i)\right], \tag{48}$$

$$\beta_2(l,z_\alpha)=\frac{l^2}{z_\alpha^2}\left(\Pi_{xx}+\Pi_{yy}-2i\frac{\Omega_i}{\omega}\Pi_{xy}\right), \tag{49}$$

$$\beta_3(s,z_i)=[f_s^2(z_i)-s^2g_s^2(z_i)+2z_i^2f_s(z_i)g_s(z_i)], \tag{50}$$

$$\beta_4(l,z_\alpha,s,z_i)=\frac{l^2}{z_\alpha^2}[f_s(z_i)(\Pi_{xx}+\Pi_{yy})-2g_s(z_i)$$
$$\times(is\Pi_{xy}-z_i^2\Pi_{xx})], \tag{51}$$

$$\beta_5(l,z_\alpha)=\frac{l^4}{z_\alpha^4}(\Pi_{xx}\Pi_{yy}+\Pi_{xy}^2), \tag{52}$$

$$\xi=\omega_{p\alpha}^2/\omega_{pi}^2, \tag{53}$$

recall that $s=1$ requires special treatment. Then, by Eqs. (45)–(53),

$$\epsilon^i_{xx}\epsilon^i_{yy}+\epsilon^{i2}_{xy}+\epsilon^i_{xx}\epsilon^\alpha_{yy}+\epsilon^i_{yy}\epsilon^\alpha_{xx}+2\epsilon^i_{xy}\epsilon^\alpha_{xy}+\epsilon^\alpha_{xx}\epsilon^\alpha_{yy}+\epsilon^{\alpha 2}_{xy}$$

$$=\frac{\omega^4_{pi}}{\omega^4}\left[\frac{\omega^2}{\omega^2-\Omega_i^2}\left(1+\beta_1\frac{\omega}{\omega-s\Omega_i}+\xi\beta_2\frac{\omega}{\omega-l\Omega_\alpha}\right)\right.$$
$$+\beta_3\frac{\omega^2}{(\omega-s\Omega_i)^2}$$
$$\left.+\xi\beta_4\frac{\omega^2}{(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}+\xi^2\beta_5\frac{\omega^2}{(\omega-l\Omega_\alpha)^2}\right],\quad (54)$$

where the arguments of the functions β have been suppressed.

Let us now consider other combinations of terms that are required in Eq. (9). Using Eqs. (11), (15), and (53), we may write

$$\epsilon^i_{xy}+\epsilon^\alpha_{xy}=i\frac{\omega^2_{pi}}{\omega^2}\left(\frac{\omega^2}{(\omega^2-\Omega_i^2)}\frac{\Omega_i}{\omega}+\beta_6\frac{\omega}{\omega-s\Omega_i}\right.$$
$$\left.+i\xi\beta_7\frac{\omega}{\omega-l\Omega_\alpha}\right),\quad (55)$$

where

$$\beta_6(s,z_i)=sg_s(z_i),\quad (56)$$

$$\beta_7(l,z_\alpha)=(l^2/z^2_\alpha)\Pi_{xy}.\quad (57)$$

Next, using Eqs. (22), (25), and (55),

$$2\epsilon^e_{xy}(\epsilon^i_{xy}+\epsilon^\alpha_{xy})=-2\frac{\omega}{\Omega_i}\frac{\omega^4_{pi}}{\omega^4}\left(\frac{\omega^2}{(\omega^2-\Omega_i^2)}\frac{\Omega_i}{\omega}+\beta_6\frac{\omega}{\omega-s\Omega_i}\right.$$
$$\left.+i\xi\beta_7\frac{\omega}{\omega-l\Omega_\alpha}\right),\quad (58)$$

$$\epsilon^{e2}_{xy}=-\frac{\omega^2}{\Omega_i^2}\frac{\omega^4_{pi}}{\omega^4}.\quad (59)$$

Equations (54), (58), and (59) complete the set of expressions required for the term within square brackets in Eq. (9). Of the remaining terms, we may neglect $1+\epsilon^e_{xx}$ in comparison to

$$\epsilon^i_{xx}+\epsilon^\alpha_{xx}=-\frac{\omega^2_{pi}}{\omega^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}+f_s(z_i)\frac{\omega}{\omega-s\Omega_i}\right.$$
$$\left.+\xi\frac{l^2}{z^2_\alpha}\Pi_{xx}\frac{\omega}{\omega-l\Omega_\alpha}\right),\quad (60)$$

where we have used Eqs. (11) and (14).

We now combine Eqs. (54) and (58)–(60) to write Eq. (9) in the form

$$\frac{\omega^2}{\omega^2-\Omega_i^2}+f_s(z_i)\frac{\omega}{\omega-s\Omega_i}+\xi\frac{l^2}{z^2_\alpha}\Pi_{xx}\frac{\omega}{\omega-l\Omega_\alpha}+\frac{\omega^2_{pi}}{c^2k^2}$$
$$\times\left[-\frac{\omega^2}{\Omega_i^2}-\frac{\omega^2}{\omega^2-\Omega_i^2}+\frac{\omega}{\omega-s\Omega_i}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_1-2\frac{\omega}{\Omega_i}\beta_6\right)\right.$$
$$+\xi\frac{\omega}{\omega-l\Omega_\alpha}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_2-2i\frac{\omega}{\Omega_i}\beta_7\right)+\beta_3\frac{\omega^2}{(\omega-s\Omega_i)^2}$$
$$\left.+\xi\beta_4\frac{\omega^2}{(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}+\xi^2\beta_5\frac{\omega^2}{(\omega-l\Omega_\alpha)^2}\right]=0,\quad (61)$$

where the functions β are defined by Eqs. (48)–(52), (56), and (57). Collecting terms in Eq. (61), we note that

$$\frac{\omega^2}{\omega^2-\Omega_i^2}-\frac{\omega^2_{pi}}{c^2k^2}\left(\frac{\omega^2}{\Omega_i^2}+\frac{\omega^2}{\omega^2-\Omega_i^2}\right)=-\frac{\omega^2}{\omega^2-\Omega_i^2}\frac{(\omega^2-c^2_{\mathrm{A}}k^2)}{c^2_{\mathrm{A}}k^2},\quad (62)$$

using Eq. (22); also

$$f_s(z_i)+\frac{\omega^2_{pi}}{c^2k^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_1(s,z_i)-2\frac{\omega}{\Omega_i}\beta_6(s,z_i)\right)$$
$$=\frac{\omega^2}{\omega^2-\Omega_i^2}\left[\left(1-\frac{\Omega_i^2}{\omega^2}+\frac{2\Omega_i^2}{c^2_{\mathrm{A}}k^2}\right)f_s(z_i)-\frac{2\Omega_i^2}{c^2_{\mathrm{A}}k^2}\right.$$
$$\left.\times\left(s\frac{\omega}{\Omega_i}-z_i^2\right)g_s(z_i)\right],\quad (63)$$

$$\equiv\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_8(s,z_i),\quad (64)$$

defining β_8; similarly,

$$\frac{l^2}{z^2_\alpha}\Pi_{xx}+\frac{\omega^2_{pi}}{c^2k^2}\left(\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_2(l,z_\alpha)-2i\frac{\omega}{\Omega_i}\beta_7(l,z_\alpha)\right)$$
$$=\frac{\omega^2}{(\omega^2-\Omega_i^2)}\frac{l^2}{z^2_\alpha}\left[\left(1-\frac{\Omega_i^2}{\omega^2}+\frac{\Omega_i^2}{c^2_{\mathrm{A}}k^2}\right)\Pi_{xx}+\frac{\Omega_i^2}{c^2_{\mathrm{A}}k^2}\Pi_{yy}\right.$$
$$\left.-2i\frac{\Omega_i^2}{c^2_{\mathrm{A}}k^2}\frac{\omega}{\Omega_i}\Pi_{xy}\right],\quad (65)$$

$$\equiv\frac{\omega^2}{\omega^2-\Omega_i^2}\beta_9(l,z_\alpha),\quad (66)$$

defining β_9. Making use of Eqs. (62)–(66) in Eq. (61), the full dispersion relation becomes

$$\frac{(\omega^2-c_A^2k^2)}{c_A^2k^2}=\beta_8\frac{\omega}{\omega-s\Omega_i}+\xi\beta_9\frac{\omega}{\omega-l\Omega_\alpha}+\frac{\Omega_i^2}{c_A^2k^2}\left(1-\frac{\Omega_i^2}{\omega^2}\right)$$
$$\times\left(\xi\beta_4\frac{\omega^2}{(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}+\beta_3\frac{\omega^2}{(\omega-s\Omega_i)^2}+\xi^2\beta_5\frac{\omega^2}{(\omega-l\Omega_\alpha)^2}\right). \quad (67)$$

Let us now compare the general expression Eq. (67) with the limiting forms derived in Sec. II. This requires that we take the high-frequency limit, where

$$\frac{\omega^2}{\Omega_i^2}\sim\frac{c_A^2k^2}{\Omega_i^2}\gg 1, \quad (68)$$

and neglect terms of order unity or less in the coefficients, together with $z_i^2\ll 1$. This yields

$$\frac{(\omega^2-c_A^2k^2)}{\Omega_i^2}\simeq\frac{c_A^2k^2}{\Omega_i^2}f_s(z_i)\frac{\omega}{\omega-s\Omega_i}+\xi\frac{l^2}{z_\alpha^2}\left(\Pi_{yy}-2i\frac{\omega}{\Omega_i}\Pi_{xy}+\frac{c_A^2k^2}{\Omega_i^2}\Pi_{xx}\right)\frac{\omega}{\omega-l\Omega_\alpha}+\xi\frac{l^2}{z_\alpha^2}[f_s(z_i)(\Pi_{xx}+\Pi_{yy})$$
$$-2isg_s(z_i)\Pi_{xy}]\frac{\omega^2}{(\omega-s\Omega_i)(\omega-l\Omega_\alpha)}+[f_s^2(z_i)-s^2g_s^2(z_i)]\frac{\omega^2}{(\omega-s\Omega_i)^2}+\xi^2\frac{l^4}{z_\alpha^4}(\Pi_{xx}\Pi_{yy}+\Pi_{xy}^2)\frac{\omega^2}{(\omega-l\Omega_\alpha)^2}. \quad (69)$$

For purposes of comparison with Eq. (28), we neglect the final two terms in Eq. (69). The first two terms on the right agree identically, as does the third term when we recall that only the cross term $\epsilon_{xx}^i\epsilon_{yy}^\alpha$ was considered in Eq. (10), from which Eq. (28) was derived. We conclude that the full dispersion relation Eq. (67) is fully consistent with what can be deduced from existing theory, which applies in the high-frequency limit defined by Eq. (68).

IV. NUMERICAL RESULTS

Even with the somewhat simplistic model for the energetic particle population that we adopted at Eq. (12), the foregoing analysis has led to a final dispersion relation [Eq. (67)] which, though relatively clear in its physical essentials, is rather complicated in terms of its quantitative properties. Furthermore, despite our efforts to the contrary, the number of free parameters has proliferated. They include background parameters (magnetic field, majority species density), ion species identifiers ($\Omega_i,\Omega_\alpha,m_i,m_\alpha$), energetic ion concentration n_α/n_i, and the key kinetic variables that characterize the two ion populations:

$$\mathscr{E}_\alpha=\tfrac{1}{2}m_\alpha v_0^2, \quad (70)$$

$$\mathscr{E}_i=\tfrac{1}{2}m_i v_{Ti}^2, \quad (71)$$

which enter Eq. (67) through the dependence of the functions β on z_α and z_i. Because the primary objective of the present paper is interpretive, we have not attempted to construct a complete picture of the quantitative dependence of the properties of the magnetoacoustic cyclotron instability on the plasma parameters already enumerated. Instead, we have concentrated attention on the range of parameters appropriate to the outer regions of JET discharges, where we believe that the ICE originates. Typical parameter values are of the order of $B\sim 3$ T, $n_i\sim 10^{19}$ m^{-3}, and $\mathscr{E}_i\sim 1$ keV.

The choice of energetic particle species, and their energy, is governed by the fusion reactivity properties of the deuterium plasma that we seek to model. As noted in the Introduction, protons, tritons, and helium-3 nuclei are produced at roughly equal rates with energies $\mathscr{E}_\alpha=3.0$, 1.0, and 0.82 MeV, respectively. Presumably, the physical arguments that were used previously to infer a model hollow shell velocity distribution for the fusion products apply equally to all three species, so that they must all be considered as potential drivers of the magnetoacoustic cyclotron instability. As we shall see, for the energies and plasma parameters considered, the triton and helium-3 populations are found to be below the instability threshold, whereas the more energetic proton population is unstable. This matches the observed absence of peaks in the ICE spectra at cyclotron harmonics of tritium and helium-3 that are not degenerate with deuterium harmonics.

Our key results are as follows.

A. Quadratic or cubic instability?

The underlying instability mechanism is reactive, and involves the resonance of the fast Alfvén wave with the ion Bernstein waves supported by the energetic species. When the ion Bernstein waves supported by the majority thermal ion species are also in resonance, they provide a positive energy loading that contributes a stabilizing effect. Recall from the discussion in Sec. III that the full dispersion relation Eq. (67) involves both a cubic resonance (for example, from the final term) and a quadratic resonance (as pointed out in Ref. 11) when $\omega\simeq c_Ak\simeq l\Omega_\alpha$. Our numerical results confirm our conclusion at Eq. (44) that, for the plasma parameters of interest, the cubic resonance can be neglected in comparison to the quadratic resonance. The real and imaginary parts of ω computed as a function of z_α for a typical set of parameters are shown in Fig. 1. Note that the value of z_α at which the imaginary part reaches a maximum corresponds to that given by Eq. (35). The

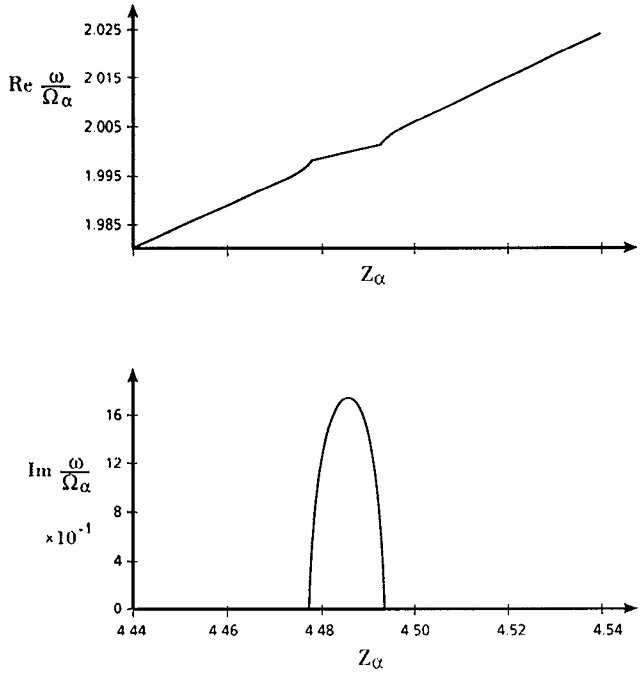

FIG. 1. Magnetoacoustic cyclotron instability of an energetic proton population in a deuterium plasma for $l=2$, $s=4$. Parameter values $B=3.1$ T, $\mathscr{E}_i=1$ keV, $\mathscr{E}_\alpha=3$ MeV, $n_i=2\times10^{19}$ m^{-3}, $n_\alpha/n_i=5\times10^{-5}$. Real (upper) and imaginary (lower) parts of ω/Ω_α are shown as functions of z_α. The value of z_α at peak instability is equal to lv_0/c_A.

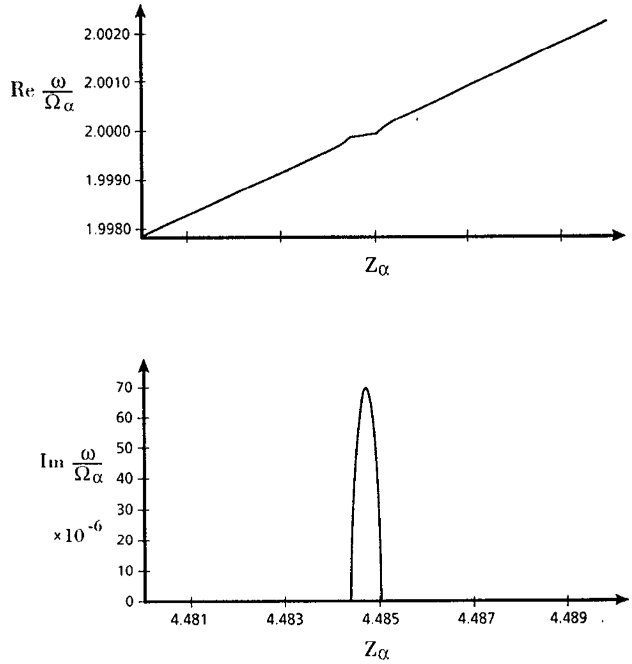

FIG. 2. Magnetoacoustic cyclotron instability of a diffuse energetic proton population, near threshold, in a deuterium plasma for $l=2$, $s=4$. Parameter values $B=3.1$ T, $\mathscr{E}_i=1$ keV, $\mathscr{E}_\alpha=3$ MeV, $n_i=2\times10^{19}$ m^{-3}, $n_\alpha/n_i=3\times10^{-7}$. Real (upper) and imaginary (lower) parts of ω/Ω_α are shown as functions of z_α. Note that instability is weaker and narrower than in Fig. 1.

properties of the instability near threshold are governed by the majority ions—a generalization of Eq. (39).

B. Energy thresholds

For an energetic particle concentration $n_\alpha/n_i=10^{-4}$, we find that an energetic proton population is unstable for $\mathscr{E}_\alpha \gtrsim 2.5$ MeV, given the background plasma parameters already listed. Since fusion protons are born at 3.0 MeV, they may plausibly drive the instability. In contrast, at the same concentration, the 1.0 MeV triton and 0.82 MeV helium-3 populations are stable. The effect of the resonant thermal ions is determined by the magnitude of $\mathscr{E}_i$. Because their oscillation forms a positive energy component of the wave field, the majority ions represent a load that must be supported if instability is to occur. Near threshold, a reduction in the magnitude of $\mathscr{E}_i$ enhances the growth rate; conversely, well away from threshold, variation of $\mathscr{E}_i$ has no discernible influence on the instability.

C. Concentration thresholds

If the energies of the triton and helium-3 populations are artificially increased to a level comparable with the protons, they become marginally stable to the magnetoacoustic cyclotron instability at a concentration $n_\alpha/n_i=10^{-4}$. Any reduction in concentration then returns these populations to the stable regime. In constrast, the resilience of the proton-driven instability is shown by the fact that it is found to persist down to very weak concentrations, $n_\alpha/n_i \simeq 10^{-7}$; see, for example, Fig. 2. In JET, observed neutron production rates of 10^{16} sec^{-1}, coupled with energetic particle confinement times of order 1 sec in discharges having $n_i \simeq$ a few times 10^{19} m^{-3}, suggest that a globally averaged value for n_α/n_i is of order 10^{-4} to 10^{-5}.

D. Multiple cyclotron harmonic excitation

The ability of the magnetoacoustic cyclotron instability to produce simultaneous growth of the fast Alfvén wave at multiple harmonics of the minority ion cyclotron frequency is shown in Fig. 3. In this figure, an energetic proton population in a deuterium plasma is considered, with $n_\alpha/n_i=10^{-5}$. The figure illustrates, in addition, the sensitivity of the instability at particular harmonics to the val-

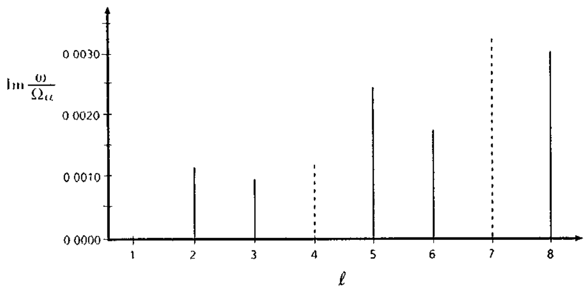

FIG. 3. Simultaneous multiple cyclotron harmonic instability for $l=2$–8, for energetic protons in a deuterium plasma. Parameter values $B=2.1$ T, $\mathscr{E}_i=1$ keV, $\mathscr{E}_\alpha=3$ MeV, $n_i=10^{19}$ m^{-3}, $n_\alpha/n_i=10^{-5}$. Instability at $l=4$ and 7 (dashed lines) is for these parameters with $n_i=1.2\times10^{19}$ m^{-3}.

ues of the bulk plasma parameters: in Fig. 3, the minority cyclotron harmonics $l=2$, 3, 5, 6, and 8 are all unstable for the parameters $\mathscr{E}_i=1$ keV, $\mathscr{E}_\alpha=3$ MeV, $B=2.1$ T, and $n_i=10^{19}$ m^{-3}, whereas $l=4$ and 7 are stable. By increasing the background plasma density to $n_i=1.2\times 10^{19}$ m^{-3}, $l=4$ and 7 become unstable (dashed line), while some of the other harmonics are no longer unstable. There is a comparable degree of sensitivity to magnetic field strength.

The key point emerging from Fig. 3 is that the magnetoacoustic cyclotron instability can possess broadly similar linear growth rates at several minority cyclotron harmonics simultaneously. Any candidate emission mechanism for the observed ICE should presumably include this feature, since the measured spectrum displays a sequence of harmonic peaks.

E. Excitation at proton half-harmonics

If we restrict attention to the primary fusion product species in a deuterium plasma, the linear magnetoacoustic cyclotron instability appears unable to explain the observed spectral peaks at deuteron cyclotron harmonics that are not degenerate with proton harmonics (that is, proton half-harmonics). However, the secondary fusion products include alpha particles at 3.6 MeV, born from the reaction of deuterons with primary tritons, whose production rate can be determined from the associated flux of 14.7 MeV neutrons. While the cyclotron frequency of an alpha particle is degenerate with that of a deuteron, so that secondary alpha particles might in principle be able to excite radiation at proton half-harmonics, the concentration of secondary alpha particles in the emitting region is not known with sufficient accuracy to draw definite conclusions, although it is presumably much lower than that of the primary fusion products. As an example of the concentration of 3.6 MeV alpha particles that is required to drive the $l=4$ magnetoacoustic cyclotron instability in a deuterium plasma, we find that $\mathrm{Im}(\omega/\Omega_\alpha)=6.9\times 10^{-4}$ when $n_\alpha/n_i=10^{-6}$ for a plasma with $\mathscr{E}_i=1$ keV, $B=3.4$ T, and $n_i=5\times 10^{19}$ m^{-3}.

We note that the excitation of waves at half-harmonics of the electron cyclotron frequency has been observed in magnetospheric turbulence,[15] and the theory of such radiation has been studied for both fusion[16] and space[17] plasmas. It is found that the instability requires a hot, anisotropic or loss-cone electron population, whose number density typically exceeds that of the thermal electrons, so that it dominates the wave characteristics. The instability of Refs. 15–17 thus differs somewhat from that considered here, where a very diffuse energetic ion population excites a wave that is supported by the background plasma.

V. CONCLUSIONS

In this paper, our objective has been to identify a plausible emission mechanism for the observed superthermal ion cyclotron emission from Ohmic deuterium discharges in JET. In particular, we have considered the possible resonant excitation of perpendicular fast Alfvén waves with ion Bernstein waves, due to the magnetoacoustic cyclotron instability, driven by the energetic products of fusion reactions. The motivation for this approach, as we have discussed, includes: the intensely superthermal nature of the radiation; its proportionality to the measured fusion reactivity; spectral peaks corresponding to deuterium cyclotron harmonics (even deuteron harmonics are proton harmonics); and kinetic arguments based on the ICE sawtooth rise time, and time delays relative to the central sawtooth crash.

On balance, although there exist significant unresolved issues, the magnetoacoustic cyclotron instability appears to be a plausible candidate emission mechanism. Qualitatively, as we have discussed in Secs. II and III, the general properties of the instability are in keeping with the experimental picture. Quantitatively, there are both debits and credits. On the credit side, the fact that the fusion proton population can drive the linear instability for the parameter range of experimental interest, whereas the helium-3 and triton populations are below threshold, is very encouraging. However, if the linear magnetoacoustic cyclotron instability were to explain the observed spectral peaks at deuteron cyclotron harmonics that are not degenerate with the cyclotron harmonics of the primary fusion products (that is, proton half-harmonics), it would be necessary to invoke concentrations of secondary alpha particles of order $n_\alpha/n_i=10^{-5}$. We also note that the observed ICE signal intensity presumably reflects the nonlinear saturated level of the instability, which goes beyond the linear treatment presented here. It is possible, for example, that equipartition between weakly coupled modes may lead to signals of roughly equal intensity at all deuterium cyclotron harmonics, irrespective of whether they are resonant with the driving proton harmonics.

Finally, we note that it appears that superthermal ion cyclotron emission may offer a natural diagnostic of the behavior of energetic particles, including fusion products, in tokamak plasmas. The exploitation of its potential, however, will require a full understanding of the emission mechanism and its parametric dependence. In the present paper, we have shown that the magnetoacoustic cyclotron instability may repay further investigation in this connection.

ACKNOWLEDGMENTS

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APPENDIX: DIELECTRIC TENSOR OF THE ENERGETIC IONS

Let us now consider the formulas for the quantities Π_{ij} introduced at Eq. (11), for the case where the energetic ions form an isotropic shell in velocity space, defined at Eq. (12). From the Appendix to Ref. 11, we may use the expressions at Eq. (A4) to write

$$\Pi_{xx}=\frac{\partial}{\partial z_\alpha}\left(z_\alpha\int_0^{\pi/2}d\theta\sin\theta J_l^2(z_\alpha\sin\theta)\right), \tag{A1}$$

$$\Pi_{xy}=-\frac{i}{l}\frac{\partial}{\partial z_\alpha}\left(z_\alpha^2\int_0^{\pi/2}d\theta\sin^2\theta J_l(z_\alpha\sin\theta)J_l'(z_\alpha\sin\theta)\right), \tag{A2}$$

$$\Pi_{yy}=\frac{1}{l^2}\frac{\partial}{\partial z_\alpha}\left(z_\alpha^3\int_0^{\pi/2}d\theta\sin^3\theta J_l'^2(z_\alpha\sin\theta)\right), \tag{A3}$$

where z_α is defined at Eq. (13). The sign of Eq. (A2) is opposite to that in Ref. 11, for consistency with the sign convention for ϵ_{xy}^e and ϵ_{xy}^i at Eqs. (8) and (15). These integrals can be evaluated using the identities provided at Eq. (A5) of Ref. 11:

$$\int_0^{\pi/2}d\theta\sin\theta J_l^2(z_\alpha\sin\theta)=\frac{1}{2z_\alpha}\int_0^{2z_\alpha}J_{2l}(x)dx, \tag{A4}$$

$$\int_0^{\pi/2}d\theta\sin^3\theta J_l^2(z_\alpha\sin\theta)=\frac{1}{4z_\alpha}\int_0^{2z_\alpha}\left(1+\frac{x^2}{4z_\alpha^2}\right)J_{2l}(x)dx. \tag{A5}$$

However, as we shall discuss, the expression for Π_{yy} that we derive from these formulas differs from that given at Eq. (A6) of Ref. 11.

It follows immediately from Eqs. (A1) and (A4) that

$$\Pi_{xx}=J_{2l}(2z_\alpha), \tag{A6}$$

in agreement with the corresponding term in Eq. (A6) of Ref. 11. Differentiating Eq. (A4) with respect to z_α,

$$\int_0^{\pi/2}d\theta\sin^2\theta J_l(z_\alpha\sin\theta)J_l'(z_\alpha\sin\theta)$$
$$=\frac{1}{2z_\alpha}\left(J_{2l}(2z_\alpha)-\frac{1}{2z_\alpha}\int_0^{2z_\alpha}J_{2l}(x)dx\right). \tag{A7}$$

Substituting Eq. (A7) into Eq. (A2), we obtain

$$\Pi_{xy}=-\frac{iz_\alpha}{l}J_{2l}'(2z_\alpha), \tag{A8}$$

$$=-\frac{iz_\alpha}{2l}[J_{2l-1}(2z_\alpha)-J_{2l+1}(2z_\alpha)], \tag{A9}$$

and this agrees with the corresponding term in Eq. (A6) of Ref. 11.

In order to evaluate Π_{yy}, we differentiate Eq. (A7) with respect to z_α, giving

$$\int_0^{\pi/2}d\theta[J_l'^2(z_\alpha\sin\theta)$$
$$+J_l(z_\alpha\sin\theta)J_l''(z_\alpha\sin\theta)]\sin^3\theta$$
$$=-\frac{1}{z_\alpha^2}J_{2l}(2z_\alpha)+\frac{1}{z_\alpha}J_{2l}'(2z_\alpha)+\frac{1}{2z_\alpha^3}\int_0^{2z_\alpha}J_{2l}(x)dx. \tag{A10}$$

It follows from Bessel's equation that

$$\int_0^{\pi/2}d\theta\sin^3\theta_l(z_\alpha\sin\theta)J_l''(z_\alpha\sin\theta)$$
$$=-\int_0^{\pi/2}d\theta\sin^3\theta J_l(z_\alpha\sin\theta)\left[\frac{J_l'(z_\alpha\sin\theta)}{z_\alpha\sin\theta}\right.$$
$$\left.+\left(1-\frac{l^2}{z_\alpha^2\sin^2\theta}\right)J_l(z_\alpha\sin\theta)\right]. \tag{A11}$$

Using Eqs. (A7), (A5), and (A4) in successive terms of Eq. (A11),

$$\int_0^{\pi/2}d\theta\sin^3\theta J_l(z_\alpha\sin\theta)J_l''(z_\alpha\sin\theta)$$
$$=-\frac{1}{2z_\alpha}\left[\frac{1}{z_\alpha}J_{2l}(2z_\alpha)-\frac{1}{2z_\alpha^2}(1+2l^2)\int_0^{2z_\alpha}J_{2l}(x)dx\right.$$
$$\left.+\frac{1}{2}\int_0^{2z_\alpha}\left(1+\frac{x^2}{4z_\alpha^2}\right)J_{2l}(x)dx\right]. \tag{A12}$$

This expression can be used in Eq. (A10), giving

$$\int_0^{\pi/2}d\theta\sin^3\theta J_l'^2(z_\alpha\sin\theta)$$
$$=-\frac{1}{2z_\alpha^2}J_{2l}(2z_\alpha)+\frac{1}{z_\alpha}J_{2l}'(2z_\alpha)+\frac{1}{16z_\alpha^3}\int_0^{2z_\alpha}x^2J_{2l}(x)dx$$
$$+\left(\frac{1}{4z_\alpha^3}(1-2l^2)+\frac{1}{4z_\alpha}\right)\int_0^{2z_\alpha}J_{2l}(x)dx. \tag{A13}$$

Substituting Eq. (A13) into Eq. (A3) yields

$$l^2\Pi_{yy}=(z_\alpha^2-l^2)J_{2l}(2z_\alpha)+z_\alpha[J_{2l}'(2z_\alpha)+2z_\alpha J_{2l}''(2z_\alpha)]$$
$$+\frac{z_\alpha}{2}\int_0^{2z_\alpha}J_{2l}(x)dx. \tag{A14}$$

It follows from Bessel's equation that

$$J_{2l}'(2z_\alpha)+2z_\alpha J_{2l}''(2z_\alpha)=-2z_\alpha\left(1-\frac{l^2}{z_\alpha^2}\right)J_{2l}(2z_\alpha), \tag{A15}$$

and using this expression in Eq. (A14) gives

$$\Pi_{yy}=\left(1-\frac{z_\alpha^2}{l^2}\right)J_{2l}(2z_\alpha)+\frac{z_\alpha}{2l^2}\int_0^{2z_\alpha}J_{2l}(x)dx. \tag{A16}$$

Equation (A16) differs from the corresponding expression in Eq. (A6) of Ref. 11, in that the integral is multiplied by a factor proportional to z_α rather than to z_α^2.

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